Nonlinear Control Design

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Let us look at an example of how this is constructed, how a simple case can be constructed. As always I rely on my double integrator example, yeah I am going to look at a control problem by the way although we were looking at this, I am going to do a control problem yeah, does not matter you can I mean the notions are same ok. So this is a control problem I want to do stabilization, so I want to go to x1, x2 that is my target, that is an equilibrium so I can always target to go there and I want to make sure that want x1 less than equal to 5 and x2 less than equal to 3 ok, is this a good enough or x2 say greater than equal to minus 3 ok, I am trying to see if this is a good specification or not, not too nice I guess, yeah I will just say yeah let us just do this ok, we will just try this I have no idea how it goes we will see ok alright. So great, great, great so I want this ok in the transient so of course I will give x1 0, x2 0 I would like to start within of course does not make sense for me to start outside, so I will start within this set which is say you know x1 I will take as 4 and x2 as you know minus 2 ok, not that this matters but we will do that right. Now how would I design a control for this system anybody, how would I design a control or Lyapunov candidate or CLF for this, what would be a good Lyapunov function? Ok you said x1 square plus x2 square some set back stepping, in this case let us start with x1 square plus x2 square see what happens it is not a strictly Lyapunov function you all understand that but let us start and see. So this is of course positive definite and all that candidate Lyapunov function all the nice things, if I took V dot I will get x1 x2 plus x2 times u and I would basically choose control as what, what would be a good control here? Minus x1 minus x2 fair enough because I will get V dot as minus x2 square negative semi definite only then of course I will try LaSalle and it will work out yeah it will be a stabilizing control ok, not a strictly Lyapunov function let us see if we land in trouble because of that unclear but we will try.

Now as you can imagine this is not necessarily going to keep me inside this set I did not no guarantees I mean if I simulate it for different initial conditions maybe suppose I start with I change the initial condition to the boundary pretty much 5, 3 that is the boundary what will happen I will get a control which is negative but that is fine, that is fine because I will get u as minus 8 units and that is fine no that is not what I want. I want to be on the boundary of this guy and I want to be give it a positive so I want to have minus 4 I am just playing so that it does not it gets out somehow at this boundary ok what am I trying I am just I think you understand what I am trying. So this is where I start suppose this is well within the set right I started within the set no problem what is the control at this point u0 is what? It is 4 minus 3 equal to plus 1 ok at that instant right so it is a positive value right so I got x2 dot is a positive number right I am already on the boundary of x2 right so I am going to get out ok I just worked hard to get a case which will happen ok. So basically it is obvious right I mean I did not really work or try to do anything to make sure that it remains inside the set so obviously I did not use it to do any designs obviously you cannot expect that anything good will come out I mean you will get out of the set even if you start in the set so this is in the set way I have defined it ok.

So yeah so sure I escape I can get out of the set ok so that is the basic point will not stop set escape as is obvious because we worked we did not do anything to actually help stop it. What I will do is I will modify this v now yeah x1 squared over x1 square minus 25 plus x2 square ok alright very ugly looking weird looking thing yeah this is what is called a reciprocal barrier function ok we have done some reciprocal construction so what exactly is happening here let us see if did I actually get this correct or should I have flipped the sign I should have flipped the sign no this is not correct. So now whenever x2 is within plus minus 3 right the way I sort then this is positive right again x1 is within plus minus 5 this is positive right so this is positive definite in this region in let me call this region C in the set C ok which is this x1 within plus minus it is a square region right x1 within plus minus 5 x2 between plus minus 3 ok it is positive in C ok what else what else happens positive I would say actually in C0 or interior of C do you understand notion of interior of a set everything but the boundary in this case plus minus 5 plus minus 3 is the boundary everything inside is the so basically if I change the inequality to the strict inequality here right here if these become strict inequalities that is the interior of the set. So in the interior of the set this is positive definite behaves exactly like your standard Lyapunov function no problem ok what happens at the boundary blows up goes to infinity right so goes to infinity at delta C delta C is the notation for the boundary or ok weird function weird ugly ok so can't deny that it is not nice looking alright great now I am going to do standard whatever I do Lyapunov like analysis with this function now because I know that inside once I am as long as I am in the interior things are ok yeah notice that this was not this is the interior by the way this is interior point this is the boundary right because it is a square right this is the boundary fine that is ok that is not a worry I could have done this with 2.9999999 and proven the same thing yeah so I could have proven this with the interior point also not a big deal yeah it is not worry about that ok.

I start with this how do I do the analysis take a V dot ok do the painful process of taking derivatives ok can somebody help me now first I have x1 x1 dot divided by 25 minus x1 square that is the first good piece then I take the derivative of the denominator right what is it yeah ok alright big mess whatever that is. I do not know I am trying this will work or not but ok then I do the second term x2 x2 dot divided by 9 minus x2 square right plus or minus again x2 square divided by times x2 x2 dot this will become plus divided by 9 minus x2 square whole square actually yes yeah whatever this mess is ok alright so I substitute for the I substitute for the derivatives yeah so this is x1 x2 minus x1 cube x2 divided by 25 minus x1 square divided by 25 minus x1 square whole square this will become a plus yeah I am just doing the computation here and just substitute for the derivatives right plus I will take the x2 dot common because that is the control so I will get x2 divided by 9 minus x2 square plus x2 cube divided by 9 minus x2 square whole square times the control ok yeah. So this is if I take if I sort of actually sum them up so I will get 9 x2 minus x2 cube plus x2 cube divided by 9 minus x2 square times u yeah so this ok yeah thank you first term is x1 x2 not x2 square I agree second term is fine I think fine no? Bolo say that again I did know where else here now last step correct similarly this guy will reduce to 25 minus x1 square whole square and you will get 25 x1 x2 yeah is that clear what not clear this addition subtraction nothing much ok alright whatever messy but it's ok now what is the good thing that happened in the control this is the denominator right it will go up right so now what should I specify my control as can somebody tell me choice of control now you can I am sure you can please tell me what is the control I just try to cancel this guy you know first for the first term and I will just try to cancel the first term ok so what is it minus 9 minus x2 square whole square divided by 9 right because that will leave x2 out here then I will take I want to get 25 x1 out here divided by 25 minus x1 square whole square ok whatever this mess is it is something yeah it's a big mess yeah and this is going to basically cancel this guy correct and then I will take minus k x2 whatever I don't care yeah because it will give me negative definite term here right give me nice negative definite term here ok so this will leave me with v dot as minus k x2 square sorry I make my life simple and probably multiply I am going to do that yeah I will make my life simple I didn't need to do it but tk yeah this gives me what I mean I will just go barbell at lemma route it will go x2 gives me 0 x2 going to be 0 x2 dot going to be 0 ok x2 dot going to be 0 implies control going to be 0 because x2 dot is the control right control going to be 0 so I have to just check this guy yeah in here I have already proved x2 is going to 0 right so this term is 0 only left with this guy yeah only left with this the only way this can go to 0 is if x1 goes to 0 yeah because x2 is already gone to 0 so this guy is not contribute positive term right x2 is already 0 so this is yeah positive term so this is just a constant this is not obviously this going cannot go to infinity don't want it to go to infinity makes no sense right so this is basically x1 has to go to 0 so from this I can prove that x1 is also going to go to 0 right but even if before I prove any of this notice before I even went to this step I have already proved this forget all of this mess to prove everything goes to 0 already prove that v dot is negative semi definite which means what which means v of x of t is less than equal to v of x of t0 right now notice whatever initial condition you started with was inside right so x t0 belongs to C right or C interior yeah I am going to say C interior so you started in the interior of the set right of this set I started in the interior of this set C ok so obviously you can see from my v construction that in the interior it is nice positive definite function and then the finite value most importantly yeah if x1 and x2 are in the interior of C are in C0 C0 then this is a finite value right at initial time yeah so therefore at future time also it is finite value correct so that's the argument x t0 in C0 implies v x t0 is finite and this implies v of x of t is finite implies x of t does not belong to the boundary that is you will never hit the boundary yeah you started at a finite value of v you prove that v dot is less than equal to 0 therefore v always remains finite and the only way for v to become infinite is you are at the boundary ok there is no question of going beyond beyond there is no question ok so you see by making this small change in the v of course I chose a different complicated control also corresponding to the v this is the Lyapunov redesign but I was able to ensure that the trajectories remain inside the set and you can verify in this case you will never you can try all these tricks that I tried but you will never get out of the set ok so this gave me a safe control this is what is called a safe control right it remained inside a set that you desired as a set C right but also notice that as you get to the boundary the control here that you see right also does bad things control also explodes at least on one boundary if not on the other ok so of course you are never going to go to the boundary you already proved it but the point is if you started close to the boundary of the set ok you started at 4.99 so you can see the denominator here in the first term of control is very small then control is big so if you start closer and closer to the boundary you are required to exert more very very large values of control as you can this may also be somehow intuitive to you you are saying that I am already working at the corner of my operating region so I have to work really hard to push it back inside might make intuitive sense right if you know you if you are you know if you are sort of working you know at the edge and you want to push it inside as fast as possible as quickly as possible but this is not always required ok the system dynamics may be such that for example right that you are naturally going back for example if I think I mean this is one of the nice example if you think a pendulum think a pendulum and suppose I don't want the pendulum to go out of this by doing my control at the my control is at the tip I don't want it to go out here ok at this edge right so at but when I come here notice this control my barrier this kind of reciprocal function based control will push it really hard back really work really hard but think about the dynamics of the system if I actually started here at the boundary I have to do zero nothing I will do nothing because gravity will push it down right but my control is agnostic to that it is going to really give it a real go here and it will probably hit at the other edge the fact is the dynamics is such that I didn't have to it falls no and never do anything it's at the edge it falls I don't have to work so these are not the best choice of barrier functions so that's what we will see how to sort of create better barrier functions ok. Thank you.