Nonlinear Control Design

Prof. Srikant Sukumar

Systems and Control Engineering

Indian Institute of Technology Bombay

Week 11 : Lecture 64 : Adaptive control: Part 2

So, like I said we are essentially augmenting the state with a new variable. This new variable is the parameter update law ok. That is what we say. So, your complete dynamics will look like E 1 dot is E 2, E 2 dot is theta tilde f x t minus k 2 psi 2. Let us see what else I did not know. I did not know.

Did I get this wrong? Psi 2 dot will be theta tilde f x t minus k 2 psi 2. Correct. So, this is actually not E 1 dot is E 2. This is equal to psi 2 plus psi 2 minus ok.

I do not know when that is happening. And I will have theta hat dot is equal to or in fact I will write it the other way round theta tilde dot is equal to minus theta hat dot and that is equal to minus gamma psi 2 f x t alright. And for this what was the V? So, this is the entire closed loop system ok. This is the entire closed loop system. This guy is of course the parameter update theta hat 0 arbitrary.

For the states obviously everything is given to you right. I mean you are given the initial conditions and everything, but for the parameter update the initial condition is arbitrary yeah. You do not you do not you should start close to the true value, but you do not have to. There is no such requirement alright. Now let us see what happens to a tracking objective.

So, what was the V? The V was half E 1 square plus half psi 2 square plus 1 2 gamma theta tilde square and V dot was minus k 1 minus half E 1 square minus k 2 minus half psi 2 square yeah. The first one is positive definite, the second one is only negative semi definite ok. So, obviously you have uniform stability in the sense of Lyapunov that is 1. Now we want to do signal chasing ok because we have negative semi definite V dot we are exactly in the domain of Babal's lemma right. So, you want to do signal chasing right.

So, how does that look? Exactly the same steps right. What is the first step? Yeah I am not going to go back. Yeah the first set of steps is to prove that everything that shows up in V dot is going to go to 0 ok alright. And you can even do it in a smarter way by the way you can in fact just I mean some people also do this they just try to prove that V dot is going to go to 0. Because if V dot goes to 0 then each of these terms go to 0.

Ok that is another way of doing it yeah. But I would say you stick to the steps I said and do not try to come up with your own steps. The first step in trying to prove everything in V dot goes to 0 was that you say that V is lower bounded and non-increasing right. So, I know that V is lower bounded and non-increasing yeah. What does this imply? Implies that V infinity which is basically limit as goes to infinity V of t exists and finite.

This was the first step remember. I have done this too many years so I know I remember the steps yeah. What was the second step? You do not remember it ok fine. We look at bounded ness of all signals in V ok. So, it is very obvious that V of t is less than equal to V of 0 because V is non-increasing by this right.

So, V is non-increasing then V t is less than equal to V 0 which means that none of these can become unbounded if they started bounded right. And they did start bounded right I mean otherwise I would doubt the sanity of the simulations ok which implies E 1 psi 2 theta tilde all are bounded signals. Now because I am going towards Barbell at sigma right I want to prove that if I want to prove that everything inside V 2 is going to 0 I want to prove that these signals have are L p L infinity and the derivatives are L p or L infinity and so on ok. So, now I want to prove that these signals are L 2, but that is pretty straight forward I mean to integrate this way. Integrate both sides V dot t dt 0 to infinity is equal to integral 0 to infinity minus k 1 minus half E 1 square minus k 2 minus half psi 2 square yeah just integrating both sides right.

Now I know what is my left hand side my left hand side is actually what is my left hand side right V infinity minus V 0. So, I am going to say V 0 minus V infinity because I am going to flip the signs on the right hand side this is just k 1 minus half integral 0 to infinity E 1 square t dt plus k 2 minus half integral 0 to infinity psi 2 square t dt right yes. Now it should be obvious to you from just this equality that individually each of these is finite yes why why is individually each of these finite from this equality I am saying this is finite this is finite why this is finite sure. Now what I am saying that this integral is finite and this integral is finite just from this how do you conclude that first thing is there anything negative on the right hand side any can anything be negative quantity here on the right hand side no right no right because I am taking integral of a square term. So everything the integrand itself is not negative therefore if I take integral obviously not negative integral is just a limit of a sum right obviously not negative not negative plus sign not negative not negative they cannot cancel each other ok this cannot cancel anything here ok which means what if the sum is actually equal to this individually each of them right has to be right otherwise yeah if this is bigger than V 0 minus V infinity there is a problem this will become a greater than yeah but that's not the case individually each of them because they don't cancel each other out right.

So obviously this individually each of them is less than equal to V 0 minus V infinity and if you notice this is enough right to claim that E 1 is in L 2 and this gives me psi 2 is in L 2 right ok alright now I can use the Babalat's lemma right oh no I am not done yet sorry what about E 1 dot and psi 2 dot I am claiming they are bounded ok I am claiming they are bound what is E 1 dot E 1 dot is E 2 what is psi 2 dot psi 2 dot is this I already know that psi 2 is bounded ok I already know theta tilde is bounded yeah I just proved theta tilde is also bounded because it appears here right. So the only requirement is that f remain bounded ok so this requires me to make an assumption ok notice just by the fact that E 1 psi 2 are bounded x will also be bounded ok it may not be obvious to you yeah but E 1 and psi 2 are basically just errors coming from the errors errors with the reference trajectory. The reference trajectory is typically a bounded trajectory yeah you do not never give unbounded trajectory so you have a reference trajectory so you are computing error between your states and your reference states which are bounded ok and if you say that your error itself is bounded then it means you are only a bounded distance away from the reference trajectory right and if the reference itself is bounded and you are a bounded distance away from the reference then x itself the states itself also have to be bounded ok. So if you say that essentially error bounded means x minus here when I say states are bounded this is basically also means x 1 minus r is an infinity right and r is already an infinity right. So this is basically the case where x 1 is an infinity right therefore x 1 is an infinity ok.

So basically you also have that x is itself bounded ok so in order for psi 2 to be bound psi 2 dot to be bounded what do you need? You need that this quantity be bounded ok not always but when the input states are bounded ok. So the assumption is typically written as assume f x comma t is bounded for bounded x and all t ok. If you make this assumption you will immediately have that e 1 dot and e 2 dot which is minus k 2 psi 2 plus theta tilde f x t are bounded ok. So this is the assumption that you will immediately have this ok. And once you have this you can use the corollary to barbell art lemma to claim that what? I can claim that e 1 and psi 2 are going to 0 ok.

I actually need no further steps although the when I showed you the application of the barbell art lemma I used further steps but right now I do not need further steps right because e 1 and psi 2 going to 0 implies what? e 1 and e 2 are going to 0 ok. So I have achieved tracking yeah rather amazing although it looks like I did very simple things some simple manipulations here and there again please go back and read so you can follow but just by introducing so what did I do in essence? In essence my controller which was a what we call a static controller became a dynamic controller. What is a dynamic controller? The control depends on some value which comes from a dynamical system right. So my control depends on theta hat and theta hat comes from this dynamics. So just by moving from a static controller to a dynamical controller it's almost like saying I added some integrator in my controller ok not a linear integrator but a non-linear integrator ok.

By adding a non-linear integrator in my controller I made my system agnostic to unknown parameters yeah I don't know the parameter I actually don't know the dynamics well at all but I exactly track the trajectory this is not an approximation ok I exactly track the desired trajectory in the absence of disturbance and all that of course. See you all of you must have at some point or the other seen or heard of robust control ok what is robust control it's all this h infinity and this kind of control yeah what is the idea in robust control? The idea is that and that's applicable only for linear systems typically there the idea is you design the for linear systems at least you design the control in such a way the control gains in such a way that it can tolerate some error in parameters ok but that error is rather limited you don't know how much error ok the error it can tolerate is not infinite not significant ok it can tolerate some error in the parameters ok beyond that you will get only bounded performance in fact even with the error you will only get bounded performance you are only guaranteed that your system will not blow up you are going to get a nice bound around the desired trajectory ok but here what are you doing here and I mean I am not saying that's a bad method or anything I am just saying that that's a different method in that method the advantage is you are not changing the control structure at all the control structure remains the same in robust control ok there is some whatever some p d k x minus k x type of a feedback it's like a state feedback ok structure remains the same here it is no longer just pure state feedback here you have a dynamic feedback right you have a theta hat dot so there is a dynamic feedback that's happening ok so we have changed the structure of the control but what have we achieved we have achieved precise tracking ok so in adaptive control you can achieve precise tracking even if you do not know the system ok and that's pretty amazing if you think about it ok now if I do the rest of the steps I told you that this steps is enough I have already achieved tracking ok let's see what the rest of the steps give me ok if I do the rest of the steps I would essentially be able to prove that e 1 dot and psi 2 dot go to 0 right that's what we have been doing we started with proving that everything that is in v dot goes to 0 then we prove that the derivatives of those quantities go to 0 and we can yeah we can can prove that this happens ok but e 1 dot going to 0 just means e 2 goes to 0 that we have already proved so nothing special there but psi 2 dot going to 0 gives me what it gives me that minus k 2 psi 2 plus theta tilde f x t goes to 0 right but again I already know that psi 2 already goes to 0 so what do I have? I have that theta tilde f x t goes to 0 ok unfortunately I have not proven anything about parameter convergence ok no evidence of parameter convergence or if you folks like this learning I did not learn squat ok I did not learn the parameter ok now that's I mean it may not seem nice to you but that's sort of the power of this method yeah it did not require you to learn the parameter I still did pretty nice tracking control if you give me a robot or if you give me an airplane or if you give me a quad rotor I am doing my tracking I don't care to learn some parameter I don't care to learn the inertia that's not my job as an engineer right I wanted to go to the way points go to the you know particular formation do whatever I wanted to do I wanted to do the control task I do not care if it learns the parameters ok but then if you do care about the learning part yeah then there are some results yeah which are connected to what is called persistence of excitation and these results are required also in deep learning by the way it may not these don't come up obviously upfront yeah you will not do good learning unless your data set is rich enough yeah and how you specify rich enough which is a very very vague word is using persistence of excitation this idea comes from system identification this got nothing to do with adaptive or adaptive control or learning or anything it comes from system identification basically it is like saying that eventually you are going to solve some linear system of equations and the senior system of equations must have a solution if it doesn't then you can't ok so that's what it comes to you write you can write this E1 E2 system so this dynamics ok I guess it's done you can write this E1 xi2 theta tilde dynamics in this you know linear system structure that you can see ok and this structure leads to some persistence excitation type results ok so basically you have this is what it will look like I guess E1 xi2 and theta tilde this bottom right minus K1 1 0 0 minus K2 F and you will have 0 minus gamma of 0 right you will have something like this I think this is correct right minus gamma F right yeah yeah yeah this is absolutely right this is the bottom right is what you will have ok whatever is in the bottom right is what you will have so this is the structure that you will have and this structure is amenable to applying some nice results on persistence of excitation which are pretty classical yeah and you can actually claim that you will achieve parameter convergence also under persistence of excitation yeah because we don't talk about it so I'm not going to go into it in much detail but like I said doing further steps in Barbalat's lemma is useless in this particular case because you cannot prove parameter convergence all you can do is that product of theta tilde and F goes to 0 ok now F is passing through 0 regularly the function F itself is going through 0s then this means nothing theta tilde is not going to 0 ok but if the function F is such that it never goes to 0 is always non-zero then yes it means theta tilde so you are asking something from F ok so if you notice there is a nice structure here so you see the 0 F and 0 minus gamma F they are transpose of each other just with a gamma multiplied right so you are asking something on F the last column and the last row ok that last column and last row has to have persistence of excitation ok and if it so happens that F never hits 0 then you automatically have persistence of excitation that is a nice assumption this is a very bad assumption notice that's why I said very carefully when I made boundedness assumption on F I did not just make a random arbitrary boundedness on F that would mean I'm only allowing function like sine X and all that I am not saying that I am saying that F is bounded if the states are bounded so polynomial X is allowed X X squared allowed yeah because if yeah that's why I was very careful assume F X is t is bounded if states are bounded if X is bounded that is allowing polynomials but if you just say F is bounded then I am only allowing sinusoid and all the trigonometric functions right pretty sad you see I you know the space of all analytic functions I went to the sines and cosines right so obviously I am significantly weakening or strengthening my requirements and weakening the set of functions that I can work with so that's the idea yeah this assumption that the function doesn't pass through 0 even sin X doesn't satisfy so I mean you can see that it's not that easy yeah on the other hand sin X is persistently exciting I can tell you the parameters will converge if F of X t is sine of X parameters will converge okay because it's persistently exciting yeah or whatever Delta persistent in this case alright so that's basically adaptive control for you in a nutshell there are of course many many more cases and so on and so forth as you can see I've already taught entire semester and probably do that next semester also but yeah yeah so but that's essentially the nutshell of what is yeah adaptive control yeah we will do some more again new modern controls and the subsequent lectures yeah alright any questions no okay we'll stop here thank you.