Nonlinear Control Design

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Week 10 : Lecture 60 : Feedback Linearization: Part 11

For this DC motor example, we are simply verifying that we have the conditions that are needed. First is that you have G, add f G and add f square G to be linearly independent, which turned out to be a little bit complicated. I will really ask you to try to verify this numerically. And then the second condition that we needed to verify was that the distribution formed out of G and add f G is in fact involutive. And that is pretty straightforward, turned out to be 0 0 0. So you have involutivity.

Now we have the, we have by Frobenius theorem that this is in fact completely integrable. So therefore we have some output beta for which you have D beta multiplied by this terms of the distribution to be 0. This is supposed to give you a partial differential equation. So I am going to expand this.

What is G? It is, I will use some 1 over L s. And what is f? Add f G is this guy. Minus KLR R, KLR x 3. And KJ x 2. So this is 0.

Okay. So how do you find beta? Just write out the partial differential equations. Okay. Let's see. What am I going to get? The first one is going to give me 1 over L s del beta del x 1 is 0.

Correct? Right. So this means what? What does this mean when you are trying to solve? Absolutely. Implies beta equal to in a very very bad notation. Okay.

Yeah. You should not write this like I mean in a paper and all. In exam paper it's fine. Not in an article. Yeah. But anyway I am just saying beta is independent of x 1.

Okay. What do I get from the second and third now? So you see that del B del x 1 is already 0. Right. So when I expand this one I don't have to worry about this term. Right. So I am going to simply write what? Minus KLR x 3 del beta del x 2 plus KJ x 2.

KJ x 2 del beta del x 3 equal to 0. Okay. So this is whatever the K goes away. Right. So I have some condition.

Right. How do I go forward now? You can guess. I guess. You can guess. Yeah. What do you think? What do you think I should have beta? I'll make this bigger if you don't see it well enough.

What do you think you can get beta from here? See there is it's very symmetric right? There is x 3 partial with respect to x 2, x 2 partial with respect to x 3. Okay. So to me it seems like there should be you know x 2 square plus x 3 square. Something like x 2 square plus x 3 square.

Yeah. Because if I have x 2 square it becomes twice x 2. This gives me x 2 x 3. If I have x 3 square I get twice x 3. I get x 2 x 3 again. Now this will cancel out.

Modulo some constants. Right. Yeah. Make sense? Yeah. That's how I'm guessing.

Yeah. It's a trick. That's all. I'm not doing anything magical here. Yeah. But you can solve any way you want I guess.

Yeah. I don't know again what is an obvious way to solve otherwise. Yeah. Typically how you do solve PDEs is you assume it to be you know some that the each the two variables appear independently and things like that. So there are some methods of solving PDEs.

Okay. Some only in some cases you can analytically solve them. Yeah. But here I guess. So what is this by the way? What is the scaling on x 2 square and x 3 square you think? Can somebody tell me? L R x 2 square plus j x 3 square.

Perfect. This will work. Why? Because it will give me you can put a half if you want but nobody cares. This will give me the first term will give me L R x 2 twice L R x 2. So L R L R goes away. So this becomes x 2 x 3. This will give me j x 3 2 j x 3.

So j j goes away. So x 2 x 3 again. So that becomes 0. That's a fair choice. Okay. So again what did we end up doing? What is this beta? What was this beta? What does all this work for? Output.

This is the output with respect to which the system is feedback linearizable. Now go back to what we had done. We had taken the system. Fine.

The friction was missing. No problem. I mean there is this friction term that is missing from here that I think has zero impact on anything. Yeah. This friction term is missing in the third dynamics.

Not here. No problem. But the output was pre-specified to some x 3. All right. Now what we are saying is that we are not going to pre-specify the output. We are going to try.

We are not given an output. We are going to try to find the best output under which the I get complete feedback linearizability. Okay. And that is this guy. L r x 2 square plus j x 3 square.

Okay. And now that we have you know this output why don't we try? Okay. Try. So H x is L r x 2 square plus j x 3 square. Okay. What about H dot? Let me put a half if you don't mind.

Yeah. I will put a half. What about H dot? This is L r x 2 x 2 dot plus j x 3 x 3 dot. Notice that x 2 dot and x 3 dot don't contain the control. Okay. Right. So this is going to give me some big mess.

I am quite sure. Yeah. I am not even going to write it. Sorry. I am not going to write it. But I am going to compute H double dot.

Yeah. This is L r x 2 dot square plus L r x 2 x 2 double dot plus j x 3 dot square plus j x 3 x 3 double dot.

Yeah. Yeah. Just product rule used. Yeah. I don't want to write this mess. So I am avoiding writing it. Yeah. Now what do I know? I know that x 2 dot square and x 3 dot square are not bringing the control.

Because x 2 dot is this, x 3 dot is this. Does not have the control in it. But when I take the double derivative of x 2 and in fact the double derivative of x 3 I have to take derivatives of these guys. Right. And here I have x 1 dot appearing.

Here I will have x 1 dot appearing. Okay and that will give me the control. Okay. So what did I end up finding? That control appears. Okay so what is it? Relative degree is what? Should I do this correct by the way? Should the control be appearing here or in the third derivative? No this is fine.

Yeah. Sorry I got all my coordinates. Yeah. I got all my coordinates right. Or should I have, no wait wait wait wait wait. I should be very careful. Did I get all my coordinates? Yeah. What would be my coordinates in this case? There will be h, h dot and h double.

No control should not appear here either. No no no no no no. Will the control appear here? This computation is so painful that I am not very keen on doing it. What is the control term? Can anybody compute it? Here and here. Can somebody tell me what is the control term? Forget every other term. What will be this term? What will be the control term coming from this? L r x 2 and in x 2 double dot I have this guy right.

I will have, so I am going to write it as minus k L s L r x 1 dot x 3. Yeah. That is the term that is containing the control, is going to contain the control. Similarly if I look at this guy, this is j x 3 x 3 double dot will be k L s over j x 1 dot x 2.

Okay. Yeah. This x 1 dot is going to bring the control. So I am just writing the terms in the control. My feeling is these two terms are cancelling. Aren't they? These two terms cancel out. Isn't it? This is minus k L s x 1 dot x 2 x 3 k L s x 1 dot x 2 x 3.

These terms cancel out. Right? Okay. That is the right thing to happen. Okay. Now not going to write it but H third derivative has control.

Okay. Okay. It looked like this second derivative contains the control but those terms will cancel each other out. So control doesn't appear. It can't because I need three coordinates right. I started with a three dimensional system. So my coordinates are now these three H, H dot and H double dot.

Yeah. Unfortunately as you can see very very ugly looking. Yeah. Well, nothing much you can do. Yeah. Doesn't look very nice but nothing much you can do.

If you folks are Nth about it we can try the other example also. Do you want to try the other example or do you want to do it yourself? Why don't you do it yourself then? You will make it an exercise. Yeah. I mean we will try. Actually we don't know. So that was the other system which was, what was the other system we were looking at? We had this other example right.

This guy. This is even worse. No. No no no no. We are not trying this. You can see that the G is not even a constant.

No no no. Maybe something else. Yeah. Yeah. I mean see fortunately or unfortunately you can do, you know, analytically you can work this out for very specific examples only. Yeah. But you can also see that you are, you know, you are sort of coming up with some rather unusual transformations. Okay. Which you would not, never have guessed otherwise without looking at the structural details.

You will never have guessed that this sort of an output would give you complete feedback integration. Right. We were working, we did work with some other, you know, y equal to x3 and some something arbitrary right. Yeah.

You would never have guessed that you would get feedback integration. But yeah with this very very crazy looking coordinates, yeah, you will get a linear system in the end. Yeah. And this is not, like I said, this is not linearization or approximation of any kind.

This is actually the system is linear in these coordinates. Yeah. It is almost like I think a lot of people say that if you, if you, if you squint your eye carefully enough every non-linear system is linear. Yeah. So if you look at it very carefully every non-linear system is linear.

It is like, you know, that is basically the idea here. Yeah. So lot of them do have this condition satisfied. As you can see it is not easy to verify. Yeah.

Some of these I would still recommend that you verify numerically. Yeah. But for systems where you can do this analytically you have a pretty powerful result. Alright. So a lot of times it is not a scale issue or anything.

It is not like it is like n becomes large. There is a computational issue. It is not a computational issue. It is just that for example if you have a thousand of the same kind of dynamical systems and you can sort of by some suitable transformation you can make each of these dynamical systems appear linear then you have now thousand linear systems that you are working with.

Yeah. So these are going to affect you in terms of scale. Yeah. If you have some, you know, robot dynamic like a mobile robot dynamics which is which you can feedback linearize. Yeah. Then you can say a lot of things right. Even if you have thousand size mobile robots you can still control them with easier control loss seemingly easier loss.

Yeah. Of course this non-linear transformation will sort of play a big role. But the good thing is you have it is just a transformation now. It is just going from one function forward and one function backward. Yeah. Everything can be transformed like that.

So yeah I mean there is a little bit of a pinch of salt associated with it. And this is also I believe yeah there is a little bit more on. Okay. I mean there is just a little bit more on zero dynamics and so on. If you notice in the last slide of this what I have done is I have looked at the rigid body dynamics.

Yeah. Yeah. And I mean although I have written it as an exercise it is almost like I have done a little bit of the work myself. You can or probably the entire bit of work actually at least a little bit of work I have done myself. So please take a look at this exercise.

Yeah. There this is the rigid body dynamics again. Yeah. In terms of some parameterization you have already seen the rigid body dynamics. It is part of the current assignment also. And this is in some parameterization. Don't worry about what this row is. But what I have asked you to do is to see if it is you know full state feedback linearizable and find the change of coordinates which is this lambda.

Yeah. So that is what I have sort of asked you to do. And I have actually checked a little bit of the you know involutivity and things like that. I don't know if I actually found the coordinates.

Doesn't look like it as of yet. So yeah you can probably give that a shot. Yeah. Basically the idea is to find an output for which you have feedback full state feedback linearization.

Yeah. Actually I think I have this.

This. I would encourage you to look at this although it's like a solved exercise. Yeah. But I would encourage you to look at this. That basically it's I have not used the integrability condition here because it life becomes very hard with that condition. I have simply guessed that y equal to rho. If I take my output as rho itself. That's going to make complete feedback linearization because the second derivative will contain the control.

Yeah. So I have sort of guessed it again instead of actually going with the integrability condition that you have. Yeah. But this also works. So a lot of there is a lot of literature in spacecraft community where they use you know this this instead of using the rotation and the angular velocity as variables they take the rotation and its second derivative as a as a sorry and its first derivative as the variables.

Yeah. Because in those variables you can claim some kind of linear system. Yeah. Though this is a little bit more complicated than that. Anyway maybe we will give you some simulations based on this. All right. Thank you. Bye.