Nonlinear Control Design

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Week 8 : Lecture 48 : Passivity based control: Part 7

Great, even if you don't completely follow the model and the logic behind the model and so on, you don't have to worry about it. Modelling is not part of this course. You are given a rotational dynamics of a rigid body. This works for any rigid body system, quadrotors, UAVs, satellites, even underwater rigid bodies. Yeah, it doesn't matter. It has to be a rigid body.

If you are flexible, then this inertia may not be constant and there is more complications there. Flexibility gives rise to bigger challenges. This is not a good model for flexible systems. But if you have any rigid body system, this is a good model.

This because it's a satellite system, so gravity etc. is missing here. Yeah, so if you are working with again something that's on earth or underwater or whatever, then you have to have gravity, you know, if you have viscous damping and things like that. Those also have to be added in the dynamics equation. Yeah, but kinematics is exactly the same.

Okay. And the dynamics is actually evolving on a linear space. Yeah, no problem. Omega is on a R3. Yeah, so nice.

Good. Yeah. What I have mentioned now is that I am going to take the output as omega. Okay. So because I am looking at a cascade connection sort of a thing.

Now if you notice, omega is an input to this system. Okay. So let's see. We are going back to our, this cascade of a passive system with a some nonlinear stable system. So omega is an input to the system.

If omega is zero, then the system is passive anyway. Sorry, system is stable because if omega is zero, there is nothing on the right hand side. Rho dot equal to zero is a stable system. Yeah, nothing to do. That's a stable system.

Right. So I am already done with my first assumptions that if this output is not there, then my system is stable. Okay. So good. So ticked one box. The next question, is this system passive? Okay, we have to check whether this system is passive.

So that's what I am saying in the beginning. With omega equal to zero, rho dot equal to zero is stable. So I can actually choose any function w. With any function w, this is stable. Later on, of course, I will specify what function w we should choose.

But because I am left with rho dot equal to zero, this is stable with any function w. I don't have to worry about what function w to choose. I can choose any positive definite radial and bounded w and I am good. Yeah, you can have rho square, rho four, anything, anything.

Excellent. So that's good. For passivity of this system, I am going to choose this. This should again remind you of what you did with the robotic system. You had the first term as half q dot as mqq dot, half q dot transpose mqq dot, which was the term corresponding to the inertia, kinetic energy. This is exactly the rotational kinetic energy.

This is the expression for the rotational kinetic energy. And this is of course radially unbounded and all that. So what, of course, I have given you this is an exercise. You have to show that v dot is actually less than equal to u transpose y.

Okay. It's too easy. It's like just you have to write one line here. Yeah. It's just one sentence.

Yeah. You just have to remember that there is this q symmetric, there is a cross product happening. Okay. You have to just remember there is a cross product happening here. If you take a v dot, you're going to get omega transpose j omega dot and you just plug in j omega dot from here.

Okay. And that's it. Yeah. So it's very, very easy to conclude that v dot omega is actually less than equal to u transpose y with y being omega itself. All right. So that you have to verify. So obviously what we have shown is that we have a driver system which is passive in this u y combination and we of course have this row system which is being driven and that is stable in the absence of the output y.

Okay. So we are done. These are the only two assumptions that we require for the previous result to hold. So what do we know? We know that the entire system is passive with this new output v and y where the u, that is the earlier output u is just v minus this guy. Right. Where am I getting this expression from? This is just this expression.

Right. I've just populated this. The original input is just the new input and this lgv type term. All right. That's what this is. This is in fact partial of w with respect to row and then this guy.

All right. This okay. This is precisely what is multiplying the omega. Yeah. And that's all. And what do we know? We know that the new system is now passive with this input output combination.

All right. And then what? Then life is too easy. I've actually asked you to do this as an exercise. I've now given you a choice of w. I'm not asking you to choose any arbitrary w. This is simply so that the structure of the feedback looks nice.

That's all. Yeah. If you choose this w which is k log natural one plus row transpose row. Okay. A weird looking choice. Yeah. But this choice of w actually gives you a nice feedback and also gives you this nice zero state.

Anyway zero state observability is anyway there. We don't have to worry about it. But anyway you have to verify that also. That you have in fact zero state observability.

Not necessarily detectability. Yeah. We in fact have zero state observability. All right. So all you have to do is pop. You know you just have to put this w in this expression.

That's it right. Because that's what is going to give you your control. Yeah. And once you have that zero state observability anyway you have to verify with this.

For this particular output omega. Right. So what does that mean? It means that if omega is zero. Then you want to show that row and omega are zero. So you know omega was anyway zero. Yeah.

So if omega is zero omega is already zero. You only have to show that row is also zero. Okay. So you want to sort of look at that because that will come from your feedback expression. Okay. Remember zero state observability has to be verified in the absence of the input.

Yeah. So so this expression. Yeah. So notice that in order to check zero state observability you typically will say that if omega is zero we also know that omega dot has to remain zero. This is how we go about checking right. Zero state observability is exactly like the LaSalle argument. So if you want to omega to stay at zero you need omega dot to be also zero.

Yeah. Which means that the entire right hand side has to be zero. Control was already has to be zero to check zero state observability. This term is already zero. So all you will have is that this term should also be zero.

Okay. And this term you will notice will bring in the row. And you'll be able to claim that this equal to zero is the same as row equal to zero. Okay. And you'll be able to claim zero state observability like you need.

All right. You just have to carefully take the partial and so on. All right. And finally once you have that the entire system is passive with this input output combination you know that I can choose my V as a function of the output that is minus phi omega. Yeah. In fact I can essentially I'm just giving an example I could for example choose V as minus K tan hyperbolic omega.

Yeah. And this is enough to give me global asymptotic stability. Yeah. So the complete feedback is not just this remember it is this along with this.

There are two terms. Yeah. Because this I already prescribed the real input that goes to the thruster or the actuator would be U.

Right. Not just V. Right. So V is only one piece. So the V piece is this. And then there is something more which comes from here. This is the actual control law or the actual control that command control command that you send to the actuators.

Yeah. Yeah. But you know that with this combination you will have global asymptotic stability. OK. So this is again you know something rather nice and powerful. So you can see that even for this you know within half a page almost I can come up with a feedback law stabilizing feedback for a rather nonlinear system.

OK. Like the rigid body attitude dynamics and this also a saturated feedback by the way. Yeah. This is bounded feedback. In fact you will notice that there is some nice properties of the first part also.

This part also has some nice properties. Yeah. You will get some nice saturated feedback. OK. So again rather powerful result within half a page if you can actually solve this problem. Yeah. Usually when I do it without knowledge of passivity you will need a little bit more work.

Yeah. It's not this straightforward. Yeah. But here in this case because you are actually employing this idea of passivity interconnected with a cascade connection with a stable system you have this nice simple construction of feedback and these two exercises that I've given you are actually rather short. OK. Anyway even for this case you will have numerics to do. OK so I will give you I will give you inertia values and initial conditions. Actually that's all I need to give you some inertia values and some initial conditions for omega and rho and you will actually be implementing this controller and seeing how it performs. OK. Make sense. All right. OK. Thank you.