Nonlinear Control Design

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Week 8 : Lecture 45 : Passivity based control: Part 4

Welcome to another session of non-linear control SC602. So we had started off with defining passivity as basically this guy. A system is said to be passive, there is a storage function such that the derivative of the storage function has this less than equal to U transpose Y thing. So this was what we had said. So the only thing is that this is sort of a little bit later in what he covered. It came up a little bit later in what he covered.

That's all. That's really the only thing. So anyway, so let me sort of try to go back to a, wow, this is a, let's see if I can go to one. So if you see, if you remember, he started off talking about all the energy balance which is actually the real motivation for passivity.

Basically if you looked at how he discussed passivity, you would understand that almost every system that you can think of can be thought of as a passive system if you don't you know sort of keep injecting too much energy into the system yourself. And if you are very careful about choosing the inputs and outputs, that is the only real trick here. Input is anyway typically fixed. You can't really decide what is your input of the system. But you can always try to determine an output with respect to which you will have passivity.

So the basic equation was just this that the energy available is the initial energy and then there is some dissipativity and supplied energy. So this is essentially what this is. And if you look at this sort of expression, the way he talked about it is that the energy difference, if you take this to the left hand side, the energy difference is less than equal to some inner product of input and output. Essentially that is what it is. Energy difference, if I take this to the left hand side, is less than equal to the inner product of input and output.

This could be any input output, right? I mean or it could be any inner product also. In this case it turned out to be the integral functions and so on. So this is an inner product in the function space. But it can be any inner product. We've also looked at.

So this is what I also mentioned. Actually it's not different. V dot is what? It's just the difference. If you look at, think of V dot, what is it? It's just VT minus V0 divided by delta T. If you think of it like that.

So this is just the difference. And it is just the U transpose Y. Or if you think of it differently, if I integrate both sides, then you have an inner product on the right hand side, typical function inner product. And on the left hand side you have VT minus V0. So it almost looks like the energy loss is essentially upper bounded by the inner product of input and output.

So the definitions that we talked about are not different. It's just that he came from a different motivation and then we looked at several examples and probably you got confused. I have no idea where you got confused. But that's essentially what it is. It's simply just the motivation is pretty much simple.

Then of course he talked about these are very useful concepts. This is why you can keep extending this notion of passivity to larger and larger systems. You can add interconnections. Essentially what he said is that if you interconnect two passive systems then you retain the passivity property. And this is a pretty cool thing.

Because you know we also discussed if you remember that if you have passivity and the zero state observability which you already defined, then you can actually design a controller very easily which is a function of just the output. The output with respect to which you have passivity. So we discussed this. This is basically the system. If the system is passive with some storage function and it is zero state observable then any feedback of this form with these properties globally asymptotically stabilizes the system.

The origin. So it's a very cool thing. For passive systems I have something nice. So in this Kahlil's text or whatever I am covering I'm always talking about passivity. So if you notice Anthony also spoke about strict passivity by the way. So where you have actual dissipation happening.

If you have actually a dissipation happening then it is strict passivity. Otherwise it is just passivity which is I mean either of them is good enough. Why is either of them good enough because I can always use the feedback term to introduce this strict passivity. So that's a pretty transition from passivity to strict passivity only requires me to introduce some it is as good as introducing a negative definite term with this feedback. See because if you see it's U transpose Y it's less than equal to U transpose Y.

If I make U as some negative function of Y you can already see that it gives me a negative term here. Right. Okay. So I have some dissipation. And therefore I will get to the strict passivity version.

I can always say that my control is some new control minus a function of Y. Which will give me a negative definite term here like a Y transpose Y if you may. And another control V transpose Y with respect to a new control V. So I will have again with respect to the control V and Y strict passivity. So going from passivity to strict passivity very easy.

Not difficult at all. So whenever I do a feedback interconnection of this passive systems I retain the passivity property and that is a very powerful sort of result to have. Yeah. But of course he mentioned that if you are in cascade it is not necessarily passive and all that. But let's not worry about that. I mean I will let the TAs get into that stuff.

Right. Which is. Yeah I mean this material was a little bit more involved because of how it is stated. Not actually not very complicated. It's just more involved in the in the in the sense that how it's stated. Okay. So this is again the same idea.

You have these feedback interconnection of passive systems. You retain passivity. Okay. Which means inherently that I can globally asymptotically stabilize some system equilibrium. The only question now is what is that equilibrium.

Right. So I have connected two different systems. Right. So one system could have one equilibrium. Other one could have other equilibrium.

Right. And in energy terms this is the same as so this is very well known by the way that equilibrium when you're looking at systems which you know sort of conserve energy and or Lagrangian Hamiltonian system equilibrium is just the minima of potential energy. Okay. This is standard. Remember this even if you don't understand this. It's almost like saying that I have my potential energy like this and this is where my equilibrium is.

Yeah. This is very standard. Okay. You may not have seen this but from mechanical systems folks this is the standard idea that equilibrium is actually a minima of the potential energy of the system. Yeah. You can think of it. I mean if I drop a ball what is the equilibrium.

It is the ground where the potential energy is zero. Systems want to minimize their potential energy. Okay. That is why yeah turbine whatever everything works because of that right.

You drop water it goes down. Yeah. Basically all systems work like this. Yeah. This is nature.

Nature's equilibrium. Yeah. This is nature's equilibrium. So you see this is also turns out to be the equilibrium in the sense of stability. For stability also this turns out to be the equilibrium of the system. Okay. So this is very standard for mechanical systems that equilibrium turns out to be a potential energy minima.

That is why he keeps talking about energy terms. Okay. So you have energy functions corresponding to these systems which you can think of as Lyapunov functions maybe if you mean. Yeah.

And you look at the minimas of these. There are two different minimas. Okay. Then if both of them have unique minima at zeros. Right. So basically what what he's saying is that if you have strict output strict passivity then and zero state detectability.

We already know what these properties are. We said zero state observability he is weakened to zero state detectability. Let's not worry about that. He actually stated it much more simply than I did. I sort of gave you a more complicated version of what does zero state observable mean. He stated it very simply saying that if y equal to zero x equal to zero.

Yeah. This is what is zero state observability. Yeah. I also stated the same slightly in a slightly more complicated way that H equal to zero set can contain only the zero trajectory. Yeah. Then the only trajectory in this set is the trivial trajectory.

Yeah. I stated it more like the LaSalle invariance that you're used to because all of this passivity based results require use of application of the LaSalle invariance. That is why I wrote it in this form. Otherwise what he said is good enough. If you remember we also connected this with the linear system observability.

Okay. Very cleanly connected with linear system. Yeah. Please.

Yes. This one. It would be strict less than. That's it. Okay. It is strictly less than. And then there is multiple versions of it if you remember.

I will go to this other lecture. He had the first slide or second slide. This slide. Yeah. The beta was actually the energy available.

So you can think of this as V dot. You can think of this as V dot. Passivity is just less than equal to. Right. The strict passivity is or output strict passivity.

Let me be more formal. Output strict passivity is this available energy or this V dot is less than equal to this minus this guy. So there is a dissipation. What is this term? This is basically dissipation term.

You just saw that example. Right. You saw electrical systems and so on. Here when output strict passivity the dissipation is a function of purely the output. Input strict passivity function of purely the input. The dissipation is purely a function of the input.

This is an unusual case. I have no idea. I never know how such a dissipation would happen. This has to be some kind of dissipation in your actuator itself. So I don't know how else I would get this kind of a dissipation. And finally you have state strict passivity where this dissipation is now a function of all states.

This is obviously the strongest sort of result. Okay. And this beta thing is whatever he used this notation beta but actually it is the energy available like E t minus E 0. So it is or V of t minus V 0. If you want to think in term of Lyapunov function it's V of t minus V 0. Yeah.

We have used the Lyapunov notation because we are used to it. Yeah. That's about it.

Okay. So a lot of there is pretty clean connections here. Yeah. So don't worry about I mean don't think that he suddenly went into something too crazy or new. He did not. Okay. So let's go back to the example and we sort of go forward here.

We saw multiple notions of getting to passivity. Right. Of course you may not start be able to start with a passive output. You may not have one or you may not know one. Okay. Though the first is if you do have a passive output we already proved that you can get this nice feedback and zero state observability will ensure that you have global asymptotic stability.

Great. We proved this result. Okay. So the first obvious thing was choosing carefully an output so that you have passivity. Right.

So that's what this is. Right. In fact he also has this if you notice. See what if you remember he talked about the KYP for the non-linear systems. That's exactly this. Output is chosen as partial of V with respect to X.

Yeah. And then there is a transpose. Right. LGV is what? LGV is nothing but. So if I take the transpose to the other side.

Okay. What did we do? Just look at this. Exactly the same thing he said. G is what multiplies the control. This is how you choose the output that will give you passivity.

That's exactly what he said. Yeah. Here. Okay. Exactly what he did. Yeah. Because as soon as you do this you will get V dot as minus alpha X plus U transpose Y. Yeah. This is actually state strict passivity.

Right. Because this alpha is now you know whatever. Class K function. It is negative in the state itself. Not just the output and the input or input. It is actually negative definite in the state.

Yeah. Because it's a class K function. And then you have U transpose Y which is what you wanted.

Right. Because you chose Y as this. Because you chose Y as this. Yeah. You chose an output. It may not be a real measurement or anything like that. We discussed this. It may be some fake unreal quantity.

But the purpose here is to just design a control law. Yeah. It's not actually to focus on what is this Y. So if you don't try to think of questions like you know why this is not a real measurement.

How are you even going to measure something funny like this. You're probably not. Yeah. This is just a way of designing a control. We figured out output Y so that you get passivity. So this was the first method of inducing passivity.

Remember by output selection. That's what we covered. OK. And that's what he also covered when he spoke about the KYP lemma for linear systems and corresponding KYP lemma for the nonlinear systems. No difference.

All right. Great. Then we went to the important concept of feedback passivation. This was a case where you may not have passivity in the original system. But you construct a feedback so that your system is passive in the new input and some output. OK. So in such a case you're already given an input and output. And you are claiming that if you construct there exists such a feedback such that if I plug in this feedback and notice V is the new control.

If I plug in this feedback here I get this system. Right. And the claim and the assumption is that with this V and with this Y the system is passive. This is the assumption. OK. And then you know that then it then you say that the system is feedback passive.

And as long as soon as you get passivity with this new control V and Y you're done. You can again apply the same theorem that you had that the control V can now be some function of Y and you will get global asymptotic stability under some simple conditions. OK. So so this is what was feedback passivity.

Where did he cover feedback passivity. That is where you guys got super lost. This one. This is where he covered feedback passivity. He considered a special case.

OK. Not he considered but that's where Kokotovich and Sussman started. He created a simpler case. If you see here there is no output yet. Correct. There was no output at all. Whereas we already said there is an output.

And then we construct a feedback so that you make the input output system feedback pass or passive. Right. So there is no output here at all.

How did he go about it. He first said that let me say that I can decompose this guy in this form. OK. That is it is somehow mean here in some output of this system. OK. For this he introduced some definitions and whatever I'm going to completely ignore the definition because they really scare you and confuse you. Yeah all this FPR and this and that. The basic idea is that he assumed that the nonlinear system can be decomposed as a linear sum of the outputs of the linear system.

OK. Pretty serious assumption by the way. Yeah. On the structure of the system. OK. You are saying that notice Y doesn't even appear here.

OK. But Y is some output of the linear system some C matrix. OK. What am I saying. I am saying that using this Y I can decompose this F into this. Where there is some drift type term and then there is some you know you can think of it as control type term or an output type term I would say.

Yeah. Where this Y is basically this and output of this guy. OK. And there are some really nice assumptions on this output already. What are the nice assumptions that this output is making this system making the linear system this guy with this is passive.

This is an assumption. OK. Passivity of this linear system that is with this psi and with this Y is passive. OK. So to get this passivity you invoke what is called the Kalman Yakubovich Popov the KYP lemma.

OK. And this is the condition on passivity. So these are the conditions. Yeah. Because C is what governs the output you designed. OK. And K is the feedback.

So basically if you have these two conditions it guarantees that this system is passive with this output. OK. And not just that this output allows you to linearly specify or linearly parameterize the drift or the or the dynamics of the X system. OK.

These are all assumptions all assumptions. OK. So that's why I kept asking him Oh God these are very scary assumptions. I mean where do you even get them satisfied. Yeah. But that's why he actually showed an example of a robot system where you can actually have these assumptions.

Yeah. And these these definitions FPR output. FPR spanned all this is just to say what I just said. Nothing these are nothing special. OK. So and then FPR stable decomposition. Basically all that is saying is that you you are able to decompose it.

You have the linear system with this Y to be already passive. On top of it you are saying that if psi is 0 then this system F 0 X 0. Yeah.

This dot is F 0 X 0 is actually globally asymptotically stable already. OK. So several several assumptions that are there. Yeah. So so this system with psi equal to 0 is already asymptotically stable.

OK. So under these assumptions you are designing a control U to stabilize this whole system. OK. All right. So what have we assumed. That the linear system with some output Y is globally asymptotically stable.

Actually this is what I think there was an error. This should be F 0. This should not be F. Yeah. The linear system with the output Y is passive. OK. The nonlinear system can be decomposed in terms of the outputs of the linear system.

OK. And the drift of the nonlinear system is globally asymptotically stabilizing if the linear system states are 0. OK under these assumptions you can design a strict passive control and so on and so forth. I mean so that's I'm not going into the details of this. It's complicated. It's confusing not complicated but it's essentially doing what I'm doing.

I'm also doing the same thing in a more general setting actually. OK. In a slightly more general setting and you're doing the same thing. What they are doing is also feedback passivation. Yeah. The only difference is they also have see why they say it's a cascade is because you see there is a the passivity is only assumed on the linear system.

Yeah. And then there is a nonlinear system which has no control in it. Right. The nonlinear system has no control variable at all. Yeah. Where does the control for the nonlinear system come in.

It comes in through this guy. You can sort of think of it as coming through this guy. Yeah. But then you're also assuming nice things on the nonlinear system already that this is somehow stabilizing. Yeah. And then you have this. So this is why it's called a cascade because you're sort of connecting a nonlinear system a linear system in cascade to a nonlinear system.

OK. This is not a feedback interconnection. This is actually a cascade connection. Yeah. I think he used the term cascade somewhere. I don't know. Cascade. This is cascade. Why? Because output of this system feeds into this system.

Because this f x i is basically summation of sorry f 0 x i plus summation of y i f i. Yeah. So this is actually in cascade. Yeah. Cascade means the output of one goes into the input of other.

Feedback is the circuit circuit as route. Right. Output doesn't go directly to the. So there is a cycle there. So here there is no cycle. Yeah. If you notice the linear system is not getting any feedback from the nonlinear system.

It is free of the nonlinear system. Right. Linear system is free of the nonlinear system. So what does what does he do. He says that if you have the you have all the nice assumption basic conjecture is that if you can make psi go to 0 fast enough then you can stabilize.

Okay. So so I mean not this one but but actually this is a counter example. That's a counter example. I apologize. So basically what if you notice what he will do is in this sort of an example all he does is he let's see he has a stabilizing term.

Let me try to point it out. I know this is again getting confusing I'm sure. Yeah this is easier. Yeah. He makes it into a sort of feedback interconnection just by the way he designs the controller. If you see for the linear system he constructed a new feedback.

There was original feedback was what. You correct. So he said you will be minus BK plus V. This is like the feedback passivation type thing that he's doing.

Yeah. And that might minus BK went in here and then the V remained here. All right. And then what is he going to do.

He's going to set the V as this guy. What is this term. Do you understand what this term is. Why do I set V as this. What is this. We just saw this somewhere in output passivation. It is just you see if you think of Y as the control it is just LGV.

If you think of Y as the control is exactly LGV. So he's essentially setting this to that. Okay. That's it. If you have passivity you are just choosing the correct output. And he chose this as his output in some sense. So he's just setting this V as you know LGV type of a term.

Here it shows up in the feedback instead of showing up in the output. Because you are doing this feedback passivation type thing. That's it. But if you see the structure of these things are exactly what you are doing. This is exactly LGV if you assume Y as the control. For this system if Y is the control then this is exactly the output that you would choose to make the system passive.

And in order to ensure that this is the output. Yeah. You choose the V appropriately. That's it. It is just a smart trick so that you get this sort of an output using this sort of a feedback structure now.

Okay. And the math of course is you can make it go through very comfortably. It's not a problem. Yeah. So this is what the general passivity based control looks like. Yeah. Where you have a system with which you have a linear system a nonlinear system with which you have a linear system in cascade.

Yeah. And under some whatever under several assumptions you sort of get this passivation. Yeah. Here we said we had a slightly simpler setting. Yeah. Let me see. We also get to the next step by the way. I think we also covered this robot example right a little bit and I think we were somewhere here.

Right. We were somewhere here and then there was this exercise. Right. Yeah. Anyway the exercise will be set today and you will now have numerical exercises in the sense you will have to do some simulations also now.

Okay. Because you have a robot. You have to make the robot move. All right. Now if you. Yeah. Yeah. Not experiments just simulations. Don't worry. Yeah. If you notice the next step was exactly this cascade connection with passive system. This is what we are going to look at now in our context which he presented sort of in the last lecture. So we will sort of look at it again. All right. Thank you. Bye.