Nonlinear Control Design

Prof. Srikant Sukumar

Systems and Control Engineering

Indian Institute of Technology Bombay

Week 8 : Lecture 44 : Passivity in control systems: Part 4(Prof. Antonio Loria)

This is the same paper that generated many other works. So essentially you have a system like this, you have a double integrator and you have this guy here. You see there is something that makes the system asymptotically stable if these were not there. But then there is this extra nonlinearity that you don't know what to do with it. So somehow you want to, the effect of this nonlinearity, you want to cancel it by designing a control input in the linear dynamics down there.

So what you want to do is first design, well I will skip this, what you want to do is to design you to stabilize Xi, but not only you want to stabilize Xi but also you want to add an extra feedback to take care of these terms here. So according to the theorem I showed you, what you do is the following. First if this is just double integrator you first use feedback to put the poles of the linear system in the right place. So let's say we choose these control gains to make the system down here asymptotically stable and you add an extra input as I said to take care of these guys here.

So this is the matrix that we saw before, so here A is just 0, what is it, 0 1 0 0 and this guy here is A minus BK which I made it Hurwitz. Then there is the extra input and my Lyapunov function that I'm going to use to figure out this input is going to be V which is a Lyapunov function for the system X dot equals F0 which in this case is just X dot equals minus X cube. So V equals X squared will do because with that you get V dot less or equal than, well actually equals to minus X cube if I consider V dot just to be partial of V times F0 and what else. And then I have this guy here that will generate this one and then I can figure out what I have to put in this input to take care of the of the external linearities that are sitting there in my system. So I just do that computation, I take the derivative, this is minus Q and this is C and what else, here I have Xi, here I have Xi and V I can choose it so I will just choose it to match exactly that with a negative sign and will take care of those nonlinearities.

I will add a minus Y here to add extra dissipation if I want and then what will I obtain, I will obtain this. If I want to continue maybe there is something that after this system there is, I will go and interconnect it with something else. So let's just say for the purpose of continuing thinking about passivity interconnections and so on, let's just invent a new input that I'm going to put in my system. So I close the loop and I add a new input. As you can see I have passivity from this new input to the output Y because I have all these negative terms here.

So I have actually strict passivity with respect to the state here, I have these negative termings Xi and I have this negative term in X so that gives me strict passivity with respect to the state and by adding this minus Y here I have a Y square in this inequality which gives me also, so this is integral of Y square. It gives me also output strict passivity. So you see with this controller I enforced the passivity of the system. So this is the picture I have. I have this system that I'm interconnecting with this one here.

This system has an output this, so this is an output that I define and I use that output to make the passive interconnection. So I'm going to use that output to inject it into this block and I'm going to define this input V exactly as being that output. That's exactly what I just did. I said my new input V is defined by this guy here. So that's this.

Then I take the output Y and then it goes back into here. So as you see it's for a particular class of systems but often with mechanical systems and such you can do that. Yeah it looks like that because you have, again, but essentially you, so think of this as imagine you will see, you will give a lecture on adaptive control. So suppose you do your control knowing all the parameters of the system and you know how to do that. So you end up with this equation X dot equals F0.

So that's the case in which you know the parameters. And then you don't know the parameters so a bunch of extra things appear. Well you can just take care of them with this feedback passivation. Yeah that's, so this cascade story is that I skipped it because we don't have time but Kokoto which was, I mean this triggered all the study about cascaded systems and as you can see also this feedback passivation is exactly at the basis of feedback of back stepping. So I mean this paper really is at the basis of a lot of things.

So the cascades thing is that you could imagine that if you have this system that is linear that is perturbing your nonlinear system. An intuition would say okay I will just stabilize this guy and once this is not there I have a nonlinear system that is stable. So everything should be okay. So they remarked that no it's not like that in nonlinear system because if you like say I'm going to stabilize it like real fast. But real fast for this system may also mean that you have a peaking so in the transient you have a huge peak.

And then yeah you stabilize it in like in one second the size of the river is small. But this peak goes to certain levels and if you inject more and more damping to stabilize it faster and faster what you will end up with is that your system essentially will explode. So the solution to the system up here is given by this equation. There shouldn't be x1 it's just x. So this is the solution to the system and as you can see here you have a bunch of terms with a minus one there.

So at certain moment all that cancels out you are dividing by zero and the solution explodes. So they were just remarking that yeah for these kind of systems it's in cascade because this guy is completely decoupled there is no dependence on x. If you just stabilize it with a certain feedback you make not only you will not be achieving stabilization also of x but you will actually make it explode in finite time. So that's not what you should do. So yeah so they figured out okay so if these if the extra nonlinearities that are sitting there have certain structure I can figure out how to inject something some extra input here in order to take care of those.

But yeah there are these structural properties right so you have to define certain output from the nonlinear system and then you have to have the xi here appears in the nonlinearities in your system but it doesn't appear in just in any arbitrary manner. It appears like this remember so you have to be able to factor out of your nonlinearities this y which happens to be exactly c times xi. So in this case well that output is this. So but then again it's really about choosing the right output in your system to define this passivity property from the right output and if you have these structural properties you will be able to do things. Okay I'll try to do this last part of the lecture on a specific kind of passivity based control that is called energy shaping plus dumping injection.

You have probably heard of this. So this method was originally proposed by a paper in 81 really long ago that was also very revolutionary paper because so 80s is you know I mean it's really a long time ago basically only Japanese had robots in factories and such so in back in those days notably in chain production of cars right so basically in those times there was very little known about nonlinear systems so they figured out how to control robots robot manipulators nonlinear systems using this method that they called energy shaping. I mean it consists in I was saying in the beginning when you have a passive system you have this energy balance equation so what they figured out is if I want to bring my pendulum to the side position all I have to do is manipulate the potential energy to bring the minimum of the potential energy to that position and my control is through my control I am able to do that I will be able to stabilize any equilibrium at that position. If on top I add dumping then I will also be able to stabilize it asymptotically using again the passivity notions. So just to illustrate this method yeah so you can also read about it in our book with Romeo Ortega in 98 but yeah I know that this method goes back to this to these guys.

So if I illustrate this method with a pendulum what the situation we have is the following. This is the potential energy the total energy of the pendulum right so kinetic energy and potential energy and potential energy looks like this right so one minus m g L cosine so it's that good. As you can see this energy has a bunch of minima so depend we are thinking that q is in R okay don't think minus p minus 2 pi minus pi pi but R so if we think of q in all variables in R then it has a bunch of minima at n 2 pi n right well if you want to stabilize your pendulum at a specific point you don't want all these minima you want to first to have only one minimum which is your equilibrium and you want it to be global so meaning that your potential energy should be look more like a parabola or something. So what you want to do is to manipulate this potential energy and you want to do that because well conserving the passivity properties and such. So the first thing to remark is that the pendulum from from the inputs u to the velocities so as we already saw torques to velocities is a passive map right so if you take this function the derivative will give you just the product of u times q dot if there is no friction so it's a passive map right in other notation it satisfies this.

So in order to change the potential energy the first thing you do is maybe add a line to the to the to the expression of the potential energy and also an offset okay so first you want to to put the minimum of your potential energy you want to put it at a two star somewhere and if you add a line then your potential energy now you will look like this so that's that would be one minus mgl plus an offset plus this line right so it will look like that. I still have several minima right but my system is still passive all I did was just basically add this this offset and this this line here and now my system will still be passive from a new input that I'm going to call u fb for u feedback so there is a feed forward and there is a feedback so the feed forward is just shifting the the minimal the potential energy to to the gravitational vector to evaluate that q star. So that this system is passive you can see it easily by just taking the derivative of the new Lyapunov function the new storage function the kinetic energy plus the potential energy of the pendulum plus the modifications I added so of course this function is zero at q equal to q equal to q star right so these two guys will eliminate will cancel out if q is equal to q star q tilde I'm defining as the difference I guess it was yeah it was somewhere in the other slides q tilde is is the difference between q out here so these will also go away and so it has it has a singular point at q tilde equal to zero and actually that singular point is zero so it's if I take the derivative of that yeah as you can see this guy goes away with this because it's multiplied there this is one this goes away with that and we only have left this this vector so all I did so far is manipulate the potential energy right now what I want to do is I don't want the potential energy to look like that I said I wanted it to be a minimum that is global meaning that it is this point is smaller than any other point of the of the function in the image of the function and also it has to be the only one okay global and unique is not the same thing yeah I could have a function that does it goes like this and then flat and then like that so all these are global minima right so what I want is something more like what I have in this picture and the way I achieve that is to add to what I already put in in my function I am going to add a parabola so if you add a parabola to that basically now the the the thing that was going like that now it will look like this this blue curve there and but this blue curve for a certain value of KP will look like this and you can see there is another minimum there okay so there is one minimum there and there is one minimum there so this minimum is global but there is it's not the only minimum in my function there is also this one so eventually my system could go and get stuck there I don't want that so I increase my KP to make my function rather look like this okay so now I'm sure that I have a unique minimum once I have done that modification to my potential energy I do that through the controller so the controller essentially will carry all these extra terms that I need to modify the potential energy and all that will cancel away so we have this guy cancelled with well it's it goes from from from this so it cancels with this this one comes from the controller so it will cancel out with this one okay so it's multiplied by Q dot here and and this one comes from the parabola that I added to my potential energy that means that in my controller I should put I should put this gain here and it's a very simple controller for a nonlinear system right it's just a proportional feedback plus constant essentially to and cancellation of the nonlinear well plus this compensation of the gravity and the equilibrium we're doing that we're doing this reshaping of the potential energy what I'm doing is that I'm rendering passive so I in the beginning I had a passive system I think these offsets only modify the minimum with the potential energy but the system is still passive and then I added this this feedback here so it's this proportional term it's a feedback of Q dot if I pass it through an integral so an integrator is a passive system right this is the simplest passive system you can think about so X dot equals U or let me write actually Y dot equals U V equals Y square that means that V dot is equal to Y times U passive system an integrator is a passive system so I have a passive system interconnected with a passive system in closed loop I will have a passive system which is expressed down there and to make the the new minimum to make it global what I need is to as illustrated in this figure I need KP so this is for a certain value of KP rather small and the red one is for an increased value of KP so that actually the condition comes to KP should be larger than the partial derivative of the gravity forces vector and that will ensure me actually that the second derivative of V is is positive and that will ensure me that my my function is strictly convex in in in Q tilde okay this is only sufficient it's not necessary it doesn't have to be a parabola it could be also function that this parabola only locally but then it grows linearly and then this position will not hold but for the purpose of energy shaping and all that that still works and then yeah so all so far what I have accomplished is I started with a passive system modify the minimum applying this gain I made sure that the minimum is global and unique I still have a passive system there from from every time I add a new input and towards the same output Q dot and remember that theorem of burn CCD or Williams he was saying that if I have a passive system and I have that detectability assumption satisfied all I have to do is to interconnect an extra feedback here which is basically a function of the output of with respect to which the system is passive and this function here should be basically a non-linearity living in the first third quadrant so if I inject that I will inject dissipativity in my system and I will make the whole thing output strictly passive so this block will be input strictly passive with the whole block here will be output strictly passive and yeah so the proof of this proposition that you have we now we have with this controller global asymptotic stability can be done using this theorem that I show you or Barbashin Krasovsky which probably you know wrongly as LaSalle's theorem but it was actually originally proposed by some Russian guys several years before LaSalle and the controller is right here so it's what I just explained is the following first I added this right to change the the the minimum of the potential energy then we added this to make it global for sufficiently KP large sufficiently large KP and then this guy to add damping right so we call this damping injection and with these two terms we call them energy checking terms and the proof is very simple you just take as the upper of function the the modified energy of the of the system in this case we only modify the potential energy the derivative of all that is is going to be it's going to be this right so you can see in UDI you just want to inject damping so that you get you get a quadratic term I hope it is there somewhere yeah here you get a quadratic term in in the output yeah so this is the proof using burn C C Dory Williams it says that all I have to do is inject this guy that will give me V dot with this it will give me minus K D Q dot square and then I have to check whether Q dot implies that Q goes to 0 in this case actually if I write Q dot equal equal to 0 that will go away that I have this equal to 0 and the only solution of this equation is is Q Q equal to Q star and it is the only solution once again due to the condition that I'm imposing on KP so it has to do with making the only minimum to be to be K star you can do the same proof using LaSalle again so that you can write the system or bar machine krasovski you write the system in this form go here set x2 equal to 0 and see that the only solution that contained in this set is the origin right so and the nice thing about this is that you have output the strict passivity so if you add another input here you will have instead of in place of these you will have the same input multiplied by my Q dot so you could continue carry on with more things let me just quickly show you what happens when you want to stabilize the system again here but remember we in the controller we were adding this term this it's the compensation of the gravity at the Q star so this is constant this is just a constant suppose you don't know that constant so you want to estimate this constant you put an estimate of the constant and then you want to figure out how do I update this constant so it's typically what you do in adaptive control that you haven't seen that yet but in this case it's just integral action and it comes back to to the method of of Kokoto Beach what we want to do is to first apply the the know that works when we know the constant and what we do next is we add and subtract that constant because we actually cannot put it there so we are actually going to apply this controller okay so we are going to use an estimate of the of that constant and we add and subtract the the real the real thing okay so but this is the control we're applying this controller can be written like this so the one that we already know how that it works it gives in passivity etc plus the difference between what I'm putting in and the real thing that I don't know okay so that again is just well this is constant and this is this is g hat so I'm going to call that psi so yeah all this to say that the the class of systems that fit this in this paper of Kokoto Beach is not that restrictive actually for instance here we have exactly that that systems here we have this this is this is the pendulum equation in closed-loop you can see here the kpq tilde term this is the kdq term q dot term and this is the gravity this is the gravity this is just g of q and this is the compensation the ideal compensation that I don't have and when I apply the the the the control with the estimate of the compensation I will have this these equations so I have psi times a nonlinearity which in this case is just this constant vector right and here we have this f0 remember that has to be gas but I know that it is gas because I know how to do things when I know that the gravity compensation so I say well what how do I estimate g well as to figuring out what to put in g dot it comes to figuring out what I put in psi right so I basically just go ahead and write this equation which is an extra integrated and these guys will tell me well just put that well I I can't really because because because what because that would involve using psi in the feedback if I wanted to put this term in in this equation it would involve using what I don't know so I will just not put that I will go ahead and test my chances with just the other part that takes care of the effect of this extra term that I have in my equation and see what happens so I we didn't see that here but there is a simple way to figure out a Lyapunov function for this guy and using that Lyapunov function you can establish passivity now not not only with respect to the velocities but actually to this output which is essentially a small constant times positions plus velocities so it's just another passive map okay and when you apply that you end up with this controller so the controller you already knew how to how to that uses gravity compensation but in the instead of gravity compensation you use an estimate of it and in place of the estimate you put this guy which comes really from from the method of of Kokovich which tells you well take the output of your system if that output is multiplying this external linearity that you want to get rid of well just put that into your new feedback and that will take care of this extra thing that you have in there so you can see that by taking the derivative of this Lyapunov function with one half of xi squared you will see that you get this guy and this guy that comes from from your equation here right so these and these go away and then you get your system and yeah it all comes just to integral action so this is what we had when we know the gravity term if we don't know the gravity term this passivity based control is saying well just take this output because the system is passive also with respect to this output not just with respect to Q dot take that output pass it through an integrator and then you will still be still be fine and it all boils down to PID control so you can see so PID control for robot manipulators is another passivity based controller these are some references in case you want to to know more about all this okay thank you