Optimization from Fundamentals
Prof. Ankur Kulkarni
Department of Systems and Control Engineering
Indian Institute of Technology, Bombay

Lecture - 1A
Introduction

So, I am Professor Ankur Kulkarni from Systems and Control Engineering at IIT Bombay. So, and I will be teaching this course. So, in the IIT Bombay code is SC 607, it is called optimization. In NPTEL it will be called Optimization from Fundamentals. So, optimization as a topic is mainly about choosing the best alternative out of a given set of alternatives ok.

That is basically that is the purpose of optimization the and this is a quantitative subject. So, when we have to quantitatively specify what the alternatives are, we have to quantitatively specify what it means for an alternative to be better than another alternative ok.

So; that means, we quantify the value or the cost or the benefit of a particular alternative. We also quantify what it means for an alternative to be an alternative ok, why is it an alternative ok. Now, all this is very easy if you could simply list down all the alternatives and run through them and select the way ok.

So, the that unfortunately is the is not the case with most problem. Usually, the number of alternatives itself is very large, how changing going from one alternative to the other, how that changes your benefit from it is also is usually complicated and unclear. As a result, then when has to you need a formal approach to all of this ok. So, that is why this that is why this is a subject in its own right.

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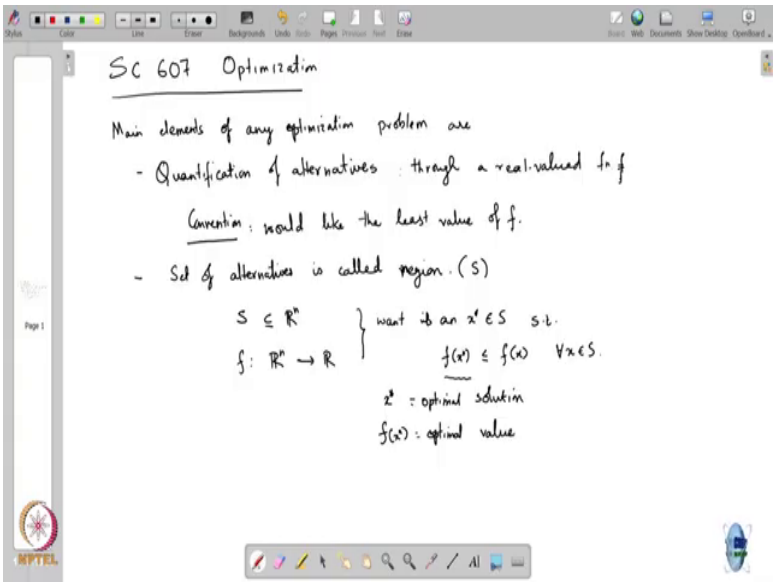

So, the main elements of any optimization problem are as follows. So, first is as I said we want to quantify what is the best alternative. So, this quantification of alternatives is done through a function, through a real valued function ok. So, we and by convention what we tend to what we want to do is get the least value of this function ok.

By our convention by convention this is just a convention. Let us call this function f ok. Now, at least value over what, over what set of alternatives? So, this the set of alternatives is called freezable region ok. So, for example so, you for example, your set of alternatives; let us call this set S. This could be a subset of the vector space R n, your function f could be a just a function that maps R n to R. And, what you are looking for is the least value of f; that means, you are looking for an alternative x star.

So, what we want is an x star that belongs to S such that f of x star is less than equal to f of x for all x (Refer Time: 05:30). This is what we want. Now, this is the ultimate goal, if you can get an x star; sometimes we are not, we do not really want an x star, we just we only care about f of x star without actually explicitly saying what the x star is ok.

So, both of the x star is referred to as the optimal solution and f of x star refers to as the optimal value. Now, the English meaning of the word optimal is some somehow unfortunately different from both minimum and maximum. The English meaning somehow optimal connote something in between minimum and maximum right.

Well, but that is not the sense in which we want to use it here. By optimal it means, it solves the optimization problem ok. Optimization problem the goal of the optimization problem is to find the least value, least value of f ok. By this, now this is by the now why did we restrict to least value?

This that is by convention it, we will see later that it does it does not matter. If you it is a standard way of posing a particular problem, you can talk of maximum value also. It is that is equivalent right ok. Now, I was saying that this is if you could list out all the alternatives, then the problem is very easy right. But, usually the number of alternatives itself is huge right and sometimes in many interesting cases it is actually infinite right.

So, let us consider for example, I want to find the shortest path from IIT Bombay to IIT Delhi, shortest path in term in some according to some measured in some way. Suppose, this may be measure it by or distance measure it by travel time whatever, in measured in some units shortest path from IIT Bombay to IIT Delhi; starting from gate of IIT Bombay to the gate of the IIT Delhi. So now, how many paths are there?

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I can. So, for this I really need to specify the what is the environment, what is a path; I will have to give you the entire map ok. What sort of roots form a path right, put together all of

that you will realize that the number of paths is huge; because pretty much at every junction you fork this way or that way and still somehow end up reaching at IIT Delhi right. So, the number of alternatives is immense as you can see.

But if you, but if you just do this in Google Maps right now, open up your app and do it in Google Maps; you will find you know it may be the matter of few seconds you will get an you will get you will get a route ok. So, it is certainly doing something more intelligent than you know cataloging all searching through all the alternatives.

It is probably searching through some, but certainly not through all because the number of alternatives is immense right. So, it is certainly doing something intelligent there, it is probably once it realizes that this alternative is bad, it is also realizing the some other alternatives are bad and therefore, eliminating all of them etcetera. The something some intelligence is built into all of this and all of that is the part of optimization.

Let us consider another problem. Since today is the first day of registration; so, the problem of assigning classrooms to courses ok, right hundreds of courses, IIT Bombay offers hundreds of courses per semester. We have hundreds of classrooms, there are about 10 to 15 different, maybe 15, maybe 20 slots I do not know, different slots that the courses can get offered in.

Now, you want to come up with a matching, you want to match courses to classrooms to and slots ok. But, then there are also other constraints like for example, classrooms should respect the registration ok. If you have 100 students registered, you cannot seat them in a classroom of size 50, whose capacity is 50 right.

So, you need classroom of suitable size for suitable courses, some courses may require say for example, lab courses can be done only in laboratories, some courses require that students be able to do programming in the class. So, they have some additional requirements; so, only certain types of classrooms can be chosen. All of these are additional requirements that may complicate the problem.

And, in addition to all of these instructors have their own preferences right, they prefer a certain slot, they prefer a certain room etcetera. Now, this whole classroom assignment problem is would be you can say well the way we can think of it is, we would like to get a classroom assignment and a slot assignment for courses; that is as close to the instructor preferences as possible without violating any of these any of these constraints.

So, you have to, but then you see the number of alternatives here; 100 courses, 100 classrooms; the number of combinations you can make out of all of and several slot. Number of combinations you can make is immense, out of that you have to come up with one kind of combination for at an institute level that will that will closest to the stated set of preferences of the of the faculty right.

So, this is an optimization problem, it comes up in day to day. We solve it every semester, similarly Indian Railways you know BEST outside on the street, every all of them they are also solving such problems in somehow the other form during their day to day operations right. So, the so, as you can see basically optimization is we essentially comes up everywhere, as you start thinking about it even without knowing at many times you are doing optimization.

When you are deciding which where to order from, where to order food from for example, you are trying to pick the best offer, you know which app should we use to recharge your mobile bills, you are trying to sort of you, you are trying to see where you will get where you can get the best cash back or whatever. All of this is optimization without knowing.