

ROBOTICS
Prof. P.S.Gandhi
Department of Mechanical Engineering
IIT Bombay
Lecture No-36
Robot Dynamics and Control

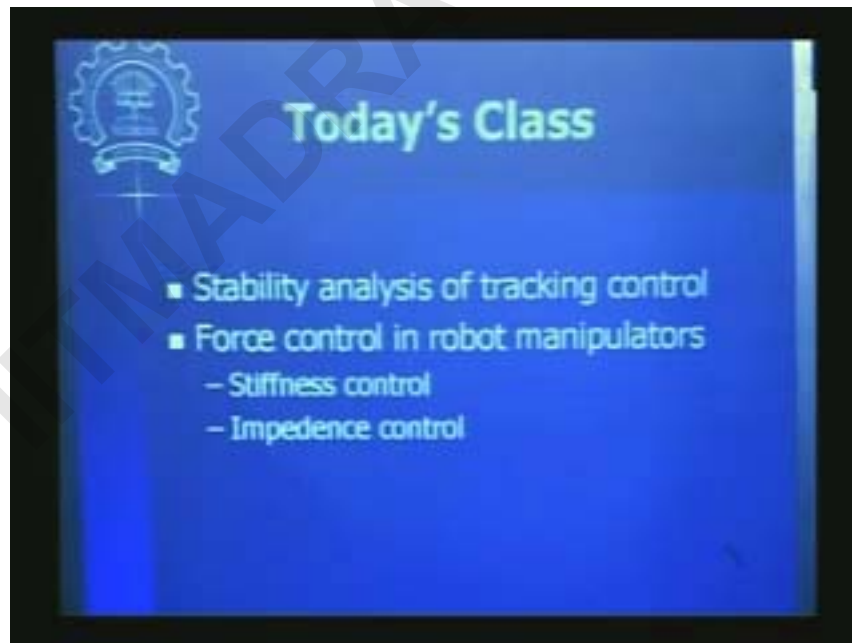
Good morning today is the lecture number six of this module robot dynamics and control this is the last lecture of the module robot dynamics and control okay

So let us see what we have seen in the last lecture we have done some something about Lyapunov theory so far we have been studying the Lyapunov theory we have applied it to robot manipulator

First pd control point to point aggression type of a control and then in the tracking control so we have given the tracking result only so we will do the analysis of the tracking part today okay

So some analysis of the tracking part we will see today and then we will move on to force control okay

So that is kind of a recap and what we will see in the today's class
(refer slide time 00:02:07)



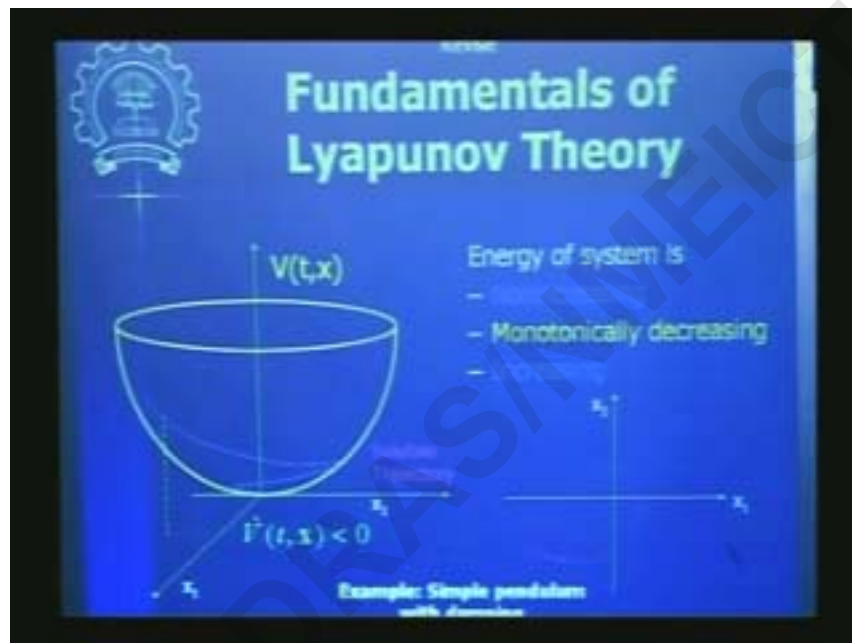
So the stability analysis of the tracking controller that we have presented in the last lecture we have seen this silly and starting kind of the controller in the last lecture

So this stability analysis we will carry out in today's class and before that we will just revise some of the fundamentals of Lyapunov theory again

It is kind of a new topic ah so we will do the revision little bit revision of ah Lyapunov theory again and then we will move on to force control

In the force control there are many strategies available what we are going to look is the strategy stiffness control and impedance control and there is something called hybrid control where you control the force and position together that we will see if if time permits we will look at that in the next class okay

So to just recap Lyapunov theory fundamental (refer slide time 00:03:10)



So basically Lyapunov captures the stability of the system in terms of some functions called Lyapunov functions or Lyapunov function candidate initially okay

This is a picture of like schematically what is happening in the Lyapunov analysis

So you have this energy based or energy kind of a function which is positive definite and then you was having a look at solution trajectory the derivative of this function along a solution trajectory this is a solution trajectory and we see that if the derivative is continuously decreasing that means the solution trajectory is having a behavior somewhat of this kind

So along this V solution trajectory along the solution trajectory the derivative of this V is continuously decreasing that means you are coming down each level on this V and then because this is a positive definite function we are driven towards the origin

When \dot{V} [noise] is decreasing okay so that is what we saw so so there is a strong correlation between the energy based or some Lyapunov function or some positive

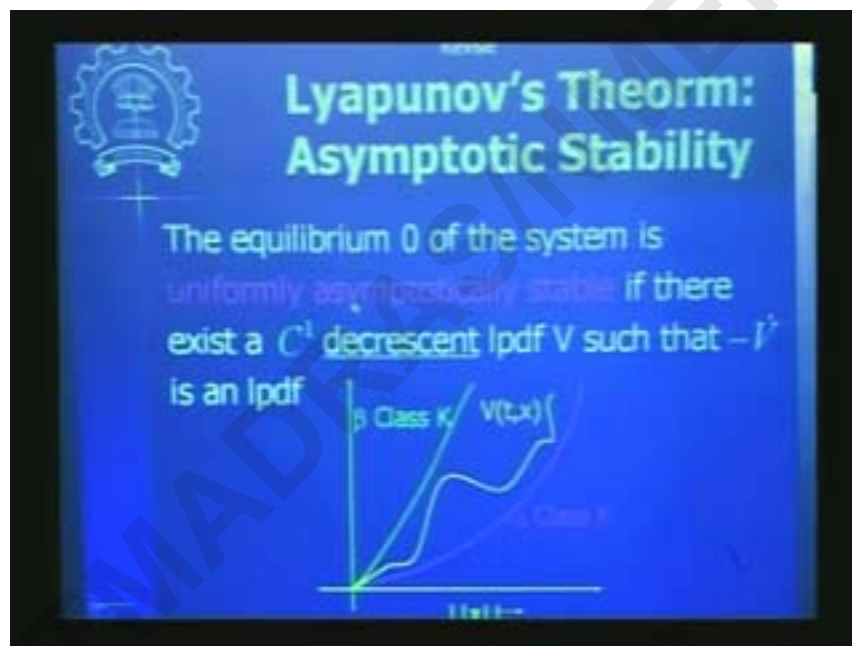
definite function and its derivative the properties of this positive definite function and its derivative and ah stability of the system okay

So this is the basic fundamental understanding of the Lyapunov's theory okay so do you do you see that this is like very strong and important to understand this point okay

So once you understand this then it is apparently clear like the the theorems that Lyapunov has presented we need not go into all the mathematical details but we can appreciate why the conditions on V and \dot{V} are there okay

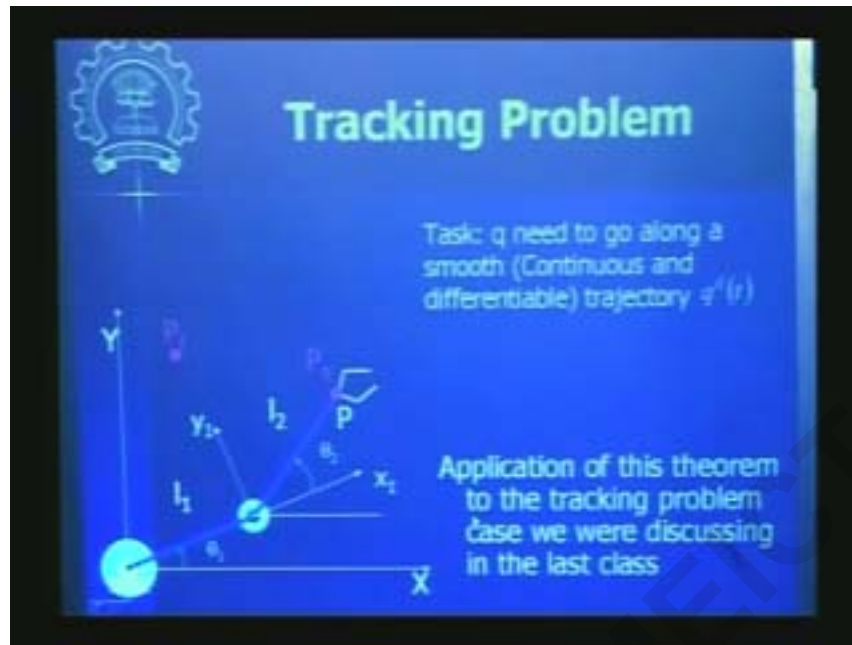
So let us have a look at again the one of the theorems it is asymptotic stability theorem

So the theorem says that equilibrium (refer slide time 00:05:41) is uniformly asymptotically stable if there exist C^1 decrescent lpdf that the three properties V has to satisfy and such that \dot{V} is negative definite [noise] or minus \dot{V} is lpdf okay



So this C^1 decrescent lpdf V can be schematically represented in this fashion

It is bounded above by class K function and bounded below by class K function okay now let us move ahead (refer slide time 00:06:20) toward tracking problem



Again we recall what we are talking in terms of a tracking problem we are given a trajectory to be tracked okay some straight line or some some curve which function is predefined okay

So based on this trajectory to be tracked we find out the joint trajectory the desired joint angles okay all that desired all that manipulated angles should be moving along this desired trajectories

So as to realize this trajectory at the end defector so we find out this joint angle desired trajectories and then our task is to drive the manipulator along this desired joint angle okay

And then this is this is in terms of time t which is a function of time so we are we need to make sure that at a given time all the joints are at the desired location then only the end defector is going to move along a desired angle okay

So that is what our tracking problem is and now we will see the application of this theorem which which is asymptotic stability theorem with the controller which we have seen in the last slides in the previous class okay

That controller will apply this ah theorem and see how the controller is functioning or how we can reach some results in terms of Lyapunov stability for the tracking controller okay and again we recall that we cannot apply Lasalle's theorem in this case because it is time varying system okay

If system is time invariant then you can apply Lasalle's's theorem there is one more class like if the systems are periodic then also we can apply Lasalle's's theorem that is slight variation

But in general when it is time varying system you cannot apply Lasalle's theorem so we have to restrict our self to whatever other theorems that are there we cannot apply Lasalle's theorem okay

So Either we can use Lyapunov's tools or you can use some more advanced tools which are based on the l_p spaces we will not go into detail case of those tools but you should be aware of okay

So let us see our previous tracking controller again you have defined this controller in this fashion so why have we defined this and all those details will now they will be clear now

So this is a is this \ddot{q}_d and this is given by this function okay so it has in some sense this desired trajectory derivative term so this is D times this term is in some sense you are computed dot you recall

The the inertia part of the computed torque similarly this C time this \dot{q}_d term will be again in some sense the computed torque okay or the C part of the computed torque okay but in addition to that you can see that there is error term which is added in both of this okay

So this additional term will take care of the thing will see as we go now but this is in some sense taking care of no need to apply lassalle's theorem okay

So we will get asymptotic stability result directly by applying Lyapunov's theorem without going to advanced tools okay

We will see that how how that fact happens here okay so this this controller developed by li and slotine and it is developed around nineteen eighty seven

So let us see the analysis with this controller now recall (refer slide time 00:11:21)

**Lyapunov Analysis:
Li-Slotine Controller**

Recall prev class

$$(D(q) + J)\ddot{q} + C(q, \dot{q})\dot{q} + B\dot{q} + g(q) = u$$

$$D\ddot{q} + C(q, \dot{q})\dot{q} + B\dot{q} + g(q) = u$$

Substitute control in equation of dynamics

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + B\dot{q} =$$

$$D(\ddot{q}^d - \lambda \dot{e}) + (C + B)(\dot{q}^d - \lambda e) - K_d(\dot{e} + \lambda e)$$

$$D(\ddot{q} - \ddot{q}^d + \lambda \dot{e}) + (C + B)(\dot{q} - \dot{q}^d + \lambda e) - K_d(\dot{e} + \lambda e) = 0$$

our previous classes the equations of dynamics with the manipulator in terms of joint variables it was given in this form

This J is the inertia of the motor transport to the joint okay so this is our equation of manipulator so this term I named it to be D one here just for the convenience so I can call this whole term as the D and preserve our basic form of the equation okay

So this D is nothing but our D one plus this J okay and still now our condition that D dot minus two C is skew symmetric matrix is preserved because this J is constant inertia so if you take the derivative it will go to zero okay

So now will will substituted in this equations their control law that we have just presented in the last slide

So this D times J which was q d double dot minus lamda into e dot plus C plus B into this term new minus Kd into r term was there is r we have seen here to be e dot plus lamda e okay

This gives us this equation now we simplify that by bringing this terms on the other side so you get here q double dot minus q d double dot plus lamda e dot and similarly these terms which finally results into this equation okay (refer slide time 00:13:26)

**Lyapunov Analysis:
Li-Slotine Controller**

$$D(\dot{e} + \lambda e) + (C + B)(\dot{e} + \lambda e) + K_d(\dot{e} + \lambda e) = 0$$

$$\therefore D\dot{r} + (C + B)r + K_d r = 0$$

Lyapunov function candidate

$$V(t, q, \dot{q}) = \frac{1}{2} r^T D r + r^T K_d e$$

$$r^T D \dot{r} = (\dot{e} + \lambda e)^T D (\dot{e} + \lambda e)$$

$$= (\dot{e}^T + e^T \lambda^T) (D \dot{e} + D \lambda e)$$

$$\simeq \dot{e}^T D \dot{e} + e^T \lambda^T D \lambda e + \dot{e}^T D \lambda e + e^T \lambda^T D \dot{e}$$

We term this variable r I mean we write the equations in terms of this variable r and then they will become as simple as this form okay

This r is \dot{e} plus λe okay so this is a form of an equation which we have to analyse for asymptotic stability with the error dynamics okay

r is basically representing the error dynamics okay it has both error derivative and actual error e and \dot{e} it is a function of e and \dot{e} okay

So in terms of this new function r we will have to we have this equation of dynamics equation of error dynamics appearing over here

Now we need to prove by using Lyapunov's stability theory that this indeed gives us error and error derivative both going to zero as time instant vary

So how do we do that so we have to select first the Lyapunov function so selection or choice of Lyapunov function will usually energy based okay

In these case since r is a new term or new variable for error dynamics we choose the Lyapunov function to be in terms of r okay

So these not just depend dependent on error this also dependent on this not just dependent on error derivative also dependent on error okay this term

And in addition this is kind of a spring energy that is there because of the this part of the term so K_d into λe is basically spring constant when it is multiplied by e

You can recall your standard spring mass system equation you compare this term again you see that this kind of spring term in the simple spring mass system okay

With step corresponding to that the spring energy is getting added here now we have to first see whether this is positive definite function or not okay

So let us expand this in terms of error now see we want to see this as a function of error error dynamics as a positive definite function okay

So our all conclusions are basically about error error dynamics we have to conclude so we have to see in terms of error whether this is a positive definite function

See in terms of r it is a positive definite function here and here so in terms of error also we can prove that it is true okay

So we just expand this by substituting for r as $e + \lambda \dot{e}$ so this $e + \lambda \dot{e}$ transpose plus D times $e + \lambda \dot{e}$ okay

So sup this r is $e + \lambda \dot{e}$ so you take this transpose inside we get this equation and then you multiply

So this $e + \lambda \dot{e}$ transpose times D the first term then this $e + \lambda \dot{e}$ transpose into $D + \lambda \dot{e}$ this multiplied by this now this cross multiplication terms $e + \lambda \dot{e}$ transpose $D + \lambda \dot{e}$ and $e + \lambda \dot{e}$ transpose $D + \lambda \dot{e}$ okay

Now this both of these terms since these matrices are symmetric positive definite matrices D and λ because of that these both terms will not be just same okay

You can just write these as twice one of these terms (refer slide time 00:18:18)

**Lyapunov Analysis:
Li-Slotine Controller**

$$\begin{aligned}
 V(t, q, \dot{q}) &= \frac{1}{2} \dot{r}^T D r + e^T K_p e \\
 &= \frac{1}{2} (\dot{e}^T D \dot{e} + e^T N^T D \dot{e} + \dot{e}^T D e + e^T N^T \dot{e}) + e^T K_p e \\
 &= \frac{1}{2} \dot{e}^T D \dot{e} + e^T \left(\frac{1}{2} N^T D A + K_p \right) e + e^T N^T \dot{e} \\
 &= \begin{bmatrix} \dot{e}^T & e^T \end{bmatrix} \begin{bmatrix} \frac{1}{2} N^T D A + K_p & \frac{1}{2} N^T D \\ \frac{1}{2} N^T D & \frac{1}{2} D \end{bmatrix} \begin{bmatrix} \dot{e} \\ e \end{bmatrix}
 \end{aligned}$$

So how does V look like now so it has I have put this substituted this as per our previous functions and then we will find that this can be simplified in this fashion

I keep collect the terms corresponding to e dot square or e transpose D e dot e dot transpose D e dot and then this is a term corresponding to e transpose something into e so this part is also added over here

And this term is represented in this okay and then these two terms are twice one of these terms and multiplied by this half is this about here

Now again we can put it in a quadratic form in this fashion in terms of this matrix okay so here this is half lamda transpose D lamda plus lamda Kd matrix

This is lamda transpose see these are two equivalent equal terms and this is half D which is the term corresponding to this

So this is a quadratic form and now we for this term to be positive definite we know that we we need this matrix to be positive definite this first matrix and then we need the determinant of this entire matrix to be positive and then if you if you multiply and see it is indeed the case this term is getting cancelled this term multiplied by this term is getting cancelled with this term multiplied with this term okay

Because of that this is a positive definite we can conclude about the positive definiteness of this entire matrix okay

Now let us see (refer slide time 00:20:57)

**Lyapunov Analysis:
Li-Slotine Controller**

$$\begin{aligned}
 V(t, q, \dot{q}) &= \frac{1}{2} r^T D r + e^T \Lambda K_d e \\
 \dot{V} &= \frac{1}{2} (2r^T \dot{D} r + r^T \ddot{D} r) + e^T \Lambda K_d \dot{e} \\
 \dot{V} &= \frac{1}{2} [-2r^T (C + B + K_d) \dot{r} + r^T \ddot{D} r] + 2e^T \Lambda K_d \dot{e} \\
 &= -r^T (B + K_d) \dot{r} + 1/2 r^T (\dot{D} - 2C) \dot{r} + 2e^T \Lambda K_d \dot{e} \\
 &= -(e^T + e^T \Lambda^T) (B + K_d) (\dot{e} + \Lambda e) + 2e^T \Lambda K_d \dot{e} \\
 &= -e^T (B + K_d) \dot{e} - e^T (B + K_d) \Lambda e - e^T \Lambda^T (B + K_d) \dot{e} \\
 &\quad - e^T \Lambda^T (B + K_d) \Lambda e + 2e^T \Lambda K_d \dot{e}
 \end{aligned}$$

how we can take a derivative and then see what is the property that V not satisfy okay

So let us have a look at versions again so \dot{V} is this form okay now we will use this form taking the derivative so it is half two times $r^T \dot{r}$ you see this r they both r differentiated and see these are the this is the three product of three terms

So we have to differentiate one at a time okay and then again using a symmetry property you can write these as two times $r^T D \dot{r}$ okay and then this is $r^T D \dot{r}$ okay and here it will become two times I am sorry this two is missing in this this part this will this will become two times $e^T \Lambda K_d \dot{e}$ or $e \dot{e}^T \Lambda K_d$ both are the same term this are the symmetric property okay

Now this $r^T \dot{r}$ or D times $r^T \dot{r}$ you recall from the previous case our basic equation this is just the negative of C plus B plus K_d plus r okay

As substituted that in this equation the two times $r^T (C + B + K_d) \dot{r}$ for $\dot{D} r$ and negative sign was here and other terms are as there

Now you can see that the $\dot{D} - 2C$ can be separated out from this equation you have this two C here and we have \dot{D} and r^T and r there same r^T and r where separated out this term $\dot{D} - 2C$ multiplied by this half over here okay

And then all the other terms are [] okay now we know that this $r^T (\dot{D} - 2C) \dot{r}$ is equal to zero because of the skew symmetry property of this matrix that we have seen in our earlier classes okay

This $D \dot{e} - 2C$ is a skew symmetric so the quadratic form with the skew symmetric matrix will have zero

If you evaluate that form it will be zero okay so we use that property here so that is why again I am emphasizing that that property skew symmetric property of all the robotic manipulators is very important

So it helps you in doing this analysis here for tracking cases also and regulating cases also okay

We have to use that skew symmetry property for a stabilization I got proof of stabilization now let us proceed and see what it results into

So you have this \dot{e}^T now I have expanded this r in terms of \dot{e} and its basically definition to this is $\dot{e}^T \lambda^T B + K_d$ into again r is $\dot{e} + \lambda e$ and then this term two times is given as this

Now if you expand this further you see that these terms are getting cancelled okay (refer slide time 00:24:57)

**Lyapunov Analysis:
Li-Slotine Controller**

$$V(t, q, \dot{q}) = \frac{1}{2} r^T D r + e^T \Lambda K_p e$$

$$\dot{V} = \frac{1}{2} (2r^T \dot{D} r + r^T \dot{D} r) + e^T \Lambda K_p \dot{e}$$

$$\dot{V} = \frac{1}{2} [-2r^T (C + B + K_d) r + r^T \dot{D} r] + 2e^T \Lambda K_p \dot{e}$$

$$= -r^T (B + K_d) r + \frac{1}{2} r^T (\dot{D} - 2C) r + 2e^T \Lambda K_p \dot{e}$$

$$= -(e^T + e^T \Lambda^T) (B + K_d) (e + \lambda e) + 2e^T \Lambda K_p \dot{e}$$

$$= -e^T (B + K_d) e - e^T (B + K_d) \lambda e - e^T \Lambda^T (B + K_d) \lambda e$$

$$- e^T \Lambda^T (B + K_d) \lambda e + 2e^T \Lambda K_p \dot{e}$$

You have expanded this this part and then you see $e^T K_d \lambda e$ $e^T \lambda^T K_d e$ here both are the same terms okay they are equal to two times $e^T \lambda^T K_d e$ okay

So do you see that because of the symmetry of this matrices $K_d \lambda$ you can have this result okay

Though you have in some cases e dot here and e dot here you can still that property will be because of symmetry you still the property will be valid like this

You can still write it as a two times one of these terms and then this term and this term will get cancelled

So what we will what we get now (refer slide time 00:26:12) is this part okay

**Lyapunov Analysis:
Li-Slotine Controller**

$$\begin{aligned} \dot{V} &= \frac{1}{2} (2r^T \dot{D}r + r^T \dot{D}r) + e^T \Lambda K_d \dot{e} \\ &= -e^T \Lambda^T (B + K_d) \Lambda e - 2e^T B \Lambda \dot{e} - e^T (B + K_d) \dot{e} \\ &= -\begin{bmatrix} e^T & \dot{e}^T \end{bmatrix} \begin{bmatrix} \Lambda^T (B + K_d) \Lambda & B \Lambda \\ B \Lambda & (B + K_d) \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \end{aligned}$$

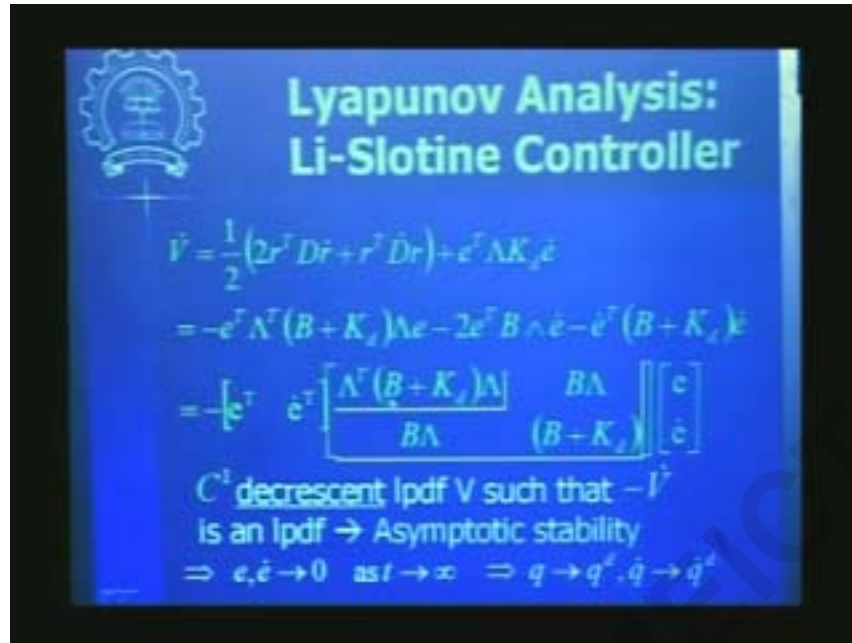
C^1 decrescent lpdf V such that $-\dot{V}$ is an lpdf \rightarrow Asymptotic stability
 $\Rightarrow e, \dot{e} \rightarrow 0$ as $t \rightarrow \infty \Rightarrow q \rightarrow q^d, \dot{q} \rightarrow \dot{q}^d$

So if we write now see this this term is ah first term this term I have written it first then this is whatever is remaining after cancellation is B times lamda and then e dot transpose this is the first term here this is the term okay that appear towards here

Now this again can be written as a quadratic form okay so in a quadratic form this is a first term this is B times lamda this is two times B lamda coming here so this B times lamda will appear equal terms in both these places cross diagonal terms and then these term corresponding to e dot okay

And then this is the negative sign taken out of all these terms now I need to prove now that these particular matrix is a positive definite matrix then I can have this result since this is a negative sign over here I have a result I will have a result of the asymptotic stability okay

Now to see that we first consider this part since it has all the positive definite matrices lamda B Kd that all positive definite symmetric matrices because of that this matrix or determinant of this matrix or determinant of this matrix is also positive okay
 So this matrix is positive definite now if you see these (refer slide time 00:28:28)



Here again if you see the product of these two terms minus product of these two terms the first term here will cancel this product okay

So lamda transpose B lamda multiplied by this B will cancel B lamda and B lamda again we recall lamda is a positive definite symmetric matrix okay

Because of that cancellation now what we we remain what you get here is for the determinant is only the determinant of the matrix which is again a combination okay of some positive definite symmetry matrices and that's why it comes out to be positive definite

So that how we conclude that this entire matrix here is a positive definite because of that now we have our result that C^1 decrescent lpdf or in this case it is a pdf V such that this negative of V dot which is now this matrix in the quadratic form is positive definite pdf

So we have the asymptotic stability result by the way of Lyapunov theorem from asymptotic stability okay

So this is how we reach the asymptotic stability results now what is this asymptotic stability means is your e and e dot are going to go to zero is a error which are going to go to zero

Now if you recall the definition of e its nothing but q minus q^d okay your joint actual joint angles minus the desired joint angle that is the error that you define

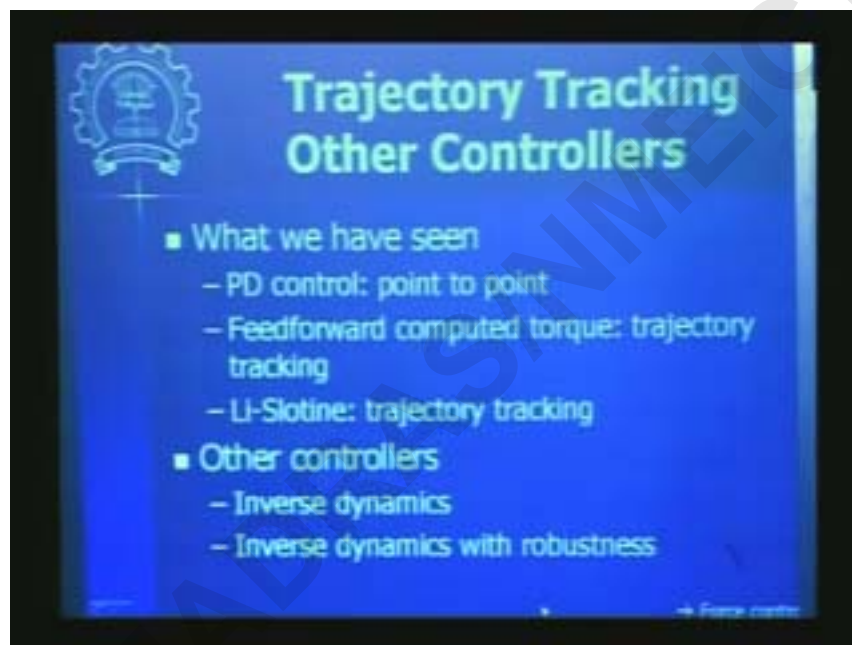
So that will also go to zero q minus q^d will go to zero which means that your joint angles will go finally to desired joint angles as time t tends to vary okay

So that is how you interpret this asymptotic stability result okay so so we have this track tracking controller which is very high performance tracking controller

If you implement this we will see like very clearly that if you see is set a gray grains appropriately then immediately your actual trajectories will converge to that desired which is tracking controller okay

If you compare the results of this simulation with your corresponding computed torque control strategy will find significant improvement for this controller okay

So is very very hyperphomous kind of a tracking controller okay
(refer slide time 00:31:56)



Now let us just recap what we have seen so far we have initially started with PD control point to point control that for regulation purpose so PD control for regulation purpose to move manipulator from one point to another point

Then we introduce feed forward computed torque strategies basically for trajectories tracking applications

We didn't do much of the analysis of these but we have seen that this controllers will result in radio tracking performance than just the pd control okay

And then we moved on to this trajectory tracking controller even by li and slotine okay

See there are n number of different types of controllers available for robot manipulators but these are only some of them right

So it is not possible to cover all of them in this class but these are the some some very basic qualities okay you should be aware of these

And then like more sophisticated can be build based on this knowledge okay so whatever is insides we have got based on these whatever controllers we have seen so far that will help you understand like more sophisticated versions of the controller okay

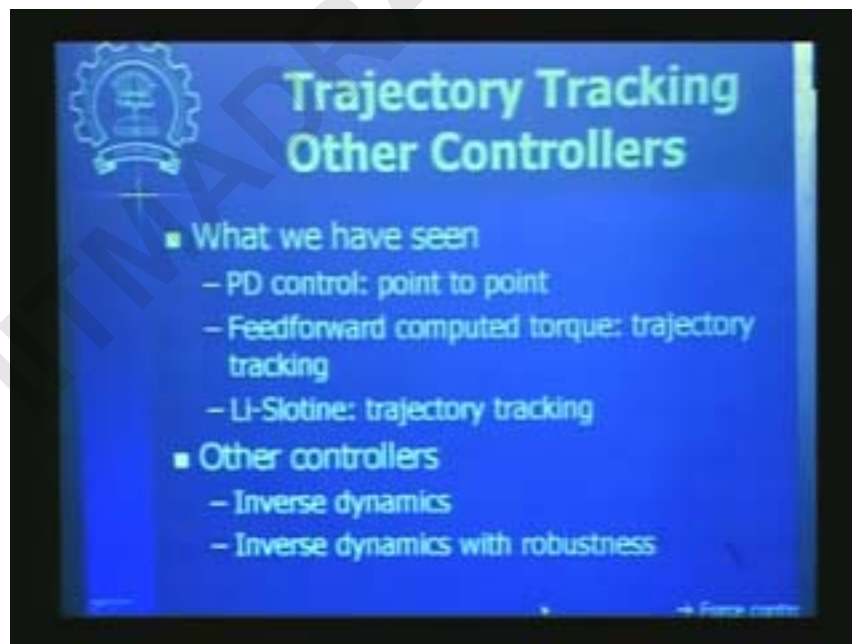
Now these other controllers are like one is inverse dynamics control and then inverse dynamics with robustness these are again some of them not all but these are the controllers

Now we have recited so for our attention to control involving only the trajectories tracking or trajectories like moving from end defector from one point to other point okay

But there are many applications which require like not just this moment but to control the force that robot applies at end defector while handling some something say for example if you want to like pickup [noise] head you want to pick up by robot manipulator without breaking it so there there comes the significance of control of the force that robot applies on its environment or objects it handles okay

So that is other thing that we will see some some inside again we will get into the force control we will see some details about a force control now okay

So what is the basic (refer slide time 00:34:37)



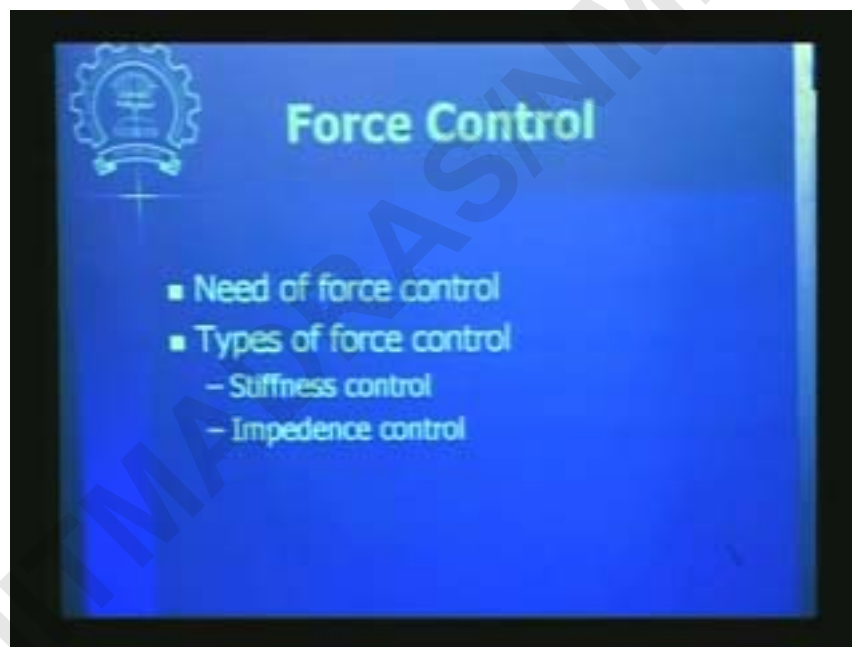
need of the force control I will first explain you that thing with couple of example so that you can appreciate why we should really go for force control okay

So whenever the robot is interacting with the environment that time the force control becomes important

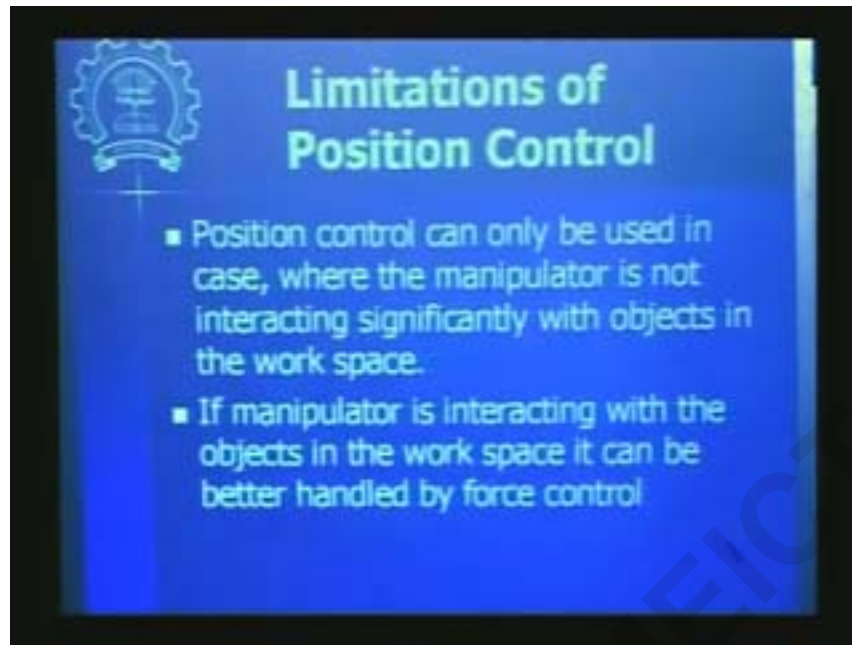
When it has to move one thing from other ah from one point to other point the thing any object from one point to other point without having to bother about if there is anything else the object is interacting with then it becomes as purely regulation or trajectory tracking kind of a problem okay

But when now that object is say stationary and say for example robot has to paint something okay by brush so there is a interaction between the environment and the end defector of the robot then the force control will come into picture

If there is to wipe something then it will come into picture so we will see the application details and how wha what is happening in this application in the while and then there are different types of force control as if you are going to see in this class which is first is stiffness control (refer slide time 00:35:58) and the other is impedance control okay



So we will we will see why this adias of stiffness and impedance control come into picture so as we have seen (refer slide time 00:36:11)

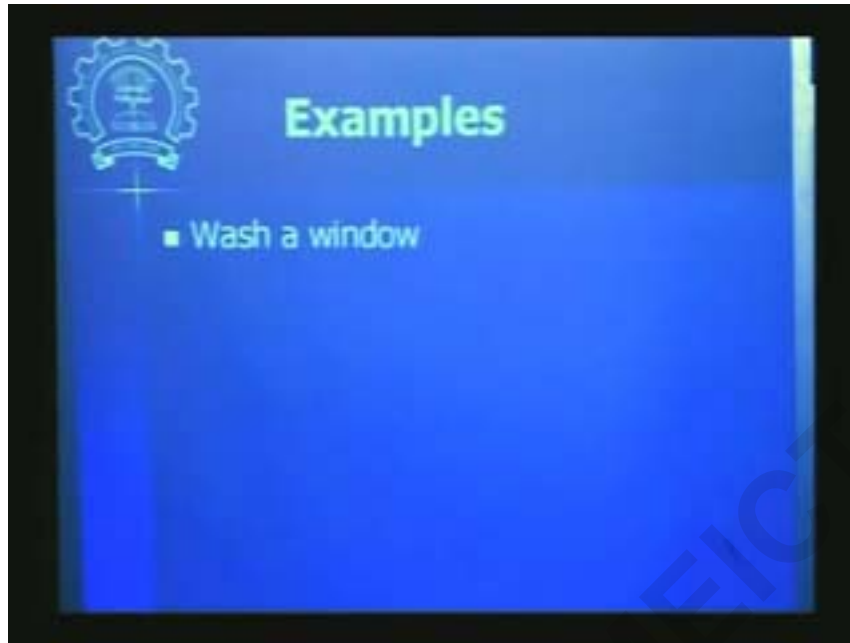


So as we have seen position control can be used only in the cases where the manipulator is not interacting with the objects in the workspace or it has there is no constraint on how much force it applies to the objects while moving okay

See if it has to move pieces for assembly purpose okay the rigid pieces for some assemble purpose then there is no not much restriction on the force that manipulator applies on the software okay

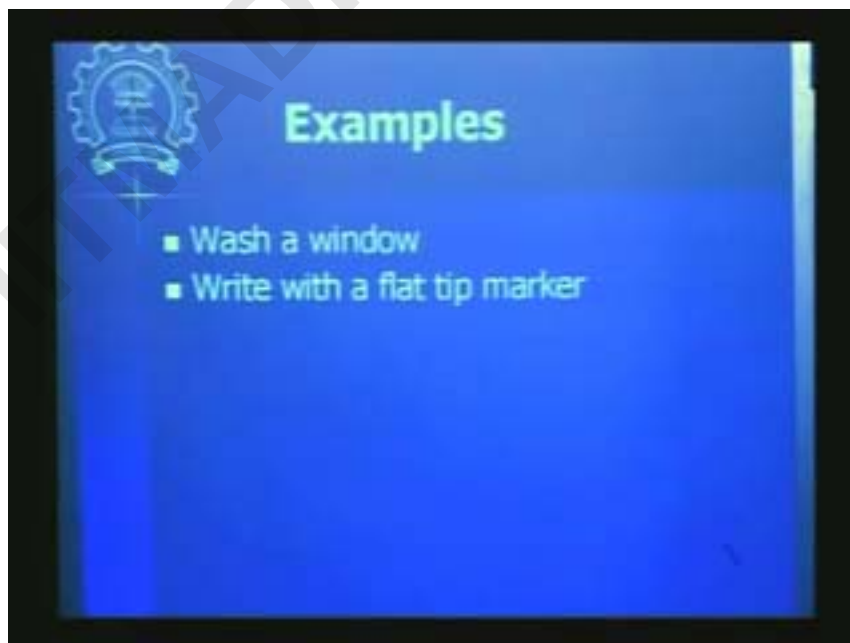
So in that case we don't need any sophisticated force control strategies to be implemented okay but when there is an interaction as I said earlier whenever there is a objects are soft and you don't want to break it by application of gripper forces that time again this force control will come into picture okay

Or if robot has to carry out any screw driving application okay then this will come into picture okay (refer slide time 00:37:26)

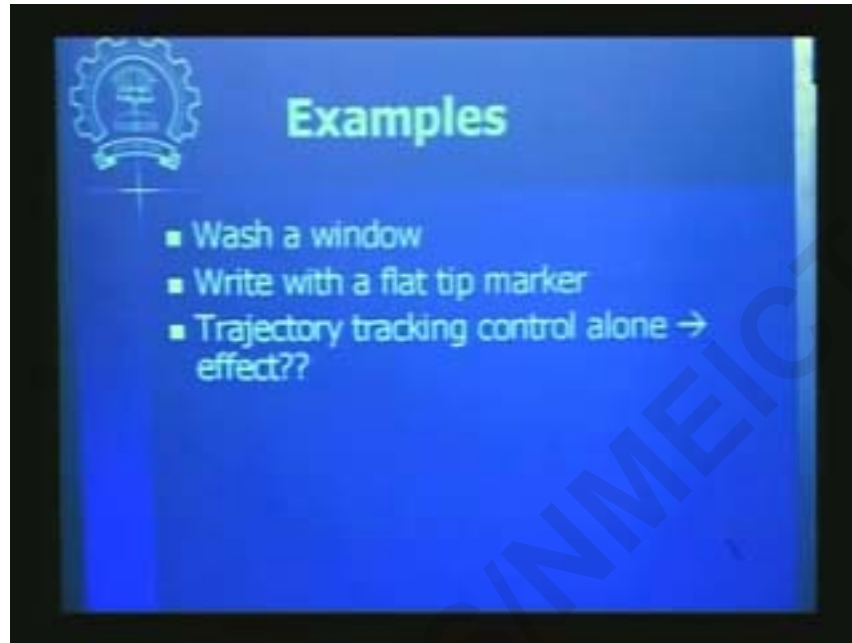


So suppose other example is washing a window or wiping the window see there is a car like car assembly line in which finally everything has been done and now robot has to give final wiping of the glasses of the car or windows of the car

In that application it should not apply force more than the certain value otherwise it will break the glass okay so how to achieve that its not just tracking like you see if we apply only tracking controllers in these cases will they work okay we will see what is the problem if we apply only tracking controller (refer slide time 00:38:14)



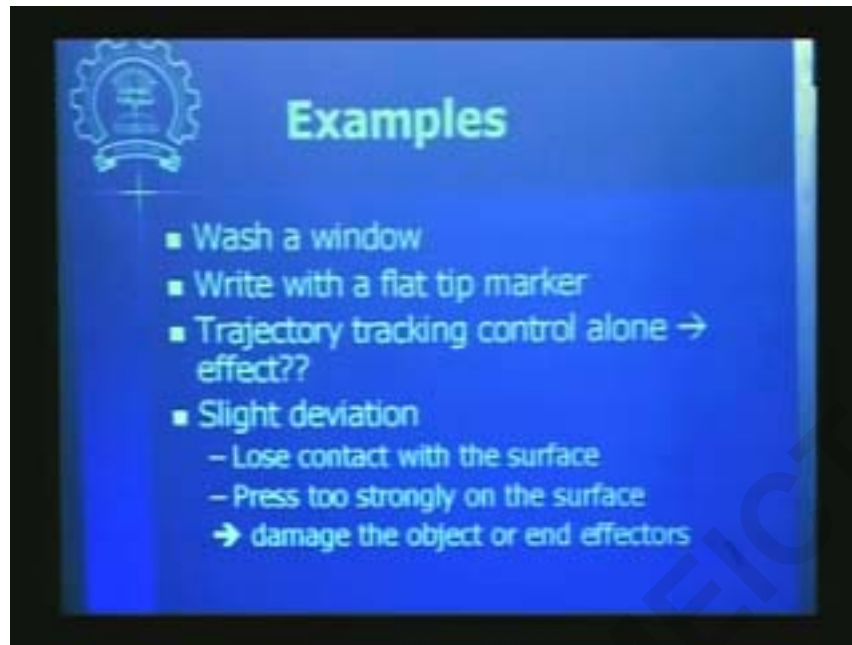
Now if robot has to write with the flat tip marker okay write some number or something on the component again it has to worry about how much force it is applying to the tip okay otherwise things will get damage like they will result into damage okay (refer slide time 00:38:39)



Those what will be the effect if we do trajectory tracking control alone in these cases okay

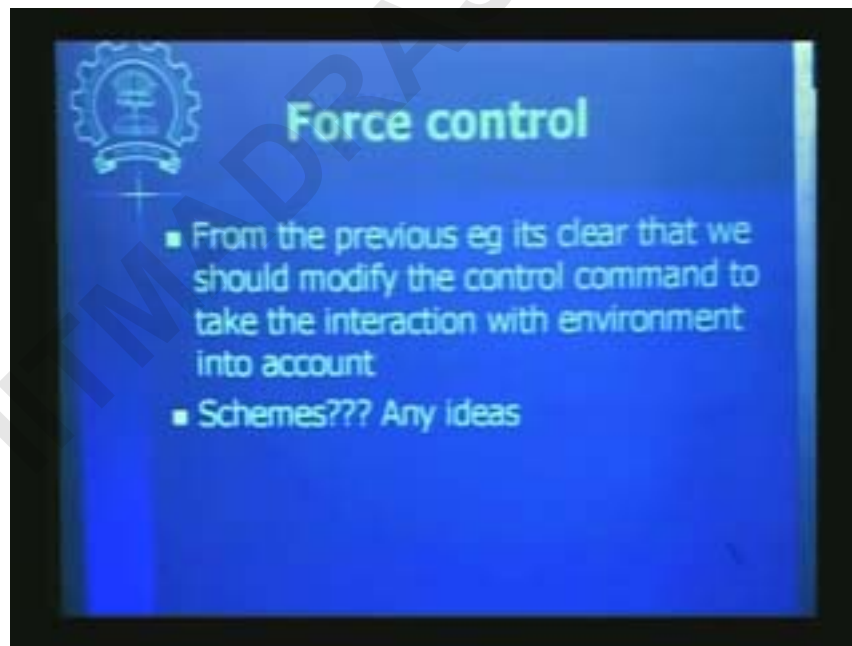
Will they work what will what will be what will be the problem if we apply only the trajectory tracking control there is no concern about force okay

Now you see that (refer slide time 00:39:08) if we deviate slightly from the trajectory then it will result in other loosessing of the contact with the surface or it will result in the excessive forces applied in the environment okay



We don't want either of these to happen that's why that we have to go for some active strategy which will involve or it will have some consideration to the force okay

Do you see that okay now let us see how how we can do that (refer slide time 00:39:53)



So we will have to modify our control command first take then take this effect into account okay effect of object touching the environment

So how do we modify our control exact this thing is accounted for any schemes or ideas you can think see its like when we are interacting with the environment what we do you just think about that and you will get many ideas how we can get to that

Say for example we have to pick up some egg from a basket what will we go about doing that what is happening in our mind when we are picking up the egg we sense in some sense the force okay with the distance like as an end effector goes closer and closer to the object we slowly decrease the force or try to see what force we are applying and then based on that we take decision how much more force to apply on so that is kind of thing that is happening okay

But how do we give this kind of a knowledge in the controller of the robot okay suppose it is given that say for example I know that what force I must apply to the egg I am picking it up then how do we use that information

To control the end effectors of a robot to not to exceed the applied I mean the required force okay

So that is a kind of an idea we have to think what schemes we can do okay when is like you can use more sensors like you can use four sensors not entire sensors where like you can know like how much force you are applying and based on that you can make a decision

But suppose we don't have four sensors that four sensors are typically very costly to implement so if you don't have four sensors what is the way what is the best thing that we can do so let us see one idea based on the stiffness and compliance okay (refer slide time 00:42:40)

Stiffness and Compliance

- Robot manipulators are mechanically very rigid by design
- One solution is to provide a device consisting of virtual spring and damper to reduce the end effectors stiffness
- Force control in this fashion can be accomplished by position control only

Idea of stiffness control

So I mean if you see by design all the robots are very rigid okay so we provide a device in a robot okay this is a virtual device okay by using what we have this control control action okay

So this device is a spring virtual spring and damper kind of a sys system so imagine that like now instead of robot end effector directly touching there is some spring and damper in between

Now if we control the position then how much after after does this spring is touching the object see after spring has touched the object which we want to pick up

Now if we change the position control the position then we can control the force we can apply to that robot because force will be the spring compression multiplied by spring constant

To this spring compression we will be able to control by controlling the position so that is the idea so we we have in control law or term which is corresponding to the spring kind of a behavior and then we have a position control strategies okay

So that is the idea that is used in the stiffness and compliance

So the advantage is like we just need to have a control over the position and then you can control the force okay

Now let us see I I have this illustration (refer slide time 00:44:38) for this idea



Say you have this end effector here see at this point it is touching the environment okay now if we move the end effector little bit then it is going to compress the environment okay

Now let us assume that our environment is very much rigid okay then robot applies force to the environment it will be the balance between the forces like reaction force produced by the environment and because of that there is see if there is some finite stiffness to the environment okay that environment or that object is going to get deformed a little bit

Most of the cases it will not get deformed to a great extent unless it is kind of a sponge that you have to pick up by the robot okay

So you see the difference between this like two cases where we have to pick up soft objects or we have to pick up hard object okay

Usually most of the applications we will find that the robots are picking up hard objects so the environment is rigid state okay

So then now this is like the trajectory along which we bond our manipulator apply a specific finite force okay right see this is a environment information like environment has this kind of a trajectory and say it is a glass window on which robot has to apply a finite force for wiping it okay or for some other whatever okay

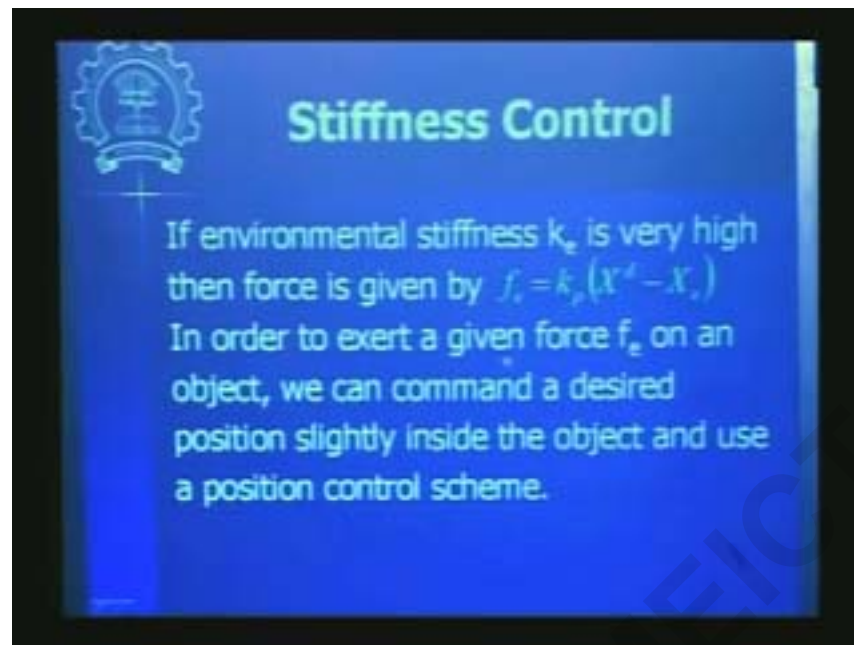
So it has to move along this trajectory and apply some finite force okay now if the idea of the stiffness control is you defined a parallel trajectory inside so corresponding to this X_e point there will be X_p point inside the environment okay

And comma and your robot has to move along this trajectory with the ap kind of a control or spring effect okay proportional control then what will happen see it is robot is trying to reach this point d but once it reaches this X_e point touches the environment now it cannot further move but it will still keep on applying a torque corresponding to the error which is X_e minus X_p multiplied by the proportional gain to the motor

So it will keep on applying that much torque which will correspond to in some sense applying the force do you see that how this stiffness credit okay so that is the basic concept of the understanding fundamental understanding for the stiffness control

You have to command robot to move along a trajectory which is inside the environment okay so corresponding to this X_e minus X_p point X_d error the proportional control or control torque will be applied which will be resulting in a force desired force in the environment okay

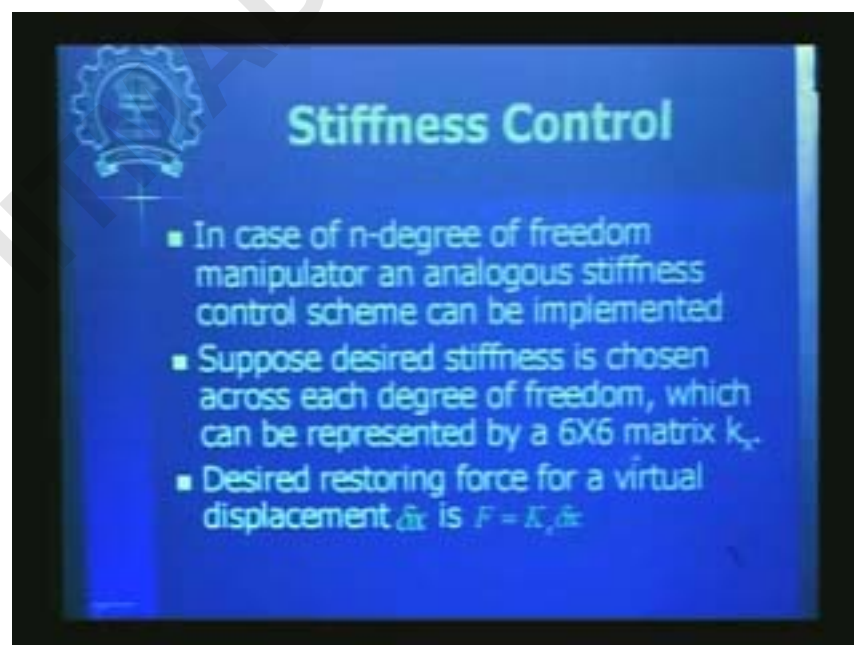
Now now let us see the detail computation for some handling manipulator how do we do that (refer slide time 00:48:47)



So this is the basic idea fundamental idea about stiffness control so in case of n degree of freedom manipulator you can design the similar scheme okay

Suppose your desired stiffness is chosen across each degree of freedom end effectors has six degrees of freedom so you can select your stiffness along each of these six degrees and then correspondingly you can get the torque or whatever okay

So so the stiffness matrix can be specified as six by six matrix k_x okay (refer slide time 00:49:23)



Now the desired restoring force for the virtual displacement now suppose virtual displacement δx is given to the end effector into the environment okay then corresponding to that virtual displacement there is this force kx into δx that will be a virtual force or restoring force okay

See why we are doing all these computation is basically we want to know how much torque we should apply at our joints or how much what is the scheme to apply the torque at the joint thus to produce this type of a force or this type of a stiffness characteristic for the end effector when it starts interacting with the environment okay

So so we are after that (refer slide time 00:50:18)

Stiffness Control

- If δq denotes the resulting joint displacement for the small displacement δx

$$\delta x = J(q)\delta q$$

Where $J(q)$ is the manipulator jacobian

Now $\tau = J(q)^T F = J(q)^T K_s \delta x = \underbrace{J(q)^T K_s J(q)}_{K_s(q)} \delta q$

Control law to implemented is given as

$$\tau = J^T K_s J(q^d - q) + \tau_s$$

Impedance control

we we convert now this delta x by using jacobian to delta q this fashion

So delta x can be rep represented in terms of delta q by using this jacobian matrix and then this torque in the similar fashion is related to the force with this jacobian

Now this force again will be replaced with x into delta x that is our desired force and then this delta X is nothing but J into delta q

So J transpose kx into Jq is now desired stiffness of the joint okay so this [symbol] is nothing but this desired stiffness multiplied by joint displacement delta q see you all see that

So this stiffness matrix at the joint can be obtained by this fashion and now if we have a control law that this stiffness matrix multiplied by the error okay then it produces the desired force and stiffnesses at the end effector along each of the degrees of k okay

So this is the basic idea about stiffness control now let us see just some case about impedance control (refer slide time 00:52:22)

Impedance Control

- Idea: Define mechanical impedance

$$Z(s) = \frac{F_v(s)}{V(s)}$$
- Specify desired impedance as

$$sZ(s) = -(Ms^2 + Bs + K)$$

$$M\ddot{x} + B\dot{x} + Kx = -F_e$$
- Design controller to achieve this impedance

$$M(q)\ddot{q} + h(q, \dot{q}) + J^T(q)F_e = U$$

Dynamics of robot interacting with environment

Now impedance control basic idea is to define the mechanical impedance in this fashion it is [ah] ratio of force versus velocity okay and then we will specify the desired impedance in this fashion

So this impedance concept is basically similar to impedance in the electrical domain

So corresponding elements like like we have spring for the stiffness control is again a like we can have the stiffness control visualize as a special case of this impedance control so that is the idea here like

You can generalize that I I will show you how exactly this stiffness control is special case of impedance but basically you specify this desired impedance in the case of stiffness control we don't have this terms \ddot{x} \dot{x} and x

We just specify that you external like F_e is equal to k times x so k is a stiffness that we desire so that is how in some instance we will come to that again little later and then design design you design a controller which achieve this kind of a impedance when interacting with the external environment

Now notice that your dynamics of robot which is interacting with the environment it will change in this fashion will have this term $J^T(q)$ into F_e which is the force of interaction with the environment appearing robot dynamics

Now let us not go into detail how we can design this controller and how we can go about then you can use whatever tools they have and we can design this kind of controller which will give finally F_e equal to this kind of a relation okay

So you have to choose this control u may be compensate for these terms and whatever like compute a torque kind of a compensation inverse dynamic kind of a compensation and then finally your equation should remain in this form F_e is now this one

That is how you have to design this controller we will not get into detail but that is this is a basic idea about impedance control okay (refer slide time 00:55:00)



So let us see some physical inside say if we have a stiffness control it can be as I said earlier it can be considered as the special case of the impedance control okay

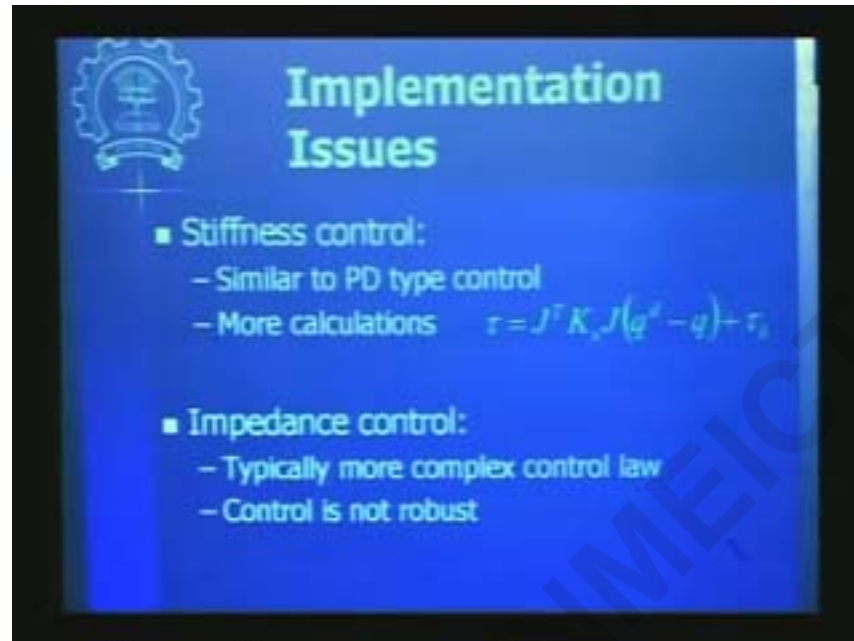
Now you see that the special case is when \ddot{x} and \dot{x} they are zero okay in the previous equation we have seen and then that means that it is a steady state kind of a analysis

So if we consider only steady state force no dynamic forces in the impedance control it will become similar to stiffness control okay

Now when the end effector is moving freely there is no interaction that means the impedance is zero okay or there is no resistance or from the environment okay so that is the zero impedance case

So that is how one can interpret physically the impedance control action okay

So both like your position control and pure force control cases they can be considered as the special cases of the impedance control (refer slide time 00:56:21)



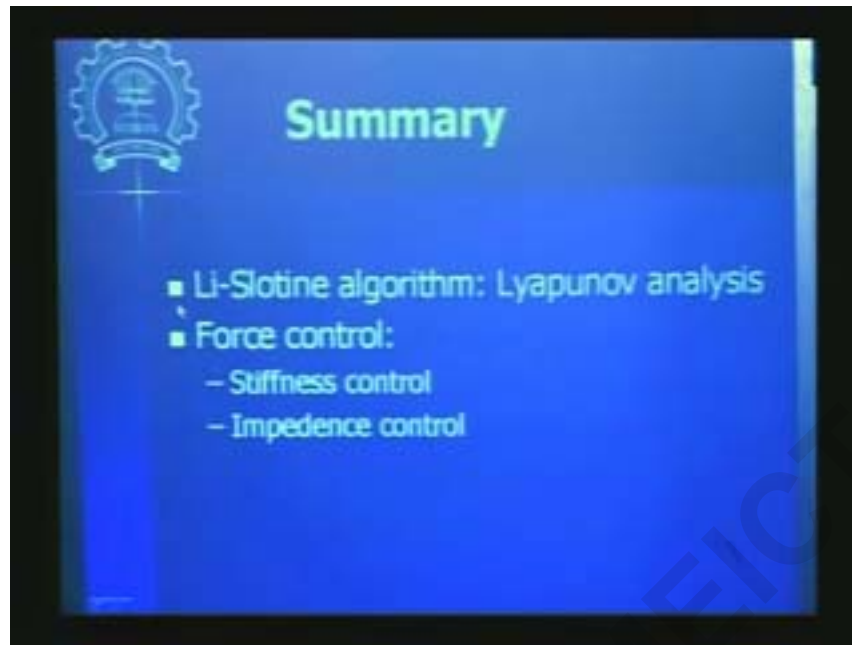
Now what we have seen is these two types of force control strategies so there are some implementation issues that that are I just want to want you to be aware of these

So stiffness control was similar to PD type of control so it is very simple to implement and but but only thing instead of in the stiff whatever the calculations they have to do in the stiffness I mean the pd control we have to do more calculations in the stiffness kind of a control

We have to do this jacobian or this stiffness transform making calculations okay and this tau b is a torque which we can add some damping in the system or something of that torque we can do do of that add or just make simply tau b is equal to zero that is also good okay

And then in the impedance control implementation is more complex typically like you find control now will be more complicated in this case and control is not robust in the impedance control okay okay

So in summary what we have seen today is (refer slide time 00:57:35) basically li and slotine kind of a control algorithm



We start the Lyapunov analysis how we can find out be \dot{b} and conclude about the stability li and slotine kind of a controller

Then we have moved to study little bit about fundamentals about the stiffness control and impedance control for force control strategies okay

And in the next class now what we will see is more futuristic topics in the robotics okay we will study little bit about micro electromechanical systems and how they are useful for robotic applications and futuristic robots okay

Thank you