**ROBOTICS**
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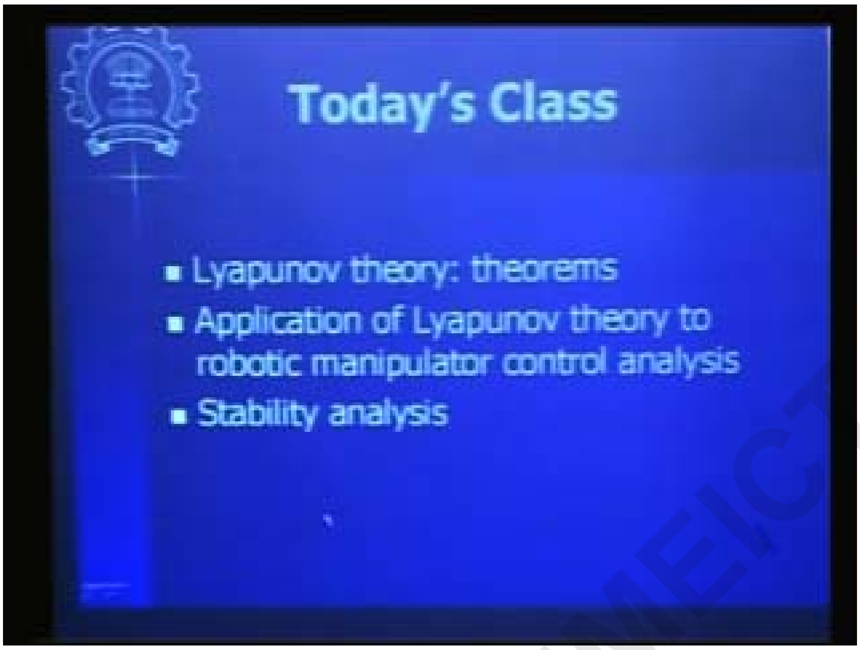
Lecture No-35
Robot Dynamics and Control

We will start with the lecture number five of module robot dynamics and control today just to see what we have done in the last lecture we have covered the trajectory tracking control ok with the specifically we have considered this case were we have linear form of the robot dynamics ok with that linear form of the robot dynamics they have developed the trajectory tracking control but we have not done much analysis of the trajectory tracking ok then we saw in that trajectory tracking control two types of control algorithm ( refer slide time 01:54)


one is feed forward control and the other is the computed torque control that's it then we moved on to some mathematical preliminaries of the lyapunov theory we will continue with that today and then we will see some applications of lyapunov stability theory over robotic manipulator ok
so that is what we will be studying today so

( refer slide time 02:19)




let us see now some theorems about lyapunov stability (slide time 2:29)

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before going to that this is the fundamental about lyapunov stability ok

so let us understand first what exactly the lyapunov stability theory is about what is the basic concepts fundamental concept behind the lyapunov theory ok suppose you have zero as the equilibrium of the system ok we saw the equilibrium of the system is obtained by like having this x dot is equal to zero and having for the x ok

so that is the equilibrium of the system and now the total energy if you see it will be zero at the origin and as you move away from the origin it will have some finite value ok

so say we are we are now having the system in such a way that your energy is at all other point is positive ok

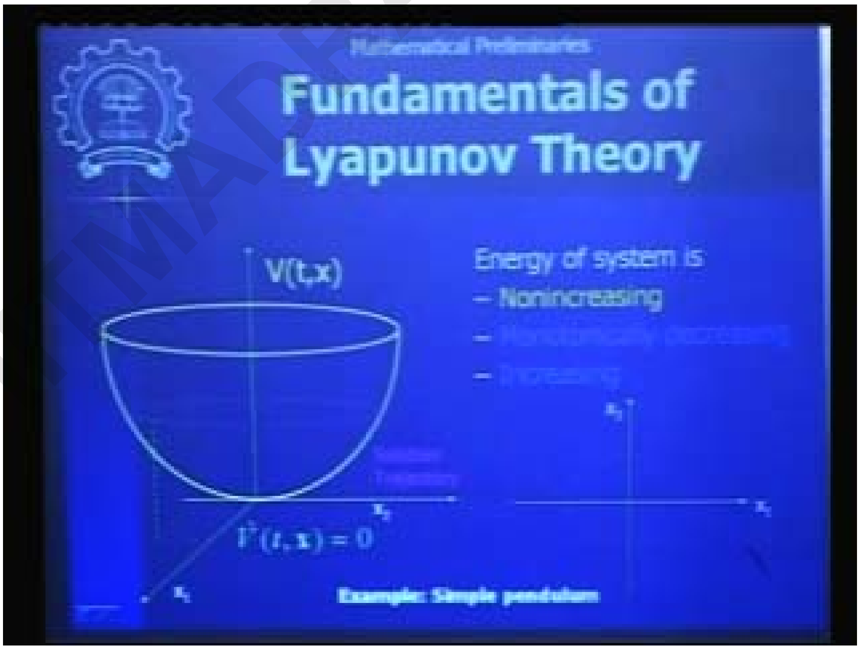
so now if see um understand what if we perturbed now system from origin what will happen ok there are different possibilities ok one of the possibility is that you have the system energy non increasing ok your energy is still remaining constant even if you perturbed the system from the equilibrium position the energy is remaining constant ok the other thing is it is monotonically decreasing the energy goes on decreasing ok even if the system is perturbed from equilibrium like energy is going on increasing then energy is increasing so these are the three possibilities that we can have

so say for example you are at this point ok or I mean you perturbed the system from the equilibrium which is here to some point over here and now you are interested in how the trajectories will evolve and how the energy levels which is this V will change ok now you recall that this v dot so

so we will study these v v dot or at energy energy derivative and the trajectory of the system ok what is the co relation between them that is what we will see now so v dot we just recall it was defined as the function ok

so you have this del V multiplied by your system f of t x t comma x ok

so now let us see how this will happen like what will happen when the energy is non increasing or the energy level is remaining constant (refer slide time 05:28)




so this is the case when the energy of the system is non increasing so you are always at this level of energy ok this is the constant level of the energy ok so you have started from this point which will correspond to this point on your x one x two plane ok remember we are considering only the r two cases like it is the second order system ok

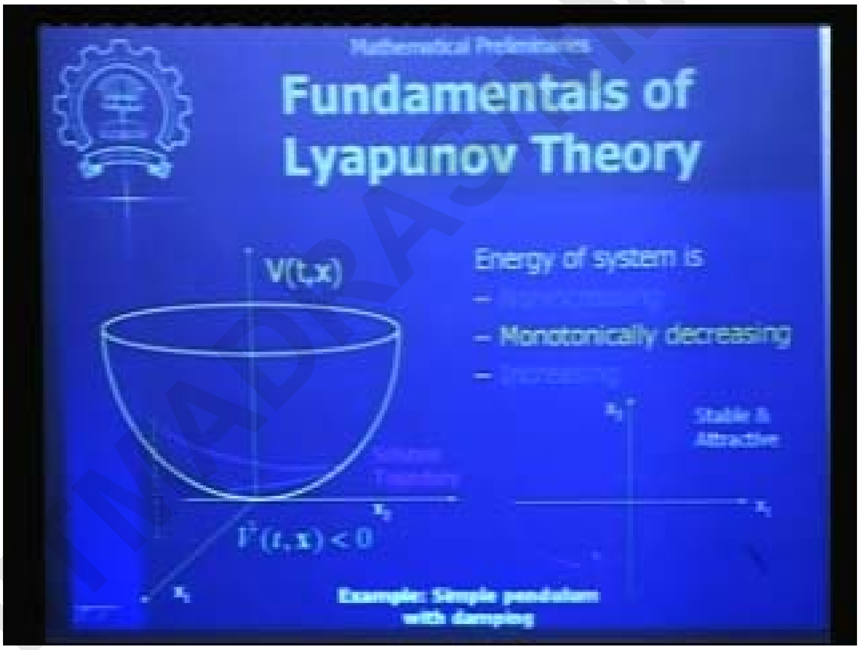
so this can be in general be generalized for hyper placed or larger order system ok but right now for our understanding purpose we are focusing on this two D case where like you can imagine very well what is the way it will behave in a geometrical factor ok

so corresponding to this energy level we have here a circle or ellipse on the face plane ok

so system trajectory is always constraint in this domain circle ok now this behavior is the stable behavior in the sense of lyapunov definition that we saw last time like given any epsilon you find out delta such that when the system starts from this delta it never leaves the ball epsilon ok

so here you can define like given any epsilon circle we can define the delta circle which will fit into that epsilon circle or ellipse or ellipse which will fit into the epsilon circle and then you can find out what is the delta ok

so that is why you can say the system is stable in the sense of lyapunov ok (refer slide time 07:39)




now let us say what will happen when the system energy is monotonically decreasing

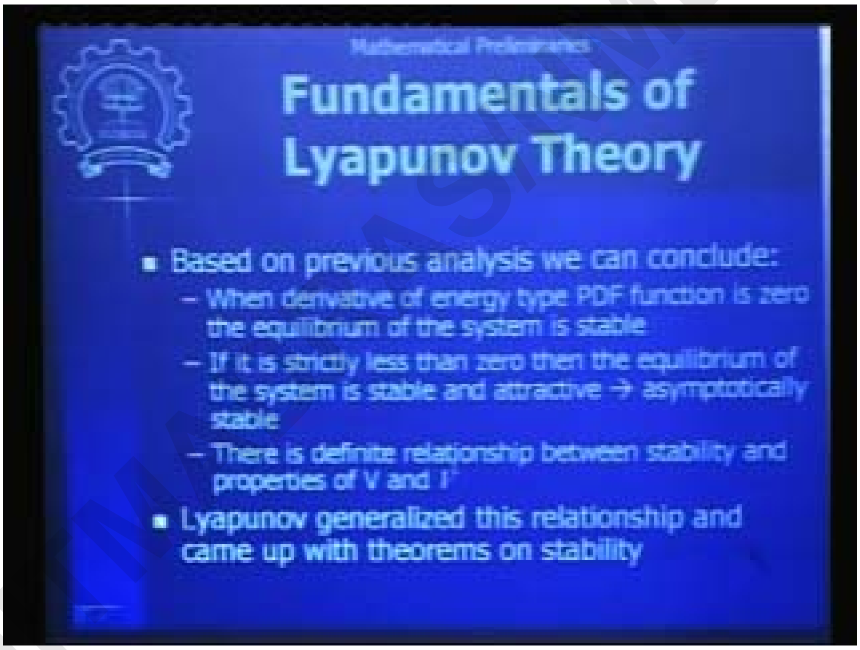
so we have again started like system was on equilibrium we perturbed the system from the equilibrium to reach at this point which is corresponding to this point in x one x two plane ok and now we are studying now what will happen to energy of the system if i release the system from this point this is the starting point and now let the system evolve and i am observing what is the energy of the system ok

you have to see what is in this case like we assume the energy is monotonically decreasing ok this is the case where energy is monotonically decreasing so how it look like on a phase trajectory yes have a look at so energy level goes on decreasing and till it reaches origin so correspondingly you will have on phase plane these kind of a behavior ok no matter where you perturbed the system like system is always going to go finally towards the origin which is the equilibrium of the system ok

if the energy level is continuously decreasing so this v dot in this case is less than zero ok

you see that in the case where the energy levels are decreasing continuously you will have in some sense like the energy like the trajectory behavior which is going closer to the origin so origin is in the lyapunov sense attractive so it is the stable also it is attractive also so we saw yesterday the definition is stable and attractive is corresponding to asymptotic stability ok

so that is the asymptotic stability result we have v dot is less than zero ok now you can think about example of simple pendulum ok you take it to a particular point and release it it will keep on oscillating and then finally oscillation will die down ok if the damping is there in the pendulum ok the previous case for the simple pendulum if there is no damping in the system like it will just oscillate to what ever amplitude you have velocity in the initial stage ok so in the second case the amplitude oscillation they will slowly die down ok so that is the case it can be represented on the phase plane in this fashion ok you see that (refer slide time 11:41)



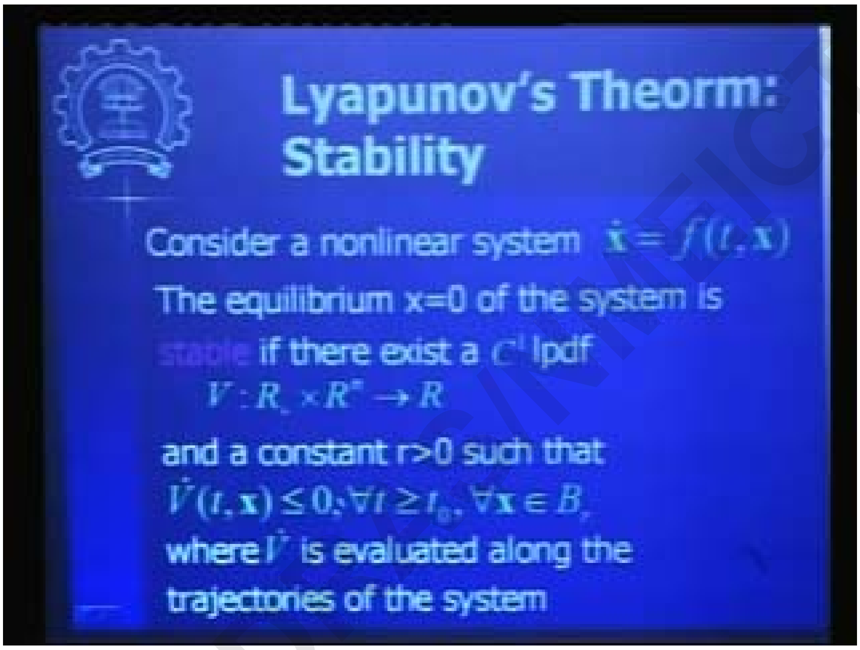

now let us consider some other conclusion that we have draw from the previous analysis the first is like when the derivative of these energy type lyapunov function ok or lyapunov function candidate may be calling ok

so some energy type PDF positive definite function if that derivative is zero we saw that the equilibrium of the system was stable ok and then if the energy level is continuously going down that means the derivative is strictly less than zero ok then we saw that the system is stable and attractive or system is system equilibrium is asymptotically stable ok

so there is a definite relationship between this V V dot and system behavior so there is some condition on V some other condition on V dot like that say for example V is positive definite function V dot is equal to zero ok that kind of the condition will lead us to conclude that the system is stable at this equilibrium is stable ok similarly V is positive definite function V dot is strictly less than zero will lead us to conclude the system can be asymptotically stable now lyapunov generalized this relationship and came up with the theorems on stability

so there is a rigorous mathematics behind what we did but whatever we have seen just the kind of a general insight into what is the fundamental basis of all his theorems ok you was a very genius mathematician in Russia and then this theory that he developed is unknown to west part like almost for thirty forty years ok

it is very powerful theory now in the in the domain of noon linear control systems the lyapunov is very well accepted theory and there are so many papers published based on this theory ok and especially in the robotic application this is used to a great extent ok we will see now how we can apply this lyapunov theory to our manipulator (refer slide time 13:31)




but before that we will see all these lyapunov stability theorems in that total mathematical definition ok

so this first is theorem on stability ok so consider a non linear system which is x dot is equal to f of x t this is our standard system we assume for the system the equilibrium equal to zero

so if it is not zero then we need to do some shifting or change of the origin and make it zero and then we apply this theorems now the equilibrium zero of the system is stable if there exists C one means continuously differentiable one time lpdf ok locally positive definite function V again you remember our definition of V how it gets t and x to some scalar value ok so this V is positive definite and C one continuously differentiable stable and constant r ok you have to find out the this r such that V dot is equal to zero it need not be just equal to zero it can be less than or equal to zero ok

and this is valid for all t and for all f belonging to this ball of radius r where the ball of radius r is where we are guarantee this lpdfness like locally positive definiteness of the function ok

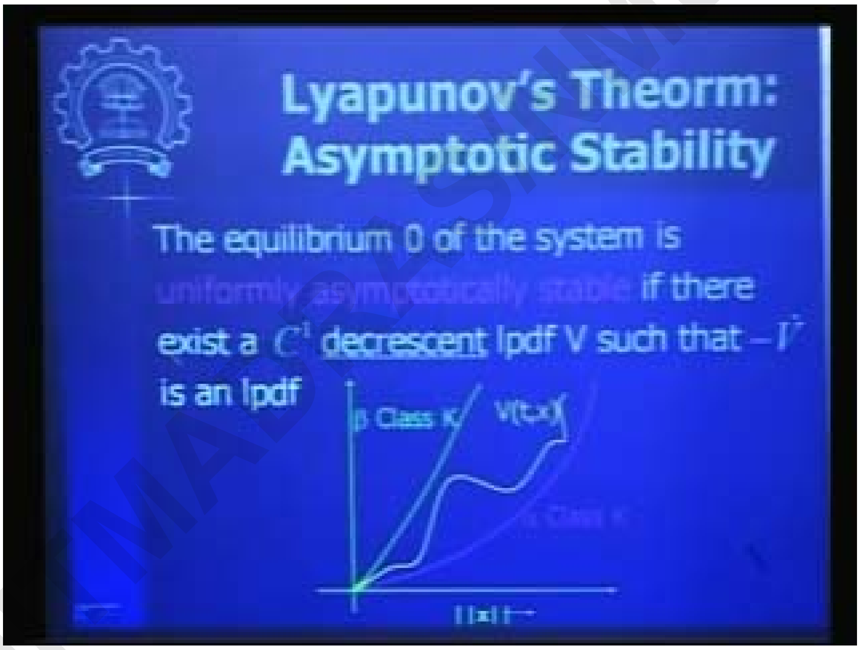
so this can be extended I mean you can see this can be easily extended to Rn space like when we don't restrict to some ball of radius r can be the entire domain so this condition whatever V dot and V in what domain they are valid if they are valid in Rn domain then it is globally stable that kind of conclusion we can draw ok if they are valid only in the local area then it is locally positive but since the stability is the property of equilibrium then therefore here it doesn't have you can say globally stable equilibrium

so that is like there is a very little difference like the equilibrium property will not change because of the globality or locality property of the equilibrium but only thing is that where you perturbed the system and it will go to equilibrium or what ever that that will change ok

so that domain will change ok the property of the equilibrium itself will not change because of the global or local ok it will be still stable or asymptotically stable whatever but globally when you say the domain of attraction or domain of domain of what we are talking when your system is perturbed that will increase ok

so this is the theorem for the stability of the system ok so you remember this condition that you need V dot to be less than or equal to zero and V to be positive definite function or lpdf function ok

so these are the condition for stability (refer slide time 17:17)



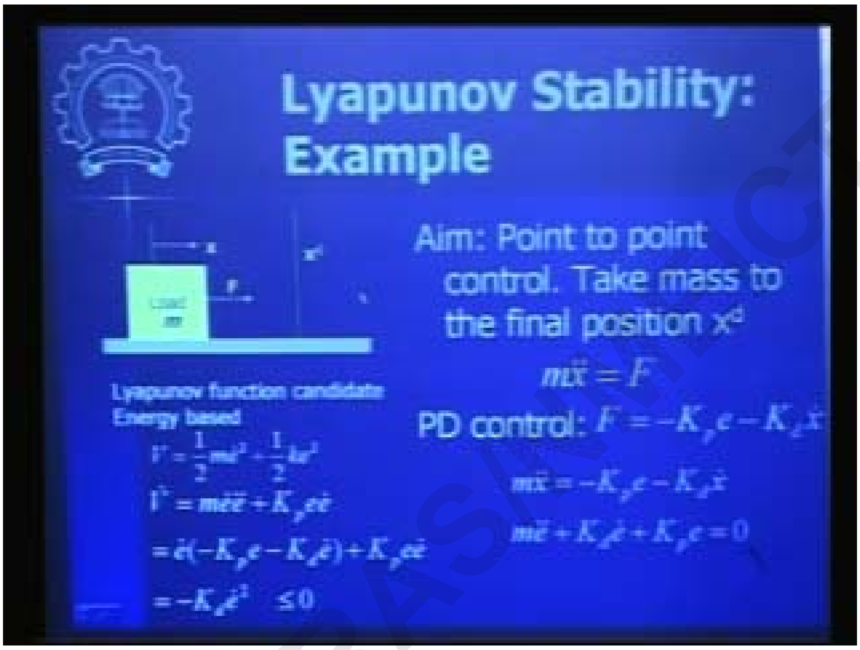

now let us see asymptotic stability what is the condition for asymptotic stability so the lyapunov theorem says the equilibrium zero of the previous system that we are talking about is asymptotically stable if there exists this class this C one continuously differentiable function decrescent function we have defined decrescency in the last class at lpdf function V such that minus V dot is an lpdf function

so here now V dot is strictly negative strictly less than zero ok so V is decreasing ok now decrecency definition and this lpdfness of v can be put together geometrically in this fashion ok this kind of a sketch to explain you what is V condition conditions on V how they can be represented so this is alpha function of class k

so for lpdfness of V V should be greater than the alpha function and for descrescency it should be less than class k function beta of class k ok so V lies in between somewhere to class k functions ok so this is like more resize mathematical manner we have this definition these condition are necessary to make sure that your system is asymptotically stable ok

so we will not get into the details of the proof of the theorem ok but lyapunov has derived all these detailed proof of the theorem from using the basic theory of mathematics ok

so we will see how they can be applied to our taste of robotic manipulator and we will just understand the gist of the theorem so that we are able to apply to any of the system ok that is kind of the understanding that we will see in this class ok now let us see one example (refer slide time 19:52)




where lyapunov stability is applied to the example ok

so you have a simple mass which is lying on the surface which is smooth and then you are applying some force F on these ok

so our aim is again point to point control as we have the we have been studying in the robotic manipulator cases so point to point or regulation kind of control and then we bound to take this mass to the final position Xd Xd is the final position where here ok

now the equation of the system dynamics is given by these expression m x double dot equal to F ok these you can reach to this expression either by using your standard Newton formulation or by using LaGrange formulation that we have seen in the first class ok

now PD control is something of this form here kp into e e is the error between xp and x ok and then Kd into x dot see final position we have to reach here is zero velocity so e dot is simply x dot ok there is no difference between e dot and x dot ok

so we just substitute this control law in the system expression and then we will find that this is our system dynamic area ok

so this x dot have replaced by e dot ok so this is our system dynamics now now we will have to see whether these equilibrium which is given by e and e dot ok you remember our definition is for system x dot is equal to f of x so we have to actually convert this system into that kind of a form and then start applying but now I mean slowly you will get practice like we can directly see what are the states with the understanding of the states we can generate this lyapunov function ok the state here is e and e dot ok

so that like with that state you will have the equations which are in the first order kind of form so e dot equal to something and e double dot equal to something this will be your two equations ok

so any way that is the way you do the state like convert your equations into the state form now once you understand that e and e dot are your space then you should have a function energy based function or we will call this function as lyapunov function candidate ok this is not till lyapunov function because it is not satisfying v dot property we are not yet proved that it is satisfying some lyapunov property so like whenever we take a travel function it is called lyapunov function candidate we are just using it for as a candidate to test whether it is indeed of lyapunov function or not ok

so this is our V which is lyapunov function candidate and then usually like see last slide we have seen that lyapunov we have seen the system behavior with respect to energy ok

so this is energy typically lyapunov function candidates are lyapunov functions they are energy based functions ok so you can take this system energy for this function and carry out your analysis but it is not restricted to that ok remember you can use there are many other type of functions that you can use especially when the system has very high non linearity's you may not have directly energy behavior or energy kind of a function for lyapunov you should remember that like you should not always that you have energy based function ok but most of the robotic manipulators system is that we will be studying in that it will be a energy based function ok and you see here how the energy has been taken so this is m into e dot square which is the kinetic energy of the system and now because of this feedback kp feedback we will have this is this k is kp actually ok just correct it

so because of this kp you have kp into e dot e square half kp in to e square is the potential energy stored in the spring in some sense ok when whenever you have this proportional feedback you will have correspondingly kp into e square kind of a term in the energy that because that is the energy because of the spring action of the control ok

so that is that is what what we have to considered in the equation ok

so this forms your total energy m e dot square half of that plus half k e square ok

so this will considered as the lyapunov function candidate and now we will see what is e dot

so take a derivative of this energy we will find that this m e dot into e double dot then kp into e into e dot term here now you substitute for e double dot so that the derivative becomes derivative along the trajectory ok so you see so for up to this step we have not used system equation ok

so this function can may for some other function other system also unless we will substitute our system equation this system become derivative along the system trajectory ok

so you substitute for e double dot whatever you get from these so m e double dot from here is minus kd into e dot minus kp into e you substitute here and then you will find that it becomes equal to kd into e dot square you see that ok now you see that this is less than or equal to zero not strictly less than zero because our state is consisting of e and e dot and this is the only function of e dot ok so as you have seen in the last example this is the case where like you have number of terms in the equation for v dot or less like they doesn't doesn't include some of states they excluding some of states when that kind of thing is happening your function cannot be positive definite or negative definite it is only negative semi definite or positive semi definite case it is negative semi definite ok

so this is very important understanding we should have so from here we have conclude the stability based on our stability definition that we have seen previously remember this definition v dot is less than or equal to zero ok and v dot is evaluated along with the system trajectory that is what we have done and then we are from these definition from these theorem we can conclude that the system is stable but now to conclude asymptotic stability yeah this function is lpdf or this is pdf this is more than that this function if you see kp and m both are positive

so this is positive definite function v dot we need is less than or equal to zero we don't need see for asymptotic stability we will need this v dot minus v dot to be lpdf but we are not saying that the system is asymptotically stable we are just saying that the system is stable ok now but from our intuitive analysis or from linear linear domain analysis you know that this simple spring mass or simple mass system ok mass moving with the force with the PD control indeed finally go to the final desired position

so it is known to b asymptotically stable but from these particular lyapunov function we are not able to prove that ok energy energy based lyapunov function may be there some other lyapunov function ok which is positive definite for that when we evaluate this v dot it is come out to be negative definite or negative semi definite negative locally negative definite ok

so v dot will come out to be negative definite or locally negative definite ok

so there can exists such a function because our we know our system is indeed asymptotically stable ok but from these particular energy based function we are not able to prove that ok so that is kind of a limitation on the energy based function you can say this is kind of a limitation on energy based function like if you use for lyapunov energy ok

so to get rid of that some one other theorem that is defined is called LaSalle's theorem that we will see in the later classes in mean later on in this class but right now we will carry out similar kind of analysis for our robotic manipulator system ok

so it is exactly the similar kind of the system this we are doing in a scalar case now we will extend it to robotic manipulator in the vector vector space so you recall our non linear equations of the robotic manipulator ok they are of this form this is the form where like we are the writing for each joint separately we can combined also this equations in the vector form vector matrix form different way of writing but main thing is that the dynamics will remain the same whether you write it in the matrix form or this form ok then this is the next equation is the actuator equation actuator dynamics now

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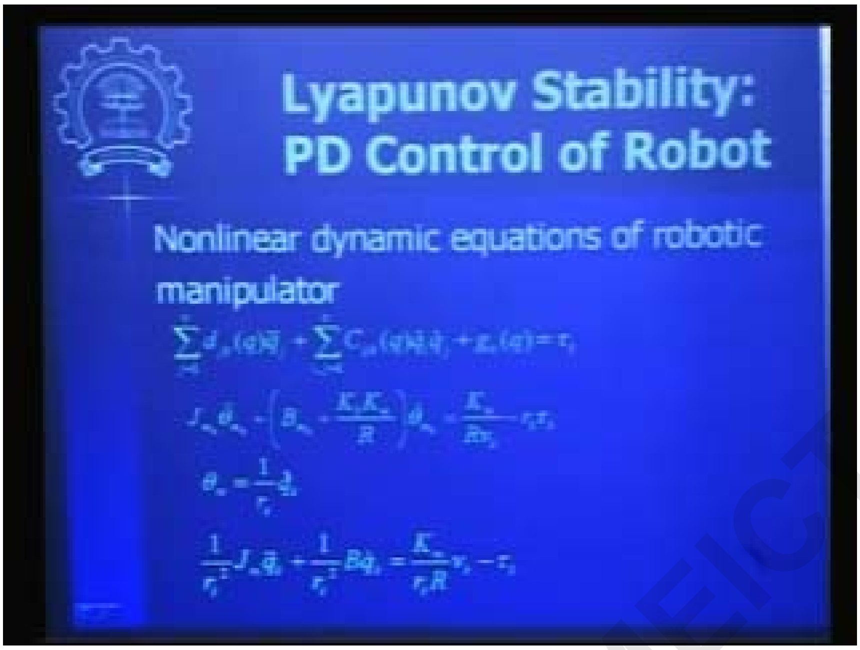
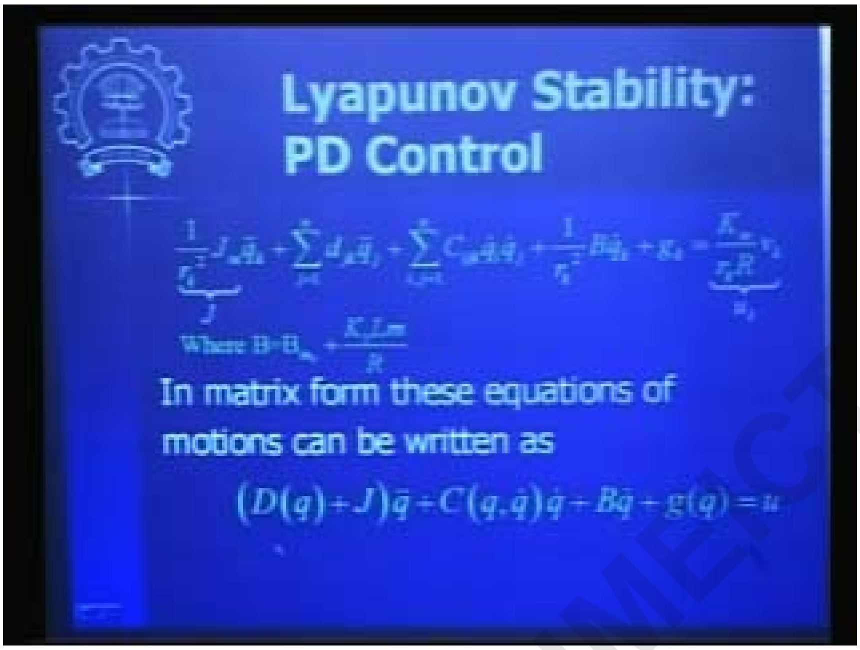



we will do transformation little different manner ok here now we like we define theta m actually we have this theta m one over rk times qk ok

this rk is the gear reduction ratio so you remember last time what when we did we convert everything to motor side ok we have j effective or b effective we find that motor side ok in terms of the motor variable now we will transform this in to the joined variable ok

so we substitute this theta m is equals to these in this equation and then we reduced this things to joined equation ok so when we do that the equation will become something of this slot we have to do that and substitute this whatever tow you get in this equation ok so that is the connection between your motor and robot dynamics ok so you substituted that and convert into single equation you will get the equation of this form ok

so we call this term as the capital J is the equivalent inertia of the motor transformed to the joined variable and then this is uk your input ok and then this is B is defined in this fashion ok this is again similar to our b effective effective inertia so matrix equations can be written in this form refer slide time 33:31




ok we have this D matrix here added with the motor inertia transform or equivalent motor inertia and then other terms and then this u is given by these ok

so now you recall again these the previous equation we have j effective b effective but in terms of the motor variable ok

so you understand now see the thing is that these equations they are like manipulated differently but they are representing the same dynamics ok they just manipulated in the different fashion so as to suit our control development ok

so previous case we want to develop a control which were totally in the linear domain and in this case we are developing control which are in the non linear domain so we have just quit these equations in a different fashion ok but they both both of these equations they represent the same dynamics ok

you should not miss this point because this is very important though we have just change the variable the dynamic is not going to change they are representing still the same dynamics ok very very important point to keep in mind ok

now next let us see the PD control scheme we have already seen what is this PD control but now we will have to do the analysis like last time when we studied this PD control we didn't do a non linear analysis we just say that this is kind of a equivalent to linear domain system ok but today we can we will do the proper non linear analysis with the lyapunov stability of this PD control system ok

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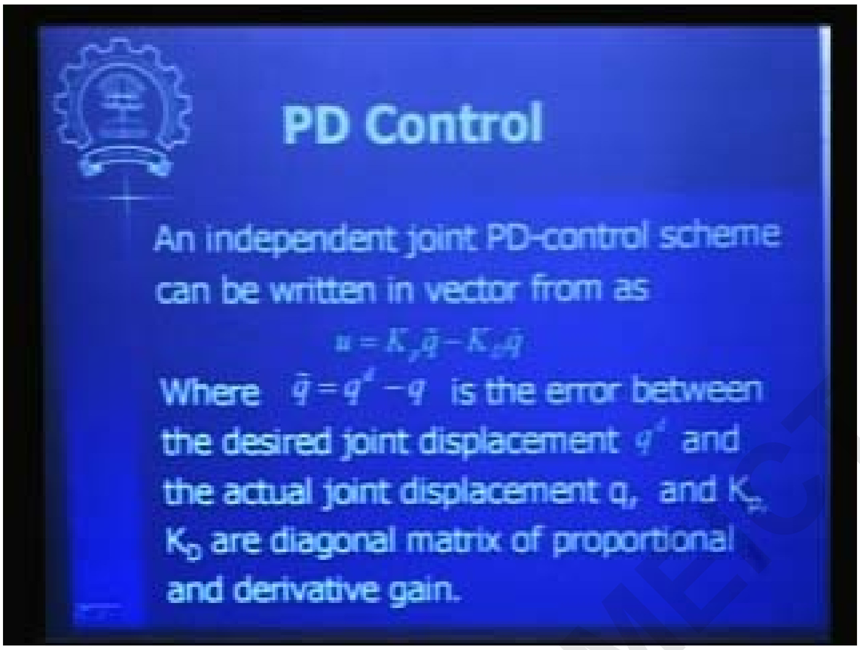


now we have this definition see you see now the PD control law is also in defining the terms of the joined variable with the motor variable ok

but it doesn't make difference because like your motor and joined variables they are related to each other through gear reduction ratio so they linearly related with each other so there is absolutely no problem only problem will be we have to change this gain accordingly what ever be the gain ok

now another point to know is that like see previously we defined for only one particular motor but here now we can defined for entire robot at a time when this u I am saying equal to kp and q theta q theta is the error and kp is the actually matrix if the diagonal matrix with diagonal entries positive ok because this q is the q theta is the vector error vector ok and then this kd into q dot kd is again a matrix

so again these are derivative proportional and derivative gain matrices now we have to show that this achieves zero steady state error we consider this energy based lyapunov function candidate as we considered for the simple spring mass system so here we have to consider now the total energy in this fashion q dot transpose your inertia matrix q dot this is similar

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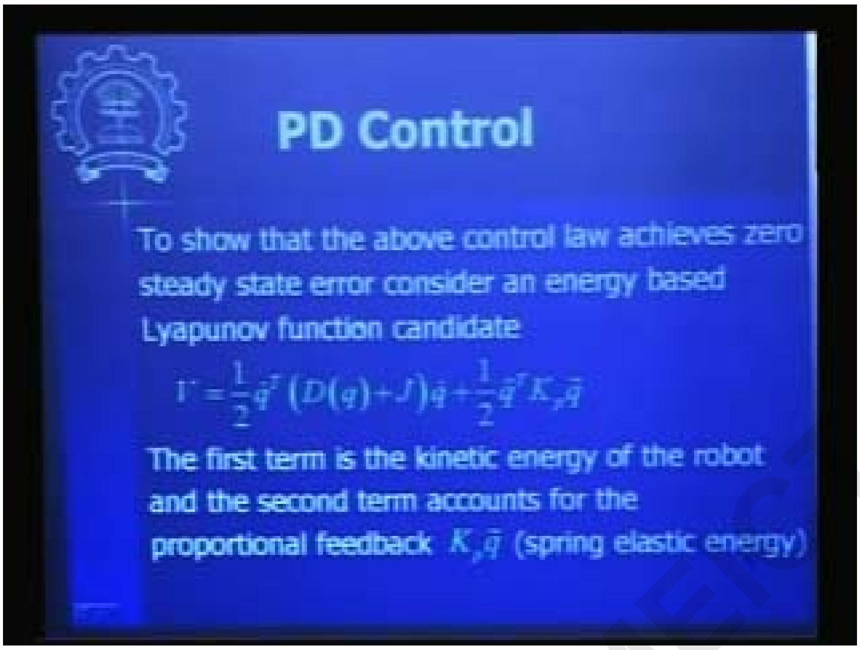


to our m x dot square term in the previous example and then this is the term corresponding to spring type of energy ok potential energy theta this was similar to kp times e square in the previous example ok

but now see when you want to represent in matrix form it have to it has to be represented in this fashion have this q d dot transpose matrix into q d dot that is the way you write in a matrix form ok and when you expand this you will get the form what ever expression you derived ok

so this is like is just a like the same thing you have to represent it in the different fashion so the complications of representing so many terms writing so many equations they get reduced when you write in the matrix form ok

now let us see the time derivative of the speed ok this v dot is given by these equations here we have not yet substituted for motor or the dynamic equations they are not we just differentiated these theta

so you see that this d is the function of q so you will have this first term where you have differentiated and then like you differentiated only q dot to get q double dot and in the second term we have differentiated d to get the d dot and q dot is as it is ok

so these are the two terms contributed by the first kinetic energy terms in the lyapunov function and these term is basically because of the potential energy

now we solve

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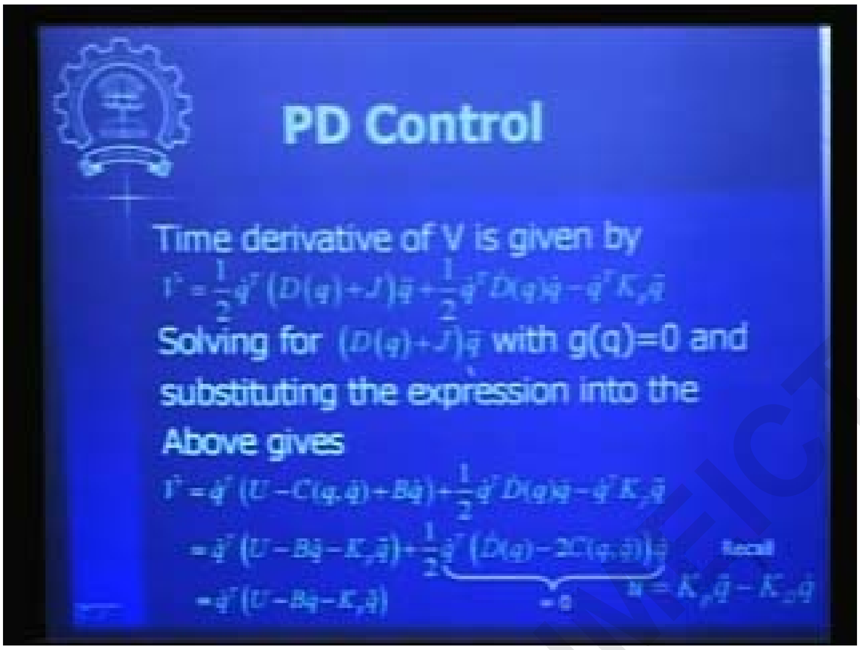



for these from our system dynamics equation which is here considering this d q term to be equal to zero and then you substitute the expression in these to get these contribution now you see that this is the q double dot and then this term d dot is coming you can combine this d dot term with this c term to get these expression

and now you recall your previous analysis in the first class first or second class we studied that your d dot minus two c is q symmetric matrix ok

when d dot minus q c is the q symmetric matrix you will get q dot transpose multiplied by d dot minus q c multiplied by q dot equal to zero that is the quadratic form with the q symmetric matrix is equal to zero ok

we can so here is the place where the property d dot minus q c is equal to q symmetric becomes very very important and useful for the applications to the robotic manipulator ok

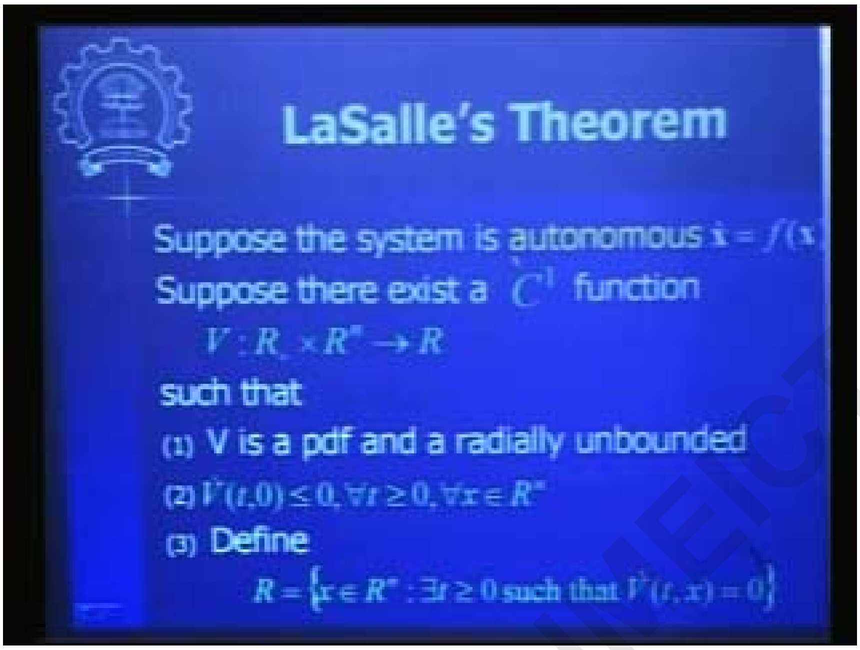
so you get this final expression see we have not yet substituted this control law u ok this u is kp and kd type of a PD control law ok so if you substituted that we will get v dot of this time this is actually negative here this term is negative ok have miss that negative time but v dot is negative of q dot transpose kd plus B they are the damping terms q dot and now since this is present contain again q term or q dot term ok this is only negative semi definite not negative definite ok

so notice that there is no term corresponding to q in this equation corresponding to q or qt dot so this analysis show that v is decreasing as long as q dot is not equal to zero but it doesn’t so this is sufficient to prove that manipulator can reach reach the position where your q dot is equal to zero ok it will gone having this decrease this v till the point where q dot equals to zero but that doesn’t means necessary that your q dot is equal to zero or q equal to desired final position ok

so this is not the sufficient result for they asymptotic stability we can say only that system is stable ok

so for asymptotic stability there is a another theorem that we need which is the LaSalle’s theorem we will see that LaSalle’s theorem now ok

I will just give you the theorem and then we will apply to the robotic manipulator system and then will not go into the details of proof of the theorem so let us see what is LaSalles's theorem (refer slide time 44:29)




so for system which are which are autonomous system ok where your x dot equal to f of x only there is no t here now ok

so this is the very important condition for LaSalle's theorem now suppose that there exists this C one function such that this V is the positive definite function and radialy unbounded again this definition we have seen in the last class What is radially unbounded function and then v dot is less than or equal to zero

so see here the condition on v dot is less than or equal to not strictly less than then condition then you define this domain R where x belongs to Rn space such that V dot of t comma x is equal to zero ok

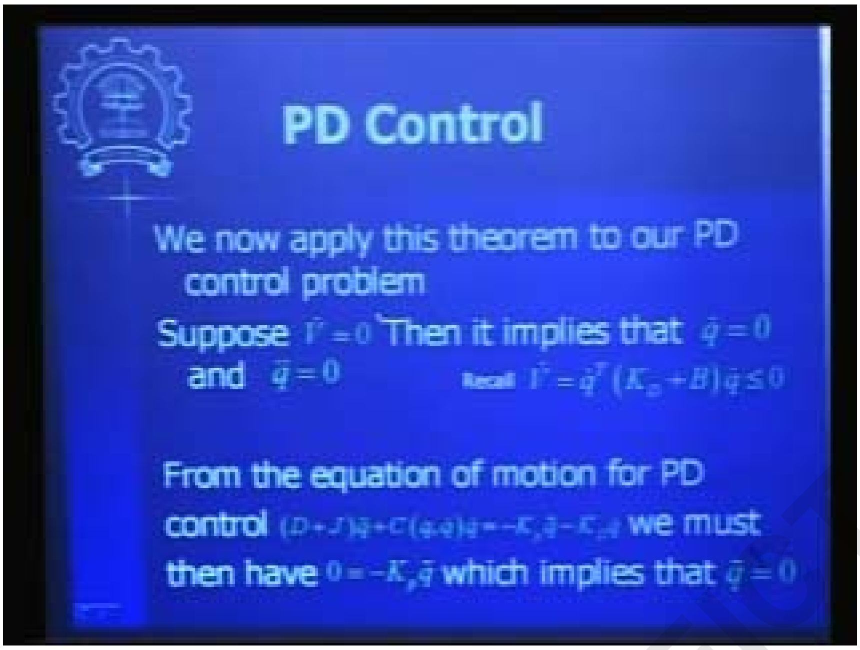
so you study what is the domain where v dot is equal to zero that basically meant by this part or in this domain where your function v dot is equal to zero ok so when v dot is equal to zero we have to see what is the what is this domain and then based on some conditions on domain like we can conclude the system is asymptotically stable

so what is the condition is that r doesn't contain any trajectories of the system other than the trivial trajectory trivial trajectory is the equilibrium of the system ok

so if you substitute v dot is equals to zero and like use the system equations and you find when v dot is equal to zero that will give you q equal to zero and q dot equal to zero or it will give you the equilibrium of your system that means there is no other trajectory than the equilibrium in this domain where v dot equal to zero ok

if that is the case then you say that your system is asymptotically stable that is the result given by LaSalle's ok

so it is it is the proof we will not worry about too much but while applying this is pretty simple to apply this theorem ok you just need to see the domain where v dot equal to zero and then based on that we can conclude about the asymptotic stability let us see how we can apply this to our robotic manipulator in the PD control case ok (refer slide time 47:14 )



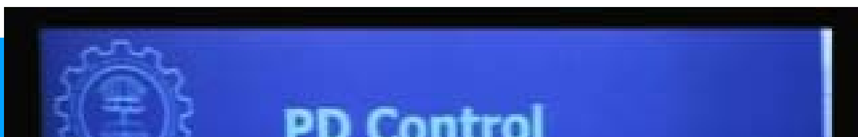

suppose we have this v dot equals to zero so you remember what was the expression for v dot it was negative this is the negative term here negative q dot transpose this term into q dot so suppose v dot is equal to zero vc that means our q dot is equal to zero now we differentiate that q so we will get q double dot also equal to zero now from the equation of the motion for our manipulator and actuator taken together this is the equation of the motion and now substitute that q double dot is equals to zero and q dot equal to zero and we will get zero is equal to kp into q theta which means that q theta equal to zero ok

so our domain of R is q theta equals to zero q dot equals to zero and q theta equals to zero so the domain is q dot equals to zero which is same as q theta equal to zero and q theta equals zero that is the equilibrium of the system and there is no other point other than the equilibrium in the domain where v dot equals to zero ok

so from there we can conclude that the system is asymptotically stable ok LaSalle's theorem gives us very nice way to prove with the energy based functions the system is whether asymptotically stable or not ok now it is important to remember that this LaSalle's theorem is applicable only to autonomous system ok

if the system is non autonomous or it is dependant on time as in the case of the tacking problems you can not apply LaSalle's theorem ok

so that is the very important point to remember ok don't do the mistakes like applying



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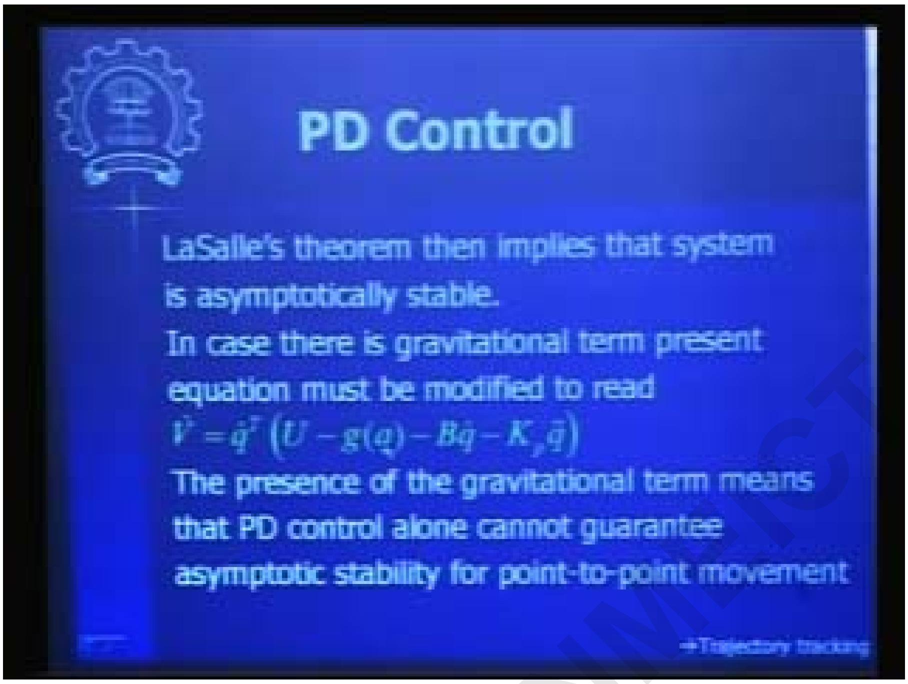
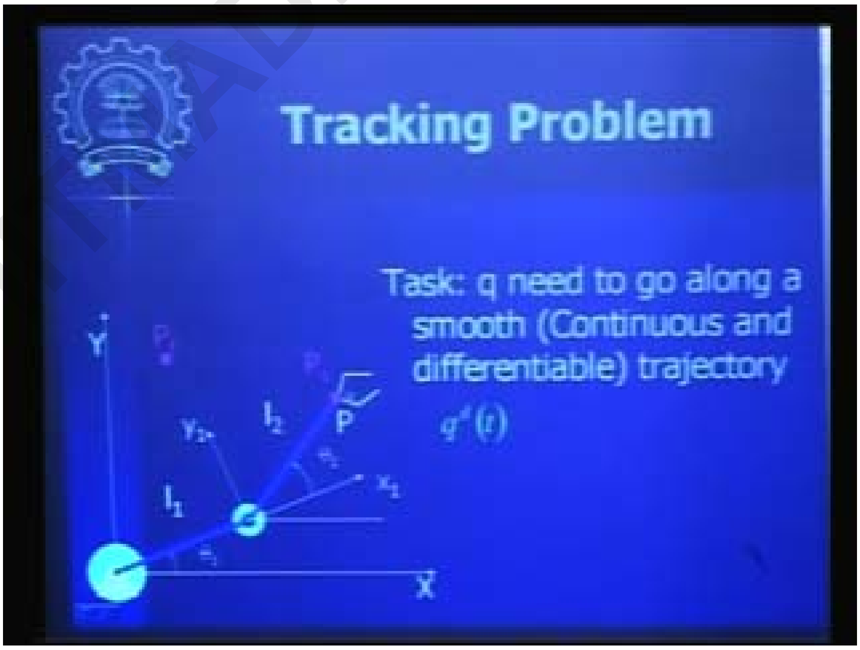



LaSalle's theorem in the case of tracking problem ok now suppose we introduce the gravitational term then equations will get modified so now we can define new control where you compensate for this gravitational term and then add so all these kinds of different manipulations we can do ok but if you have just PD control and a gravitational term then you cannot guarantee this asymptotic stability from these results ok now let us move on to trajectory tracking cases ok (refer slide time 50:13)

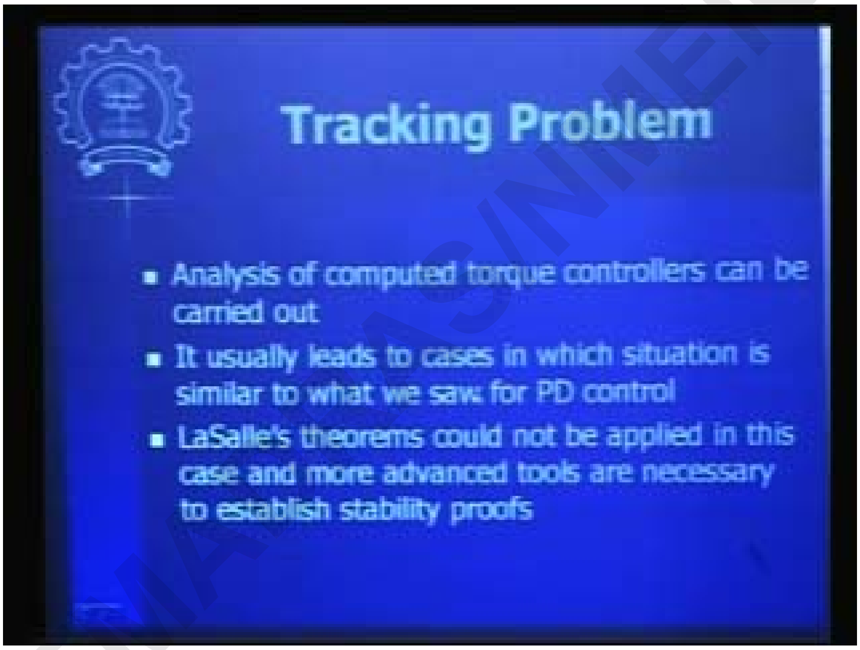


we just we will not get into the control development or control proof in this case I just give you the result then will stop and you think about the result till the time the next class ok

so let us revive what is our trajectory tracking problem again so we have to move from the start point to final point in a specified manner ok may it be straight line or it may be some different kind of curve ok

so it may be straight line in this case or it may be some different kind of a curve we have to move along this trajectory like you have to take the end defector along this trajectory and corresponding to that you will get all the desired joined position as a function of a time ok desired joined trajectories will be available to you and then we assume in our analysis that it needs to go along a smooth trajectory ok

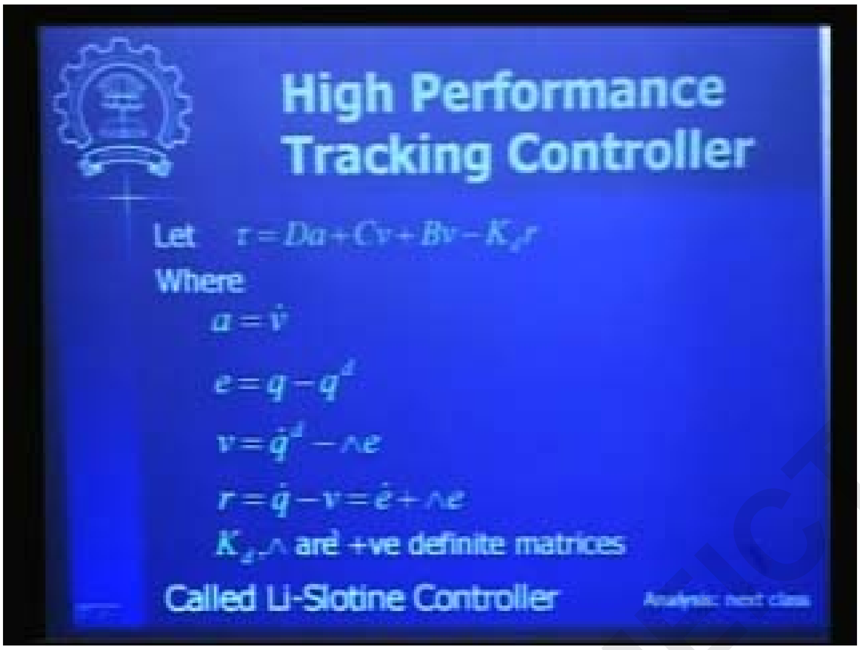
so it is continuous and differentiable trajectory function in terms of the joined variable qd ok now you can do this analysis in a similar manner as we did for the PD case for computed torque control (refer slide time 51:46 )




ok the computed torque control that we have seen you can carry out some analysis in the similar manner but as I said earlier you usually end up in the case where the situation is similar to what we saw in the PD control and there where you have only semi definiteness condition on v dot so in that case for the PD or regulation kind of a problem we applied LaSalle's theorem to get the asymptotic stability but in this case we can not apply that there are some additional tools set are there which will not get into details of that but then these tools are there to like the system to be asymptotically stable what ever the computed torque algorithms we have seen in he last class ok

we will not get into those details like you can repair to books for that and only thing you have to remember that you should not apply LaSalle's theorem in any case for a tracking problem ok as it is there only for the autonomous kind of problem so we will see one controller which is very high performance kind of a controller ok which has been developed by the Li and Slotine ok

(refer slide time 52:54)




this is the kind of the expression for the controller ok

so I will I will explain you all these terms in the next class but we will see these li and slotline type of the controller which is which is very high performance tracking controller which will give us asymptotic stability

so the basic idea is here you are compensating for these terms but not directly in the Dq d double dot fashion ok this is not D multiplied by qd double dot

so it is defining some intermediate variable so this new is qd dot minus A e is minus lamda e so you have introduced this error term as new term from the like as compared to the computed torque control see in computed torque control you should we will have like d multiplied by qd double dot now it is d multiplied by v dot which is qd double dot minus lamda e dot with this error terms have been introduce here there are for some purpose like because of these terms we will get very high performance in this controller ok

so how that controller becomes high performance controller and all those things we will see in the next class proper lyapunov analysis of the controller we will see in the next class and then we will move on to application for of these so let us see first see the summary of these class

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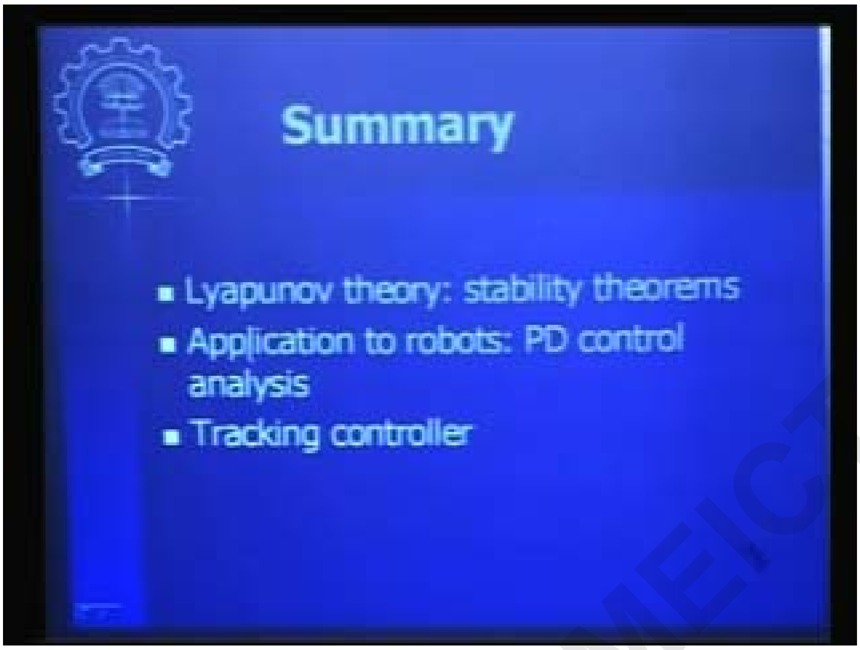


what we seen is the lyapunov theory stability theorem basically then we move on to the applications of this theory to robot manipulators we seen some examples before going to that and then these examples of the PD control kind of the example then we have seen tracking controller we have not done the analysis of that but we have seen the tracking controller which is li and slotline kind of a tracking controller which are high performance controller carry out this analysis in the next class and then we will move on to some other advanced robot control tools then we will not do a detail analysis about these other tools we just see some fundamentals about the tool like a force control ok

so that's what we will study in the next class we will stop here now ok (refer slide time 55:48)