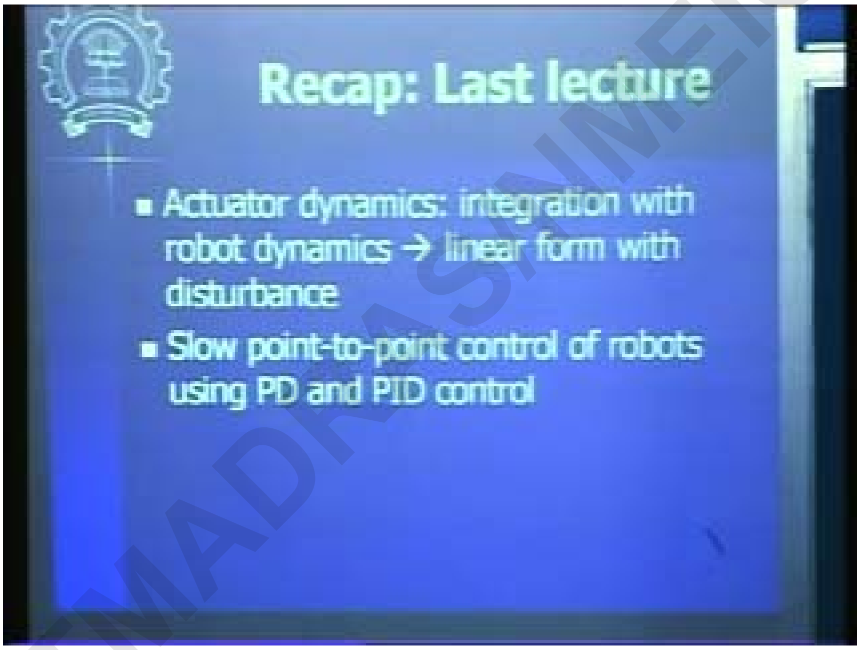
ROBOTICS
Prof. P.S.Gandhi
Department of Mechanical Engineering
IIT Bombay
Lecture No-34
Robot Dynamics and Control

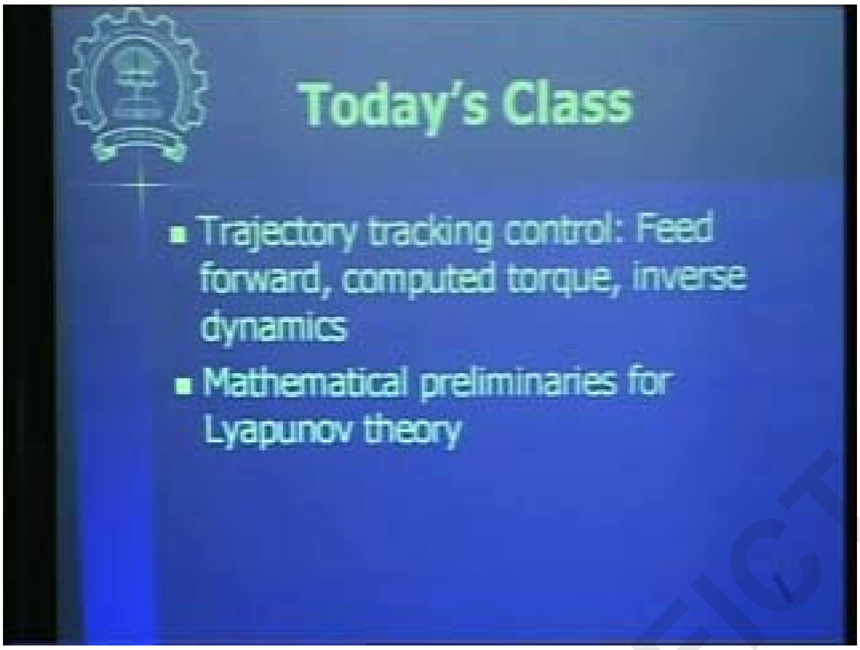
Let us start with the lecture number four of this module robotic um dynamics and control ok robot dynamics and control so let us see first last like what we are seen in the last lecture

so we had a look at the actuators ok how to model the actuators and how to integrate the actuator dynamics with the robotic dynamics and finally we came up with the linear equations which was now on the system dynamics ok it has it had a linear form with the non linear disturbance acting on it ok you recall that form (refer slide time 01:54)



so that is that is the first thing we did then after that we put that form and like we develop some control algorithms basically for slow point to point regulations problems ok

so those control algorithms like we studied them what is the steady state error we did some analysis about that ok (refer slide time 02:32)



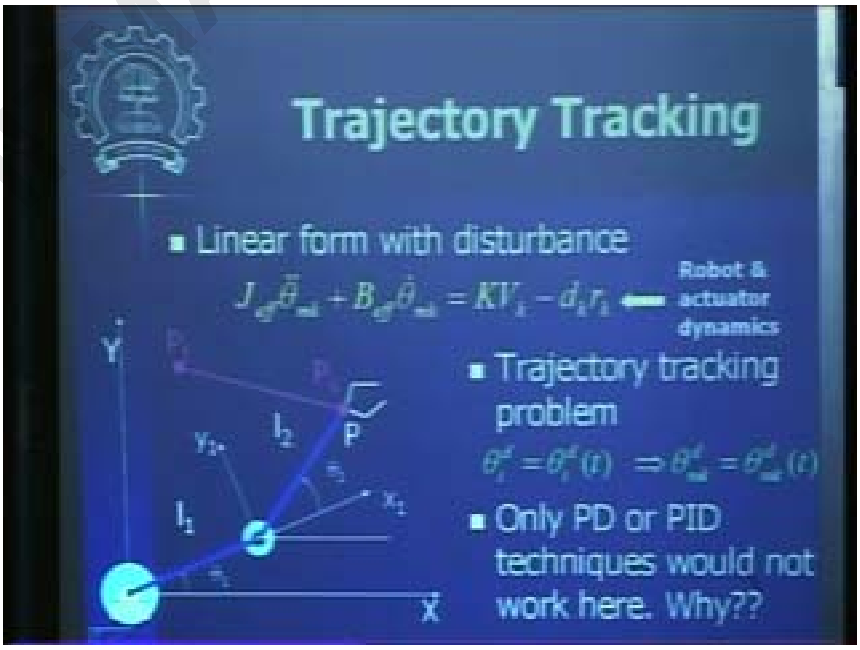
now in this class we see suppose we want to track a trajectory what kind of control algorithms we can come up with using the same form for the equations of the robot dynamics along with the actuators ok

so we will use the same form but now will concentrated on developing some control algorithms for trajectory tracking kind of a problem ok and then like we have to see it will give you some improved performance over the previous algorithms which are regulational algorithms but still to improve the performance further you will have to go for an non linear control ok the linear form of the equation would no longer give you very high performance kind of a control algorithm ok

so you have to study some part of a non linear theory ok specifically lyapunov theory that we are going to see ok

so for that we will just build up our mathematical preliminary understanding for lyapunav in the today's class ok

so let us start with the trajectory tracking problem (refer slide time 07:35)



you remember this is our linear form with the disturbance or robot actuators robot and actuator dynamics taken together ok

so you have this effective inertia and effective damping term ok this effective inertia again we recall that it is not a constant we just assumed it to be like a some value which is averaged over the range of the motion of the manipulator ok

so it is a function of q in general ok and then this d effective is the constant damping kind of a term and then this is our control input which is going in to the system fine and then this is the disturbance term dk which accounts for all these corialys centripetal all these acceleration or inertia term um and like coupling terms in the d matrix ok

now we have take a look at what is the trajectory tracking problem again so we had our like standard example of two link manipulator ok you recall that now trajectory tracking problem is to go from you take the end defector from these point Ps to this point Pf ok in some pre defined trajectory fashion ok so in this case say for example you are having a linear trajectory like going in a straight line from one point to another point ok

that is the trajectory tracking problem so in general you will be given this trajectory in terms of a um end defector co ordinate like how like in a global co ordinate you are given a path along which we have to go ok

and then that is your trajectory tracking problem now you do the inverse analysis inverse kinematic analysis to get in this case the desired joint angle ok

so this is joint angle for theta one and theta two for this particular manipulator ok now these desired joint angles you you want to then convert them in to motor angles desired motor angles ok

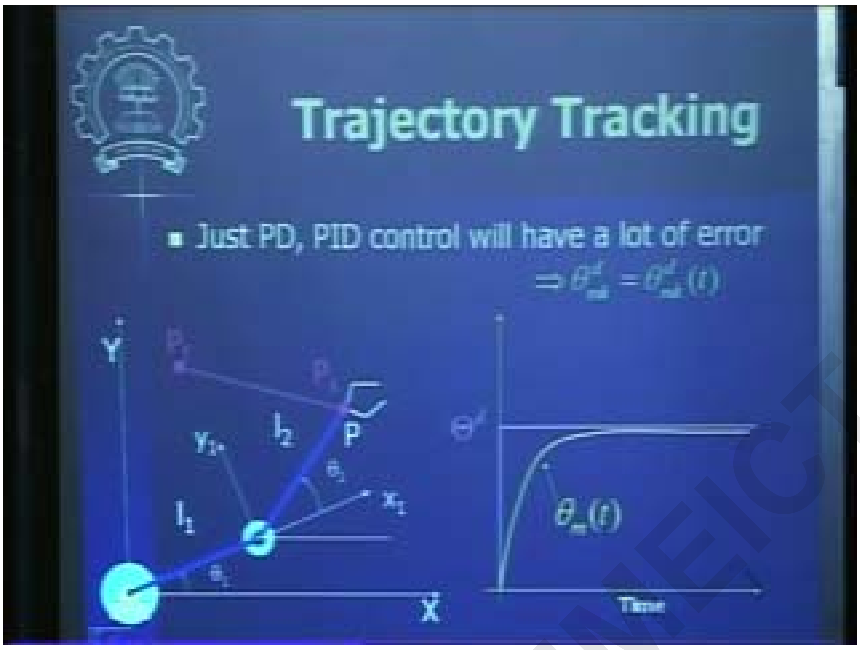
so this is the way you will find out desired motor angles by using the gear reduction ratio you know that this there is a gear reduction from this theta one to like theta m one ok

so that ah using that you will convert the trajectory to the desired trajectory into desired motor motion trajectory ok and then that we will use for the trajectory tracking problem in the in this equation so so now you have this equation and you want to have here theta m theta motor position to follow the trajectory defined by theta dm ok

so that is your trajectory tracking problem in the mathematical sense um now um we will see now we have seen already this PD and PID control techniques ok they are for the regulation problem ok now what is we apply them for this problem can we apply the first phase and you will find that if we apply this PD or PID controller for trajectory tracking problem it will not work can you think about why it will not work just think about why it will not work

so in general this regulation kind of control algorithms PD PID control algorithms will not work here for the tracking no over shoot we can we can have a gain tuning such that like there is no over shoot theta is equal to one critical damping case but the problem is that now you imagine you see this response

(refer slide time 08:17)



ok this is your desire position ok

the actual response in the regulation problem where you have point to point motion is something of this slot ok

now what is happening in the tracking case this theta d is not no more a constant ok is actually varying ok this theta is actually varying

so like for this motor response to catch up with theta d in very short time is not possible so just PD PID kind of a control will not work because of this reason because we are talking about the tracking problem now ok

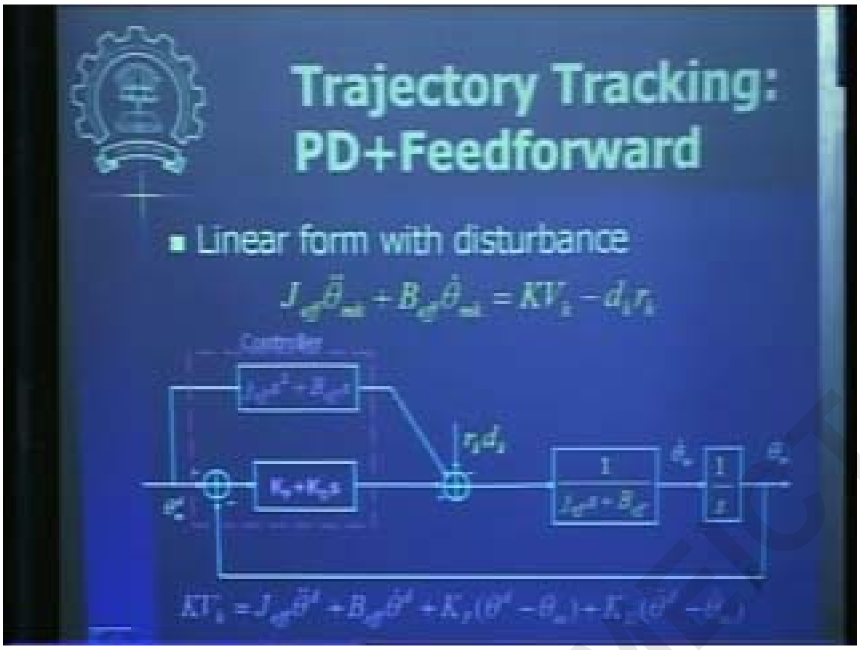
so that kind of the control algorithm will not work here for the tracking trajectory tracking problem ok

so what is the solution so we see like why this is not catching up because the disturbing term which are this inertia and damping term they are not in some sense feed forward in the control down is this ok

so what we can do is we can estimate both these what are these term along the trajectory and then that estimation we can compute then put in control ok because we know that it has to overcome for the given trajectory whatever is theta md we have to overcome these forces you know that there will be inertia force corresponding to j effective multiplied by theta double dot md ok and d effective multiplied by theta double dot md dot ok

that those effective force if we pre compute and fading our control law then it becomes the feed forward control

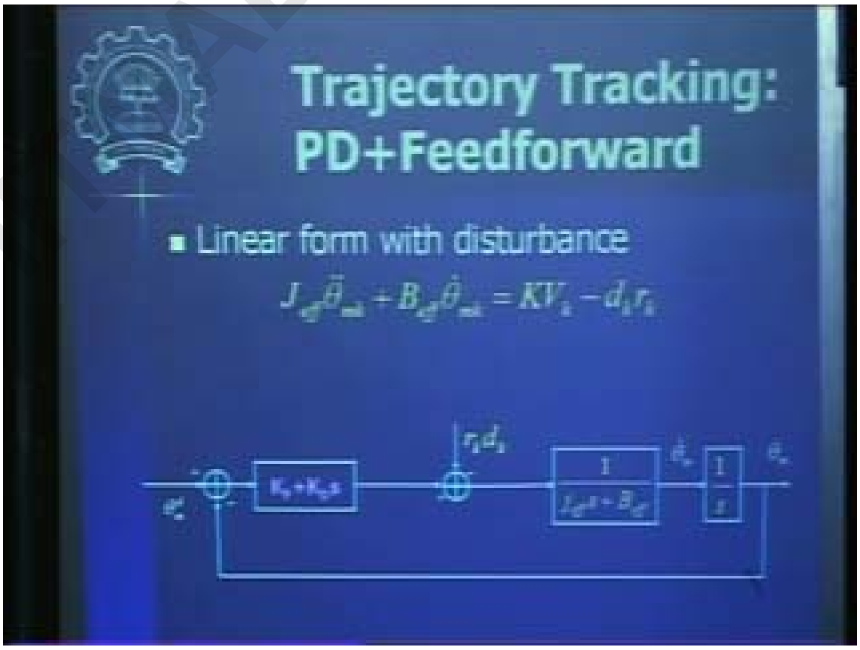
(refer slide time 10:48)



so this is the block diagram of our standard PD kind of control now we compute this feed forward term is the feed forward term based on the theta d desired motor position ok and then that becomes our total control ok

the rest of the part like this this system dynamics or actuator and robot dynamics taken together these three part will remain the same ok now let us see what is this this is the control expression

you have this PD part and then this is the feed forward term now let us see what is what does it mean in the mathematical equations ok to explain again why we use feed forward term let us see (refer slide time 12:01)

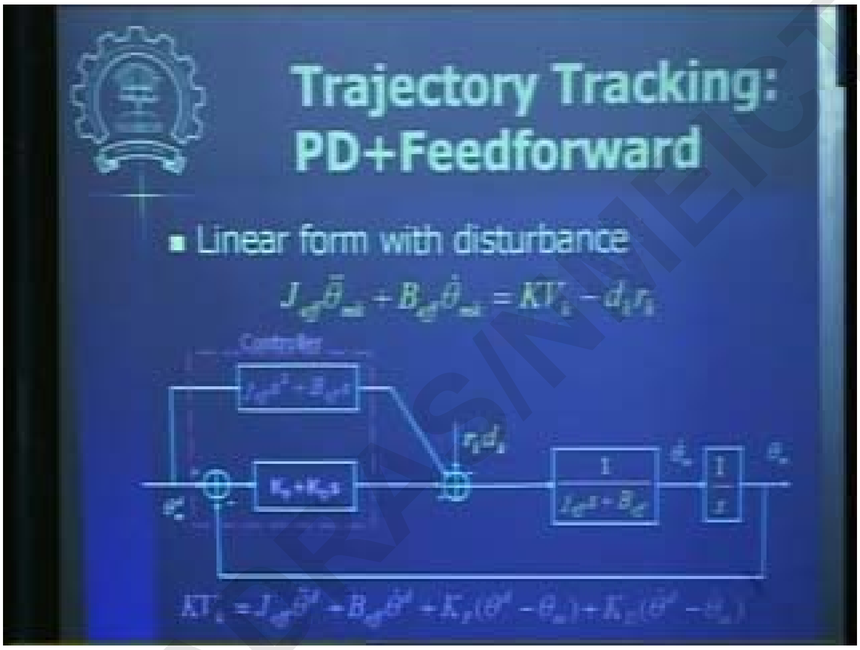


suppose we have just see we have this equation and we are not using feed forward term ok we have just PD control then we know that see this PD control is acting in such a way that it is what ever error that is getting multiplied by proportional gain ok and that error is produced now because of these dynamics and your dk dynamics ok

you understand that ok you agree with that that the error will be produced by these dynamics because of the k effective and b effective terms and this dk part which is the disturbance ok now now suppose now we know that when we track um desired trajectory suppose there is no error then already we know that the term corresponding to the j effective multiplied by theta m double dot d and b effective multiplied by theta m dot d they are already known that this much of torque we have to overcome

so why rely on your just PD error kind of a control strategy why not feed these term a

(refer side time13:36)

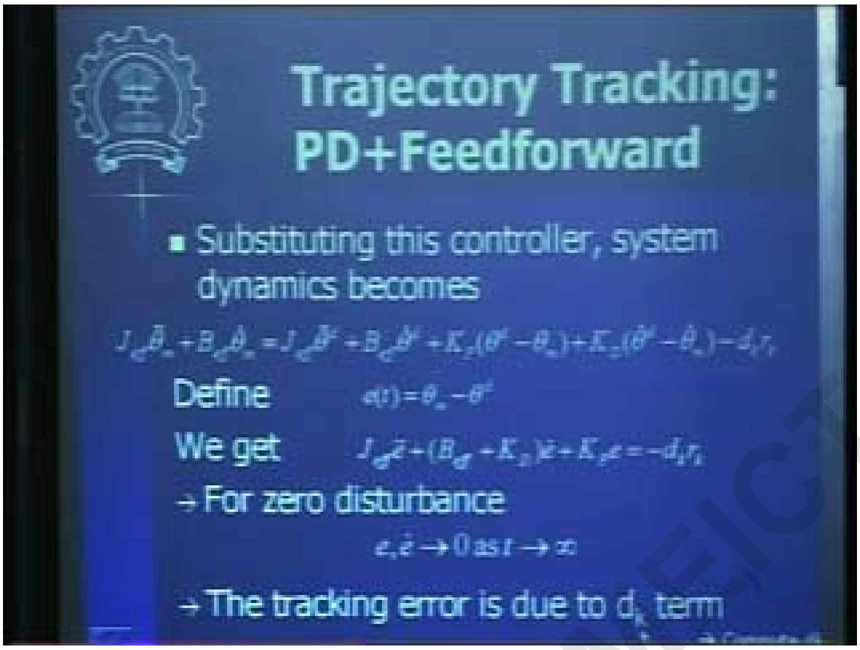


priory because we know we have to overcome that part so that's why we we use this kind of a control law this feed forward control ok

so there then that error will be reduced then we see now when we do the analysis how that is happening ok this is our controller and this is our control expression next we will carry out some analysis of this controller and see what we get for the tracking trajectory error like error in the tracking the trajectory ok

so tracking trajectory error dynamics or dynamics of error in the following of the desired trajectory ok

(refer slide time 16:00)



so now what we do is we substitute this control expression that previously shown in the system dynamics ok and then you will get this expression ok

now this is the feed forward term and then this is our normal PD term and then this is the disturbance term that is already there in the dynamics ok

so now we will define the error in tracking this is the function of t you can see here ok as theta m minus theta dm ok so then we will get these as our tracking error dynamics now you notice here that given that disturbance is to be equal to zero we will find that this is kind of a second order standard second order spring mass type of a system

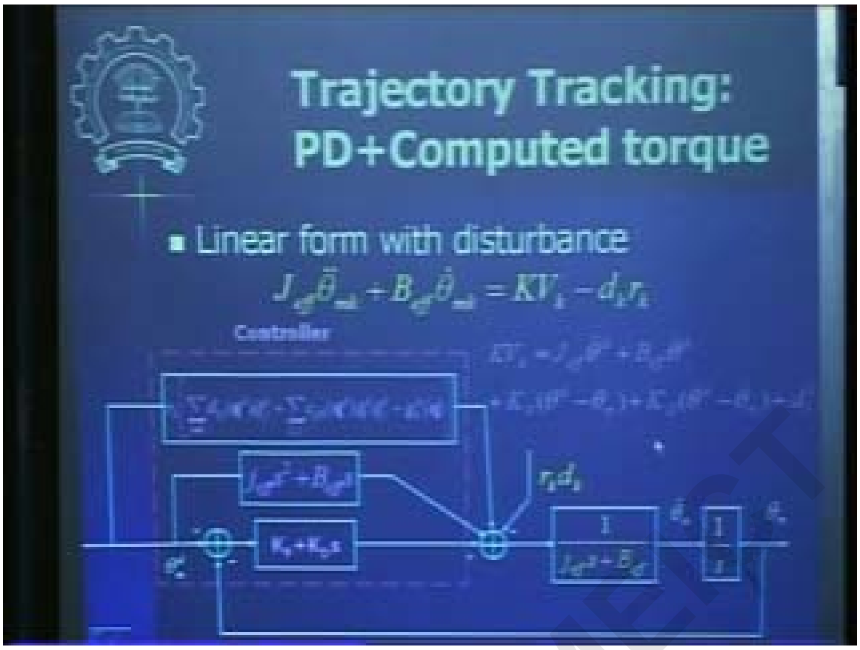
so we can always choose this Kd and Kp gains to have this error going to zero with what ever settling time we need ok so last lecture we have seen already how to do these computation for Kp and Kd gains ok for a given like type of a response like you want to move fast how how will you define omega and how omega will get this Kp and Kd for the system ok and now you see that tracking error is only due to tis dk term this disturbance term ok most specifically dk multiplied by r theta ok

so basically now only the disturbance term is remaining the immediate solution now is that to reduce the error further in tracking you compensate for these dk term by pre computing that is immediate logical thing that comes into the mind to do ok right so

so next we will do that and then see that becomes the computed torque kind of a control strategy ok

so again our linear form of the equation

(refer slide time 18:21)



with disturbance look some thing of this term Now our feed forward control the block diagram can be given in this fashion now we have to add the computed term corresponding to these so these becomes something of these block to recall what was your dk so this is the disturbance term dk remember this j not equal to k all the coupling terms in the inertia of the matrix then corialys and centripetal acceleration term and then your gravity term ok

so these multiplied by rk with from this computed term and then you will find now that your error will error in tracking will further go down or either you will able to achieve faster tracking i mean faster in the sense the actual motion will converse to the trajectory faster ok

then the transient period will also get reduced here reduction in the error ok

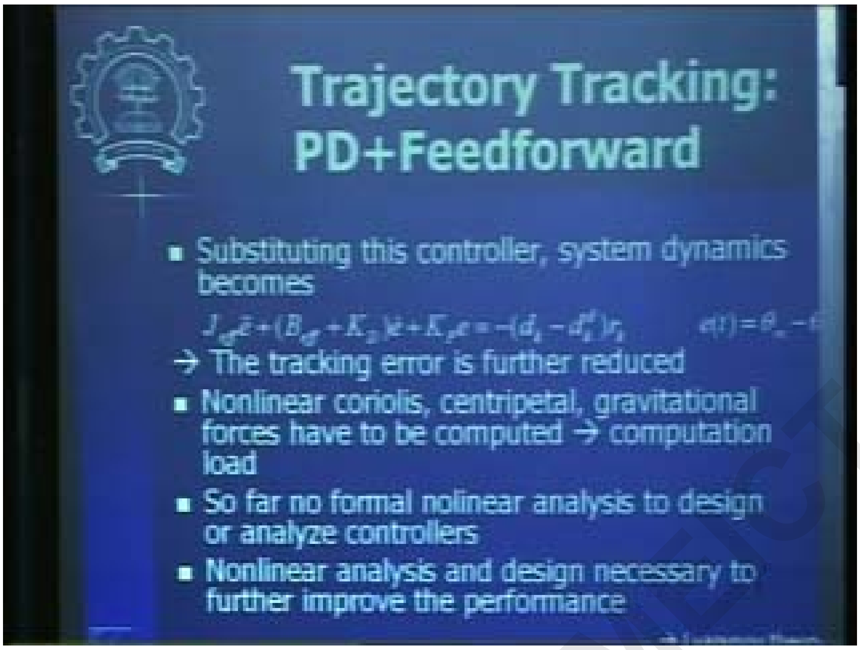
so what will get for the controller is now the total this expression ok

so you observe that now we have to compute these term and we have to compute in addition these term notice that there all functions of the desired trajectory ok

which we know priory we will know what is our desired trajectory that is we follow ok

so this can be computed now the total computation will look like this of entire control law the important thing to note here like that we have to have this lot of computation done for this control law ok as we see here now system dynamics with these

(refer slide time 22:32)



now we can substitute the control law and find out the system dynamics and you will see that now this dk term previous term has reduced by these amount so it will be dk minus dk desired ok

so in general you have once the tangients are gone the error between the trajectory and the actual motion will be lesser will be very less and because of that this term will also be very less so that that's how we like we can manage better trajectory traction in this case ok

but this is also not perfect you have not guaranteed zero error in the tracking ok it works very well for practical systems I mean there is no problem but still like theoretically some some betterment of the performance can be done ok

the main advantage or the main advantage is that is error gets reduced but now what is it at the cost of why the error error gets reduced you have to pay for something like so

so when you are you are talking about reduction error you have to do all these control term computation like non linear carioles centripetal gravitational forces in this d term they have to be computed and then computation means like see you remember like you have sampling time in which you have to implement this control law in digital domain you have seen in the fundamentals of the control that there is a sampling time issue

so here like if we have to do these computations we have to finish of within on sampling time ok

so that means if you are able to finish off you have to use a faster microcontroller to compute ok

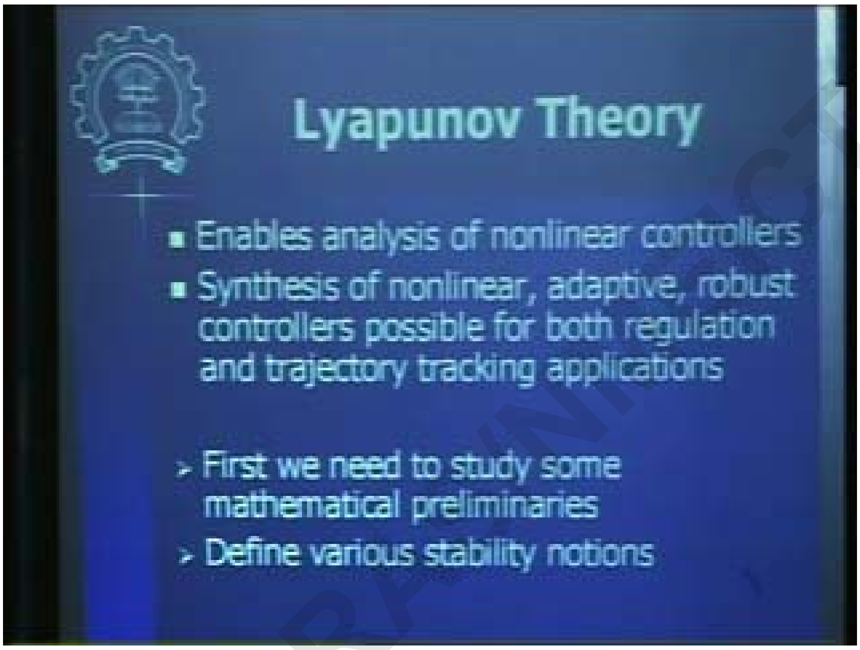
so like we that is the additional cost that we are paying to reduce the tracking error we have to do these computation ok and then compensate for this d term and get our tracking error reduced ok

do you see that ok next we have not done so far any formal non linear analysis we have been working in the mostly in the linear kind of the domain you have seen intuitively how this PD control algorithm are good to reduce the error and then like now we have seen how to extend further for the tracking kind of a problem ok by pre computing some terms which are there in the equation

so next we will study little bit about about lyapunav theory which is an non linear analysis and deign techniques ok and there are there are several non linear analysis techniques that are available in the literature ok

but will not be able to cover all these in this class or in this course of the robotics we will touch upon many important of them ok

so lyapunov theory forms an very important domain of non linear control tools specifically for robotic kind of application ok we will will study some fundamentals about the lyapunav theory now (refer slide time 23:09)



ok why we should have why we should study lyapunav theory what it enables it enables the analysis of any non linear system ok and you know some robot dynamic equations they are highly non linear in the nature they will have all sine theta cosine theta terms or square terms of the velocities ok

so in general these equations are highly non linear in nature ok and then also these theory helps you synthesize like many different controller ok to improve our enhanced performance more than that what linear theory energy ok

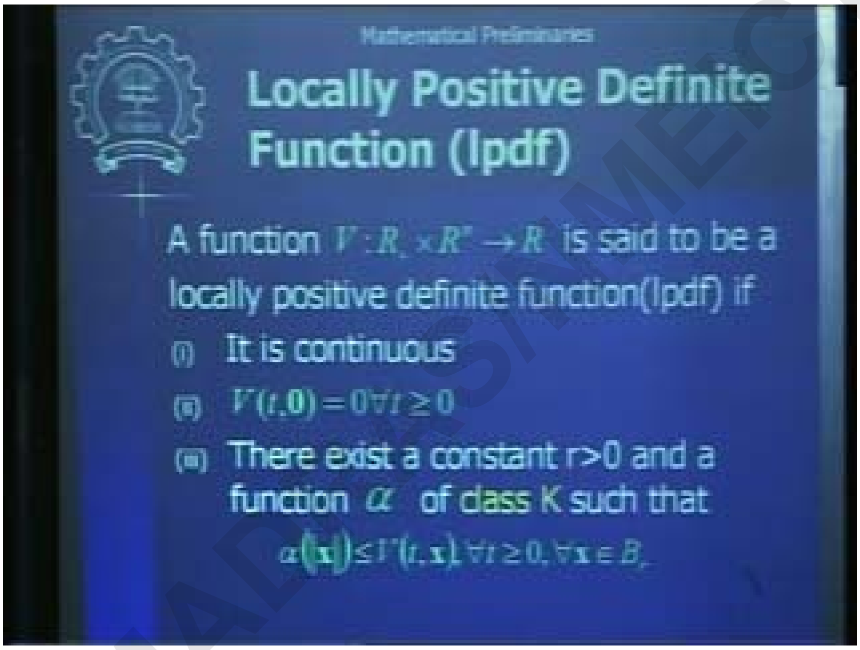
we can improve the system performance more than that what linear domain tools or linear controllers can by doing the two non linear analysis philosophically if you see all these linear or non linear things they are approximation is to what is the real world ok

in the real world the systems are non linear in nature highly non linear in the nature in when you say the system is linear you are making linear approximation on that non linear world ok and doing all the analysis for the sake of understanding more the designing better some controllers for your system ok when you say non linear now it is little bit better approximation to the real world but again its not capturing the entire gist of the real world itself have the philosophical understanding of the back of your mind when you say in non linear system it is a better approximation than the linear approximation to any of your real world systems but its not the end like actually you have like always you have some room for improvement ok but when you go for the non linear models the the um like control analysis becomes difficult ok you sure as simple as the linear domain analysis ok

so here you have to study couple of tools you have to get keep yourself these tools to really do the analysis for the robotic manipulator ok ok

so let us see two things today we will study some some mathematical preliminary and then we will see some notions of stability which are not there in the linear domain in the linear domain we have the other stability or system is not stable ok but when you go for the non linear domain you will have different notions of the stability so we will see one or two of them here so as to get you feel of that but there are there are many of the different notions which in the interest of time cannot be covered all the in this class ok

so first important understanding is what is called as locally positive definite functions ok LPDF functions(refer slide time 26:09)



now understand the way the functions has be defined here the function V is taking some element of a real line positive real line and some space element of the vector space rn and producing an element which belong to a real line again

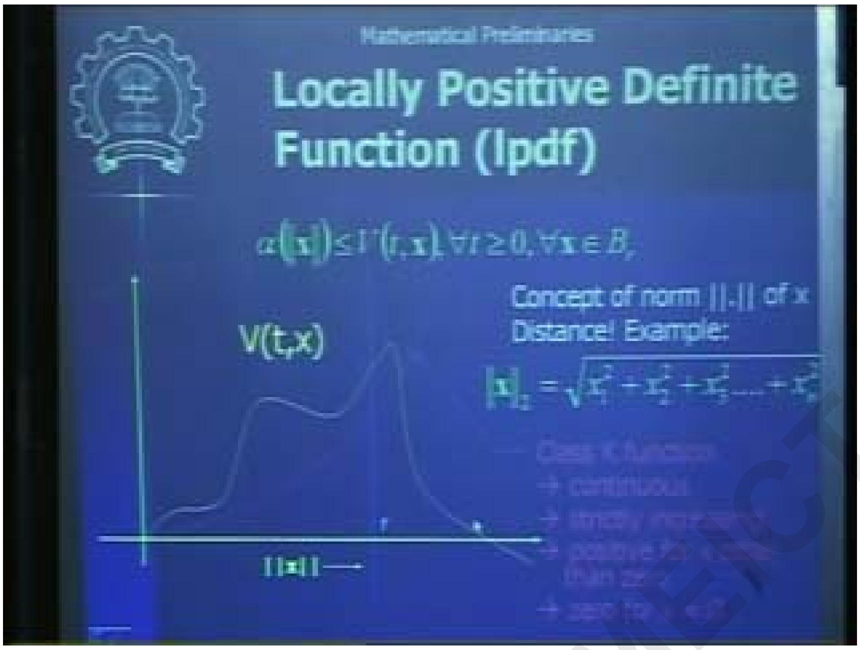
so this V is a function of say some time t and some state x ok

so V is a function of time t and x x is belonging to this vector space rn and v is producing a number which again belongs to this R or it is producing a scalar number in a simple term ok

now it is said to be locally positive definite function if it is continuous and then v of t comma zero like when x is equal to zero we will get a zero numerator what the value of t ok that is important numerator what the value of t is when x is equal to zero we will get v is equal to zero and now there exists another constant r which is a positive constant and the function alpha of class k will come to these definition what is the class k function very soon such that this V is always greater than the or equal to this function alpha of norm of x there is other new concept here this is a norm of x

so you see some of the concepts then definition of the class k in the next slide

(refer slide time 28:18)



now this is our condition

so let us see first what is this norm the concept of norm is very similar to what we talk about distance in our geometry ok

so if you are given two points in a space how do you find distance it is basically square root of x square minus y square ok

so x and y are basically defining two points in a vector form now we have from the origin this norm is the distance measured from the origin ok

so you can have this kind of a concepts like I mean this is like to give you just intuitive feel about what norm is so this is specifically two norm to is given by this expression ok

so this is the this is our true distant concepts but in general in the n dimensional space this is not two dimensional or three dimensional space can be n dimensional space depending upon what your n is ok

so that is the concept of norm now class k function has these properties it is first continuous then it is strictly increasing ok and then positive for x other than zero so all these places where x is not equal to zero it is positive

so it has this kind of a nature and it is zero for x is equal to zero ok

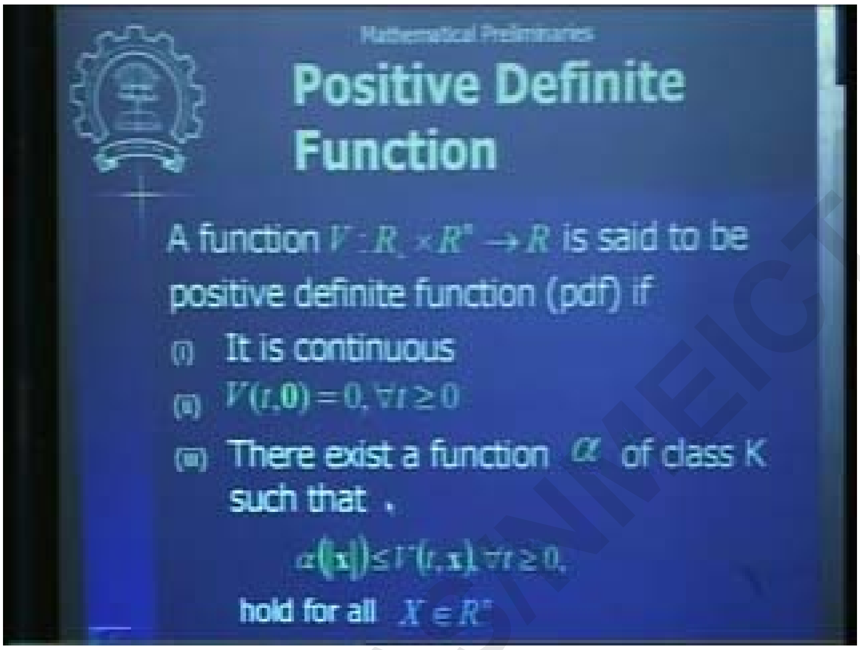
so this is the point where x is equal to zero so this function is zero and now what this condition means alpha of x alpha of norm of x is lesser than equal to this V

so V is some function which can lie only on this side class k function ok it is always greater than the class k function of this norm ok so you see this is plotted again the norm of x ok now this definition says that it is for all x belonging to this ball of radius r ok now in general this ball will have like in two dimensional it will be a circle in three dimensional it will be a sphere and in general n dimensional it is hyper sphere ok so here in one dimensional you can put that limit as r over here ok so these number r the norm should be less than that number r or x remaining the ball of radius r ok

so nor of x is less than or equal to r that is the condition mathematically then you say that x belongs too the ball of radius r ok so x should be less than or equal to this r so beyond this line you are not interested like what happen to the function the function may go below class k function it may go even zero go negative you are not worried about that but as long as you are saying that x is belonging to this ball of radius r the function has its condition satisfy ok

so that's why we are calling it as a locally positive definite function we cannot have it everywhere it some local domain which is defined by this value r or ball of radius r ok you understand that um now let us see next definition

(refer slide time 32:31)



which is positive definite function ok

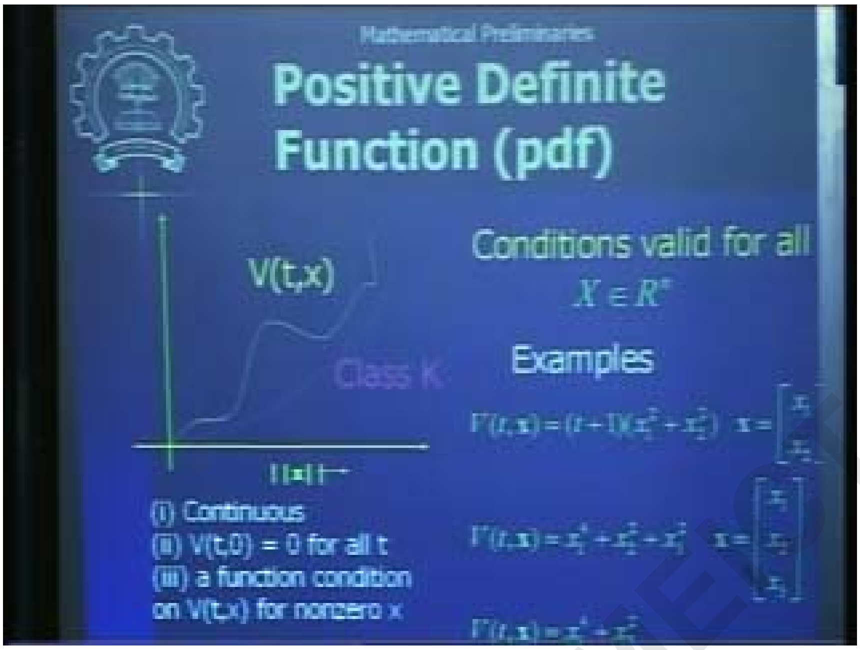
now we have the similar definition function V taking some value in R plus domain or positive of the line and some value in this Rn vector space to R this is the scalar valued function so you can call in other words V as the scalar valued function as the scalar and the vector ok

so this is said to be positive definite function or just pdf if it is continuous again the same condition V of t comma zero vector here you notice that zero is the vector zero vector it gives us scalar zero as the output of this function for all t values ok and then there exists this function alpha of class k again the same function we where is satisfy now this condition and this condition also again the same condition but it is holding for all x belonging to Rn it is not restricted to ball of radius r now ok

now if everywhere or whole of the Rn is included or in the other words like in the small r in the previous definition is checking value infinity ok

so it is everywhere now this function is positive so that is how this definition of the positive definite function is so you can have now here in our same picture we

(refer slide time 35:01)



can see that this V will never come below this class k function ok it will always be above that function since the condition is valid for all X belonging to Rn ok

now let us see some examples that will clarify this matter little bit more ok so we have first function let us see whether now you can do any analysis to tell me if it is a positive definite what we have to do is to verify this three condition the first condition is it continuous for this part like this V one it is continuous ok second condition if we substitute zero for x one equal to x one and x two x one equals to zero x two equals to zero this going to be give you zero and no matter what the t value is it is always going to be zero so second condition also satisfies and the third condition is whether it satisfies or not this this function itself is not a class k function ok

it is always positive no matter what so for x not equal to zero it is positive and strictly increasing ok for more higher and higher values of x the function will take higher and higher values ok

it is strictly increasing function so this is indeed positive definite function ok and this is the positive definite not the locally positive definite function because this is valid whatever condition we have seen valid for all x ok

so now let us come to this function here we have x two square x three square and x one and x four ok now similar to this part this part is positive definite now this part x one to x four is it positive definite or not it is right we can see that so then this x here is given as x one x two x three ok this x vector is having all these x one to x three terms and this function is also having x one x two and x three terms ok

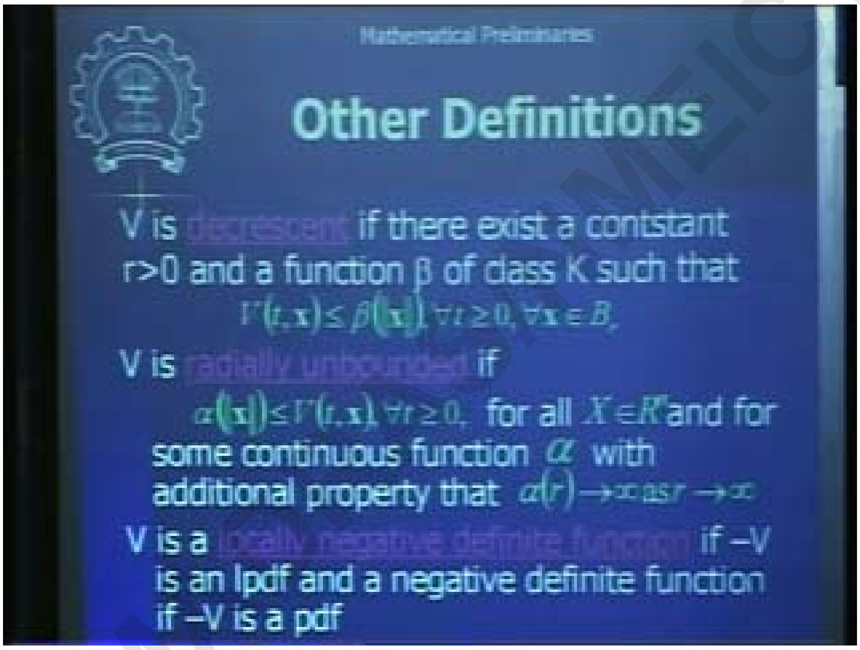
i will tell you why I am telling specifically in this case so we have this result for this function again that it is a positive definite function now let us come to this function where are x is specified as these ok

so it contains x one it contains x two but it is not containing x three so you see which condition is now gets violated here second condition gets violated ok V of t comma zero will be zero but it will not be that zero if we have x other than zero till we have need to be zero which is not valid ok so far any value of this norm of x somewhere over here also in this diagram you can have Vt to be equal to zero because if we have some some condn some vector like this zero x two equal to zero x three equal to some value then the norm will turn out to be some value over here and the function V is zero

which is violating the condition on the function condition on t it is the third condition ok so the third condition will get violated not the second ok this is not positive definite function ok you see that it is very important understanding very important like you should never never forget this understanding we will find at your lyapunov function then you do the analysis you will use this pdf function for lyapunov theory analysis

so we will see at that time also I will tell you but this is the very important understanding for then the function does not contain some terms ok then it can be a pdf function ok

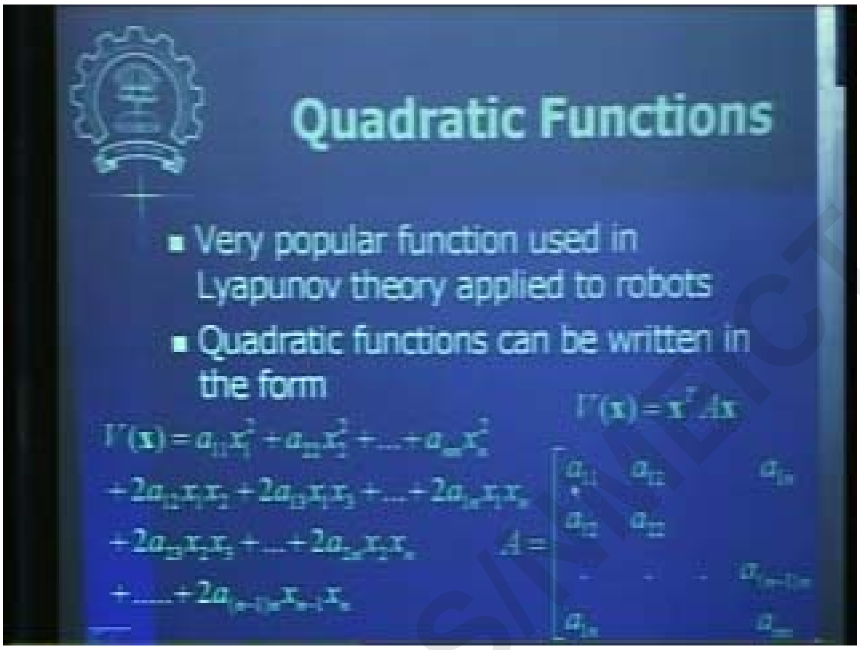
so it should contain all the component of your vector x ok if one of the component of the vector x is missing ok then the function can not be a pdf function ok you see that um now let us see some other definitions (refer slide time 39:59)



see the first definition is something called decrescent the function is said to be decrescent if there exist a constant r greater than zero and function beta of class k such that V is now less than equal to is beta you remember the previous condition of the alpha function V was greater than or equal to alpha now this V is less than or equal to beta function of class k and again now this is either local condition or it can be a global condition if you restrict this x to ball of radius r then it will become a local condition and if you don't restrict this x which is globally decrescent or locally descrescent you can say that now function b will be radially unbounded if again our condition alpha like alpha condition is satisfy ok V is less than or equal to V is greater tan or equal to this alpha of class k and this condition is higher for all the domain that we are talking about globally it will be valid for all x belonging to Rn the condition to be valid and then for some continuous condition alpha with additional property that alpha r tends to infinity as r tends to infinity ok so the function alpha is to go on increasing as r go on increasing till infinity ok

so that's how it is called radially unbounded function this is the radially unbounded function now another definition v s locally negative definite function if minus p is lpdf function and a negative definite function if minus v is a pdf function ok so locally negative definite function again locally or globally follow just negative of pdf ok

so this is other definitions so let us we will see later how we use this definition for our analysis purpose but just have patience to see this definition and understand them carefully ok later on we will use them robot [{42:47}] ok (refer slide time 44:43)



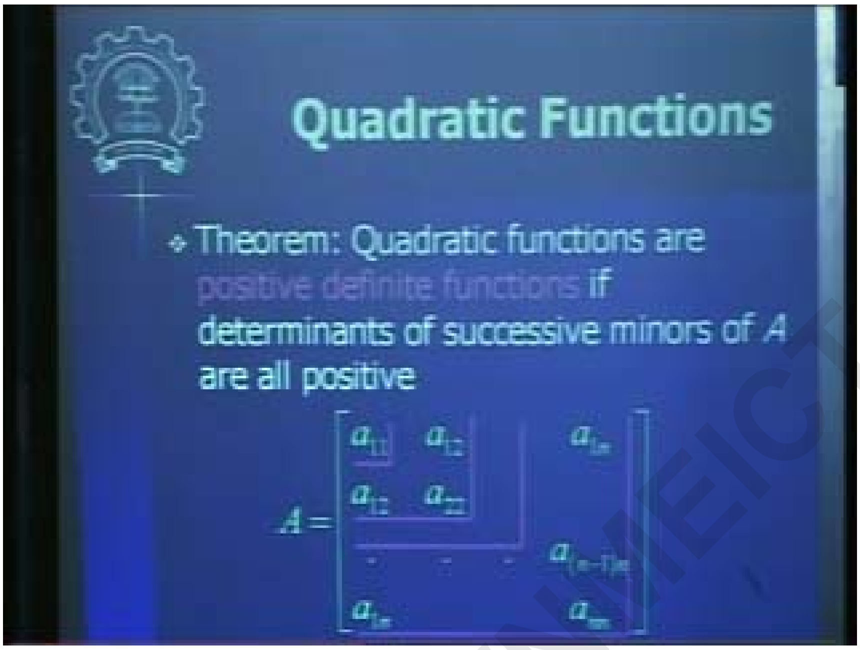
now this quadratic function let us study little bit more about this quadratic functions why we have to study them because they are very popular functions used in this lyapunovs theory ok especially theory applied to the robotic kind of application ok this you will find at this function quadratic lyapunov functions they are very popular ok

and then now let us see what is the quadratic function is the quadratic function can be written in this form you understand this form you have power or degree only two here not more than two ok the polynomial kind of functions and then you have these terms coming crossed coupled way then square way ok

so like this it can be represented as matrix form in this fashion you see that remember your x is the vector or your state ok x will consists of x one x two x three up to x n component where n is the typically degree of freedom number of degrees of the freedom or number of degrees of state equations number of state equations what ever

so x has a n component ok x belongs to Rn state seeking in pure mathematical form and then this A is the matrix given by this expression so A is a symmetric matrix you can see that the element a on two is same as the element a two one ok so because of these these term two a one two x one x two come here ok given given this kind of a matrix you should be able to write this quadratic form equation or given any kind of a quadratic expression we should be able to convert it into this form x transpose A x so it is very easy to do that then you need to look at the co efficient multiplied at the x one square x two square and coefficients multiplying x one x two x two x three and x one x three these kind of terms and then like you divide these terms with these co efficient of cross terms by two and put them in a appropriate place in this matrix then you construct this matrix A ok this how you go up on this ok now next we will see how like suppose we want this quadratic function be a positive definite function what are the condition ok

so this is the theorem we will not see the proof of the theorem I just tell you the theorem and explain you the theorem (refer slide time 46:19)

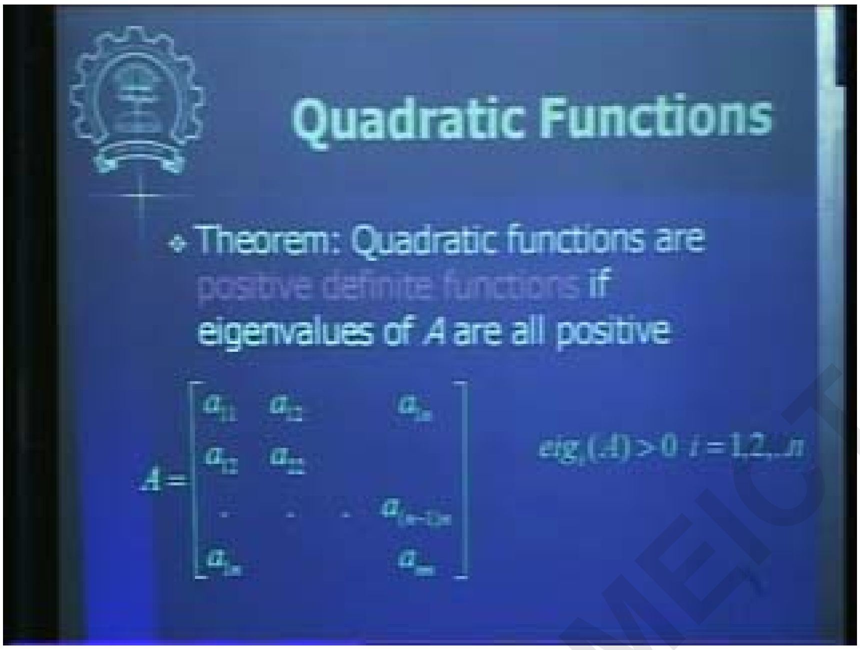


quadratic functions are positive definite functions if determinants of successive minors of A are all positive ok so you have this matrix A you take this first minor ok it will be formed by only the first element a one one determinant of it is just the magnitude of this element ok it should be positive so this element should be positive not the magnitude is the determinant is the element is that ok

now lets take the next minor ok it looks something like this slot now you take the determinant of the a one one a one two a one one multiplied by a two two minus a one two square ok you check whether that is positive like that all you find the successive minor and see whether they are positive if they are all positive then it is guaranteed that your quadratic equations function form is the positive definite form ok

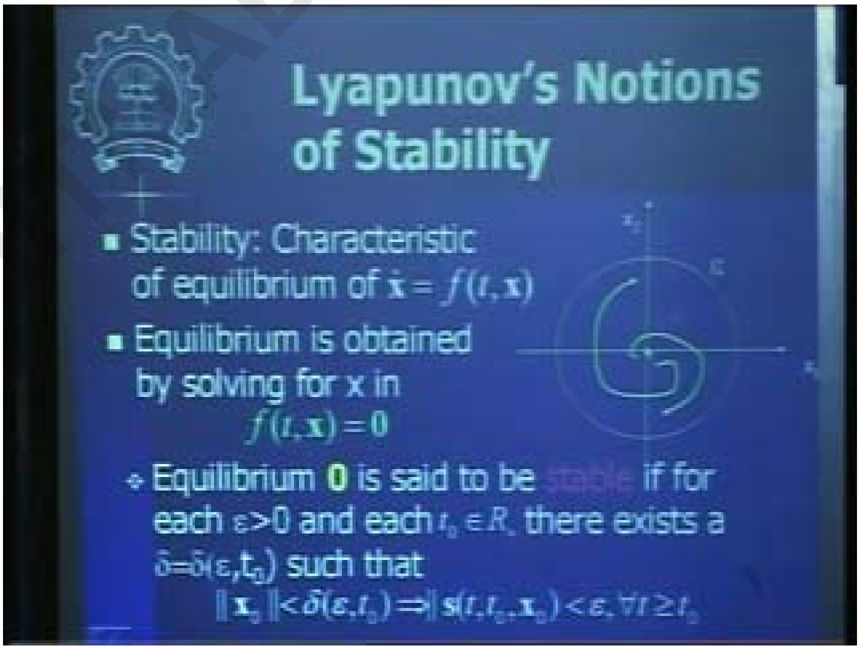
so this is how you verify whether the function is positive definite or not ok there is other condition also for verifying that the function is positive definite function or not based on the same matrix it will it is in terms of the eigenvalues you can see this theorem

(refer slide time 48:03)



this quadratic functions are the positive definite functions if the eigenvalues of A are all positive ok so you take this matrix A find it eigenvalues and see whether they are positive and you can conclude about the positive definite form of the quadratic function which is formed by using this A matrix ok so this is the important understanding about the quadratic function being positive definite

now let us see some notions of stability in the lyapunovs sense lyapunov has defined has different notions for stability and let us understand try to understand couple of them so first of all now (refer slide time 54:27)



stability in lyapunovs analysis is the characteristic of the equilibrium ok in general the system x dot equals to fx ok this is the like non linear system here we represent it in this form ok if you have x double dot coming into your any of your equations you can convert them by using the state of transformation ok you introduce one more state x dot equals to say y and then you transform it into this kind of a form ok so that you can represent that system as first order non linear differential system of first order differential equation ok

this is possible for all the robotic manipulator exist we can be put in this form ok now the equilibrium of the system will be obtained by solving for x when you substitute of t comma x this term equals to zero ok

so this is the now zero vector this is the vector valued function and this is the vector so your x is a vector so substituting this equal to zero all the equilibrium of the system there can be in general many equations of the given system ok then now you can do some transformation ok to make the equilibrium like say you say transform the variable so that your system equilibrium is zero always so our assumption will be for this definition will be that your system has the equilibrium at a zero position ok it just like you have too shift the position of your x one x two up to x n to appropriate place then you will have the zero as the equilibrium for this case ok

it is always possible to do that transformation so it is it is possible to do like the transformation for each each of the equilibrium ok and when we talk about stability we talk about one of such equilibrium like for each every equilibrium there will be some kind of different stability possible from lyapunovs analysis ok

so you should remember that equilibrium like the stability is the characteristics of the equilibrium of the system so that you should remember ok I repeat again if the stability is characteristic of the equilibrium of the system ok now this is the definition for the stability in the lyapunavos sense so equilibrium zero of system x dot equals f of x t is said too be stable if for each epsilon greater than zero and t zero belongs to R plus there exists delta which is the function of epsilon and t zero such that when ever you start x zero is your starting position ok

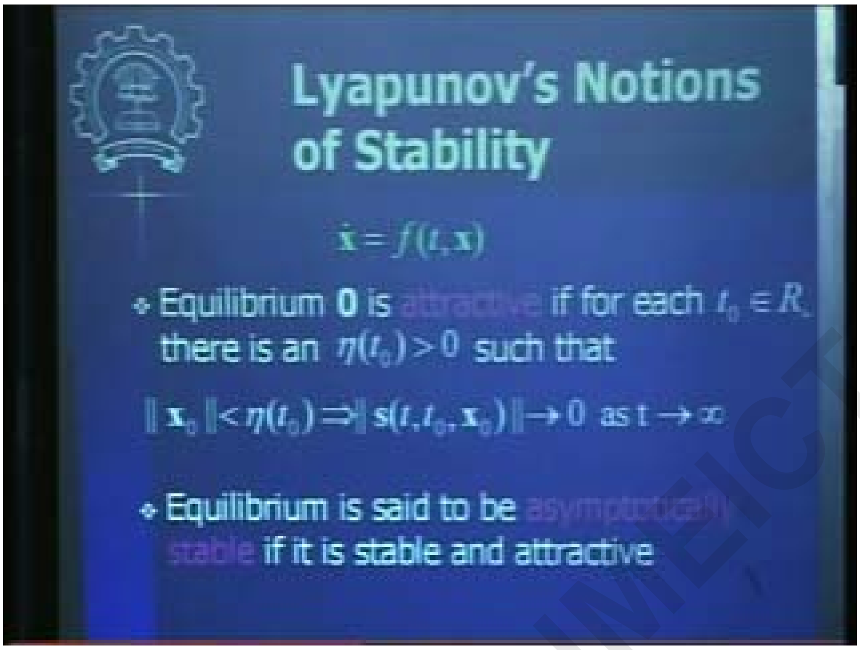
you consider any differential equation it will evolve from given this is the differential equation system ok your n ordinary differential equation ok so if I specify x zero and this equation for different values of t we will be able to find out how the system is evolving right ok

so given x zero and this differential equation you will be able to find out solving this differential equation how this system is evolving so when x zero initial state in this ball of delta radius then it is guaranteed your solution to this differential equation then the norm of the solution will always less than epsilon ok

so that is the definition of the stability in the lyapunovs sense now let us see geometrically for say R two space ok how this will look like we have this R two space we have only x one and x two here ok

so here you are given epsilon some epsilon number so it will transform into like suppose we want to say the norm of the x is less than the norm of some something is less than epsilon norm of this x x one and x two is less than epsilon then it will then your x lies on within this circle ok defined by the circle ok

now given this epsilon circle or epsilon ball or epsilon hypo sphere in general then we should able to find another circle delta such that whenever your trajectory is starting in delta circle they should not what ever the evolution is ok it is revolving like this and it will go on continuing but it will never leave the ball of [{54:07}] epsilon if it is leaving then in the in the sense of lyapunov it is not stable but if it is not leaving then it is a stable kind of thing and then that is valid for all such points in this data neighborhood of the origin so in the other words it is meant that if your system is at equilibrium and you prod up the system from equilibrium little bit it will not go wide ok it will contain itself to some limits ok that is kind of the notion stability (refer slide time 54:49)



now lets see what is the next notion we have to define first first attractiveness to see what is asymptotic stability

so we will see mainly this two functions like stability and asymptotic stability and for asymptotic stability first we have to understand what is attractive equilibrium so equilibrium zero is said to be attractive if for each t zero which is belonging to this R plus there is this neta function of t zero such that when the ball when x zero starts in the ball of radius neta ok r or this number neta then whatever solution is generated is tending to zero as t tends to infinity ok

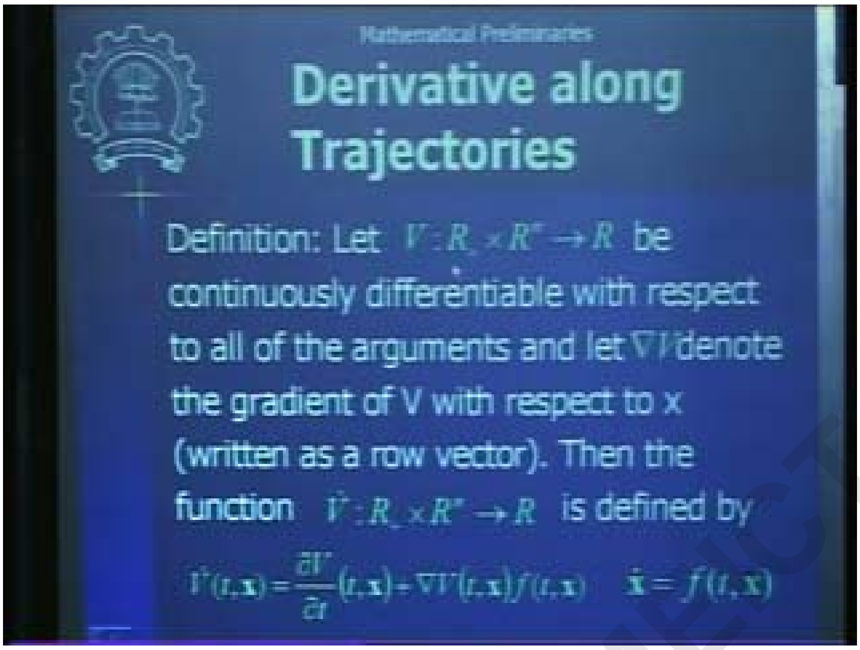
so any solution if it tends to zero as t tends to infinity ok then that particular equilibrium it is called an attractive equilibrium ok

so if you are in the neighborhood of this equilibrium if you know cut of your system from this equilibrium and the system behaves in the way such that your solution of the system norm of the solution goes to zero um goes to zero as t tends to infinity then that particular equilibrium is called attractive so it is attracting all the trajectory where ever you start in the neighborhood all the trajectories attracted to the equilibrium so that it is called attractive kind of a equilibrium ok

now if the equilibrium is combination of these two it is stable and it is attractive then the system is called or the equilibrium is called asymptotically stable equilibrium ok

so that is the asymptotic stability notion ok

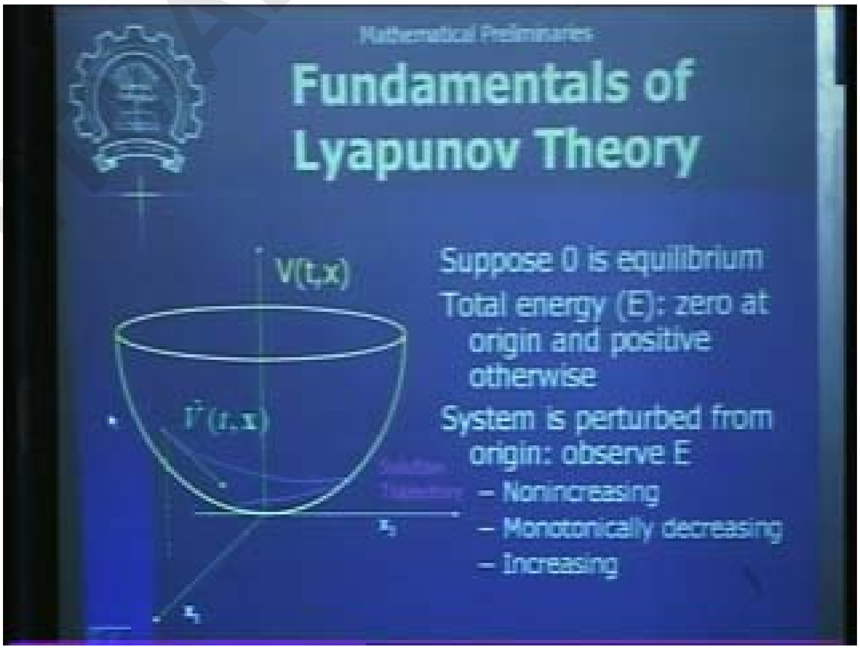
(refer slide time 57:02)



now next definition is derivative along trajectories this is pretty simple

so we have this V again same as of the previous V and then it is continuously differentiable with respect to all its arguments these arguments are this time t and x belonging to R n vector space then its derivative is defined by these so this is the standard derivative only difference here is that we have used her del V multiplied by x dot where x dot equals to f of x t so when you substitute this x dot equals to f which is corresponding to your differential equation systems then you are getting the derivatives along the trajectories ok

so geometrically we can explain this in the following way (refer slide time 59:57)



you can look at the figure so you have this V say in the x R two space we are representing here so we have this V as the function of x one and x two so it will be in general say say some positive definite function and then you plot the solution trajectory the solution trajectory will be some function of x one and x two ok in x one and x two space it will be some curve so in this plane consist of x one and x two axis some curve now this curve is projected on this V will look something of this slot vision trajectory ok there is no time axis representing in this kind of geometrical expression ok we are not giving importance to time representative this is the representation only in the x one x two plane with V plotted on the g axis ok

so now the slope of this line ok slope of these solution trajectory which is predicted on V at a particular point we will give you v dot at a particular point ok that is the physically interpretation of the v dot derivative along the trajectory derivative of the v along the trajectory ok

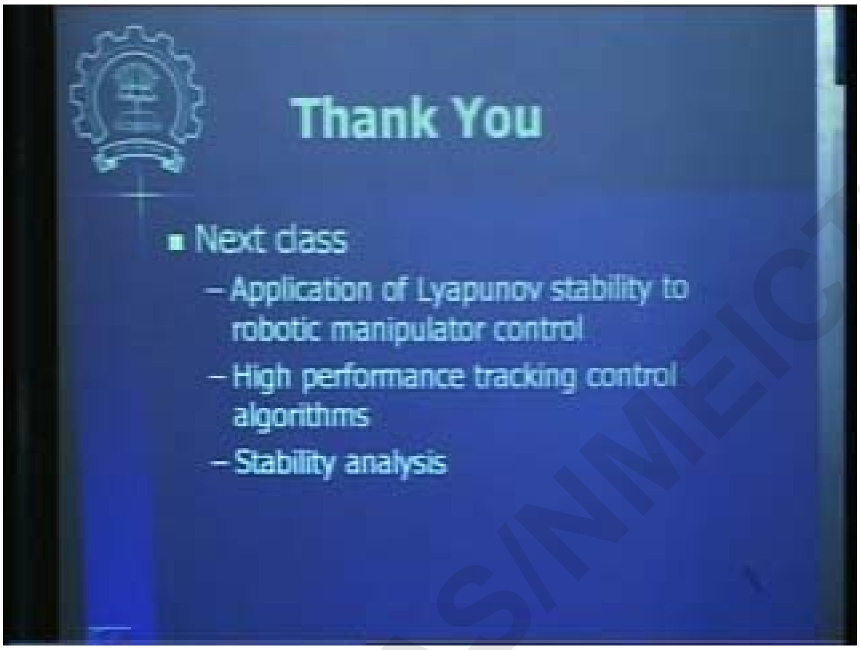
in general the derivative of this at this point wil be along x one direction and x two direction at when you substitute your equation this part is equal to f of x that the derivative come along this along the direction of the solution plane ok

now this is how I can explain the question that has come that is how this can help us in doing the analysis so suppose 0 is the equilibrium and your total energy is zero at the origin and positive at other places that is the positive definite function and then you perturbed the system from origin and observe this energy function ok or the positive definite function now depending upon these three possibilities of function you can have the different conclusion about the stability of the system ok

so we will see this things again in the tomorrows class but we will stop here right now so you just close this other thing we will se tomorrow class we will stop here right now so just summarize the thing (refer slide time 1:00:54)

QuickTime™ and a
decompressor
are needed to see this picture.

we have seen the trajectory tracking kind of a control algorithms which have in which we included feed forward and computed torque algorithm then we start some mathematical preliminaries for lyapunov stability and tomorrow (refer slide time 01:01:23)



we will see the same mathematical preliminary and then how they are applied in your robotic manipulator ok so

so we will generate some high performance control algorithm based on this theory then see that performance in the tomorrows class ok