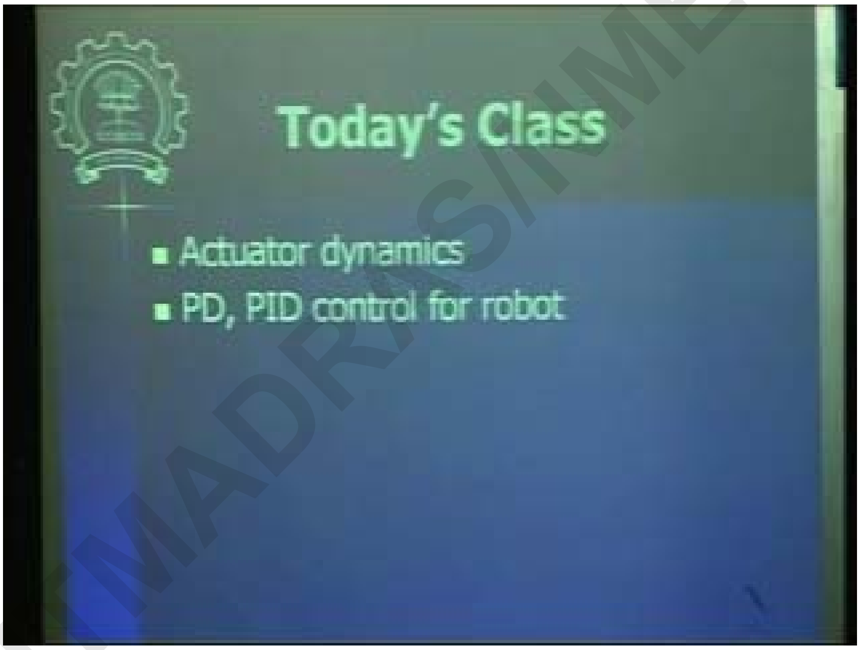
ROBOTICS
Prof. P.S.Gandhi
Department of Mechanical Engineering
IIT Bombay
Lecture No-33
Robot Dynamics and Control

Good Morning today we start lecture number three of the module robot dynamics and control let us see what we studied in the last class we have seen in the last class the dynamics of robot manipulator a general case for n link manipulator what we saw basically the procedure how to how do we get dynamics of the n link manipulator ok

we saw we have to first do kinematic analysis and then based on the kinetic energy and potential energy of the manipulator we use Lagrange formulation to get the dynamics of the manipulator then we had look at introduction to control ok some basic fundamentals of the control we reviewed ok in terms of control implementation what are the different types of controllers available and thing like that ok in today's class we are going to study (refer slide time 02:17)



Actuator dynamics ok how to integrate that actuator dynamics with the dynamics of the control dynamics of the robot manipulator so that integration part will we see and then we will use our understanding about controls to develop some simple control algorithms for basically point to point movement of the robot ok that is what we are going to do in the today's class ok

let us first start with the actuators now what are the different kinds of the actuators that you know for the robot there are motors electrical domain actuators then in the hydraulic domain we have hydraulic actuators ok these are the most popular actuators for the robot ok so

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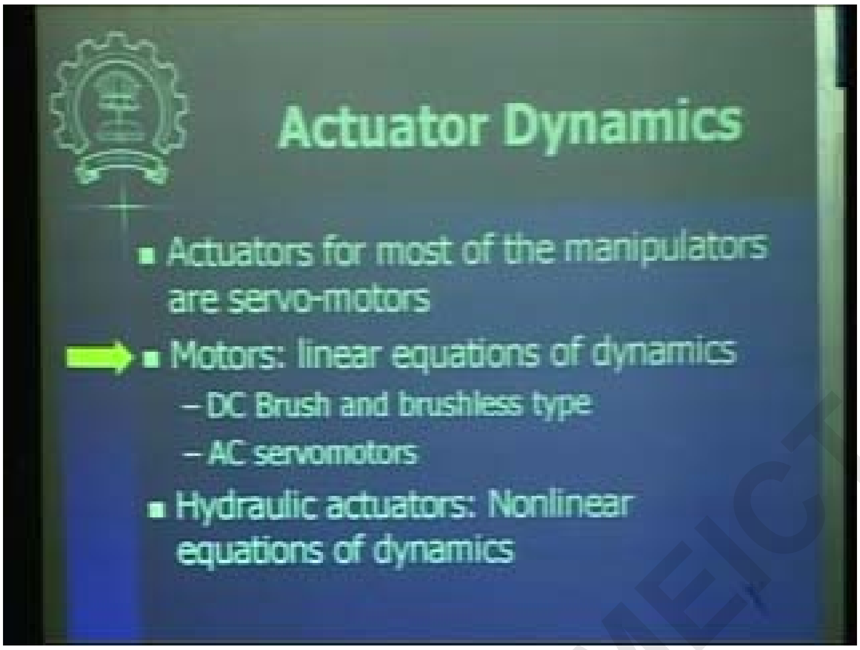
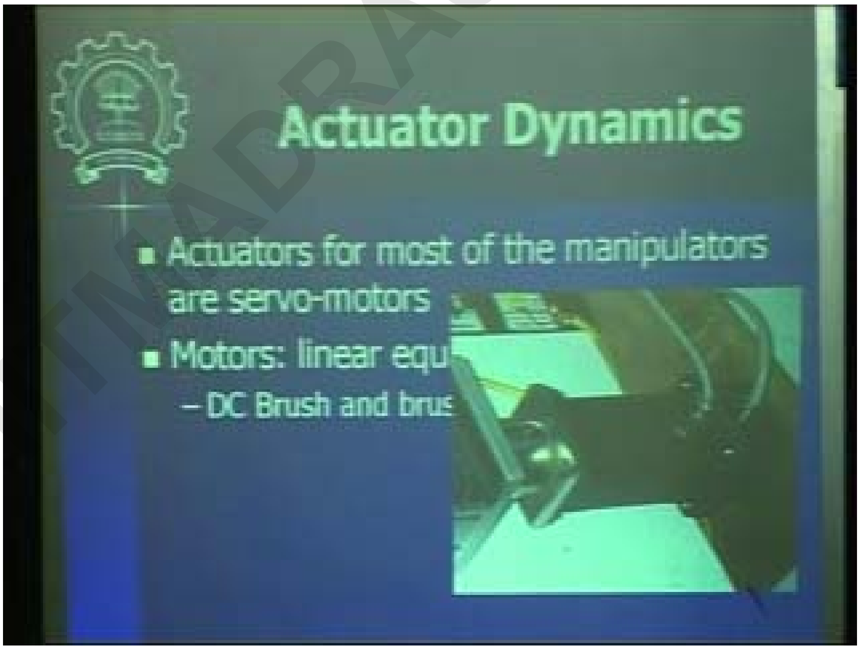
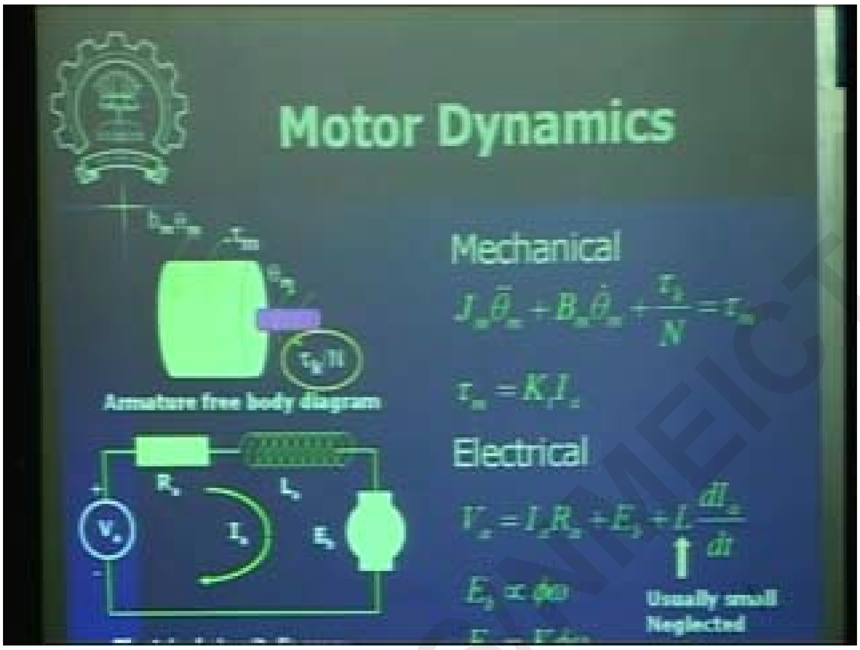


now in most of the robots like the motors or actuators that are used are servo actuators ok they can be hydraulic servo or they can be electrical servo motor ok now in terms of motors there are different types of motors possible one is DC brush and brushless type of motors so Dc motors DC motors is one class and AC motors is the another class now here is an example (refer slide time 03:43)



of one of the motors in our robotic labs at IIT Bombay then there are AC servo motors possible so depending upon the application like different motors can be used ok then you have the hydraulic actuators and then for hydraulic actuators you know the equations of dynamics they are non linear so we we will not get into the get into the dynamics of hydraulic actuators will first see today the dynamics of the motor which are most popular in many robotic applications the hydraulic actuators are used in the cases where you need very high power robot ok or else you need very high performance high speed in the case of the robot

so that is the place where you use hydraulic actuators so will not get into the dynamics of the hydraulic actuators in the interest of time today but we will see the see in detail the dynamics of the motor ok so let us come to the motor dynamics there are mainly two parts for the motor dynamics (refer slide time 07:10)



one is mechanical part of it and electrical part of it ok now we see this is the free body diagram of the armature of the motor ok you see there are terms which are damping in the motor then this is the electrical torque generated in the motor then the this is motor motion theta m this term tow k oven N is the tem which is the contribution from the robot dynamics ok where N is the gear reduction ratio now if you do the balance torque balance from the armature we get these are the mechanical domain equations form the motor dynamics so so the contribution from the robot manipulator comes over here ok

and this is the motor torque generated which is proportional to the current which is flowing in the motor now let us come to the electrical part of the motor dynamics so we have these resistance of armature inductance of armature and then this is the motor which generates the back e.m.f corresponding to the speed so here if we do know the circuit voltage balance we will get these as the equations of electrical part of the motor so your voltage given are supplied to the armature is balanced by IaRa this is the heat loss then back e.m.f and conductance inductance of the motor ok

this back e.m.f is proportional to flux and speed ok if this omega is nothing but theta m dot is the motor speed then this back emf will be given as k times flux into theta now usually this inductance term is very small hence we can neglect that term so with that our equation will get reduced to this form now from here

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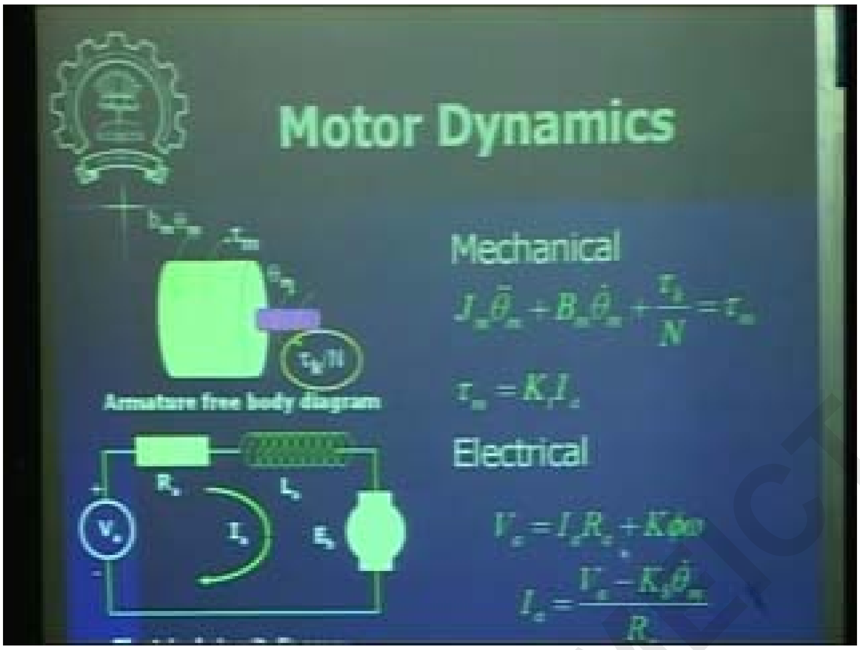

we can find out this current value i to be equal to Va minus this back emf now usually this flux is constant flux ok this flux is basically the property of the field field of the motor so in field like you have a constant voltage or in the permanent magnet motor this field flux is constant ok so because of that you can observe this constant into this constant into new constant Kb which is corresponding to the back emf and then multiplied with theta m dot ok so this is how your current will look like now this current you can substituting here and then these whatever torque you get you substitute in these equation then you will get this following equations of the motor dynamics so you have

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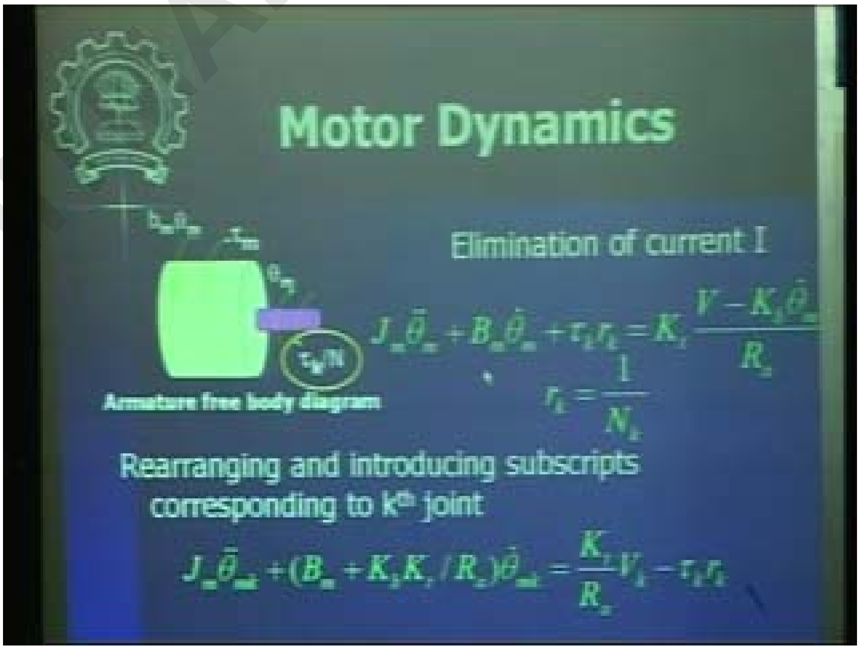
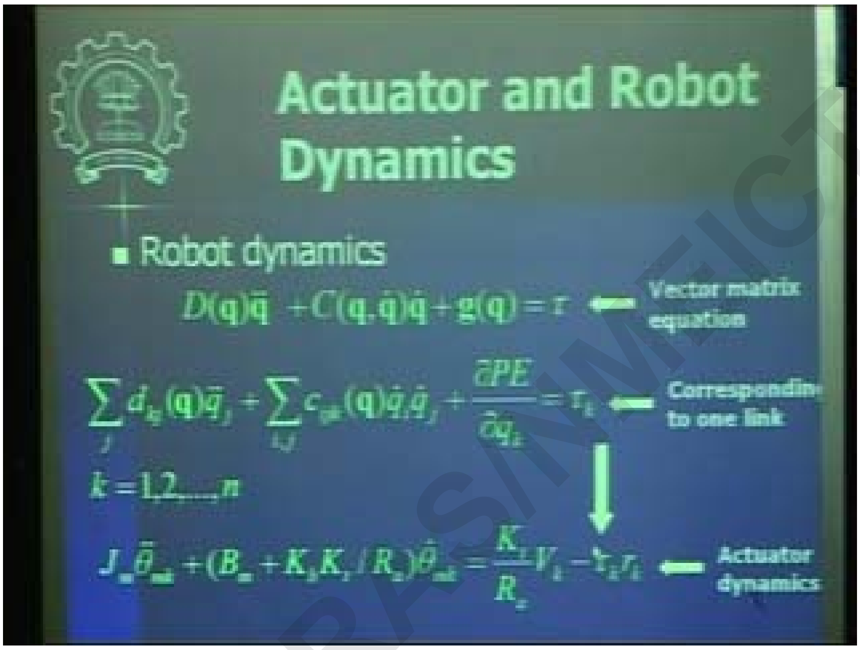


now both mechanical part and electrical part of the dynamics coupled together in this equations ok this rk is the new term introduced corresponding to the gear reduction ratio and then this rk is given by one over Nk ok now we can rearrange these equations and introduce these subscripts for the case joint for robot manipulator to get these equations of motions ok you observe that these terms are kind of effective damping terms ok then this is your input which is something which you can control or you can command ok so this were your actual robot command is going input so now how we can combined this equations into robot dynamic equations ok so look at the robot dynamic equations what we had seen they look like of these form (refer slide time 11:24)



D q double dot plus C q dot plus g q is equal to tow ok now this is the vector equation ok this is the vector matrix equation these are the matrices D C and this g tow they are vectors in this these two r vectors q double dot and q dot now this is the corresponding form for the one of the joint or one of the links of the robot ok you remember these form we derived by using Lagrange formulation basically kinetic energy potential energy of the robot

so this is the standard form of the robot dynamic equations ok now we have to combined these with these motor dynamics these are the equations of the motor dynamics as we have just derived ok this is the actuator dynamics now these can be like equations of other actuator also what ever actuator is there in your particular robot that you are considering so this can be like hydraulic actuator equation also ok but but the way in which like the robot dynamics and the actuator dynamics they combined that remains the same ok so you just understand the way in which these two are combined together ok so now what we have to do is we have to substitute this tow k from robot dynamics into the actuator dynamics and see what we get

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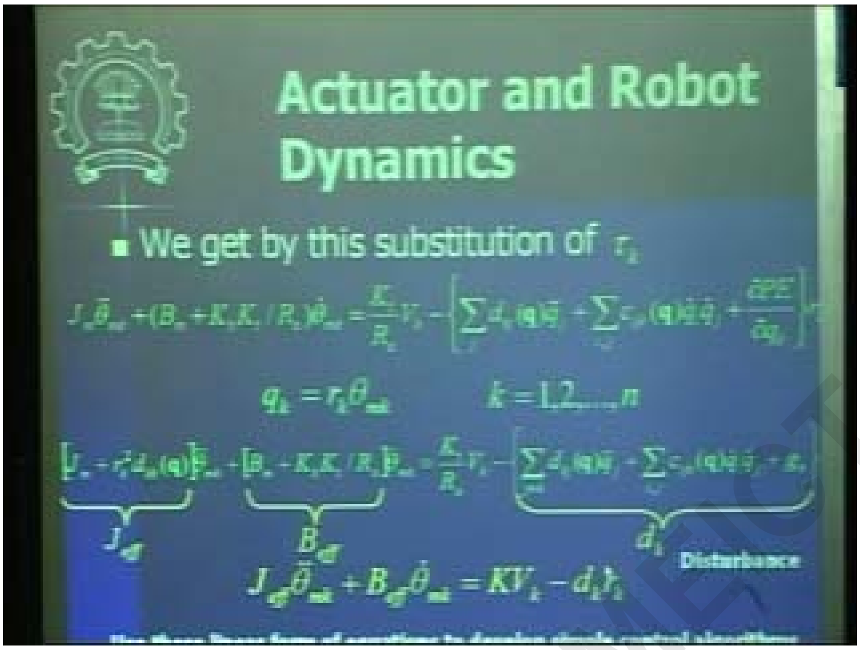
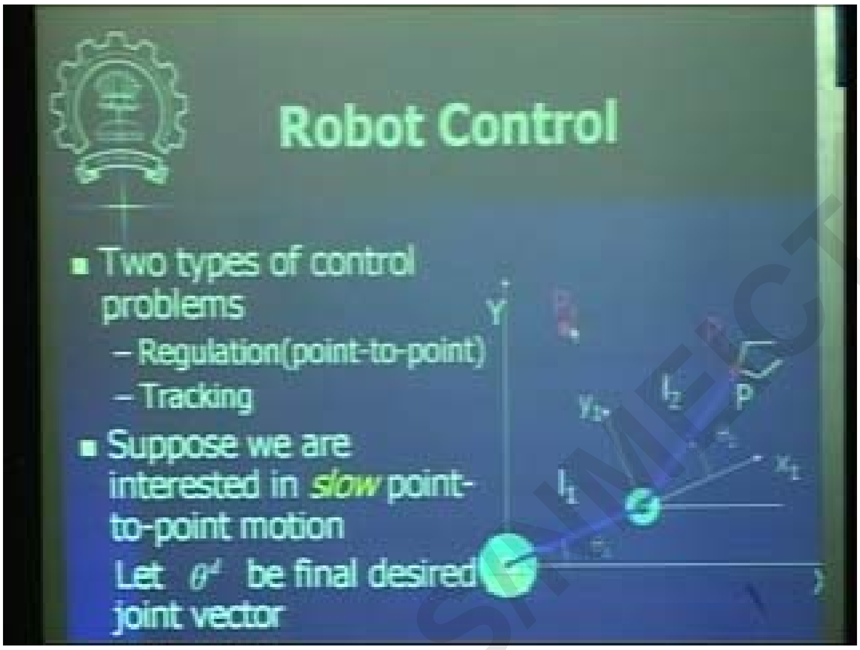


by this substitution we will get these equation where now these terms are remaining the same up to this point this input is also remaining the same only in place of tow k which was kind of low torque on the motor now the motor load is basically because of these all these terms in the robot dynamics ok physically you can interpret that this is the load coming because of the dynamics actuators dynamics of the robot link ok

now we can have we can make use of this relation your generalized co ordinate of the robot is given by gear reduction multiplied by actuators rotation ok so if you make use of these and convert everything into theta mk equations we get something like this now here you observe that what we have done here is we have taken the term corresponding to k equal to j out of these equation these part of the equation robot dynamics equation and then use this formula and plot over these term over here ok

so you see this rk square into dkk of q is the additional terms to the inertia of the motor you see this will come to this track why we have done this thing but this is this is the mathematical manipulation we have done to get some simplification now what remains here is the summation for j is not equal to k

ok now let us see we call this inertia as a effective inertia we call these matrix term this b matrix or b damping term would be equal to b effective ok remember we are doing this for a single link of the robot so these these can be done for each of the links ok we have to do this for all the links like these then you can develop the control so next thing we do is we have this dk term substituted for all these non linear term ok these dk is nothing but the disturbance you can imagine this as the disturbance on the on this system now is you observe that this system is getting reduce to linear system with these disturbance ok we will question that these these term here has been the non linearity in the terms of q ok so as your robot is evolving ok this term inertia term is changing ok but but we can actually to be some constant inertia term ok for some average value of the inertia ok or whatever range your robot is applied ok so finally we will get these equations governing the actuators and robot dynamics both combined together so see this k term is get your array and these type of equations will get for each of the robot link and now we will just focus on one link and develop the control for that and entire procedure remains the same for all the other ok so we will do it for only one joint hence for ok now let us see first couple of control strategies which are very simple which are very intuitive juristic kind of control strategy ok (refer slide time 18:17)



actually we have two types of problems in in all the robot manipulator first is the regulation problem and second is the tracking problem we are seen this before but in the regulation problem to move from one point to another point you don't fear how do we move like or what is the trajectory trace by the end defector moving from one point to another point but you have to just go from one point to other point ok that is the regulation problem or it is called point to point control problem ok another type of problem is a tracking problem ok where you have to track this trajectory you have to go from starting point to the end point in a specified manner ok in these particular case we have seen the straight line trajectory but it can be in general any trajectory it can be circle it can be parabola it can be any any thing ok and now what we are considering is that we are interested in slow point to point motions ok we want to move to the final point in a very slow manner ok

and then your term theta d as the final desired joint vector now you know by using inverse kinematics you can find always the final joint positions corresponding to final end defector position ok by using inverse kinematics ok so you have to do that before you proceed for development of control ok so this theta d is basically final desired joint vector ok

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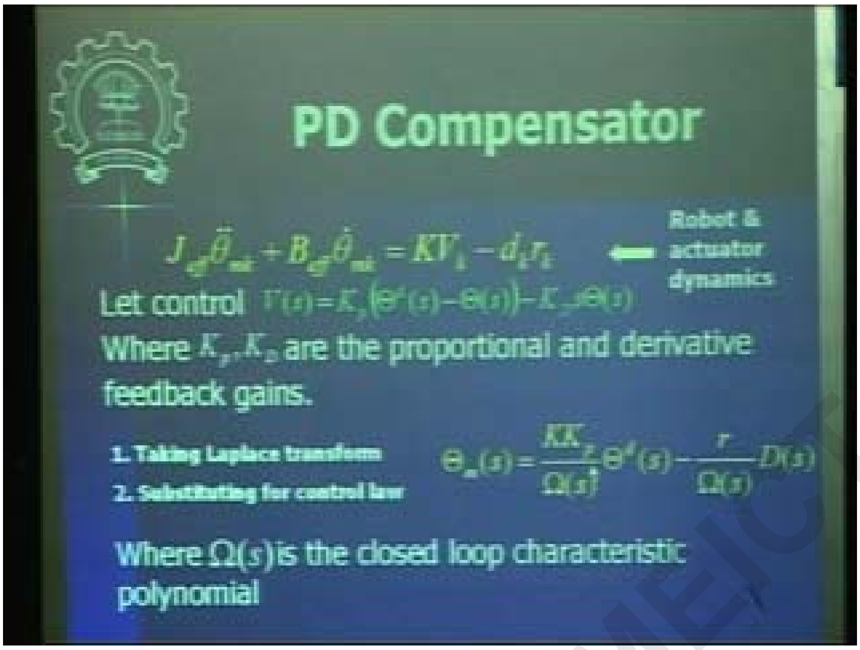

now let us see how we design control so recall that our robot actuator robot and actuator dynamics equation combined together looked something of this form ok now this is the most simple kind of a strategy one can think about

you have these desired position you subtract from the actual position and then that is what you get as the error you multiplied this error with some proportional constants ok that plus similar thing if you done here are velocity the final desired velocity is zero you subtracted the actual velocity of the motor s theta s the actual velocity of the manipulator or motor joint and then we have used it as the another term in your control expression so these called the proportional and derivative feedback so in simple term it is called PD compensator

ok with these now we will see how we can analyze our original system so we have to take the laplace transform of the original system then substitute for this control law in the expression then will get these kind of expression for motor dynamics motor dynamics revolves according to this expression now recall that your d term has lot of non linearity ok it contains lot of non linear terms so these part is acting as the disturbance part and this part of the equation is governing theta m kind of a transfer function between theta m actual motor position and the desired motor position ok so you see that so actual motor position will not be equal to the desired motor position because of these terms because of the disturbance it may not be equal to but when the disturbance is equal to zero you will have actual terms matching with the actual position of the motor matching with the desired position ok now these omega s is the characteristic equation this is given by this characteristic polynomial this characteristic polynomial is dependent on j effective b effective plus this k factor coming because of the control now ok now these whole thing can be seen we can represent this in terms of a block diagram in the following slide ok

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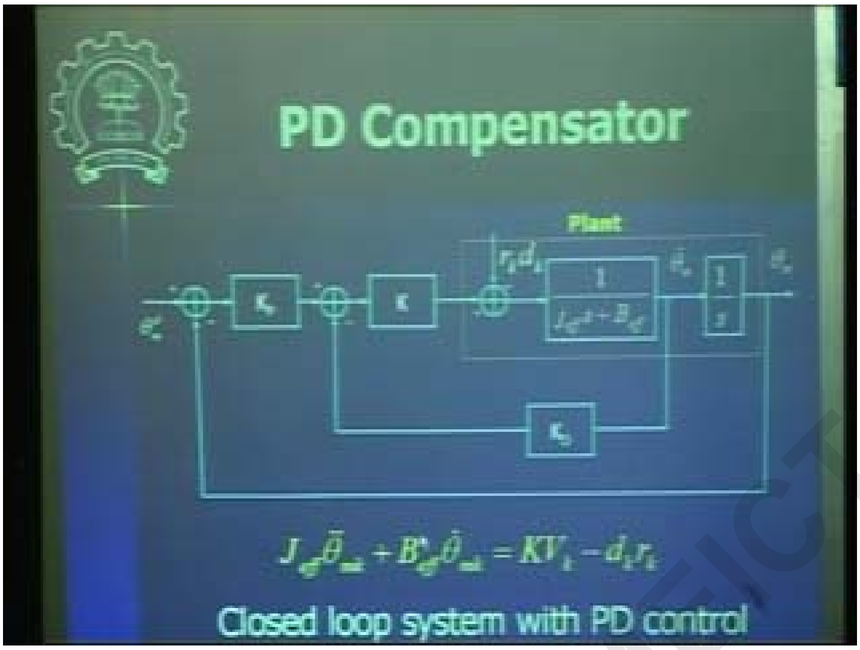
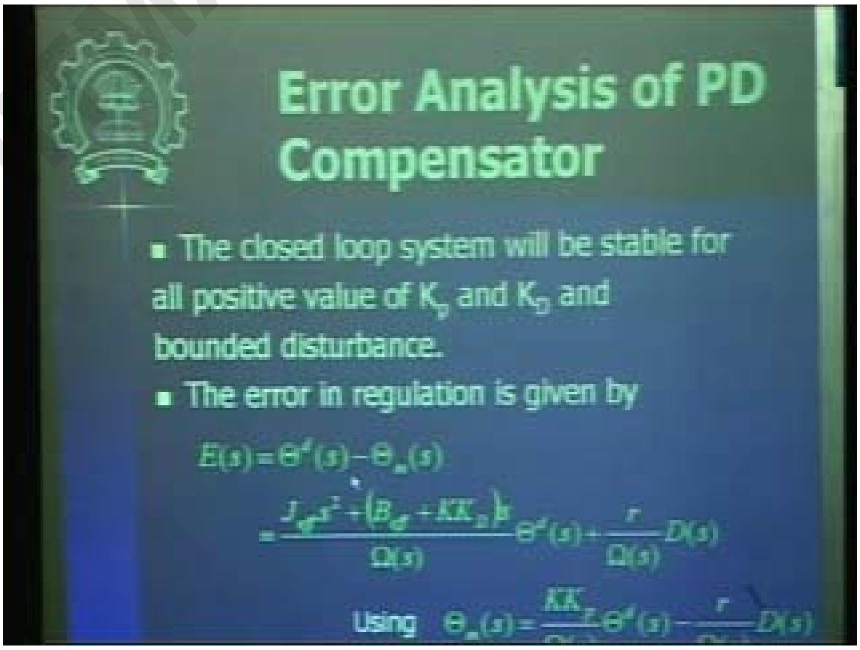


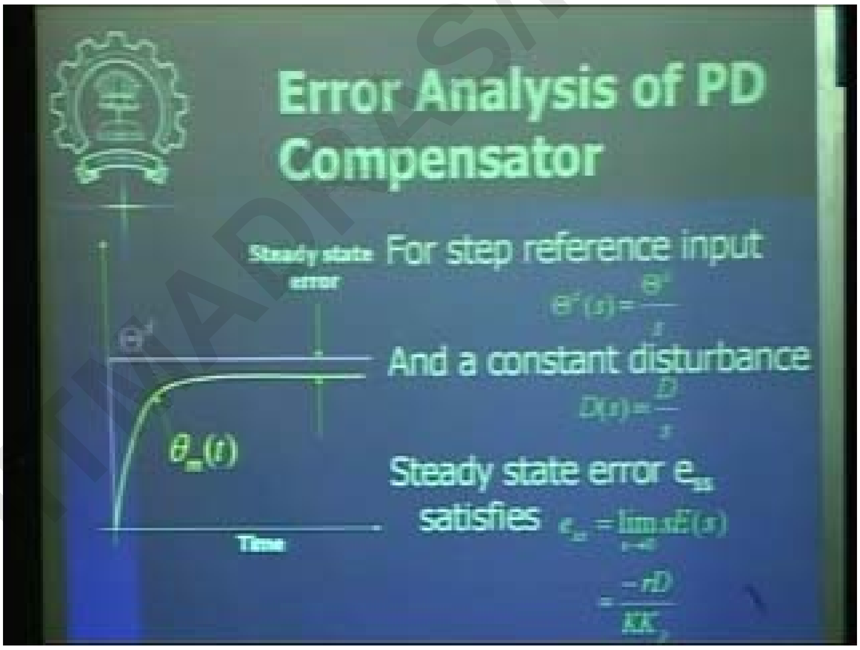
this is the block diagram you can see here this is your plant or this is your robot manipulator ok it has the disturbance term coming in here and then these part is like a linear part of the equation this is the equation this linear part is represented over here by these block and next integral block ok here the output is theta dot m then you integrate it to get theta m now you can see that kd gain and Kp gain they are constructing your control law Bk PD control law in this fashion like you take theta dot m multiplied with kd ok add that into the error theta m minus theta d m desired function that error multiplied it with Kp you add to the derivative part ok so this is output here is PD control part ok then that multiplied by this K will give you term over here that term from that term you subtract this disturbance term and then you generate this part ok that is how this block diagram is generated you see that ok now let us do some kind of the analysis (refer slide time 26:17)



with whatever we have developed so far ok now if you use some theory about linear systems transfer functions basic fundamentals of automatic control when we find that this closed loop system which is form because of the combined motor and robot dynamics with PD control

ok that is your system now ok you see important thing to note that for different types of controllers we will have these different closed loop systems ok you have to do separate analysis for each one of them or all the controllers it is not like you will get a same equation or different controllers will get different equations and then different systems and you have to basically do analysis for each of the system separately ok

so you see for this particular case when you are having PD control now if you see the error in the regulation it is given by these expression so now you substitute for theta d from the previous expressions or you can use the previous block diagram ok to see how the error at this point will look like ok and then you will get this expression ok do you see that beside this characteristics polynomial omega and the term corresponding to this part is show about here now ok so this is here error expression in the s domain ok so now let us say if you are interested in what is the steady state error what happens in the steady state like we we allow enough time for the manipulator to take to go from initial position to final position and what is now the final position steady state whether there is error in the steady state final position of steady state that is called we are interested in so let us see the steady state analysis (refer slide time 27:49)



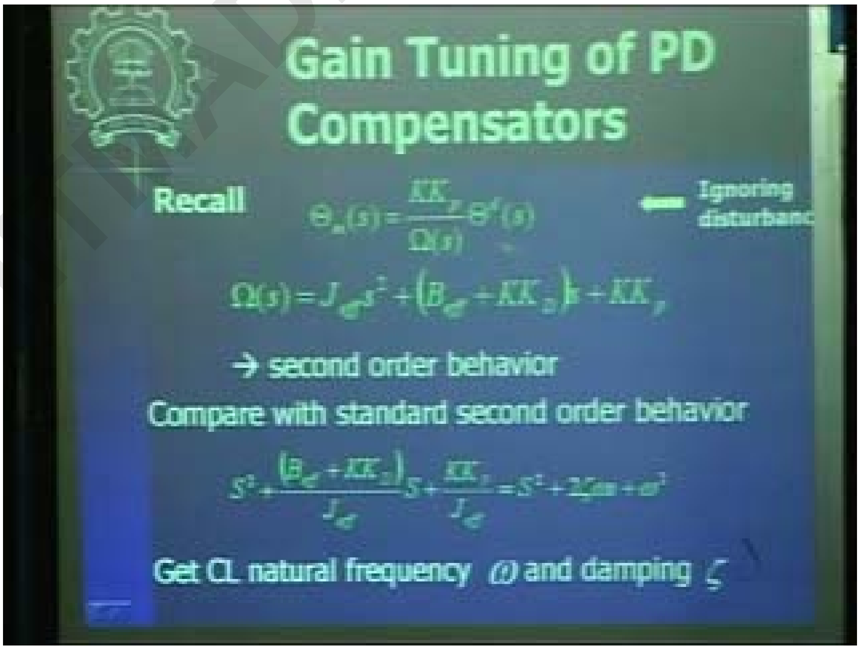
with this so commanding manipulator to go from initial position to the final position means that you are giving some of the step input in theta d ok the desired position you have from from whatever initial position we have to go for the fixed final position it is kind of a step we have to go from initial position to the final position ok

so now let us have a look at what is the steady state error now before that we have made here one assumption that disturbance is constant ok we will will worry about like when what happen when disturbance is not constant it later but just assume for the sake of analysis at we have some constant disturbance term coming into the picture ok with these constants disturbance what is my steady state error so if you do the analysis again by using the laplace domain tool we know that some laplace analysis that steady state error will be given by limit s tends to zero of s into es es is the expression for error which we have derived previously and if we now plug in see this is the previous expression for the es ok now let us plug in that here and the steady state error will get is of this form ok so the steady state error in response curve is represented by these

so when when time or when you allow sufficient time for things to be settled down and see now what is the error between your initial i mean your desired position and the actual position will say that is as steady state error this is theta d commanded step response and this is evolution of the theta m with the PD control and then this is finally steady state error so we see that there is a finite steady state error when you use PD control now these error is smaller for higher gear reduction ok higher the gear reduction the smaller is the error ok another thing you observed here is that it can be made small by increasing the gains ok so you go on increasing the gain of PD control and then you will keep on reducing the error ok

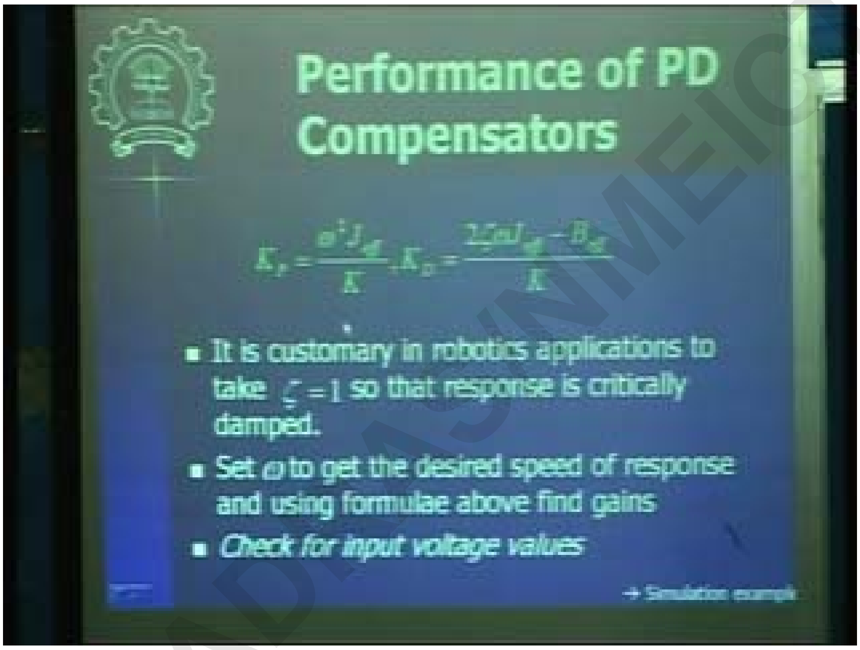
so these are the two ways suppose if you want very small error then we have to use very large gains if you are using just PD control strategy when we use larger gains then it may lead to system instability or it may have un decidable tangent response of the system ok certain movements are oscillations of the system when we increase the gain we are basically increasing the natural frequency of the closed loop system ok this may lead to increased oscillation in the system ok so when we are thinking about robotic applications we never or interested in having these oscillations in the system ok we don't want link to go and oscillate about the final position and come to the final position we want it to go steadily achieving the final position ok

so that is the very important control point of view we cannot have like very high gains so how to now eliminate this problem that is the question that we will see later ok let us see how will select the gain first now ok so you recall now (refer slide time 32:15)



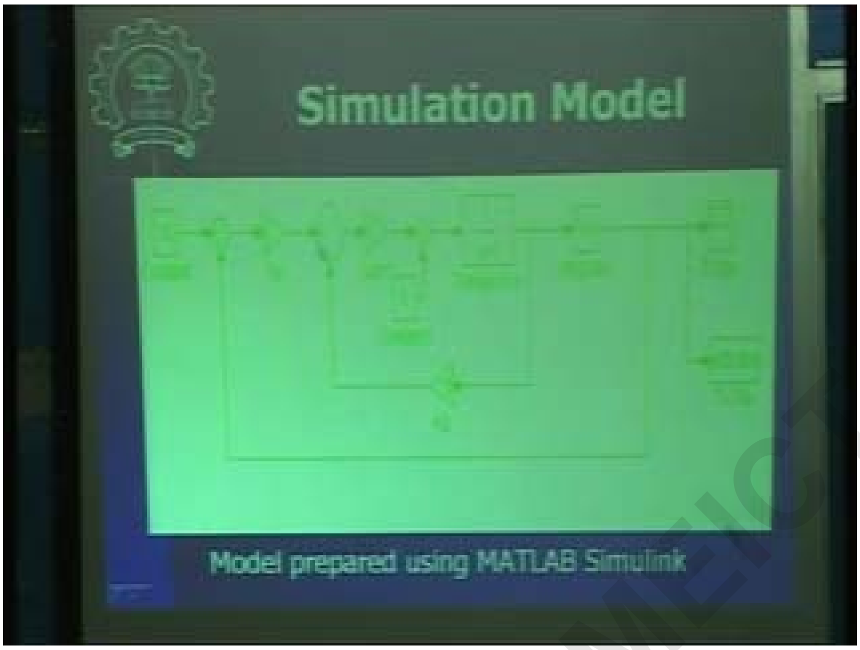
again the transfer function between theta m and the theta d ok is given by this expression here we have assumed that disturbance is zero ok now this is the characteristic polynomial and from this characteristic polynomial you observe that this is a second order kind of a behavior ok of course here you again recall the assumption that our J effective effective inertia has some terms which are non linear but we are assuming them to be some constant terms ok averaged over the range of the operations of the manipulator ok

so if you compare this behavior with our standard second order behavior we recall the standard second order behavior s square plus two theta omega ns plus omega n square ok that is the standard spring mass system equation second order standard behavior ok now if we compare this two we will get the expression for the omega and theta in terms of your gain parameter gain and j effective d effective this parameter ok so so we will get this expression for omega and theta which are closed loop natural frequency and damping ok these expressions are given here in next slide and then the these expressions we can find out values for kp gain kp and gain kd ok now what we need to do is to select your theta and omega (refer slide time 34:26)

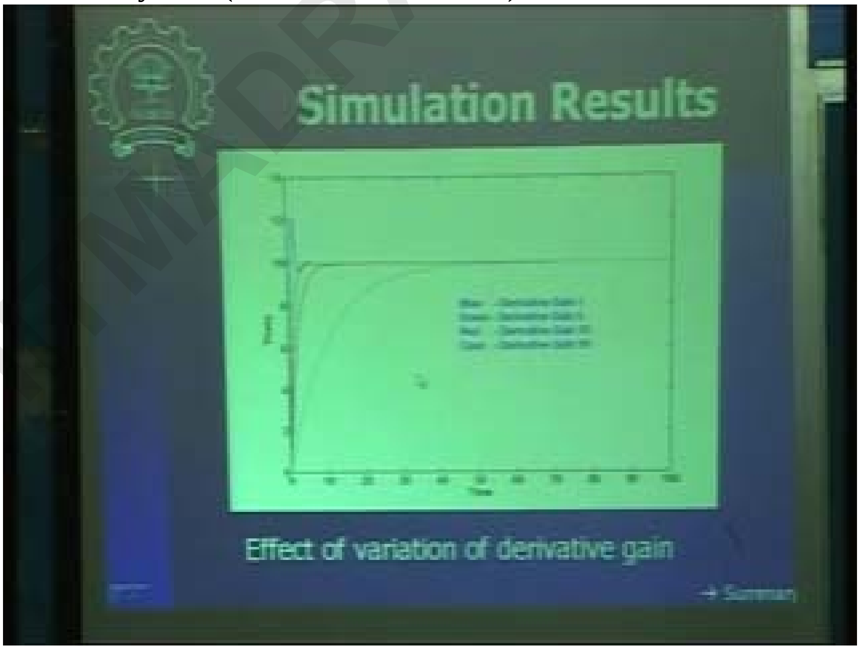


see theta is as i said earlier you have to have non oscillatory response ok so best way is to choose theta to be equal to one ok that is the critical damping we can choose for over damping also but then like that is not much desirable because the response become slow then ok sluggish response will be there so so it is not desirable to do that so basically you choose theta corresponding to critical damping ok so that way your theta gets fixed and then um from these expression you can find kd given omega now omega selection you can do based on what how fast you want to move ok now again recall the triad i told you already that you can ask that robot to move from one point to other point as fast as you want but you have a limitations on the actuators ok

so there is a always balanced between how much energy you have ok and then how much how fast how much performance we are desiring out of your robotic system ok so these triad you have to balance and accordingly choose the defensive omega and then your your gain that is will be in place ok so so always after choosing your gain values you have to see over the response like in the simulation like over the range of response of your system going from the initial position to the final position you see whether your input is any time exceeding the saturation limit ok there will be always a limit put by the motor so see whether the system is exceeding those limit ok now let us see one simulation example not for the actual robot but kind of using the similar equations we have to carry out the simulation (refer slide time 36:49)



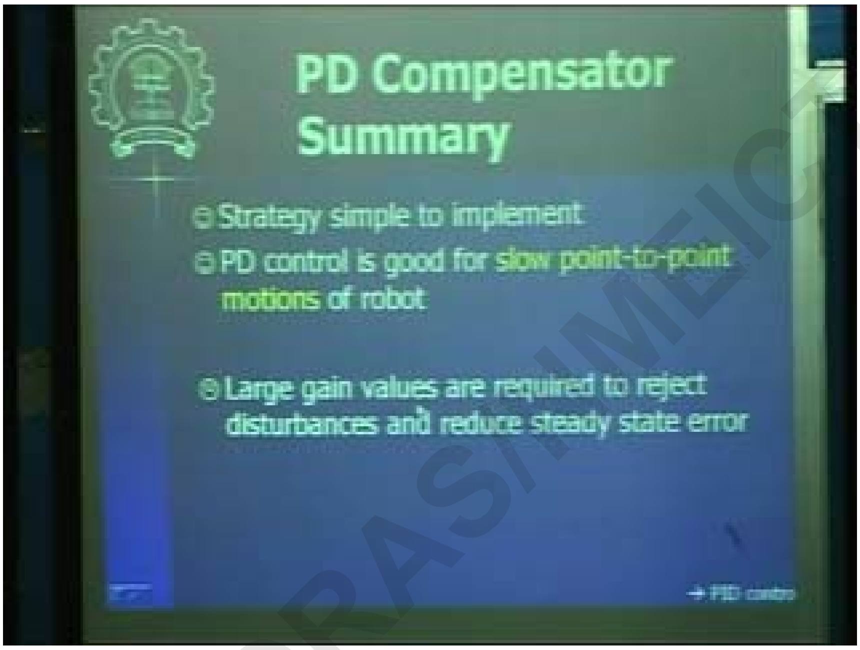
so in simulink in MATLAB you can construct the model of this kind we are here this is the integrated block your j effective and D effective in this transfer function are assumed to be one ok and then this is your k gain this is your kp gain and this is your kd gain ok so this is PD control example simulation and then we want to study now if we vary this kd gain how the response is going to look like then the next slide shows the response of the system (refer slide time 37:36)



for various variation of derivative gain for various derivative gain we have this response you can see as you increase the derivative gain the oscillation first die out then your response goes on becoming sluggish goes on becoming slow and slow ok

so it takes the system takes like more time now to reach the final position when the derivative gain is high ok so your settling time will be increased in the derivative ok so you have to have write the derivative gain um not to have (noise) system not to have system go in the oscillatory fashion also not to have system going to slower to sluggish

so these are the two criteria we have to used to choose the derivative gain ok next let let us summarize what we have seen for the PD compensator first we saw the strategies very simple to implement ok then PD control is good for the slow point to point control ok so if you want to go very fast from one point to the another position this is not the good control strategy ok we will come to that as we go along you will understand why I am saying this this is not a good strategy for that purpose (refer slide time 39:16)



next the larger gain values why large gain values are required basically to have lower error steady state error ok then steady state error in PD control can not be eliminated altogether inherently PD control will have some error if we have dist disturbance corresponding to that we have some error depending upon the gain values you can control that error values but there will be some error so that ideally there is a zero error in PD control in steady state when you have a disturbance next is PD control often leads to saturation of the input voltage ok especially especially when you are going for very large movement of the robot manipulator like your starting point and final point they are located far apart ok

then initial error we can imagine that initial error will be very large that is may deployed by kp value so that gives you very large term which will typically exceed the saturation limits of the motor ok so this PD control will often lead to saturation so now how we overcome some of the efficiency the answer is PID control ok so let us look at the next strategy which is PID control

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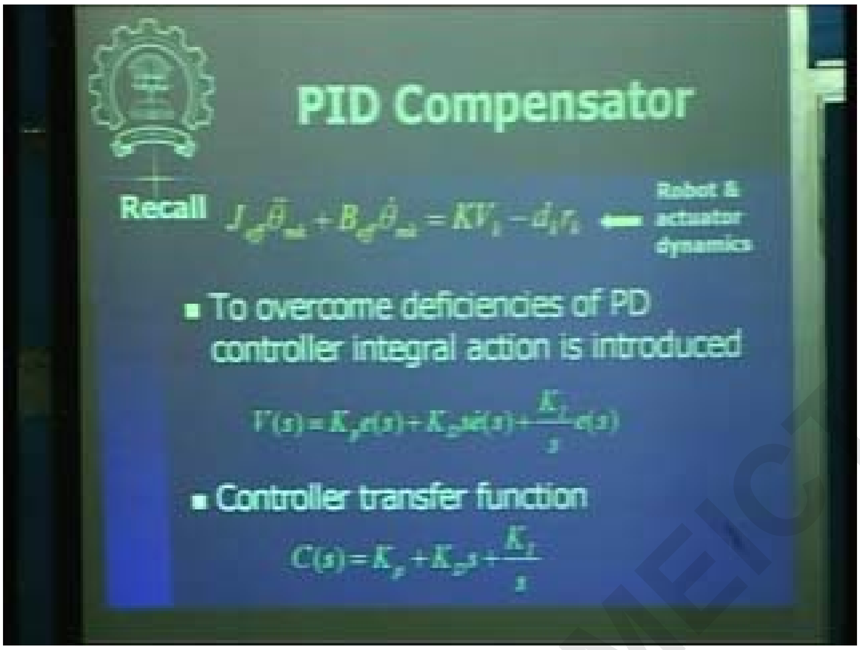

now recall again the combined equation of the robot robot and actuator dynamics in this fashion now to overcome the deficiencies of the PD control the most important steady state error are to introduce this integral control action so what this action does is basically takes the error value it is integrating it over the time so in the steady state you imagine that you have this constant steady state error value now we integrating that and we are generating integrating is nothing but the area so as time proceeds this integration is going to increase its area is going to be increased because of that like we are now implementing that that thing on on the robot control so because of that the error slowly slowly start decreasing so for higher higher time like you are doing this integration like larger than the larger value your time to apply on the robot manipulator so that will eliminate the steady state error so this is how the steady state error is reduced in the integral control action ok so that is why we introduced integral control action that is the basic motivation now with these these controller transfer functions is given by these expression this is the controller transfer function this is the controller transfer function now let us see how it looks in a closed loop action so closed loop system will be now of a third order this is the omega two which is the next different characteristic polynomial in PD control this also omega which was second order characteristic polynomial now this is third order characteristic polynomial and as i explained earlier the steady state error in the PID control will be eliminated ok so this is the basic advantage of using PID control and now you can tune again the gains of the PID controller suit your applications required ok this is how the PID controller will look like in a block diagram fashion so you have entire thing same as the previous block diagram for PD control

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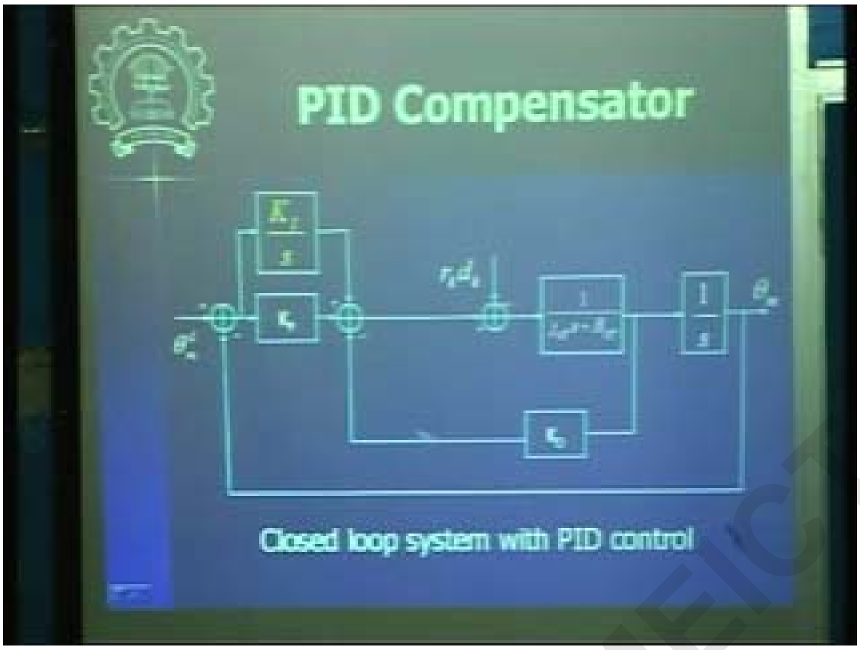

ok this entire thing is same and now we have added one more term in your control expression corresponding to integral control action ok this is how your PID controller will look like and then this disturbance is still entering here ok robot equation now let us see some analysis so PID control you can apply loop stability criteria which you have studied in the automatic control course and with this stability criterion where when we have the zero disturbance there is no disturbance acting on the system we get this expression for stability of the system ok the system will be stable if this integral gain is less than particular value which is in general dependant upon the derivative gain proportional gain and motor or your manipulator parameter ok now this implies that if ki is increased beyond certain values your system is going to go unstable ok that is that means like your integral control action in some senses is introducing some kind of instability in your system ok we have to be very careful when we use the integral control action because it is always leading to some kind of the instability into introducing some kind of instability into the system though it takes care of steady state error it has its behavior ok one has to be careful about introducing integral control action that too very large integral gains small integral gains always we will take care of your steady state error but when you bound to introduce large integral gains you have a deal with it careful about ok because it is introducing some kind of instability ok next we will see the simulation similar to what we did for PD control now here is this simulation model

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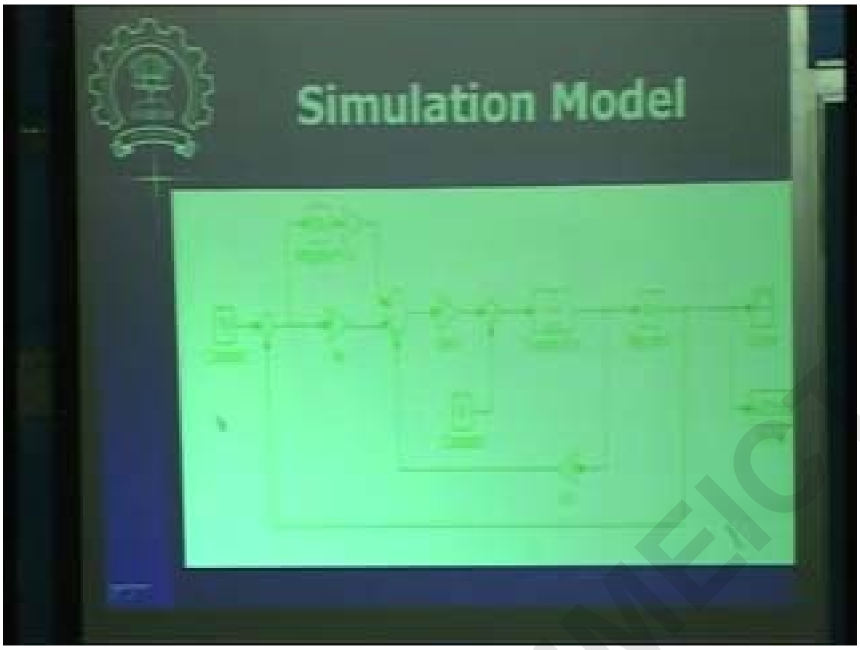
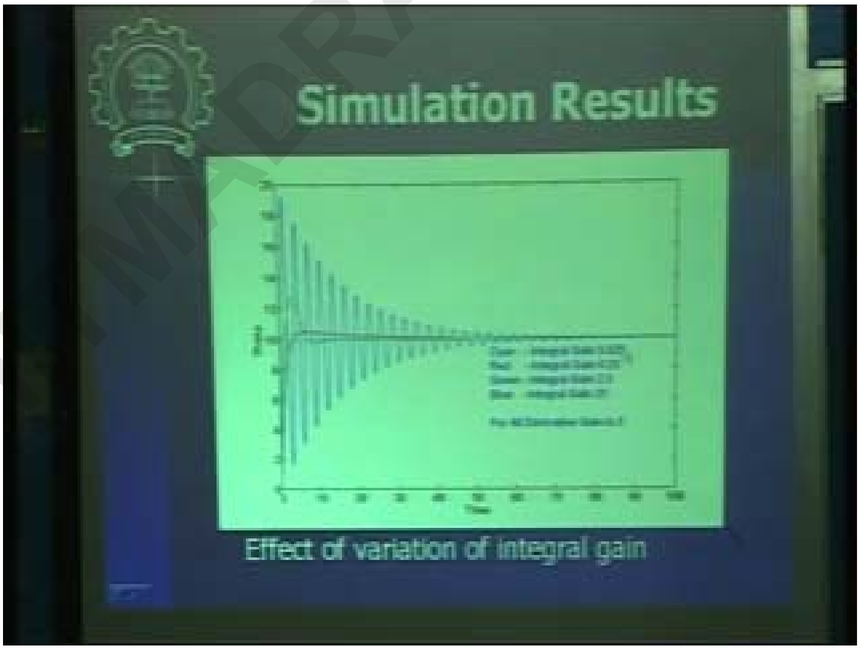
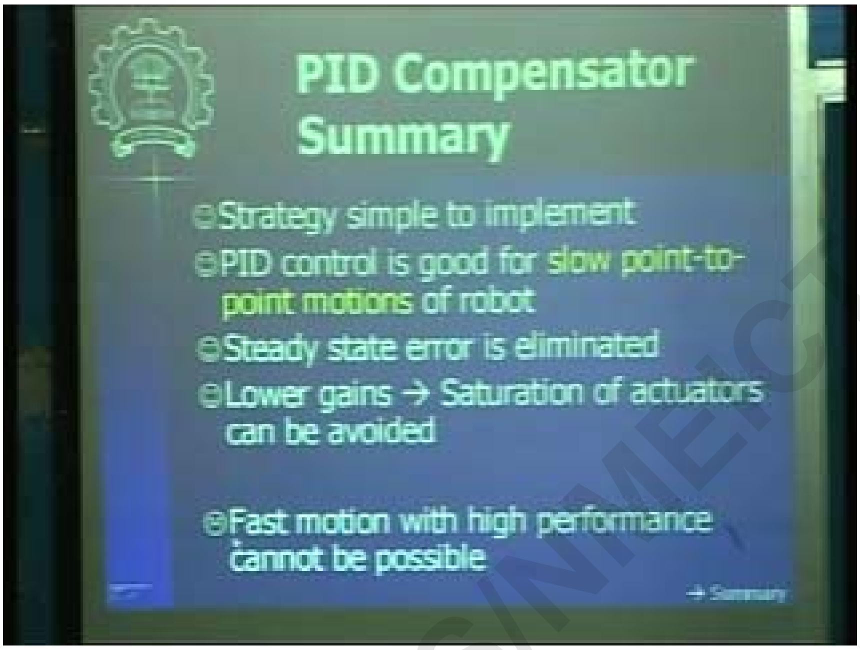


all the other parts are remaining the same in a simulink ok only these new block diagram is added to simulink model corresponding to integral action of the control ok now with these we would like to study what is the what will happen if I go on increasing the integral control again ok so that is the simulation carried out in the next slide shows you the simulation results (refer slide time 47:06)



so you can observe clearly here that the first steady state error is taken care of your reaching the final position ok but when you increase the gain values from these kind of order of magnitude change then you see that your response become oscillatory in nature and again the settling time goes on increasing the time it requires to take the final position fixed final position is on increasing so this is not desirable ok so we have to limit our integral action to a certain values so that there is no oscillatory response of the system ok this system does not go in the oscillatory mode that we have to ensure and most most or most of the robotic system that is too like a [{47:07}] that should not have any kind of oscillatory response ok now let us summarize our PD control um PID control of the motor (refer slide time 49:34)



so again this is the strategy which is simple to implement ok very simple to implement you don't have to have any non linear competition done we will see next why why we have some other strategy to do this non linear computation in PID control we don't need to have a non linear computation it is very good for slow point to point motions of robot mainly because now it is taking care of the steady state error ok now again these steady state error steady state error is eliminated then lower gains we can use now lower gains because of the integral control action we don't have to use very large gains to have smaller steady state error ok as in PD control here we can use lower gains so it can avoid saturation of the actuators ok this is this is very important in terms of control implementations now the problem with these is fast motion with very high performance is not possible ok because this non linearity's which are there are the disturbance d term we are not doing anything about that term we are just taking care of that term by using this PID control gain so because of that term you will see that there is always going to be slow response like the response will not be very fast or very high performance we cannot expect from PID control gain ok you can tune the gains to the maximum extends possible but then still like because of this disturbance that is not taken care of there will be some kind of [{50:22}]available for the performance improvement if we take care of this d ok now let us summarize what we have done today

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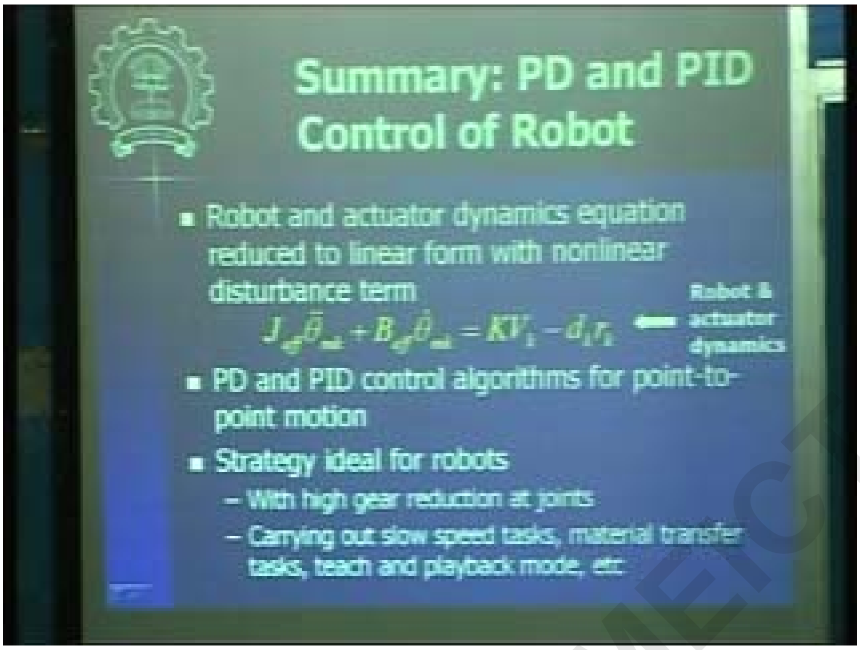
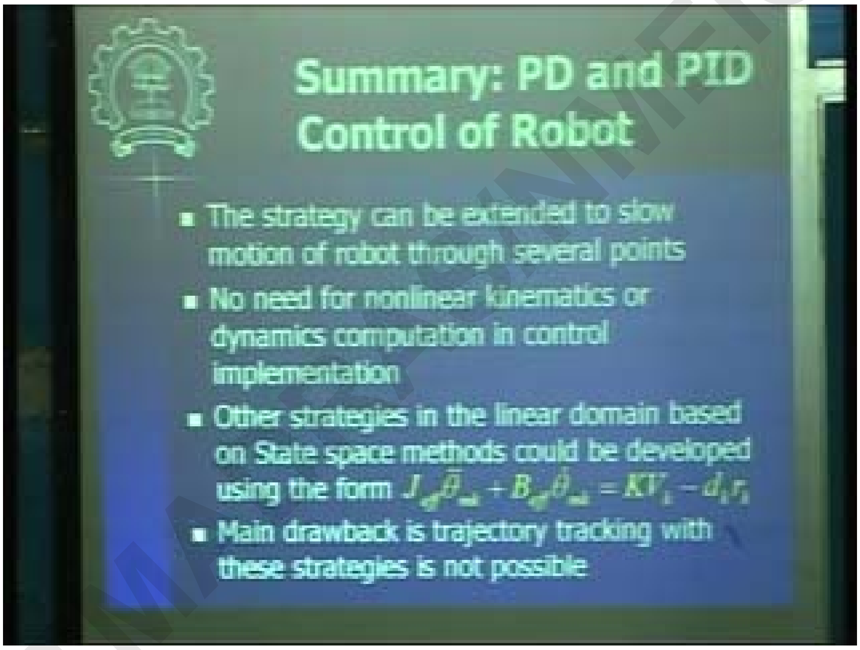


so PD and PID controller basically we found at first the robot and actuator dynamic equations and then we have reduced them see see you can observe now what we done with the we have reduced these equations to a form which is very similar to the linear form with non linear disturbance term this is um surely from the point of view of development of control ok we wanted to develop good control based on some linear techniques that we know already so because of that done this manipulation ok we we see now the system robotics system as a linear system with some non linear disturbance ok so these kind of a manipulations or this kind of a variations we have to carry out to have different perspectives of looking at your robotic systems that to develop different kinds of controller ok so next like in the in the classes to come we will study like how we can develop the purely non linear controller based on the same equations ok without like neglecting couple of terms like without considering the terms to be non linear or acting as a disturbance ok so so we will see that in the later classes so this is our equation that we got by considering these linear form with the disturbance term here now when we start different PD and PID control algorithms for this point to point motion regulation motion then this strategy is ideal for robots where there is a very high gear reduction at the joint because then without the high gear reduction the non linear term disturbance term that we neglected that we have considered as the disturbance will be smaller ok so basically because of that very high gear reduction this is the ideal strategy if we have a robot where like the motor is directly connected to the link ok here there is no gear reduction then I think this strategy should not be used or this will not give very good performance ok even for performance point to point motion so we have to be careful when we using this strategy you have to see whether the disturbance term how much is its contribution ok then when you are to carry out task which are slow like we have to move slow like we have to transfer from one place to another place we can place object from one place to another place in a slow fashion then basically you can use this kind of strategy and then there is a um one more kind of a mode of robot operation where you take the robot physically to different points end defector to different points yourself then record those points with the in the memory of robot then it you can command now to go from whatever points you have taught the robot ok so this is the teach and play back kind of mode of robot operation where now there is no like you don't have to worry too much about robot dynamics or the non linear dynamics you not not to worry about too much or inverse kinematic analysis you don't have to do in this case because like you are physically taking the robot to through different points you want to want it to be carrying out the motion ok so that is the important advantage of these teach and playback model so you will find in the industries like many industrial robots they are operated in this fashion there will be teach dependants with the robot then they recorded take the robot to different desired positions then record those values of the joint angles in the memory of the robot ok now this strategy like whatever the strategy PD and PID strategy we have seen can be extended to slow motion to several points ok you have to you van specify several points next to each other and then like you slowly move the robot through these points ok so this is in very crude fashion like we are tracking some trajectory but very crude fashion here the tracking trajectory points were not very closely placed they are kind of far apart ok then you use (refer slide time 56:32)



PID control strategy is to locate through this point now there is no need for computation of nonlinear dynamics ok in this case then you can use other strategy in the linear domain based on state space tool will not go through these strategies but as we mentioned in the point that we can use so many other strategy that are available in the linear domain you can we can plug in this strategies we can apply here for the this particular linear kind of a form of a robot manipulator equation its disturbance and then main drawback is you can not use this for trajectory tracking kind of application so these control strategy is mainly for point to point kind of a control ok so here we will stop now in the next class we will study the trajectory tracking algorithms will um have look at these algorithms like method of computed torque speed forward controllers then also will have some fundamentals of non linear controller studied in the nest class so like we will prepare ourselves to understand now non linear domain of control tools ok many s there are many non linear strategies I that are very efficiently available there very efficiently strategies available for robot control in the literature so we will have look at couple of them that

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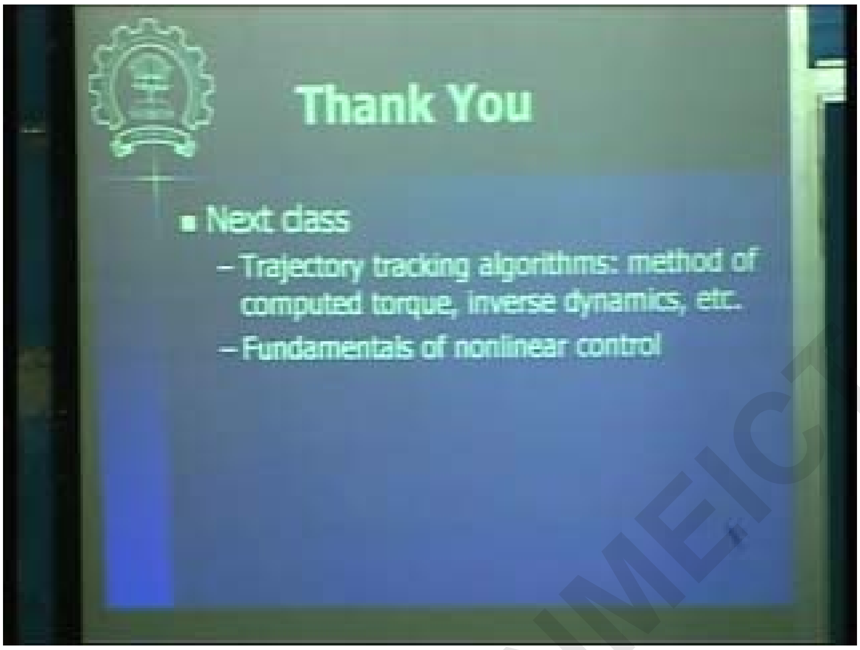

will like close our chapter on robot dynamics and control ok so we will stop here .