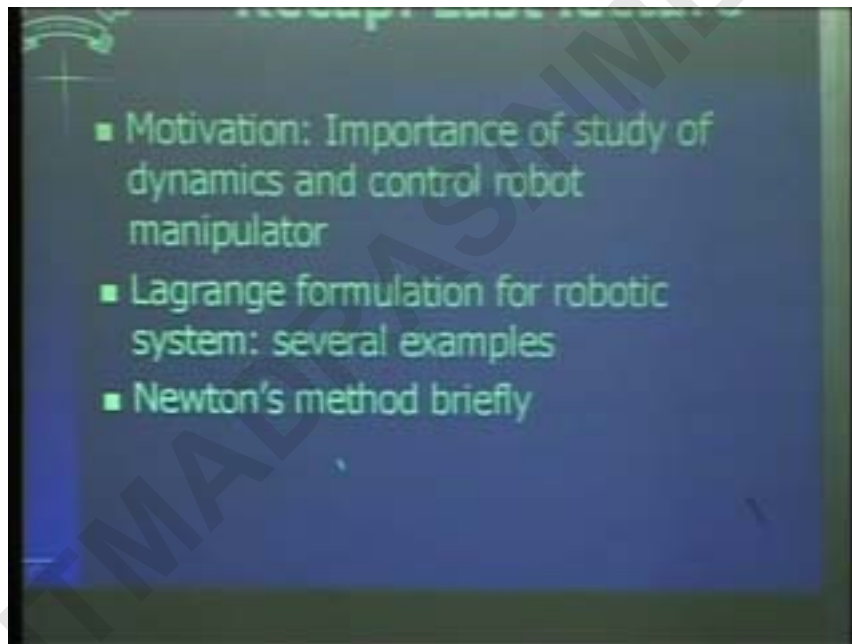


**ROBOTICS**Prof. P.S.GandhiDepartment of Mechanical EngineeringIIT BombayLecture No-32Robot Dynamics and Control

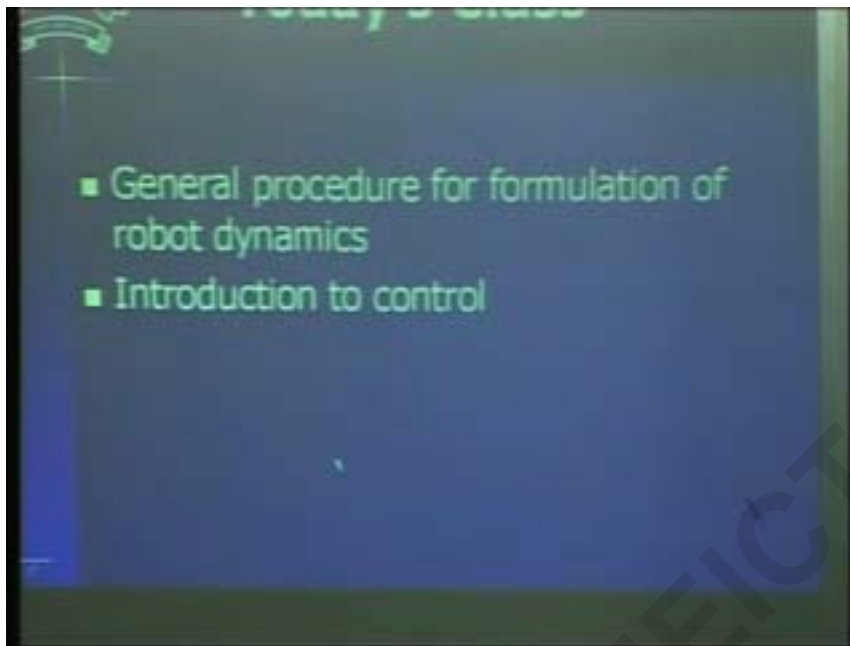
Good morning today we will have see the lecture number two of the module robot dynamics and control first we will see what we have already seen in the last lecture so this is the recap of the last lecture first is we saw what is the motivation why we need to study robot dynamics and control then we saw some fundamentals about robot dynamics ok in particular we studied the Lagrange formulation and we went through one example

Actually several examples one example of the robot manipulator to actually see through how the Lagrange formulation can be done then we saw briefly how to use the Newton's method for formulation of the problems of the formulation of robotic (refer slide time 02:46)



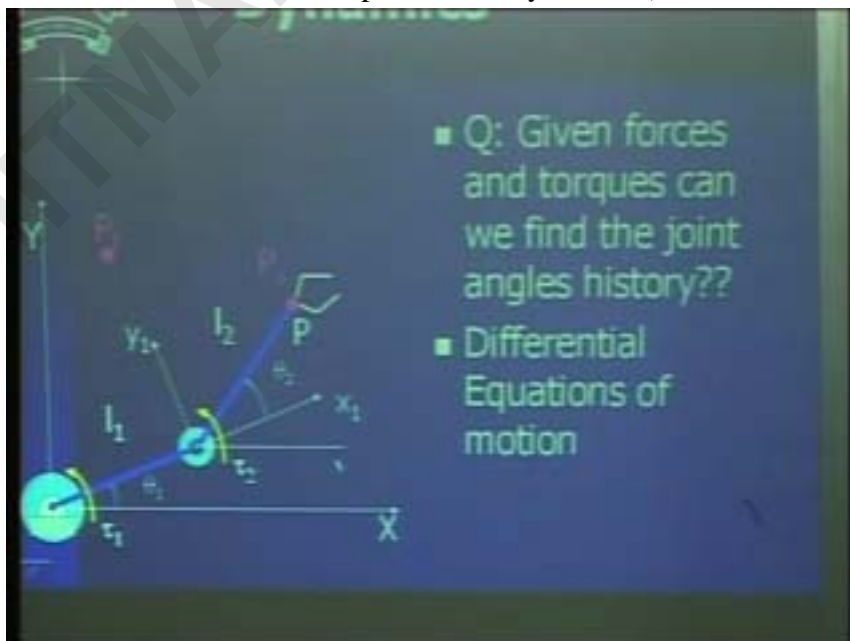
so any task that you carry out for the robot has to be done with understanding of dynamics and control ok especially this become important when we are talking about the robot which are working at fast speed ok when robots are working at slow speed then the dynamics will not have a lot of importance it will have importance but it will not have lot of importance but when we are talk about robots we can working at very fast speed you want higher performance then the dynamics and control they become complicated

(refer slide time 03:39)



so you need to have fairly good amount of understanding about this issues now in the today's class what we are going to see is general procedure we have seen the procedure in terms of one of the example yesterday

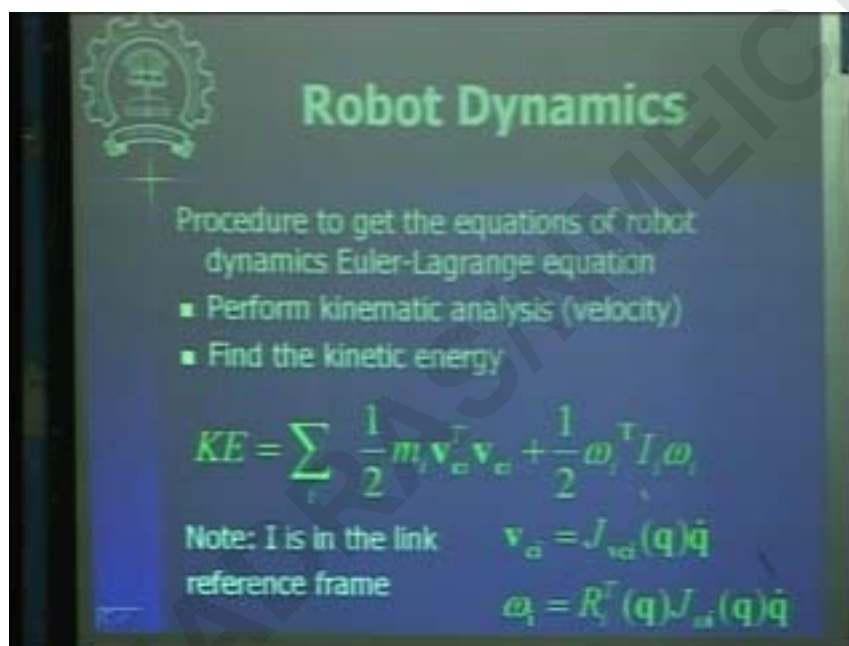
so we will generalized the procedure to get the dynamics of the robot manipulator and then we will go onto some introduction of control some fundamentals of the control we will understand ok so these procedures that am talking about is already like we seen yesterday but now we want to make it more general for the this is having like a general configuration of a robot manipulator we have seen this for 2-R manipulator ok we studied the Lagrange formulation we studied this procedural for 2-R manipulator and now we want to generalize this so how do we do that so first let me recap about the dynamics (refer slide time 04:53)



the basic dynamic question that we again raise is given the force forces and torque on the manipulator joint how will we find angle angular history or history of joint angle so you basically are interested in getting the differential equations of motion for the robot manipulator ok and we did this by using Lagrange formulation yesterday and

now I will give you a generalized procedure that you should follow to get this dynamic ok so this is very important kind of procedure you will follow to get robot dynamics ok

first thing you have to set up your configuration right for robot then perform kinetic analysis by cinematic analysis i mean basically the velocity analysis ok then these velocity analysis you have to use it for finding out kinetic energy and you remember what expression we use for the kinetic energy this is basically the expression And then you can see that this  $V_c$  is basically the velocity of  $V_g$  of the link that we are talking about(refer slide time 07:05 )



then  $\omega I$  this is a this term is basically angular velocity  $\omega I$  and the kinetic energy because of the rotational part ok

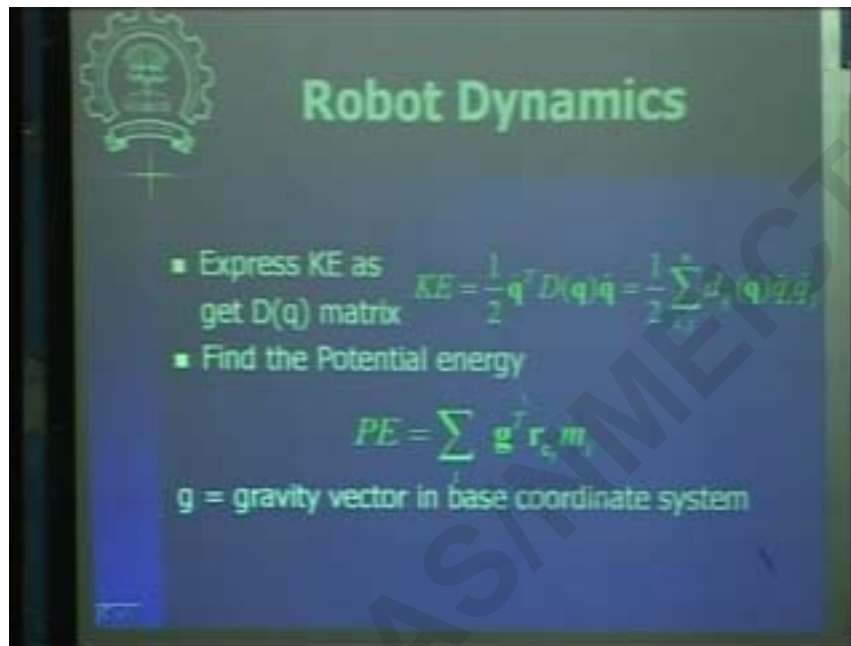
now these can be further given by Jacobean multiplied by your generalized product so this  $V_c$  is the translational velocity of the link which is given by Jacobean in the translational case multiplied by your generalized co ordinate ok and then  $\omega I$  here is given by this formula this is the Jacobean for  $\omega I$  now important thing to know is that your inertia is in the link reference frame ok

typically when you have a body attack reference frame your inertia parameters are known better ok so so they are constant in the body reference frame if you transfer them to base reference frame then inertia parameter speed will change because it involve the rotational track so we want to avoid that so here this link inertia is expressed in a reference frame which is attached to the link hence you have to introduce this rotational matrix over here to take care of its transformation like see the transformation of  $\omega$  to the linked reference ok

you understand that because you can not multiply here  $\omega$  in one reference frame and  $I$  expressed in other reference frame you have to have all the three

expressed in same reference frame so your omega should be in a reference frame which is same as the reference frame of I ok

this is the important thing very important understanding but see the thing is that see this is the energy whether you expressed in a like omega link reference frame or in the base reference frame is going to remain in the same so since omega I is in the majority in the link reference frame we have to multiply it by this R matrix this is rotational matrix transforming from base to the linked reference frame ok next we will move to the next step (refer slide time 08:58)



To express this kinetic energy in this form now remember this is the total kinetic energy ok this kinetic energy of all the links taken together is not just kinetic energy of one link you can see that there is an summation over here ok this kinetic energy is of all the link ok so this kinetic energy you have to express in this form separating that of D matrix ok

so you get this D matrix from here and corresponding elements this  $i, j$  for the D matrix next you find is potential energy so this is the  $r_c$  is the link  $c$  ok the coordinate of the link  $c$ g and then this  $g$  is the gravity vector expressed in a base coordinate system ok  $m_i$  is the link mass ok so summation of these for all the links will give you the potential energy of the

(refer slide time 10:39)

**Robot Dynamics**

- Apply Lagrange equation (simplified version) to get the robot equation

$$D(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

$$c_{ij} = \sum_{k=1}^n \frac{1}{2} \left\{ \frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{jk}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_k$$

$$\mathbf{g}(\mathbf{q}) = \frac{\partial PE}{\partial \mathbf{q}}$$

Note: Generalized force may contain damping terms

ok next we will consider this lagrange equation and applied to this system ok using this potential energy and now we will use the simple form simplified form of the Lagrange equation ok

so these simplified form we use the D directly which we have gotten from the kinetic energy of the link and then we will estimate this C parameter using again the D terms and this formula ok we derived this formula in the last class ok we will use this formula to get your C matrix and then this g matrix here is depends on the gravity gravity vector ok this g vector is given by differentiating potential energy ok

this tow is the generalized force acting in the direction of the co ordinate generalized co ordinate so tow is the generalized force acting in the direction of generalized co ordinate ok now this tow may contain your damping term also so you should remember like if any forces you have to consider external forces other than your input then you have to consider them in the tow ok so in general damping forces friction forces they will show up in this tow term ok (refer slide time 12:37)

**Robot Dynamics**

- The equation is 2n order ordinary differential equation in general nonlinear

$$D(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}$$

- Can be solved for  $\mathbf{q}_s$ , given the torques applied
- Numerical solution using MATLAB

next we will see how this equation looks like it is the second order ordinary differential equation you can see there is a double dot over here and there are  $n$  such equation for  $n$  link manipulator so i have expressed this in a vector form already for all  $n$  link ok so we will in general we get two  $n$  order ordinary differential equation this is ordinary differential equation then there are no partial derivative terms and it can be it can be solved for this generalized co ordinate  $q_i$  ok

how will you solve this you can solve it usually if it is in simple form you can solve it by doing simple analytical techniques for solving the differential equation ok if it is in complex form then we can go for the numerical solution and the numerical solution you can use mat lab for that ok so you are already conversant with using mat lab to solve ordinary differential equation so you will use the same fundamental in and solve these equations

now what what we need for solving this equations in addition to these equations you need initial conditions ok you need to specify what are the initial condition and you need to specify the input tow ok what is the input that you are applying so those two things along with the equations that will give you the solutions what is the trajectory or time history of the generalized co ordinate ok understand that um so for 2-R manipulator we saw the whole procedure yesterday ok so we we finally ended up in these equation (refer slide time 14:50)

**Robot Dynamics**  
**2-R Manipulator**

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

$$D = \begin{bmatrix} m_1 l_1^2 + m_2 (l_1^2 + l_2^2 - 2l_1 l_2 \cos q_2) + I_1 + I_2 & m_2 (l_2^2 + l_1 l_2 \cos q_2) + I_2 \\ m_2 (l_2^2 + l_1 l_2 \cos q_2) + I_2 & m_2 l_2^2 + I_2 \end{bmatrix}$$

$$C = \begin{bmatrix} h_1 \dot{q}_2 & h_1 \dot{q}_2 + h_2 \dot{q}_1 \\ -h_1 \dot{q}_2 & 0 \end{bmatrix}$$

$$h = -m_2 l_1 l_2 \sin q_2$$

$$g_1 = (m_1 l_1 + m_2 l_1) g \cos q_1 - m_2 l_2 g \cos(q_1 - q_2)$$

$$g_2 = m_2 l_2 g \cos(q_1 - q_2)$$

for these equation this is the D matrix and this is C matrix now you can notice here this D matrix this term and this term they are equal ok

so in general for all the D matrices that you get for all the robotic manipulator the matrix will be symmetric ok that is the property of this D matrix and get it from Lagrange formulation ok another thing we will see some more properties and we will generalized this properties also so C matrix as this kind of the form where  $h$  in this C matrix is given by this expression and this  $g_1$  and  $g_2$  they are gravity gravity terms in this vector to this  $g_1$  and  $g_2$  are gravity terms in this bold  $g$  vector ok

so now next we will see one more property ok now what if we differentiate  $\dot{D}$  and find out what is  $\dot{D}$  ok this is our D matrix and we find out  $\dot{D}$  now

(refer slide time 17:38)

**2-R Manipulator Properties**

D Matrix will be

$$D = \begin{bmatrix} m_2 l_{12}^2 - m_2 (l_1^2 - l_2^2 - 2l_1 l_2 \cos q_2) - I_1 + I_2 & m_2 (l_2 - l_1 \cos q_2) - I_2 \\ m_2 (l_2 - l_1 \cos q_2) + I_2 & m_2 l_2^2 - I_2 \end{bmatrix}$$

$$\dot{D} = \begin{bmatrix} -2m_2 l_1 l_2 (\sin q_2) \dot{q}_2 & -m_2 l_1 (\sin q_2) \dot{q}_2 \\ -m_2 l_1 (\sin q_2) \dot{q}_2 & 0 \end{bmatrix}$$

Taking  $-m_2 l_1 (\sin q_2) = h$

Matrix  $\dot{D} = \begin{bmatrix} 2h\dot{q}_2 & h\dot{q}_2 \\ h\dot{q}_2 & 0 \end{bmatrix}$

ok when doing the differentiation only these term will contribute all other term are constants so this is  $m_2 l_{12}^2 - m_2 (l_1^2 - l_2^2 - 2l_1 l_2 \cos q_2) - I_1 + I_2$  and cosine its derivative is sine  $q_2$  and  $q_2$  dot ok likewise in all the term we take a derivative and we will get these  $\dot{D}$  dot ok now if we use the same  $h$  as we defined in the previous case we get this  $\dot{D}$  dot matrix of this form ok

now do you see any kind of a similarity between  $\dot{D}$  dot matrix and  $C$  matrix you see what is your expression for the  $C$  matrix these are the  $C$  matrix and your  $\dot{D}$  dot matrix is this ok (refer slide time 21:40)

**2-R Manipulator**

Matrix C

$$C = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_2 & 0 \end{bmatrix}$$

$$\dot{D} - 2C = \begin{bmatrix} 0 & -h\dot{q}_2 - 2h\dot{q}_1 \\ +h\dot{q}_2 + 2h\dot{q}_1 & 0 \end{bmatrix}$$

Which is a skew-symmetric matrix

We will generalize this result now

so you will find this matrix  $C$  and  $\dot{D}$  we just multiply  $C$  with two and subtract it from  $\dot{D}$  dot so you will get  $\dot{D}$  dot minus two  $C$  you will find that this diagonal elements are

zero and elements which are half diagonal they are negative of each other like  $a_{ij}$  will be equal to minus  $a_{ji}$  ok so you know what is this property of the matrix skew matrix this is skew matrix property of the matrix and it is very important property for all the robotic manipulator ok from the control perspective this has a lot of hypercation so we should have very firm understanding of this part to as to use this for the development of the control later on ok

this is the skew symmetric matrix property then in combination with this skew symmetry if we see now quadratic if we take a skew symmetric matrix ok and then you form a quadratic of that matrix by multiplying by  $\dot{q}$  dot transpose and again multiplying prefix to the matrix  $\dot{q}$  dot transpose and after the matrix expression so that is something like you take this matrix see i can not write over here so basically you take the matrix C multiplied on on the on the one side on this side by  $\dot{q}$  dot and on this side by  $\dot{q}$  dot here it will be  $\dot{q}$  dot transpose and here it will be  $\dot{q}$  dot ok rather for these matrix D dot minus two C

so you take this D dot minus two C matrix on this side you multiplied by  $\dot{q}$  dot transpose and this side you multiplied by  $\dot{q}$  dot and then you see what is the expression that you get ok we will find that it will be zero so it is the property if we form a quadratic by multiplying these matrices by vectors then you will end up with the zero ok

so quadratic expression for skew symmetric matrices is null is zero ok that is the property that gives many in of the control development ok control development control stability proof every where this property is up lot of terms um so we will see that in the later like when we talk in detail about robot control we will see how to make use of this property but before that we need to develop our understanding regarding some fundamentals of control ok that understanding will generate now so so this property first will generalized and then we will use it for control ok and then we will see first we generate generalized these result first we will generalized this result then will go for some fundamentals about control

(refer slide time 21:54)

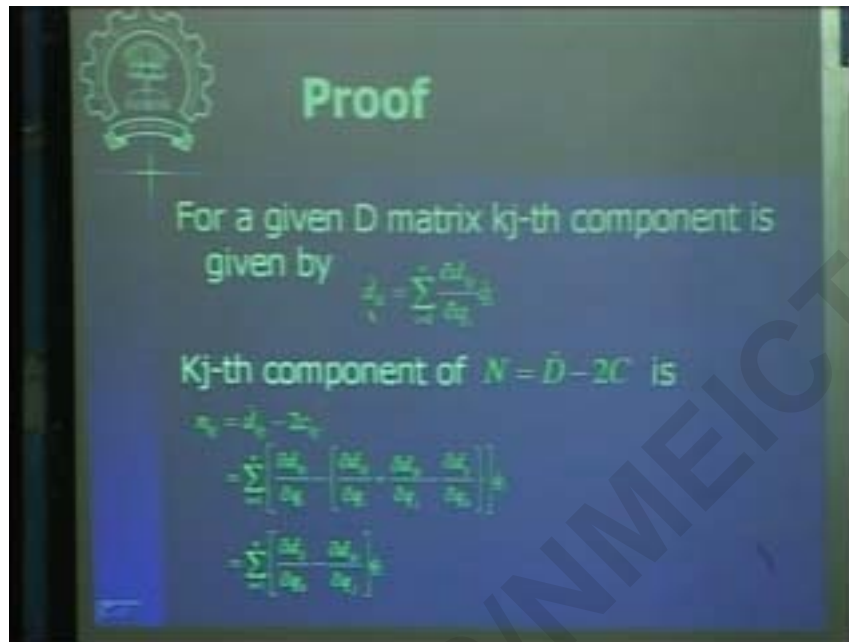
**Theorem**

Define the matrix  $N(q, \dot{q}) = \dot{D}(q) - 2C(q, \dot{q})$  then  $N(q, \dot{q})$  is skew symmetric matrix, that is, The component  $n_{jk}$  of N satisfy  $n_{jk} = -n_{kj}$

$$c_{kj} = \sum_{i=1}^n \frac{1}{2} \left( \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right) \dot{q}_i$$



ok to generalized these results we have this theorem you define this matrix N which is D dot minus two C then N is skew symmetric matrix so component in jk will be equal to minus Nkj so for that to prove this property we will use this C matrix and this (refer slide time 22:34)



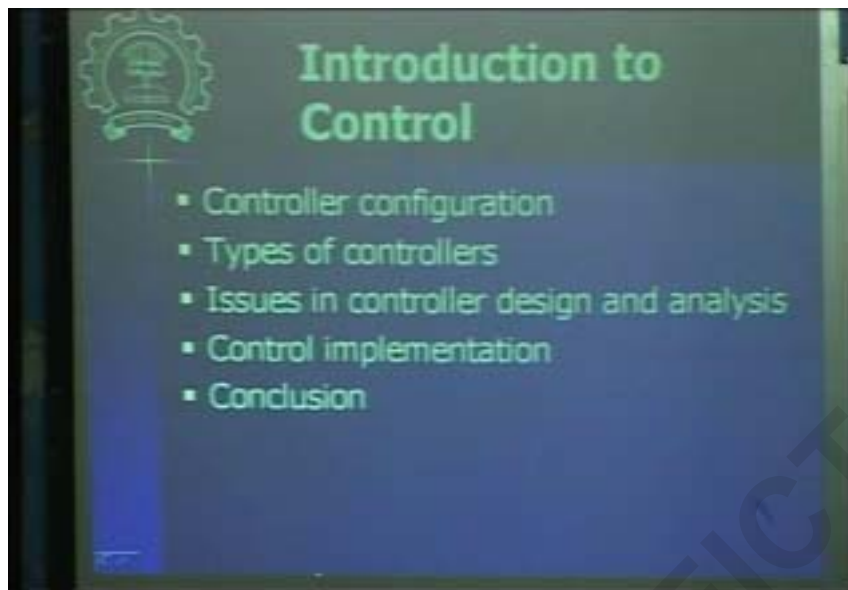
D will differentiate in this fashion each element will be differentiated like this ok and then now if we take D dot minus two C you see what is happening here for this matrix expression so these terms will get cancelled out and you will end up with this final expression ok so you see observe carefully this expression suppose now we put in place of k and j you just put j and k and see what is the expression that you get

so you will find if you replace this k and j you will get njk is equal to minus nkj do you see that so we have generalized this property in general for n link robot manipulator

so all the manipulators whose expressions like the equations of motion are derived from the Lagrange formulation they will all have this property ok and then these property will be used as i said earlier in the control in general the non linear control of the robot ok so now we will stop here for the dynamics part of it and then we will see first some of the fundamentals of the control ok

And these fundamentals of control then later on we will develop on them to again come back to this property and talk more about robot control ok but first you need to have firm understanding what is meant by control what are the different things that go in the control so let us see what we are going to see in this robot control introduction

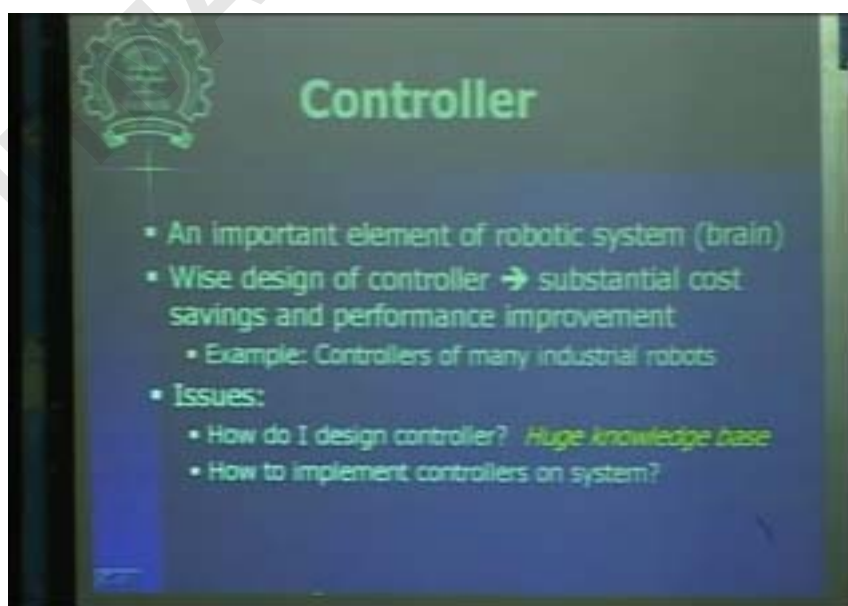
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first we will see what is control configuration typical control configuration then various types of controllers i will give you some brief idea about what all type of controllers available ok

And then there are many issues in the controller design and analysis to the issues we will touch upon some of the issues so you have this background of control develop in your mind and then you can use it later when we talk in detail about robot control and all in a robot control you can see from this perspective like the fundamental background perspective then you will get better understanding about how to implement how to select controller and thing like ok

so we will also see some control implementation issues and then we conclude the lecture ok so controller forms an important element of the robot manipulator its like a brain of the robot so whatever sophisticated tasks robot performs ok that task like controller has has to be deigned to carry out the (refer slide time 26:01)



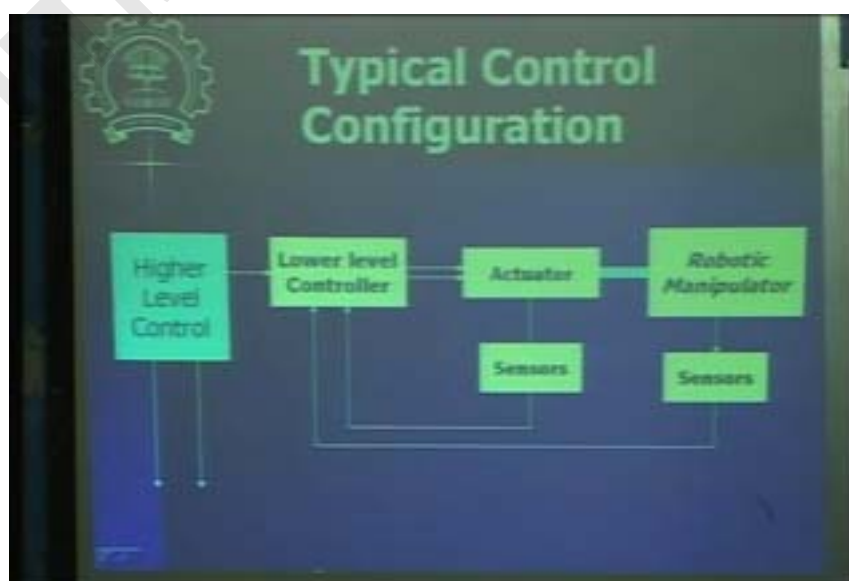
particular task ok controller is designed designing in a very sophisticated manner all the high performance task robot it like a brain of the robot and it really gives that intelligence to the robot

so suppose you designed controller wisely then it will improve the performance of the robot and if you are know fundamentals understanding about control and you are not designing the control properly then it may ampere the performance of the robot ok it may badly affect the performance whatever task you want to achieve out of the robot may not be achieved

so it is very important that you think about the controller in the depth for any type of robot at you design or you design ok there are other issues like how do i design controller now ok there is a huge knowledge base about these how one can design the controller ok to give you just an idea in general this control theory has been developed over the years so much i mean there are so many different varieties of controllers available in the literature and just give you current status current result status like there are two important conferences that are held internationally very important control conferences there one is conference on decision and control conference on it is called CDC conference on decision and control and the other one is ACC American control conference ok

so there are lot of papers like can you imagine number how many papers are published every year in his conferences they are in the range of about two thousands two thousands to two thousand five hundred so that two thousands to two thousand five hundred new research papers ok which are referred i mean that which are like gone through scrutiny ok gone through like rigorous review they are published in this conferences so that so much of new literature on control is being generated every year ok so this control is really a huge vast area and in these course will not be able to cover it in entire this so for robot control we will give you some like fundamental understanding and the directions where you should go for more sophisticated kind of a control ok

so it is really a use knowledge base ok now other important issue is regarding the implementation of controller on your manipulator how do we implement controller on your manipulator ok so we will touch upon these issues um in a little bit passion then i will like later on when we talk more about robot control in specific that time I will give you some more hints how to implement control ok (refer slide time 29:53)



so this is the typical control configuration for the robot manipulator ok i mean these part you can say it is a general control configuration for any dynamical system but typically each axis of the robot will have its part ok so you have this load level controller which is actually controlling the actuators the sensors can be giving the feedback from either actuator or actual robot manipulator ok

so it can be either either of this and then you have the higher level control now this higher level control has it generates some commands and then implement it give it to lower level controller ok then this higher level controller can be operating many such axis at a time ok it has to co ordinate between the different axis of the robot ok

say for example typically robot will have like many joints ok so all these joints need to be coordinated with each other then the co ordination tasks is done y the higher level control ok (refer slide time 31:24 )



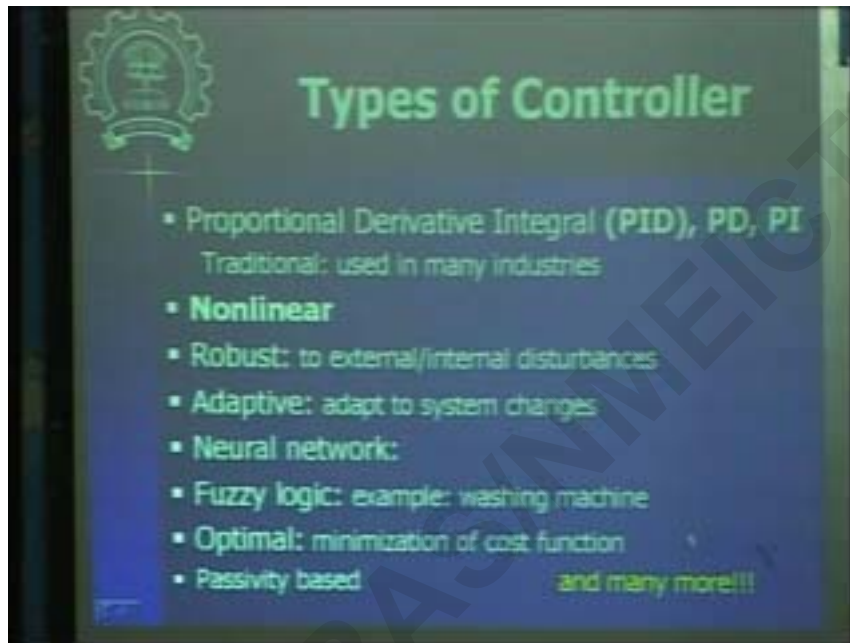
now what are the design issues in the control first is the stability the controller to be stable what ever controller you design should be stable both in the numerical implementation and in the actual performance of the robot ok

then how what is the performance of the controller so it is high performance controller or low performance controller like that like that we can have various levels of performance that can be that can be given by different types of controller ok so depending upon the your application needs you have to select appropriate control strategy which suit your performance requirements ok

and other very important issue usually when you study the control theory this equation is neglected when people don't need enough attention to these part which is how much the energy which is i need too put in the system to acquire the level of performance that i am get it ok it is a it is like a balance like you have on one side your performance you want better and better performance on the other side we have limitations on the energy that we can give ok

so your controllers are always optimally balanced on these two issues depending upon the needs of the particular application task that we are talking about ok so this issue is the energy required to achieve very high performance is very important issue so kindly you pay attention to this like whenever you design the

controller first simulate and see what is the control input that you required to achieve whatever quality of output that you are getting out of your robotic system ok then there are issues regarding implementation so you can implement controllers in analog domain by using electronic circuits or in the digital domain by using microprocessor or computer most of the modern robot you will find the control implementation is done with is this the other use computer or you use microprocessor and there are in general you will find many microprocessor use in a typical robot ok for each axis you will find there is a separate microcontroller ok (refer slide time 34:15)



next we will see what are the different types of the controller ok there are several types as I mentioned it is very vast area so I have listed only a few of them here ok first and foremost important is PID control ok PP PI or any combination of these PP PI PD you can use it for controlling the robot now these are traditionally used in many industries ok and the important thing is here is that like these controller when ever when when will you use this controller basically when ever your task needs have not have very high performance are you want to move it very slowly you have to pick and place the object very slowly or you can you have to traject the you have to do the trajectory tracking very slowly in those case you have to use these type of control ok so these controllers PID controllers they will not give very high performance ok

but they will cover up your needs ok then next is non linear control non linear control is again very vast speed and is is actually developing it has not been saturated like like lot many new things coming up in this field ok and you see all your equations of the robot that you get for example we saw two link manipulator those two link manipulator equations where in the non linear formula ok

for the non linear form of the equation in general we require non linear equations ok so what ever you will study controller mainly will be these two classes like proportional traditional way we will see see some high performance non linear controllers ok

there are other types of controllers robust control adaptive control neural control fuzzy logic controls then optimal control these are these are various different types of controls that are available and they can be used for robotic application also

but unfortunately because of the time restrictions we are not able to get into the details of these control there are more modern some more modern passivity based controllers are also there then there are controllers based on the lee bracket lee derivative that are available i mean you see in the latest literature they will be of that type ok but but unfortunately we will not have time to go in the details of all of these controller so we will give you some fundamentals understanding about these two classes like these PID class and the non linear class and then will have these other things for your own studies (refer slide time 37:47)

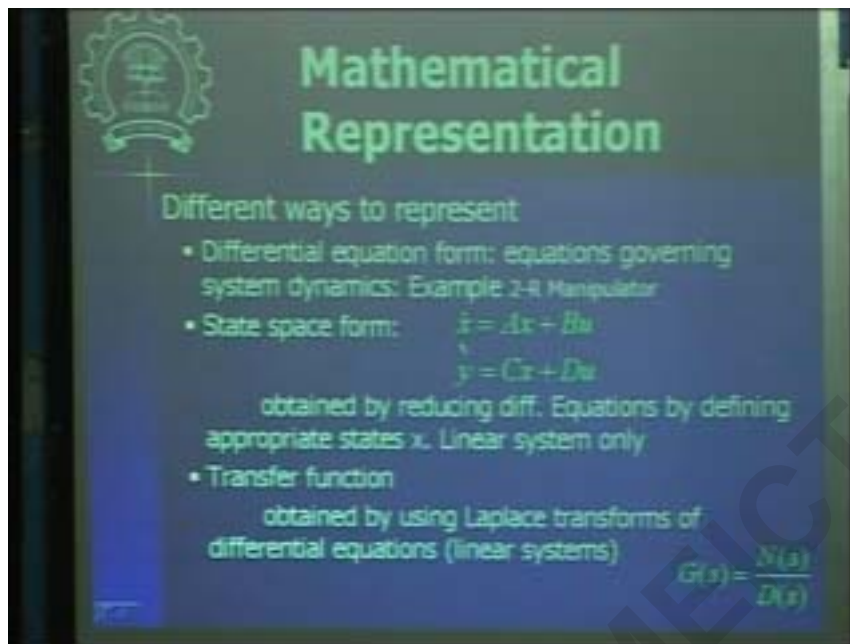


now what are the design steps that you will follow to design a controller first very important step is modeling we have seen this modeling in the robot basically getting the mathematical representation of the robot dynamics in the differential equation form most of the cases now these we have seen already by using Lagrange formulation in the last class ok and we have also outline the procedure to get the mathematical representation of the robotic manipulator ok the second thing you see is selecting the control strategy

now here is where like you have to put in lot of efforts i mean you have to do first you have to have first good understanding of the dynamics so carry out some simulation understand the dynamics of your robot ok step response or some kind of response you see how for the given input of the robot respond and then based on that you can say like what kind of the controller you will have to select ok

and then you carry out the selection of the controller now next is once you select the controller then if what you have to do is like controller parameters there are many controller parameters you have to select them then that selection are tuning will be based on again the performance requirement how much performance you want out of the system and how much energy you give that these two constraints will always be there like any time you select or make the choices these are the two right areas you will use to make those ok then you will carry out once you selected control strategy finally you will carry out simulation using this strategy and do the experimental implementation to test it ok so these are the typical steps you will follow in control design ok

(refer slide time 40:15)



next we will study what are the different ways of mathematical representation we have seen one way for the robotic manipulator ok that way was by using your LaGrange formulation and getting the differential equations ordinary differential equation governing the robot manipulator dynamics ok

but there are some other representations also you should be aware of i mean the they may not be directly useful for the robotic manipulator cases but you should be aware of that there are some other mathematical representations mainly for the linear systems ok one is these state space form as you see in the slide is the state space form basically you reduce the order by changing the variables and put in the matrix form to reduce the order of the each equations like so you in the robot we got second order differential equation for each joint ok

so here we will reduce that second order differential equation to the first order by introducing some new variables ok and then we can put that equations in this form  $\dot{x}$  is equal to  $Ax$  plus  $Bu$  and  $y$  is equal to  $Cx$  plus  $Du$  so this is the called as state space form and then these form in these form this  $A$  matrix  $B$  matrix  $C$  and  $D$  matrix they are characterizing this form and this matrices are constant for the linear systems ok using these state space form you have lot of control tools that have been developed ok

and then again like when you say this  $A$  and  $B$   $C$   $D$  constants are the matrices are constant matrices then it is you are talking about the linear systems the tools that have been developed in the literature that for the linear systems ok

so if you have the linear system or if in some sense for like simple robotic system you can represent it in a linear fashion or approximately it in a linear fashion then you can probably use the tool which are in the state space ok

then you see the next form is the transfer function form ok so the transfer function has typically this kind of a form it is in the laplace domain so your it will be like numerator over denominator polynomial for the laplace and then his  $G_s$  will be basically the laplace transform of the output divided by the laplace transform of the input ok

we have to represent the equations like we have to take the first laplace transform equations in the differential equations and then you have to separate out the output and input and take the ratio ok name it as the transfer function and find out the numerator and denominator polynomial this is how we will go to find out this how we will proceed to find out your transfer function ok

this transfer function analysis is mostly in most of the robots like you will not find these kind of analysis you need to do i mean you need to do these kind of analysis mainly for the actuator dynamics so because actuators dynamics we can say that it is in a linear domain ok but again it is not independent of your robot itself so if you are taken care of the taking into consideration the coupling term because of your robot manipulator joined with your actuators then it doesn't really become remains linear domain so but you should have this understanding ok we will see some details when we talk about PID type of control how we can represent PID in the laplace domain and use it for robotic manipulator ok now there are many important notions in stability I mean in analysis but there are many important notions in analysis of control so first foremost important thing is stability (refer slide time 45:00)

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decompressor  
are needed to see this picture.

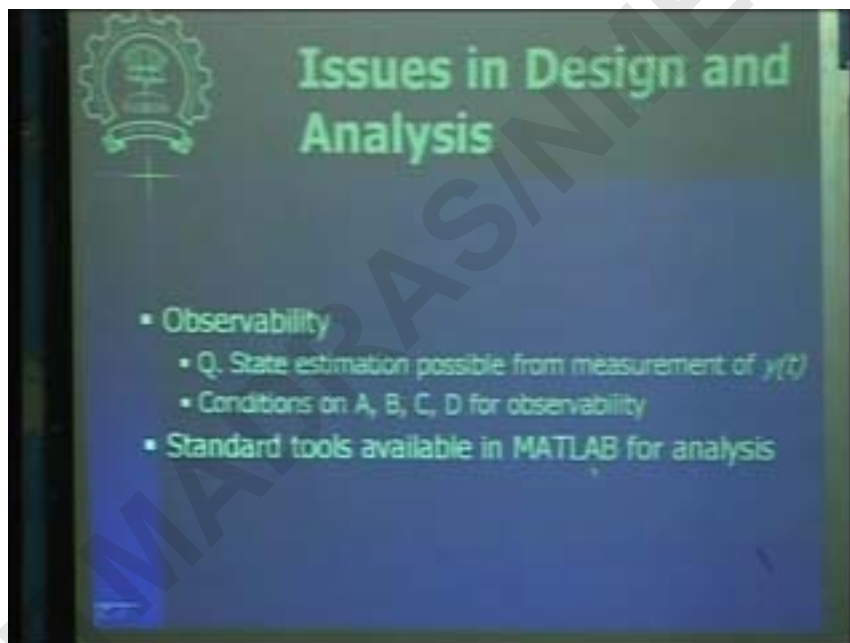
how do you define stability one of the way is to define is bounded input leading to bounded output if we have a bounded input given to the system a finite input which is given to the system then it produces a finite output that is one of the notion there are many other notions like asymptotic stability exponential stability to these exponential stability is basically for the linear system all the linear systems whether it is stable when when you say it is stable when you say a linear system is stable it can be only exponentially stable there is there is no other notion for linear system but when you go in the non linear domain then you have the other notions of the stability possible say for example asymptotic stability it means that your output goes to the equilibrium ok in asymptotic factor meaning output goes to equilibrium as time  $t$  tends to infinite that is the kind of the notion we talking about ok

then exponential stability is output goes to the equilibrium in a exponential factor there is no restriction in the asymptotic stability for in what fashion it goes but it goes to infinity as time  $t$  time  $t$  time it goes to zero or equilibrium as time  $t$  tends to



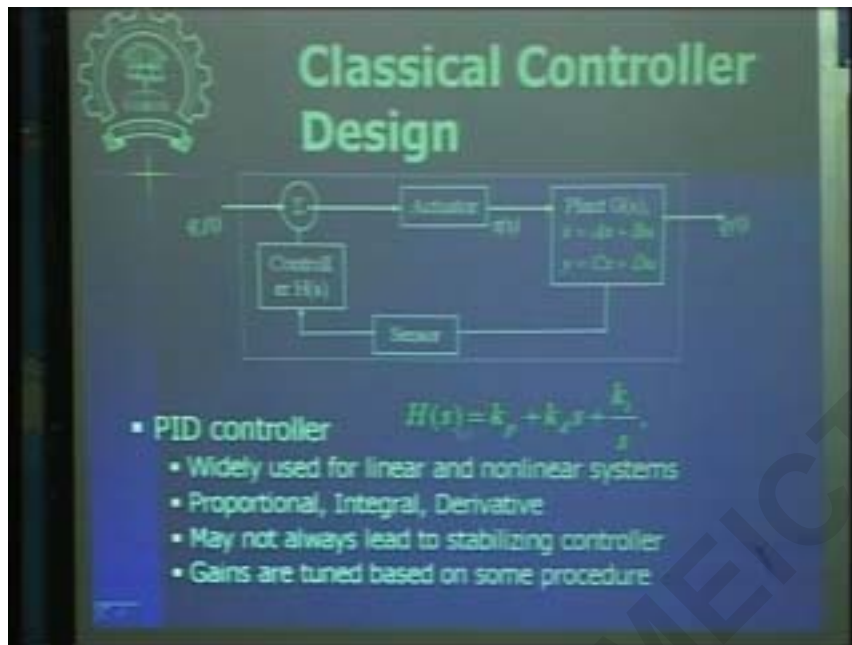
infinity but in the exponential it will go to zero in the exponential factor ok that is the difference between these two notions then there is a notion called controllability so far that i say we have state see we have plotted here  $q_1$  and  $q_2$  space which is like a cordial space ok so this is the state kind of space am just using just two co ordinates here but in general further robotic manipulator manipulator that they are may be number of such co ordinates and you move you want to move from this point to to any other point in this space ok

so controllability refers to whether a robot can be take from these point to these point in a finite amount of time ok so robot should be we should be able to take robot from one state to the other state in a finite amount of time then we can say that robot is controllable now in case of linear systems there is conditions on these  $A$   $B$  and  $A$   $B$   $C$   $D$  matrices for controllability there is other notion is called observability which will see next ok observability basically refers to whether what are the sensors we are using are the sufficient for our control application control purpose so observability answers this question ok so whether what ever feedback we are taking are we able to construct the state out of the feedback completely ok (refer slide time 49:09 )



and again there are conditions on the matrices  $A$   $B$   $C$   $D$  for observability and then there are several standard tools available in the MATLAB for us this analysis either in the linear domain or in the non linear domain ok next

(refer slide time 49:32)



we see how classical controller will look like ok we will typically have this plant it is in the linear domain then there is a actuator sensor and controller now this is the expression for PID controller in the laplace form ok

this PID controllers are used widely for both linear and non linear system then they may not always lead to the stabilizing controller because you see this integral action here is in some sense destabilizing the system ok the derivative action is kind of you are introducing like more damping in to the system in integral action we are introducing one more pole here zero in the s plane we will talk about talk about these thing little bit more we actually see robot how to use this PID PD controller but you just remember that the integral action will have a destabilizing effect ok they are used integral action is used when you want that the steady state error to go to zero ok

we don't want any steady state error when it reaches some final position in that case you use integral action ok and then if you want to feel any oscillation like robot link vibrating and settling down do you want it directly to go to final position in that case you use derivatives of controller ok you just remember this

now what are the techniques that are available for doing this analysis so this slide list all the techniques

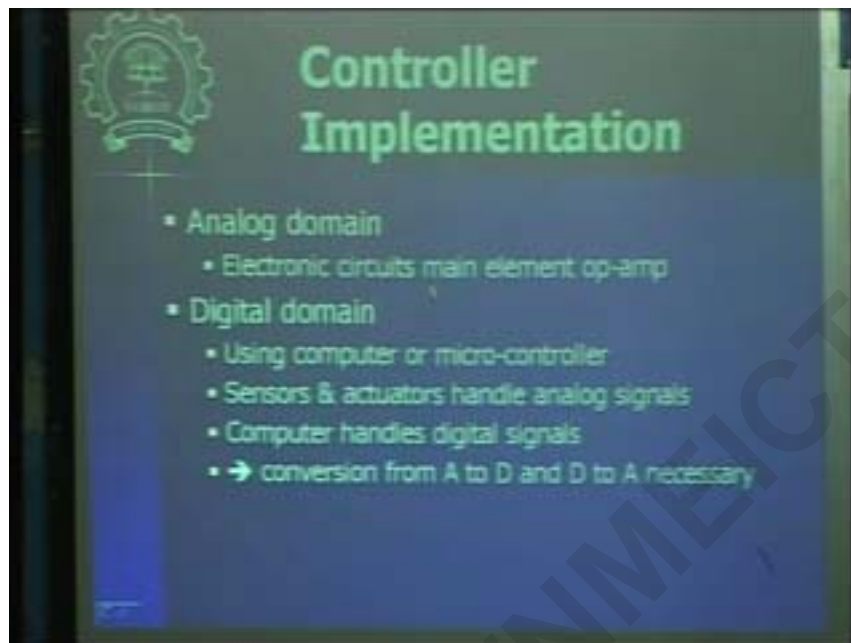
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most of the techniques actually to do the robot analysis the techniques in this first class they are classical techniques control techniques ok this is the root locus bode plot nyquist plot ok these are all graphical method for linear systems you cannot talk about root locus for the non linear systems so typically n degree of freedom manipulator you should not say what is this root locus that that is not a right question to ask because it is in general non linear system and you cannot talk about the root locus for the non linear system ok

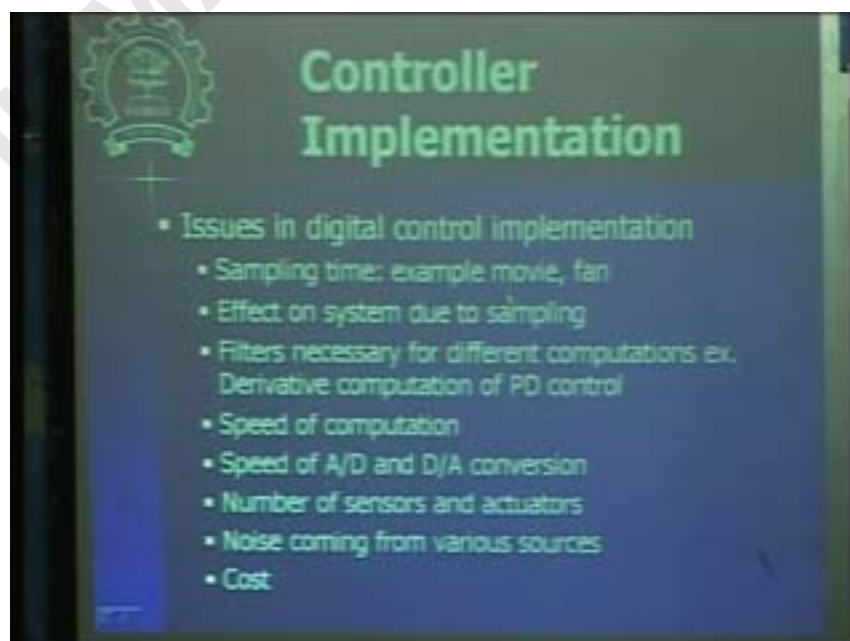
then other tools in non linear domain you see this slide Lyapunov method will use for control development so we will go through about some fundamentals about the Lyapunov method then there are singular perturbation tools then feedback linearization these are all non linear tools or tools in the non linear domain we will not able to have like a in a time to go through all these tools but you should be aware that such tools exist ok in case the tools that we have seen are not sufficient for you for your application of robot then you should be going for the other tools so you have to study them by yourself will not be covered in this course ok next we will see some fundamentals of control implementations so in the control implementation

(refer slide time 53:24)



in the analog domain is basically using the electronics circuits as i said earlier and then in the digital domain they are using computers or microcontroller in digital domain these sensors or actuators are also use which are in the analog domain ok

so sensors and actuators signals there in the analog domain you are processing your control in digital domain ok so you need this convergent to be done from analog to digital domain when you move from sensors to your controller you use the analog to digital domain converter and when you use from the controller to your actuator ok that time you use digital to analog converter ok so what are the issues in digital controlling implementations let us see them one by one (refer slide time 54:36)



first is the sampling time ok

so we will talk about this little bit in detail when we actually come to controlling implementation in the later classes but you should have observed this when you see movie sometimes like if there is car which is going in the movie its wheels they appear rotating in the opposite direction to opposite direction to direction the direction of the movement of the car ok

that is the thing that is the problem because of the sampling you choose ok this not pass enough pass sampling that has not been done so that like it appears like the motion is gets distorted like that your signal will also has get distorted if we don't do enough pass sampling of the signal ok

then if i done the system due to sampling this is gain a very important issue like because of the sampling what is its effect on the actual robotic system so if you are the best way like sample with right speed so that you don't get into the effects bad effects of the sampling on the system ok

then you need filters to filter out your sensor signals typically your sensor signals sensor signals will have the noise so you have to filter out the noise from the sensor signals so you have to use this filter either in the analog domain or in the digital domain then other issue is speed of computation like what ever microprocessor you are using is sufficient to compute your control law in a given sampling time ok in six sampling time your control computation should be over for the particular time today ok

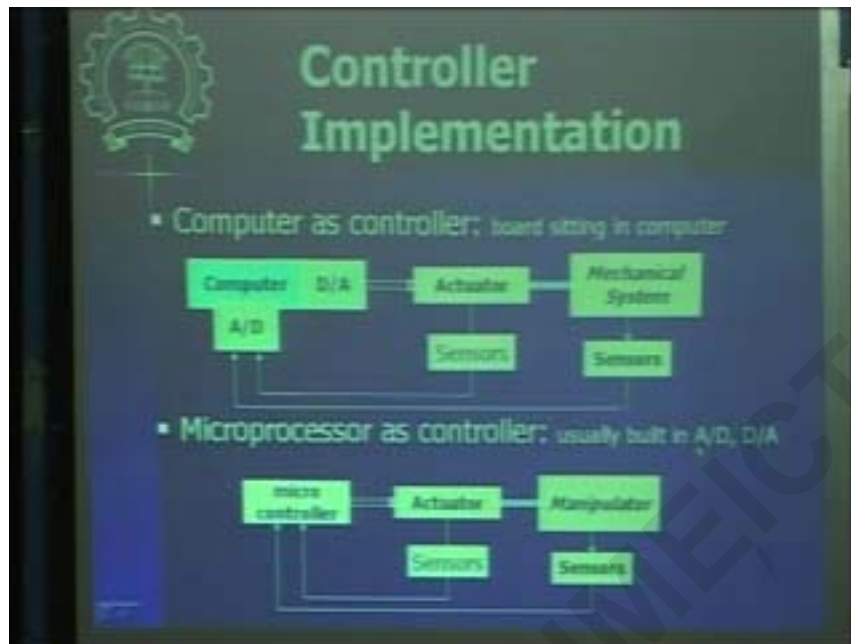
these are all the issues when we come to practical implementation you have to deal with these two if it is not sufficient then probably you will go for the high speed microprocessor to achieve this same task in the same controller ok or else change the controller to some low order or less complex controller that we can implement it easily with the what ever available hardware you have ok

then speed of A to D and D to A convergence this is not very important issue it will be usually much higher i mean much faster speed the convergent will happen as compared to the speed at which you are moving your manipulator ok these these time we are talking about in some some microseconds of convergence time ok

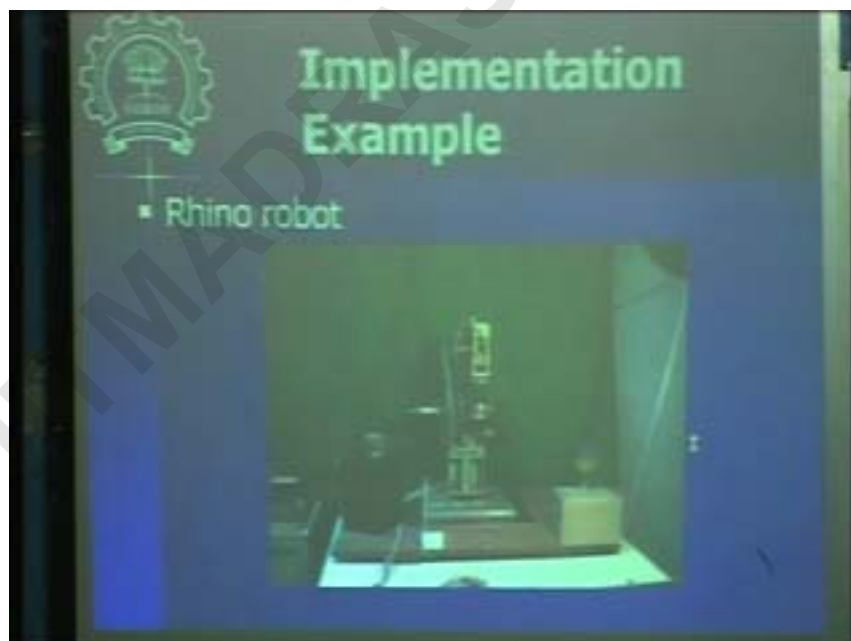
then number of sensors and actuators if you have number of sensors and actuators like your amount of processing is going to increase filtering and all other ok then the noise as I mentioned finally the most important is the cost ok you have to see what what cost you are entering to get this performance ok without consideration of cost nothing will accepted you have to give good consideration to the cost ok it is if it is at research level when you carrying out the application then you can like slightly be flexible about the cost issue but when it comes to the industrial application then the cost issue assume the lot of weight ok

this is the kind of the scheme that you see in the slide for the implementation in a digital domain there we have these two schemes possible you use computer or you use microcontroller then other thing remain the same then

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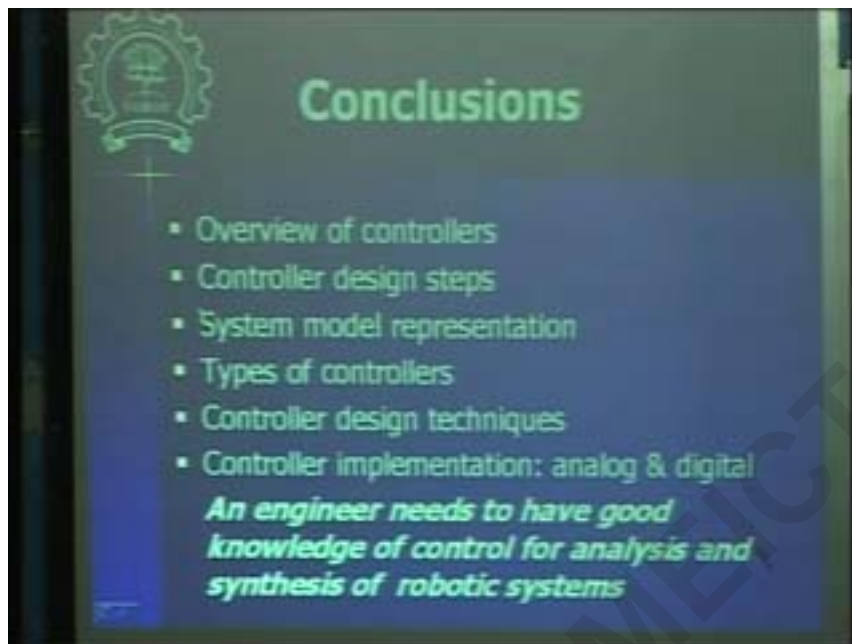


micro controller we will have usually built in A to D ok (refer slide time 58:56)



this is the little picture about the movie um of this Rhino robot performing some task but this is the interest of time may be we have look at it some later ok so what we have seen today is basically

( refer slide time 59:22 )



overview of controller then several design steps you need to follow for development of controller types of controller we have looked at and then some issues in the implementation of control basically regarding to controller implementation is analog and digital domain and some issues regarding digital domain implementations ok so it is very important to have the fundamental understanding of the controller right for any robotic application if we realize ok so we will stop here for this class and in the next class we will continue the discussion with the controller ok thank you.