ROBOTICS
Prof.P.S.Gandhi
Dept. of Mechanical Engineering
IIT Bombay
Lecture No-31
Robot dynamics and Control

Good morning. We will start the module on the robot dynamics and controls today. This is lecture number 1 of that module and this is a recap of the previous things what you have covered so far in the class. (refer slide time 01:15)

First we started with the robot anatomy then you saw their actuators, sensors and controllers then how to specify the task of the robot if you are given the specific task how to convert that in a mathematical form.

Then you studied little bit about robot analysis, basically DH notations, dynamitic analysis, forward and inverse dynamitic then we saw little bit about motion planning and obstacle avoidance then last module was on robot vision where you saw something on image processing some segmentation algorithms and things like that. Now today we will start with the robot dynamics and control.

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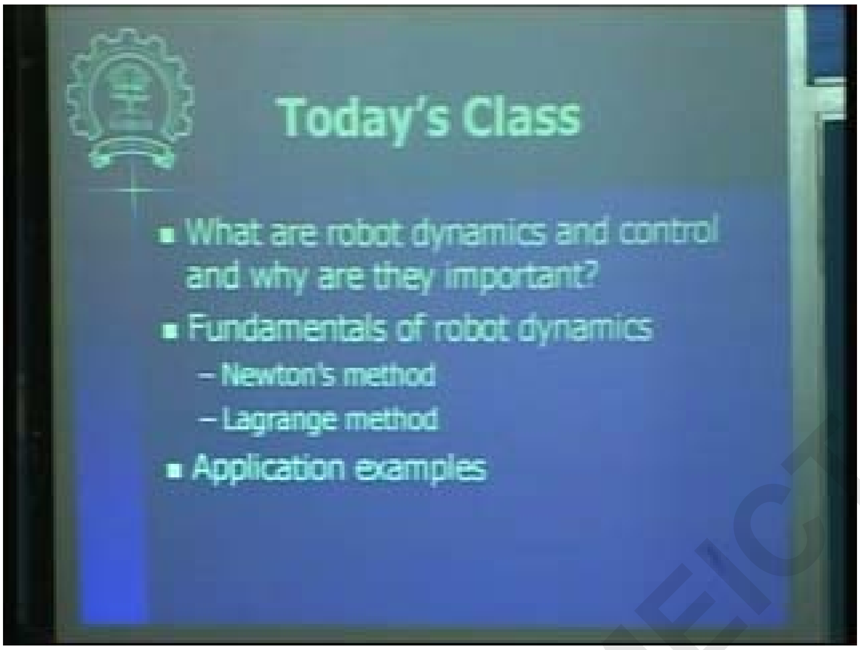
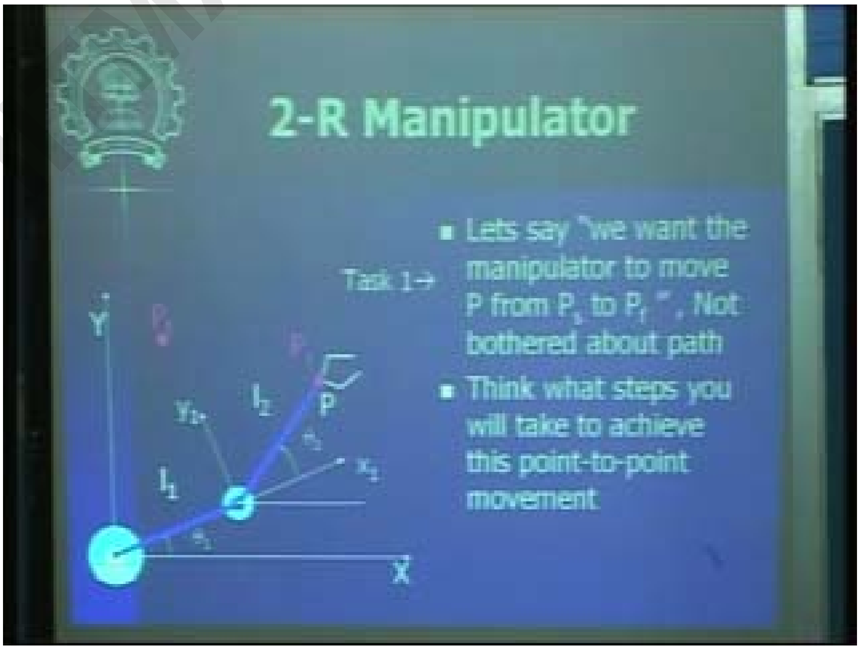
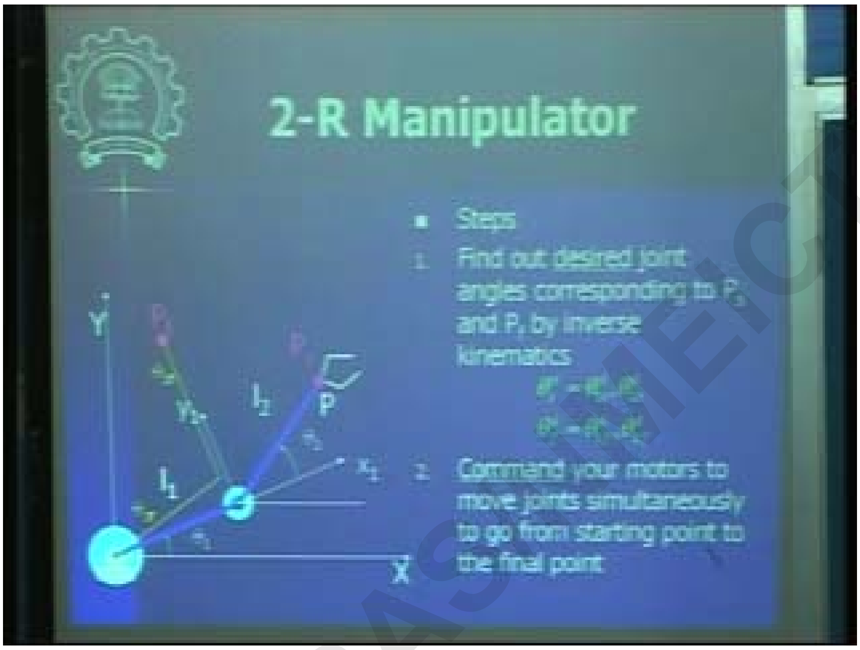


In today's class what we are going to see is what is robot dynamics and control, why it is important to study the robot dynamics and control and then what are the fundamentals of dynamics we will study first and then we will move on to some application examples. So that is what we are going to cover in today's class.

So let us think of this example 2-R Manipulator. you have here these are the two links link number 1 and 2 these are the two links of the manipulator, this is a revolute joint and this is another revolute joint so this is 2-R manipulator. So revolute joint corresponds to this r. and then you have one motor a bigger motor which is carrying which is moving this link and a smaller motor which is moving this link. And then at the end there is an end defector. Now let us say we have a task that we want the manipulator to move point P from Ps to Pf. (refer slide time 04:15)



So we are not really bothered about what is the path taken to go from point Ps to Pf but we just want the final position Pf to be reached. So this is the task number 1 we will consider then we will consider some other task also then I will demonstrate why you need to really worry about this robot dynamics and control. So we want this task to be performed. So let us just think about what are the steps you will follow to achieve this task. Any guesses? Could you could you see what are the steps you will follow based on your inverse dynamitic analysis? (refer slide time 05:41)



So you will find at the first step that will follow is to find out the desired joint angles corresponding to these points Pf and Ps by using your inverse dynamitic analysis. You have studied the rotation metrosis and how to do this position analysis so you do the inverse analysis and then find out these angles. Now in this case like your theta 1 and theta 2 in this position will correspond to theta 1 and theta 2 at the start and then you see here for this final position corresponding to this position we will find theta1f and theta2f which are corresponding to this position . So these are the equations I mean these are the angles corresponding to the initial and the final position. then you will command your motors to move the joints to go simultaneously from starting point to final point, not bothered about the path that you will follow. You see that now there are some of the questions: what is
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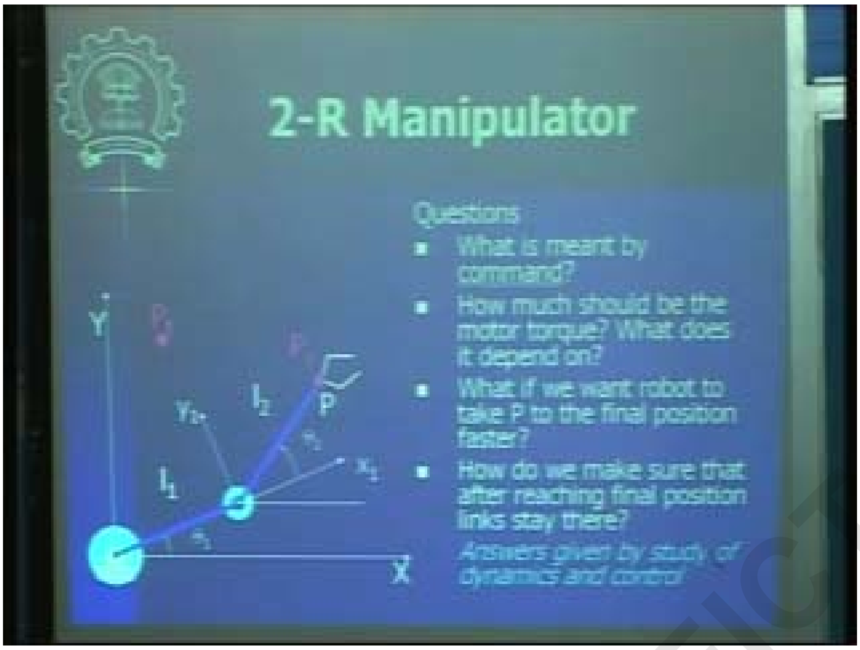

meant by really this command what I wrote as a command what is meant by a command? So this can be like some kind of voltage given to the motor or some kind of a torque given to the motor. But what is the torque, what algorithm you will follow to really apply this torque? Then how much I should have this torque like what are the ranges what is the value? What all it depends upon? What if we want to take this point P from the starting point to the final point in a faster way? So naturally you will see that we need to have more torque if we want to move faster. So we see that like if we specify this faster movement then because of the dynamic effects like we have some torques to be overcome and because of that like you need more torques to be applied to the motor or more voltages to be given to the motor. So in general the power requirement will be higher. Now, after reaching the final position how do we make sure that the links stay there, they will not move again from that final position. So these are all the questions that will be answered by the study of dynamics and control. So as you see here there are two parts; one is dynamics and the other is control.

So now let us consider another example, is again the 2-R manipulator.

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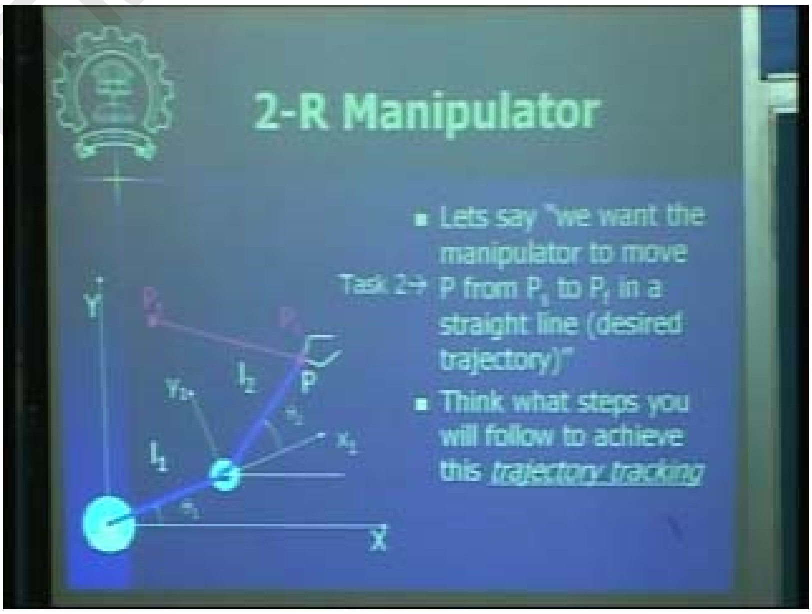

But now the task specified is different. Let us say we have to move the manipulator from Ps to Pf in a straight line. Do you understand the difference between the previous task and this task? So we want to move these in a straight line fashion as you can see in the screen. So that is the desired trajectory we want to follow.

So, in the previous task we were not bothered whether it goes along the straight line or it goes along any other curve. We just wanted it to move from one point to the other point. In this case we want it to go in a straight line fashion. So now again you can think about what are the steps that you will follow to achieve these tasks, what are the different differences in the previous and these tasks. So what you will do typically is you will find the desired theta in terms of time theta1 and theta2 as a desired time trajectory so they are the functions of time here. You can see their functions of time. And then again you will command your motors to move the joints
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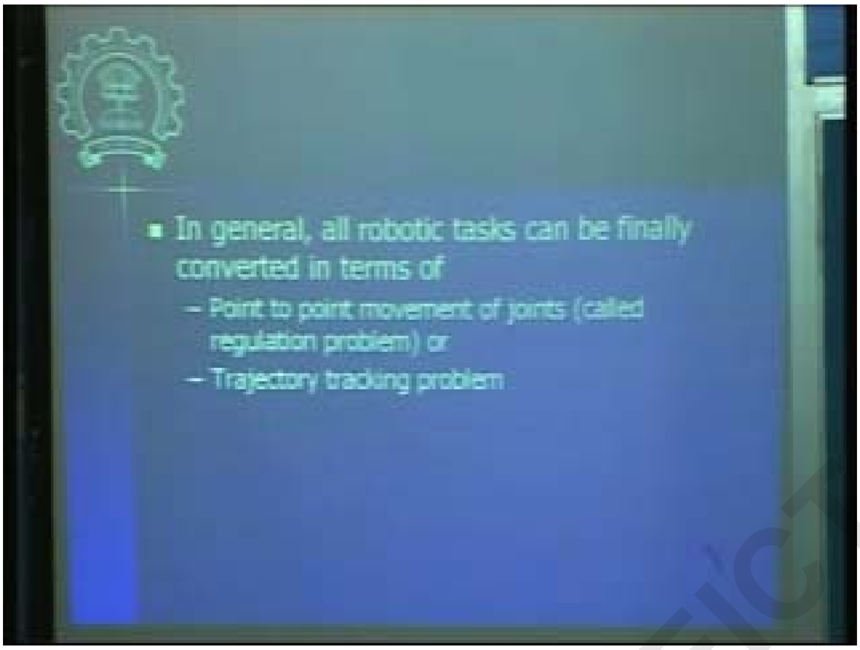
simultaneously to follow the desired position trajectory now. In the previous case you were not bothered about the trajectory, you were not bothered about the time like what time theta1 should be this value and theta2 should be this value. So here you will have a trajectory to follow. So this problem is different from the previous problem.

And again, here we can ask the same questions. What is meant by command and then how much should be the motor torque, what should be the motor torque to take it from the starting position to the final position in a straight line fashion and then what if we want to take to this position faster like we want to move along this straight line trajectory in a faster way, how much deviation from the straight line will occur is some of the questions that you will ask. Now, how much will be this deviation will depends upon so many factors. One of them will be really your control algorithm, what control algorithm you will use. but you can appreciate that to again find out what control you have to apply in this case or what torques you have to apply in this case you will have to study dynamics and control. so you have to study dynamics and control to get desired performance, desired task to be performed by your robot in a nice fashion. Now let us get into some fundamentals. (refer slide time 11:18)



in general with this example we can see that all robot tasks can be finally converted in terms of either point to point movement of the joints which is called regulation problem or a trajectory tracking problem. Now think of any application that comes in your mind say your robot is needed for painting purpose. You have to paint some car or something. So, on the surface it should move from one point to another point. And then like depending upon the motion trajectory of the spray or a spray nozzle you have to have some joint angles evaluated and then you have to command the motor joints to move along those desired trajectories. So again that problem is converted into a trajectory tracking problem. So any task whatsoever you imagine for a robot to perform it has to finally get converted into the in terms of some joint commands either a desired trajectory tracking type of a problem or a regulation problem.

So you have to move either from one point to another point like in a fashion not bothered about the trajectory and in the other case you have to move from the same point or like some from different point to follow a particular trajectory. So these are the two important classes of the problem we will come across in a robot manipulator. so any task which is specified for the robot will get finally converted into one of these two kinds of problem.

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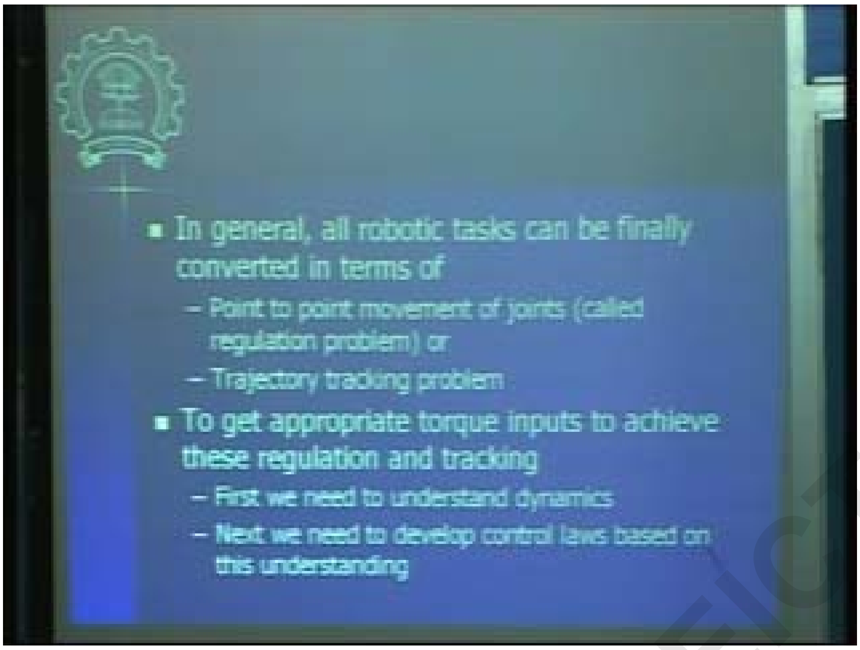
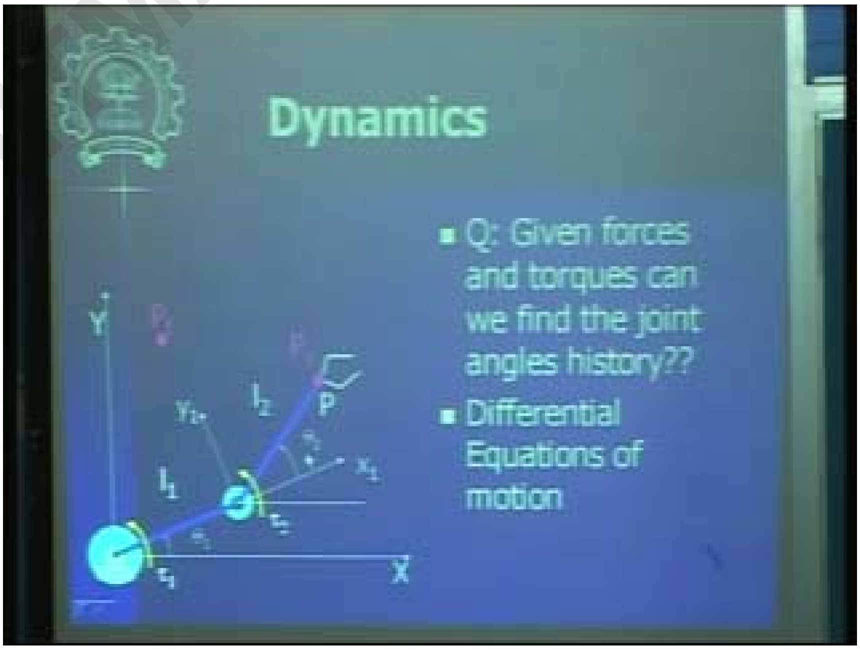
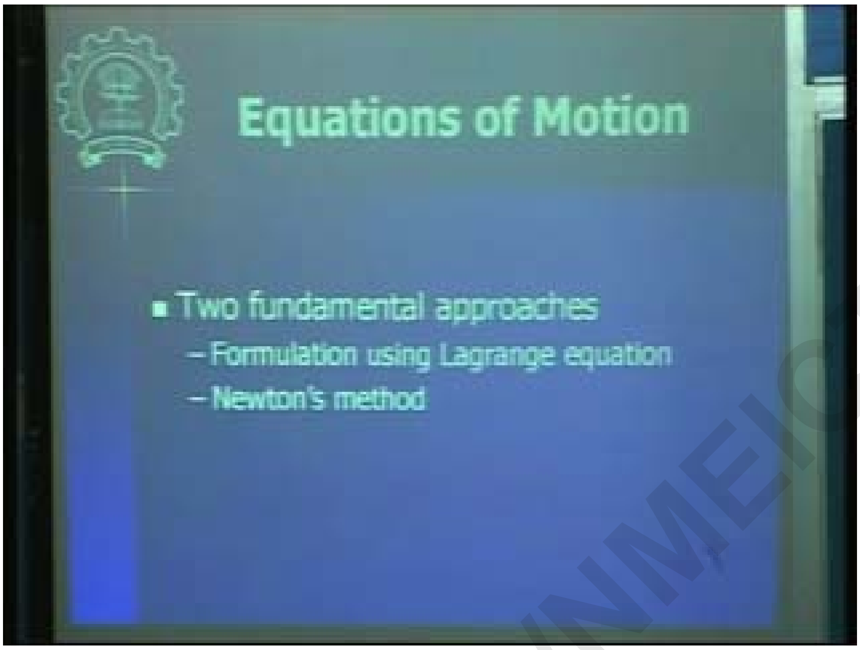


Do you see that? Now to get appropriate torque inputs to achieve these regulation and tracking tasks we first need to understand what is dynamics. And next we need to develop control laws or control algorithms. These are some kinds of algorithms which will specify your torque values in terms of whatever sensor feedbacks you are getting. And you have to use understanding of the dynamics to develop these control laws. So next we will move to the understanding of what is dynamics? So we will consider again the same example 2-R manipulator.

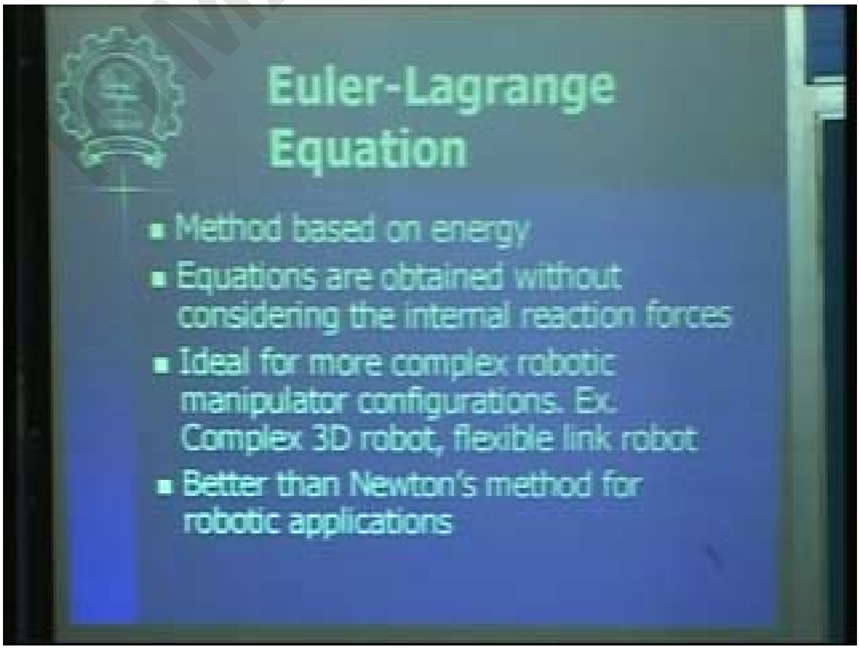
In this case you can see that there are these torque one tow1 over here and torque tow2 over here, this torque is applied by the motor. Now the question that we ask is given the forces and torques can we find out the joint angle history. Or in this particular case given these tow1 and tow2 is tow1 and tow2 can you find out angle theta1 (refer slide 14:49)



and theta2? Now to do that, we need to have the differential equations of the motion for this manipulator. So the whole idea is to get these equations of motion. So now will we will move from what we want in the actual robot to how it is represented mathematically. So we have to represent the robot dynamics in a mathematical way, so how do we do that? (refer slide 15:18)



There are two fundamental approaches for doing that. One is formulation using Lagrange equation and the other is Newton's method. You are familiar with this Newton's method right? You all studied so far in your curriculum this Newton's method. Now we will go in the details of formulation using Lagrange formulation in the next couple of slides and then Newton's method I will just give you the steps how do we approach to get these equations of motions using Newton's method for a particular case. So we will study the Lagrange formulation in detail now. For that (refer slide 18:15)



first thing is this method is based on the energy, next the equations in this method are obtained without considering the internal forces. This is a very important point for the Lagrange formulation. All the forces are reaction forces they are neglected in this case you do not have to worry about them. In the Newton's method you had to first do the free body diagram and then you consider this balance of the forces by Newton's method and then you are obtaining the equations of motion. But in Lagrange formulation we do not bother about the internal forces or the reaction forces. Next is, this method is ideal for more complex robotic manipulator configurations. you imagine how if we have to apply for a 3D complex robot manipulator the Newton's method there are many forces that you have to imagine and you have to get your force free body diagrams correct. but in the Lagrange formulation you avoid all those complications. Suppose other example is like say if you have a flexible manipulator there are flexibilities in the manipulator and then you want to find out the equations of motion in that case it becomes very easy to get the equations of motion based on the Lagrange formulation. And Newton's method application becomes very tough for flexible manipulator kind of problems. So in numeral this method is better than the Newton's method for robotic applications. Next we will see what is this method, what do we do in the Euler Lagrange equation. So this is the basic form of the equation. You can see it is d/dt of towl/towq. – towL/towq – towi. Here L is legrangian and it is given by kinetic energy minus potential energy so KE and PE are basically kinetic energy and potential energy. And this towi that you see in the equation is the external force in the direction of generalized coordinate qi so qi is the

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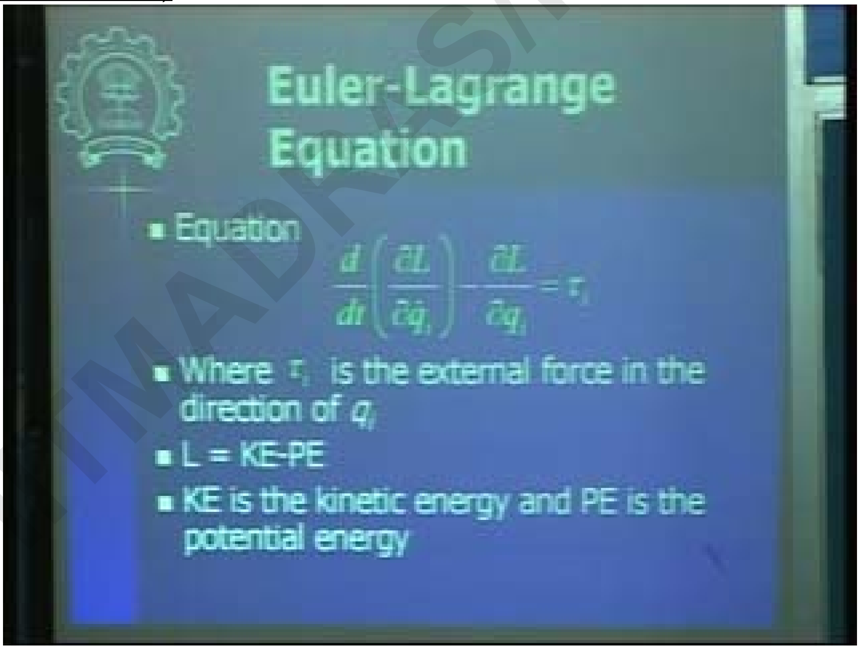
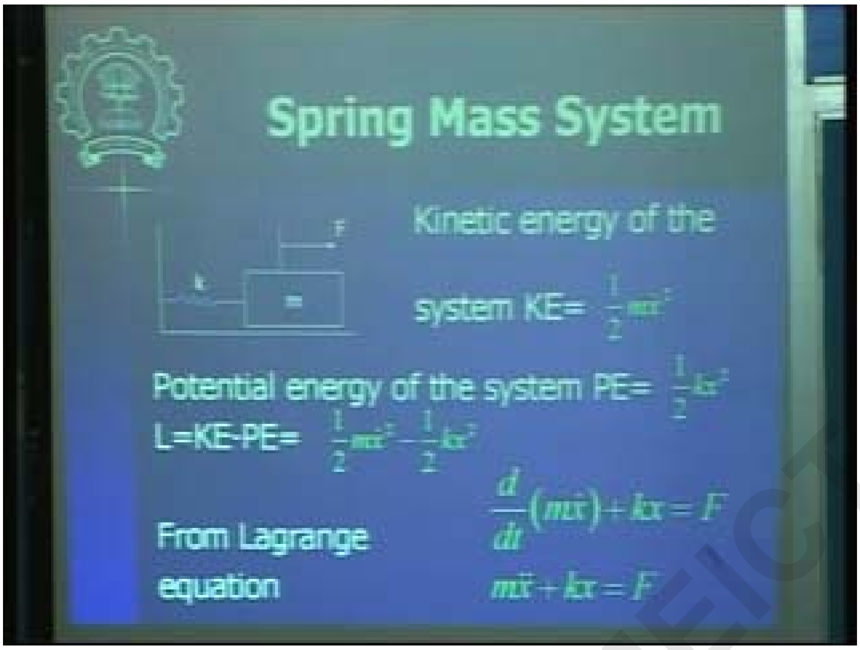


generalized coordinate. So typically qi will correspond to the number of degrees of your robotic manipulator. So if you have a 2 link manipulator you will have two qi's typically they will be theta1 and theta2 for our manipulator that we have seen the example of. (refer slide 20:04)



Next, we will take some examples and apply this Lagrange formulation some example problems and then see the beauty of how you get the equations of dynamics. So you can see here the dynamitic system kinetic energy is given by this 1/2mx. square for this simple spring mass system. Then potential energy is given by 1/2kx square system here the lagrangian L will be given by this kinetic energy minus

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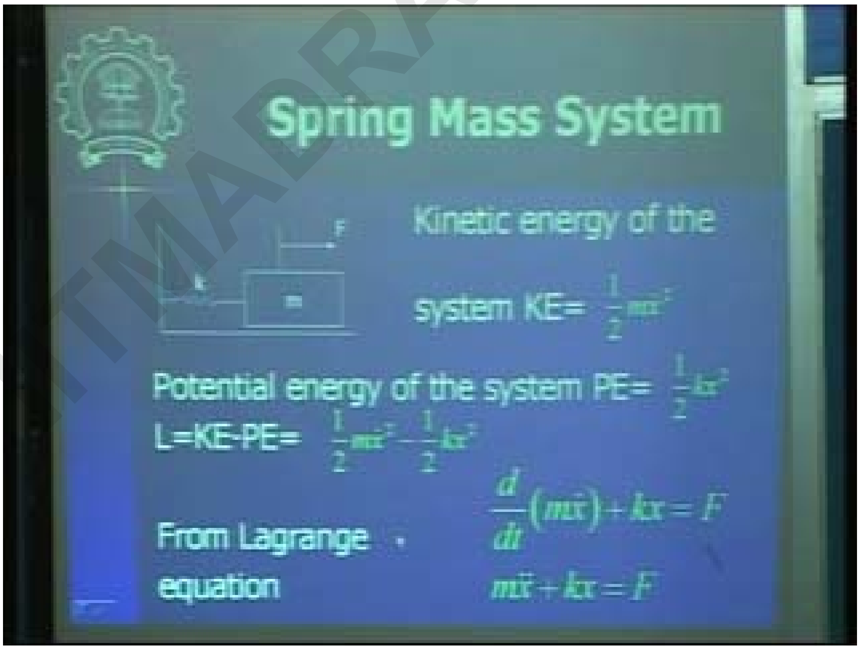
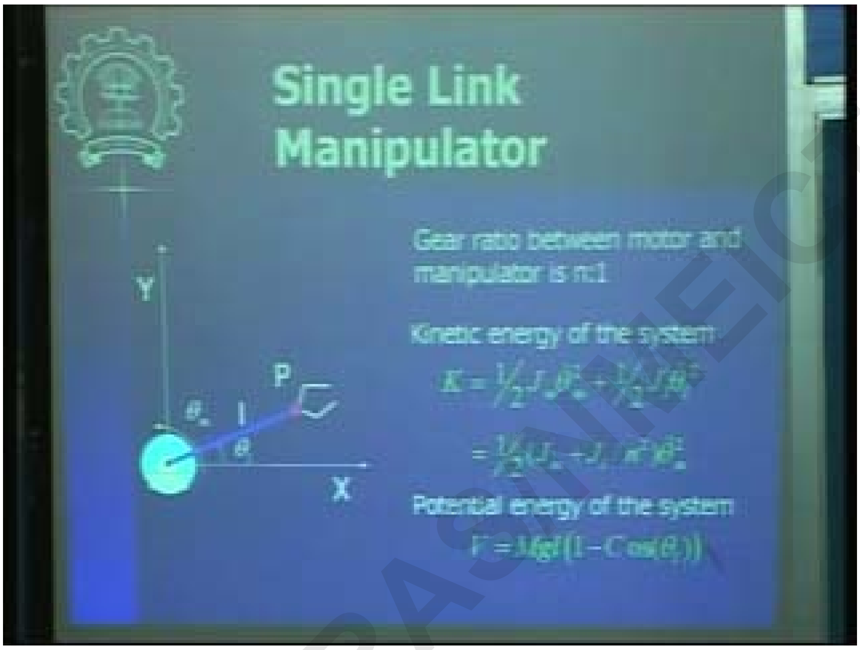


potential energy and your LaGrange equation you can see now is d/dt of towL/towq. Now q. in this case is x. so you differentiate this partially with respect to x. and then you will get this mx. here so d/dt of mx. minus you differentiate this term again because you see that you are taking here towL/dowx and your potential energy is only dependent on x, kinetic energy is independent of x. So you will take only this term this part of the equation which is towL/towx so you get this Kx and that is equal to F which is a force applied in the direction of x.

So that is the way you write this Lagrange equation and you simplify by taking this derivative and you will find out your standard spring mass equation is obtained. So you have not drawn the free body diagram in this case. You have not considered any forces force balance etc. directly from the energy you have got the final equation. So this is the way you will get the equations for robot manipulator R-2. Now let us consider the second example. (refer slide time 22:15)

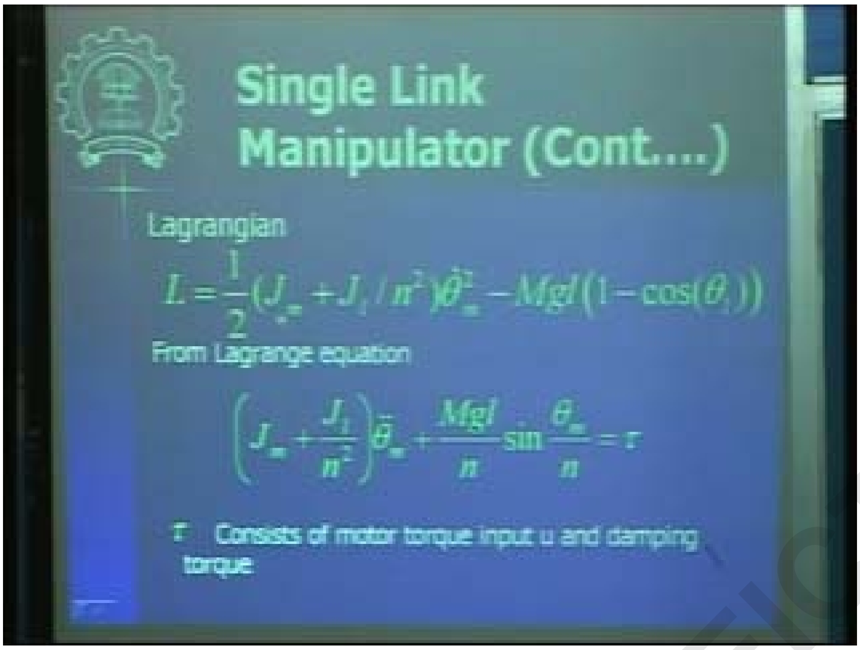


We have the other robotic manipulator but a single link manipulator you can see here that we have the single link connected to the motor to a gear box now. So in general theta m and theta l they are different so theta m may be for this particular theta l you get larger theta m

So there is a gear reduction in between. So the gear reduction n the ratio is n:1 so now can you write down the expression for the kinetic energy? You can see that kinetic energy of this manipulator is 1/2Jm theta. m square + 1/2 jl theta. l square.

Now you need to see that there is only one generalized coordinate. You may use it either theta l or theta m because they are both dependent on each other. so you need to express theta l or theta m in terms of other variable and then you should use it for the equations. So that is a very important point to note. Here theta m in this case is a generalized coordinate we have considered. So theta l needs to be expressed in terms of theta m.

So by doing that you get this equation. So now, 1/2Jm + Jl/n square theta. m square. So theta. m is the generalized coordinate. Now potential energy is the next term so you will get potential energy by Mg × l × 1 – cosine theta l. Now let us use our Lagrange equation. (refer slide time 24:33)



First form this lagrangian L = kinetic energy – potential energy. And then you apply the lagrangian equation which is this equation and by applying this equation you will get these terms now. So you see that when I take derivative of l with respect to theta m. partial derivative towl/tow theta m. I will get contribution from this term and that when differentiated with time I will get this term. Now next contribution is dowl/dow theta mk so if I differentiate this l with respect to theta m partially this term will not contribute anything and this term will contribute this part.

This is how you get the equation. And then tow is a external force. We can have tow consisting of like motor input u and the damping torque. So you can see now what we have as tow. So this tow has u part and then this is the damping part. So these are the damping coefficients Bm and Bl on load on motor side and the load side and then you have to take this divided by n square because

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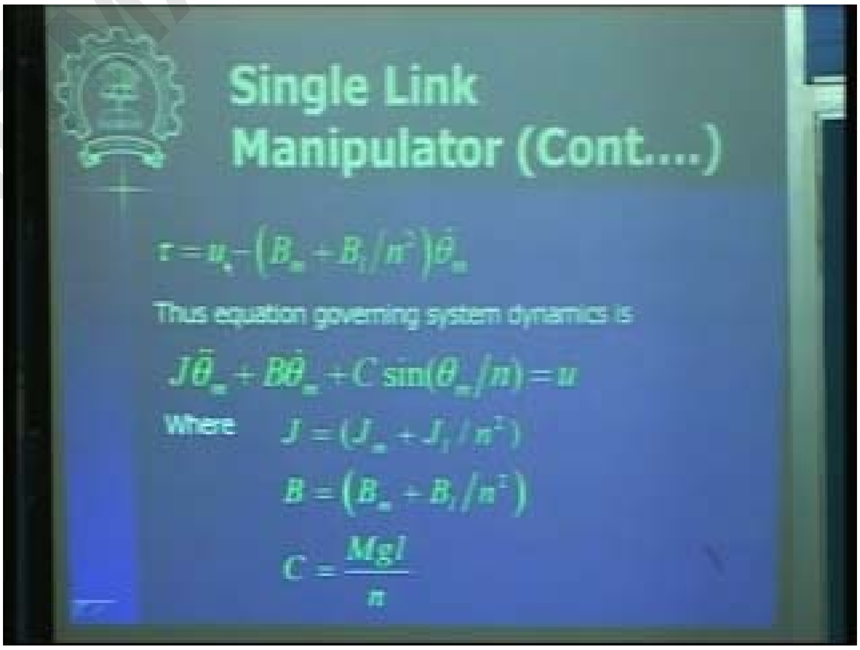


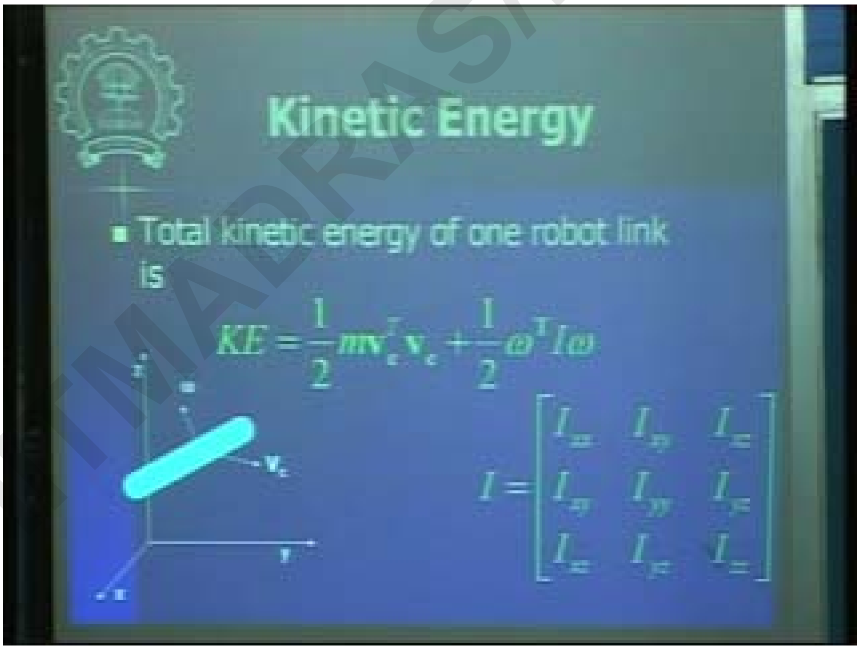
it is Bl × theta. l is the damping factor. So you will get in general this equation for tow. now the second part is, you see that there are n square terms here, Bl × theta l is a torque so where is the second term coming from is basically when you transfer that torque to the motor side you have to again divide by another L because of that you get this n square term.

So in general your equation then can be written as, you have J times theta m double dot + B times theta m. + capital C times sin of theta l which is theta m/n that is = u. now J here is the inertia which is transferred to the motor side.

So you can this Jm + Jl/n square term. And B is the damping which is transferred to the motor side so you will get Bm + Bl/n square term. And then this capital C is given by this gravity force.

Now let us again come to our basic equation Lagrange formulation. Now what we want to do is to extend this understanding so that we are able to apply to our robot manipulator, we want to use this understanding for the application of our robot manipulator.

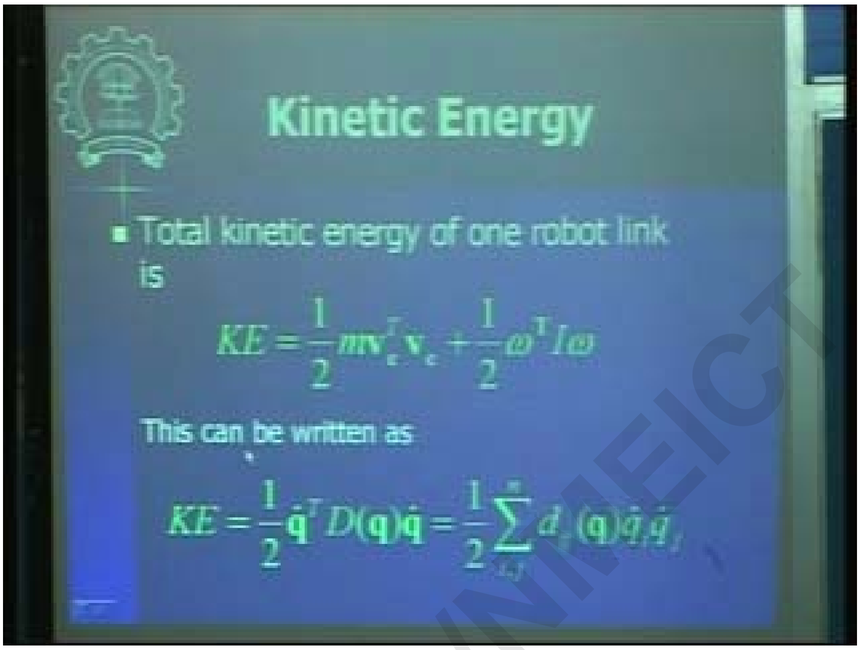
So, for that you just imagine what you need for applying these techniques to the robot manipulator. So, first thing is how to find out the kinetic energy of your robot. So what is the general formula for the kinetic energy. You see now that the total kinetic energy of the robot link will be given by this formula. For that matter this is valid for any rigid body. Any rigid body in space will have this formula for kinetic energy. So it consists of two parts you can see, the first part is because of the translation (refer slide 30:07)



and then the second part is because of the rotation. Now this translation part has this Vc term. This Vc is nothing but the velocity of the rigid body of the robot link. This Vc vector is given here. So, all these bold quantities are vector quantities.

So this is our familiar 1/2mv square kind of a term which is generalized for the rigid body case. Now, what is the rotational part? The rotational part consists of 1/2 I omega kind of a term but in this case you will observe that your I which is the inertia matrix is in general given by all these elements.

So you have these diagonal entries for inertia of the rigid body about the axis of the coordinate frame and then these are the cross coupling terms. And then this omega is a vector of the angular velocity of this rigid body or the rigid link of the body. (refer slide time 32:03)



Now this kinetic energy can further be expressed as this part; you have kinetic energy is q. transpose Dq q. Now this q's are generalized coordinates. So you see here that we have expressed these velocities of the cg of robot link or rigid body in terms of the generalized coordinates and then this D matrix as all the terms corresponding to the translational part and the rotational part also.

So in general this will be the expression for the kinetic energy of the robot link. And then the total energy you will find it as the summation of all these kinetic energies. Now this can further be written in terms of the elements of this D matrix as Dij times q. i q. j. So you see that this is a quadratic form q. transpose Dq q. this d is in general n/n matrix for all n generalized coordinates of the robot. And then this is the expression which is the expanded form of the expression for this quadratic form. So you can see that the D matrix is symmetric in nature. In general you will have for the robot manipulator that you are talking about this D matrix will be always positive definite symmetric matrix. So this D is positive definite symmetric matrix. And its quadratic form can be expanded as 1/2Dij they in general depend upon q and q. i q. j.

So next we will see how we can manipulate these terms so as to get now in general lagrangian equation. So now we want to use lagrange equation with this form of the D or this form of a kinetic energy, so how do we do that? So now we will first construct our lagrangian L = KE – PE. So I
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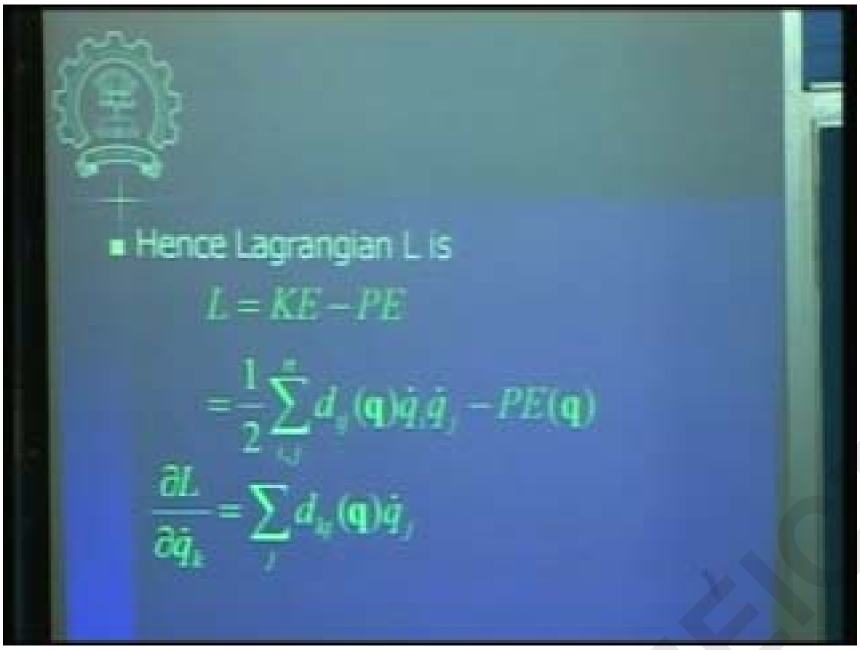
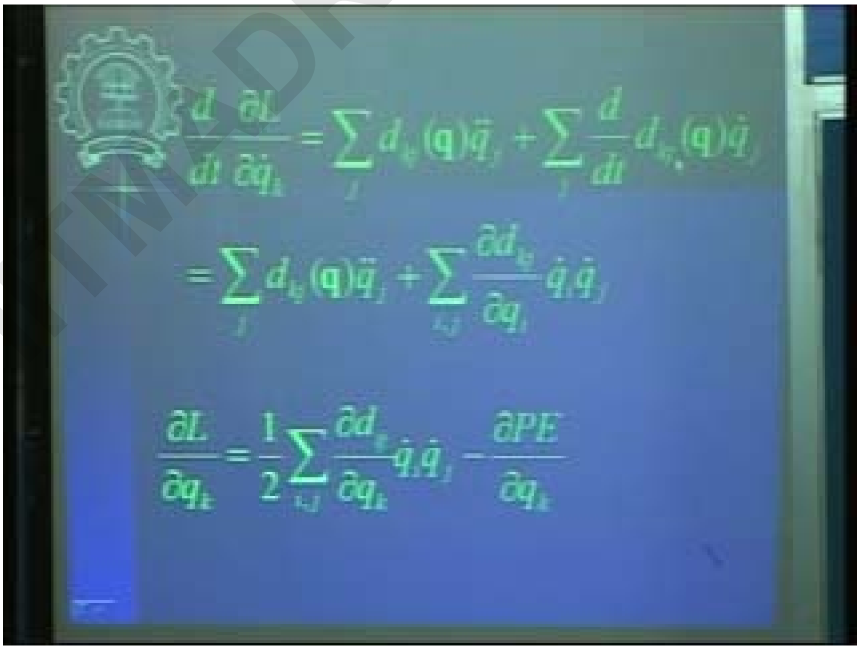


have substituted here for my kinetic energy the term that I obtained in the previous equation, you can see the same term over here. And then PE is the potential energy which will in general depend upon q. It is not dependent upon the q. So it is only a function of your generalized coordinates not their derivatives. Now from there you will get now dowl/dowq. if you differentiate this l with respect to q. you will find out it can be expressed as this value. Did you see that? So only one q. is remaining here other q. is gone (refer slide time 36:00)

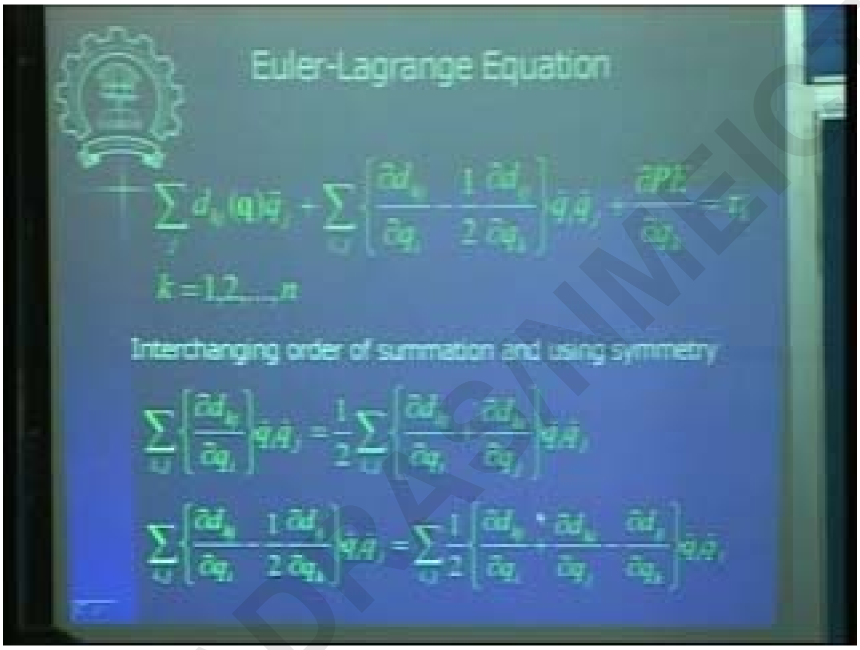


now because of the derivatives. Next we have to take d/dt of dowl/dowq.

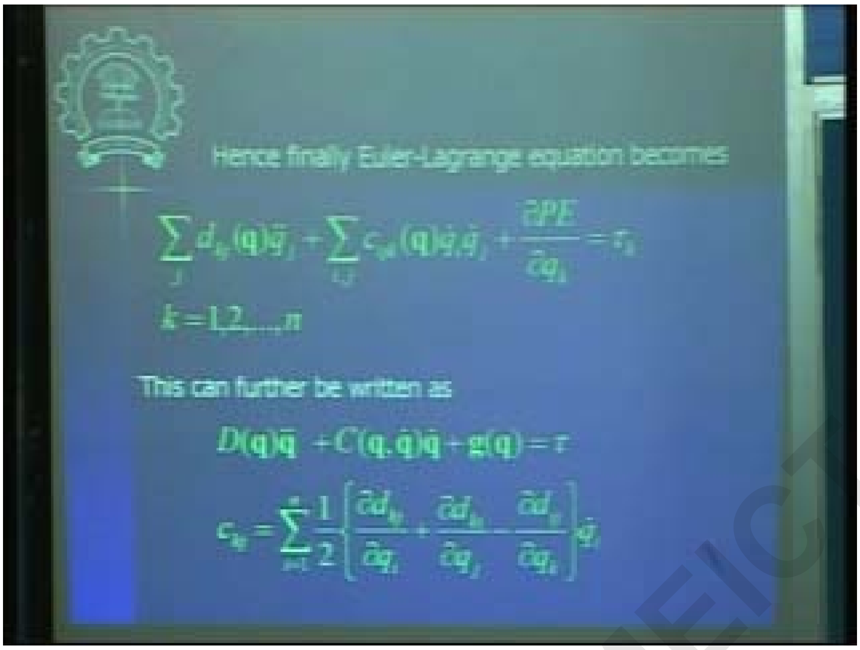
So you take this derivative and you will find that you get this term. Now you will observe that this is a product of two terms q. and Dkj. So you have to apply this derivative of the product then you will get first term as q double dot j times Dkj. And the second term you have to again apply d/dt to Dkj and multiply it with q. j. Now you can see that I can simplify these further, this term remains as it is, the first term will remain as it is, the second term I can differentiate first partially with respect to qi and then dqi/dt which is qi.

Now this will appear as a summation over i also because there are many number of coordinates so you will have to in general carry out summation over i from i going to 1 to 10. So this is dow Dkj/dow qi × qi. that is this d/dt ki.

So this is a first term in the Lagrange equation. Now the second term which is dowl/dowqk will be simplified as these. Again we use the same thing here and then we get this equation. So you can see from the previous form of the kinetic energy then we differentiate partially with respect to qi we get this part here and this part is our potential energy contribution. (refer slide time 39:02)



So our total equation now taking both the parts together can be written in this form. They have just taken the first part which is this up to this point the first part and it is added with the second part which is basically this term combined with this term. You see this simplification, how we have done that, we have just combined the terms involving qi. and qj. You have combined the terms containing qi. and qj. together this single term from both the parts and we have formed this equation. Now we will interchange the order of summation and use the symmetry of this D matrix. And then from there we will get this part like the first part which is del Dkj/del qi × with qi. and qj. can be split into these two terms by using the symmetry of the matrices. The matrix D is a symmetry matrix so I have this Kj and Ki splitting here and then I will take a derivative with respect to qi and qj here and then I will get this final term. Now the next term, so this local term summation of this part that can be written now as summation of these three parts, summation over ij because we have simplified the first term in this fashion we get this total summation as this, you see what is happening. (refer slide time 42:08)



Next we will get the final equations in this fashion. So you get here the contribution in terms of acceleration terms.

So, these acceleration terms you can see they are all basically because of the parts in the kinetic energy expression, Dij where the parts in the kinetic energy expression. So they are the elements of the kinetic energy matrix D. You remember how we expressed our kinetic energy because basically 1/2q. transpose D times q. and then this D matrix has this Dij terms which is appearing in the acceleration as a term to multiply the acceleration of generalized coordinates.

So you see in this equation the term Dkj multiplied by acceleration term is directly got from your kinetic energy. So you see that once you get your kinetic energy you can directly write this first term. So now what we are aiming at is writing this equation directly without considering these differentiations in the lagrangian formulation.

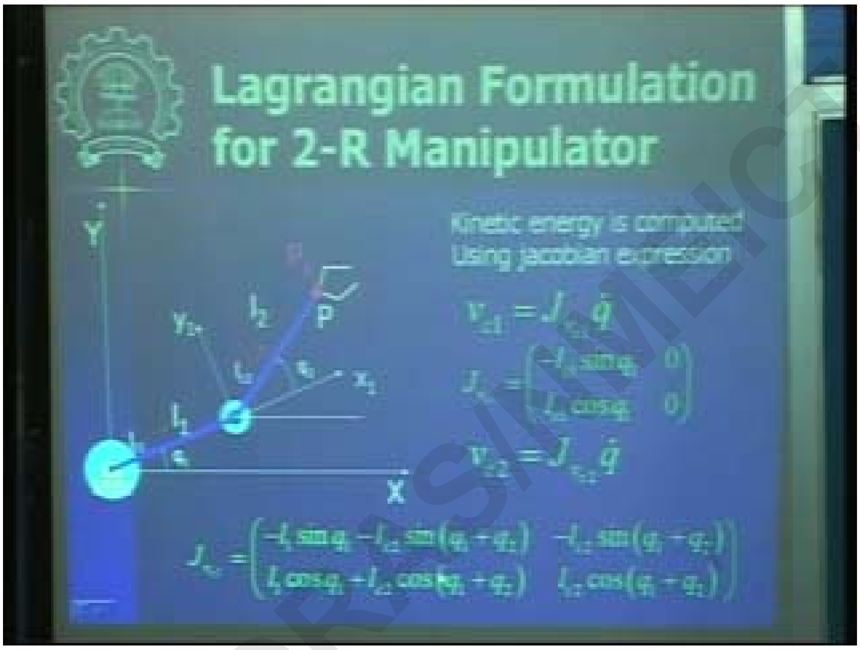
To do that first part of your equation you can directly see is coming from the kinetic energy contribution term basically this Dij matrix or Dkj in this case. Now, the next term can be expressed as this summation. They are basically separated out q. i q. j terms multiplied by some C elements. And these C elements we have just derived and they are expressed to be of this form.

So in the previous slide we had seen that your C elements are of this form. So the final equations of your manipulator are obtained in this form. So you have this standard form of the equations of the manipulator dq q double dot + Cq q. l in general will depend on both q and q . but not q double dot times q. + g times q which is the gravity vector that will be equal to tow. Now let us see what is the physical meaning of each of these terms. These D terms are basically inertia terms and this is a inertia because of the translational acceleration q double dot.

Then your centrifugal acceleration terms or centripetal acceleration terms and coriolis terms are combined in this term C the term Cq q. the term C q q. and then your gravity vector is in this D term and that will be equal to your external forces. Now these external forces can be torque applied by the motor or it can be damping torque factor. The other thing to notice here is that your C terms are dependent only on D matrix. So once you know your D matrix you can use this formula to get your C terms.

So it all boils down only to find what is the kinetic energy and potential energy and then by directly using this formula you can find out the different parts of the Lagrange equation or the equation of the dynamics of your manipulator.

So you do not have to carry out all this differentiation every time. You can just you can find out this D matrix for your energy and then use that in this form to get the final expression for the equation. So that reduces your task a lot. So now what matters is to find the kinetic energy and the potential energy of the system that we are talking about. (refer slide time 48:27)



So let us now see some examples, again our 2-R manipulator example. We apply these concepts and then find out the equations of motion of this manipulator.

so for this manipulator you can see that the kinetic energy here, you have studied the Jacobean in the previous classes so we can make use of Jacobean for finding out the velocity of CG of these links. So velocity of the Cg for the first link will be given by this Jacobean times q. vector and then this velocity is again Jacobean of that link times of q. the velocity vector. And these Jacobeans are given with these expressions. Now you can see that velocity C1 which is this CG of link one will be given as – l c1 sine q1 × q1.

If I expand this in terms of its components and then the Y component of the velocity will be lc1 cosine q1 × q1. You know that the velocity of this link CG is in this direction and it will have in general x and y components the y component is coming out to be positive and x component is coming out to be negative, you can see here.

Like that you can analyze these expressions and bring down into the different velocity component which are constructing the velocity of link two. So you will get these velocities first to find out now the kinetic energy you make use of these velocities so you will find now this is the expression for translational part of the kinetic energy based on the velocity expression that we have seen in the previous case. So in the general fashion you can write this velocity as Jacobean transpose multiplied by Jacobean into mass and then this is m2 again multiplied with the Jacobean transpose and Jacobean and then this matrix will be the part of the D matrix contributed by the translational energy. Now how about the rotational part of the energy? We will see that, in the rotational part you have to find out from the omega. So what is

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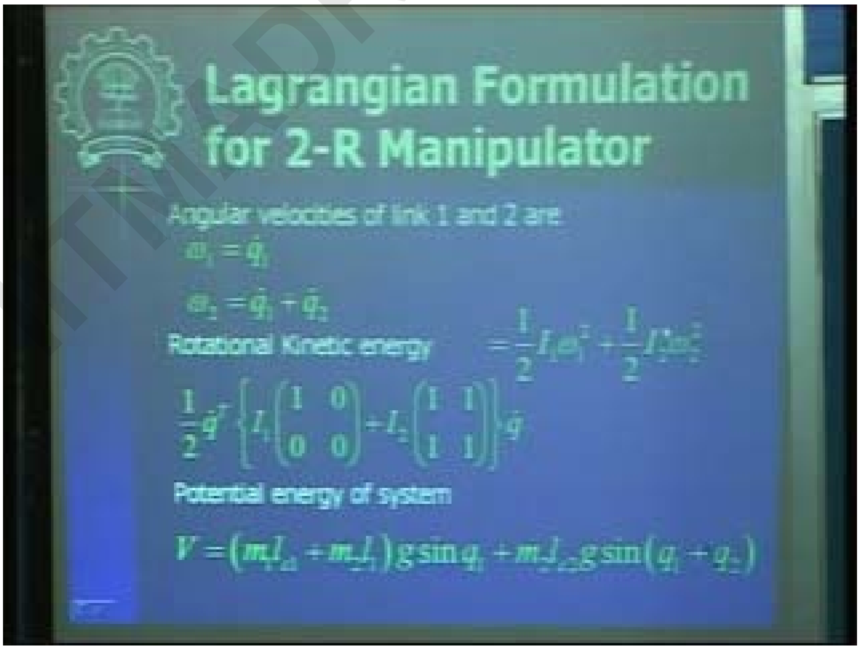
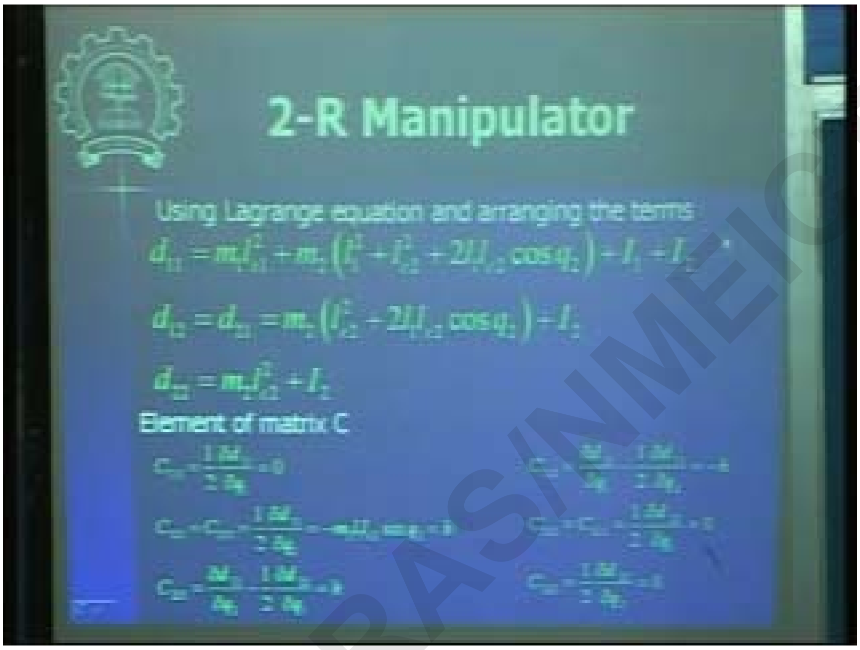


Omega 1? Omega 1 is q1. and omega 2 is q1. + q2. So rotational kinetic energy will be 1/2 I1× omega 1 square + 1/2 I 2 × omega 2 square and you plug in these values for omega 1 and omega 2 and you will get this expression for rotational kinetic energy. We have expressed in the same form as the previous form which is q1. transpose multiplied by some matrix into q1.

So that way it will be very easy now to identify the terms in the D matrix by taking summation of this part and this part in the translational case. So you sum up these two and you will get your D matrix. Now potential energy of a system is given by these expressions. These we can find out by seeing where are the locations of CG and in multiplying by the corresponding mass and gravity term. So height of CG with respect to your reference you take for first mass and second mass and then you will get this final expression for potential energy of the system. Now using the previous formulation that we have seen (refer slide time 53:49)



we get these expressions for d11, d12 and d22. Notice that d12 = d21 and then these elements of this matrix C are obtained in this fashion by using our previous formulation. So these are all the elements of C and combine these things together
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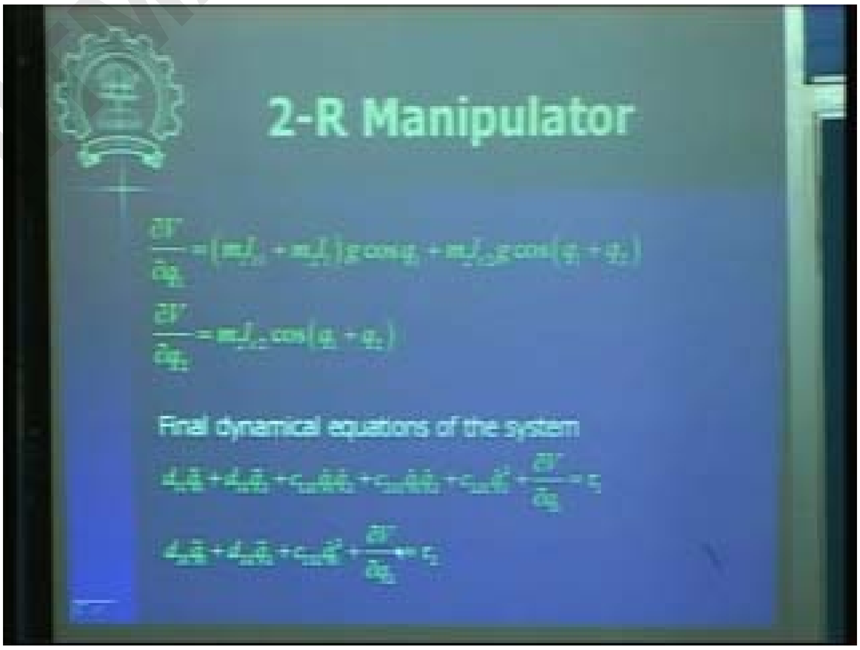

next you will get the final dynamic equation of the motion for the 2D robotic manipulator or 2R robotic manipulator. Next we will see some properties of
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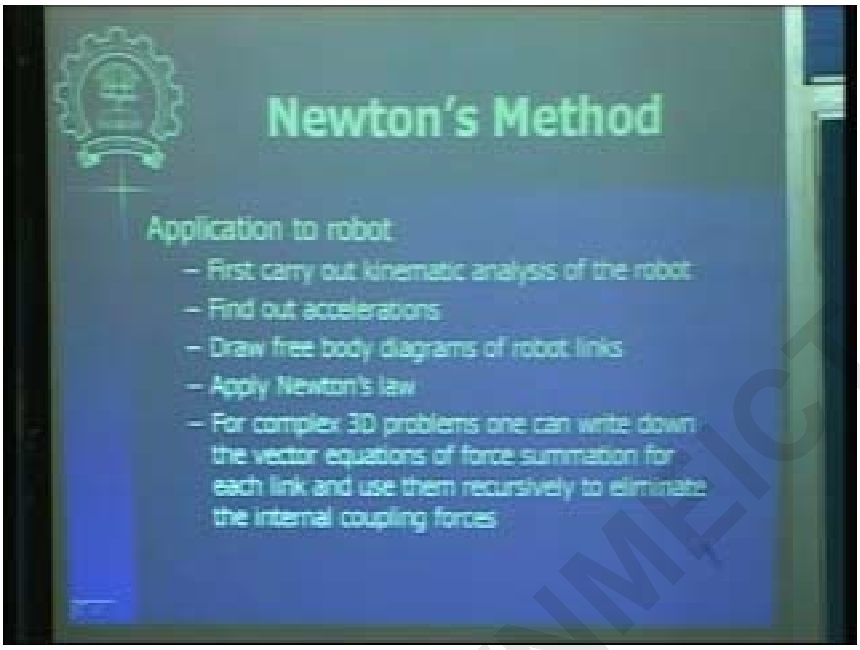

these in the next class.

So let us first see if you want to go for the Newton's method how will you go about it? These are the steps that you will follow to apply this Newton's method. First you will carry out kinematics analysis of the robot then you will find out the accelerations by differentiating your velocity terms. Then you will use these accelerations for finding the inertia forces like inertial forces and then draw free body diagrams for your manipulator and then you apply Newton's laws which is sigma fx, sigma fy, sigma fz directions all the three directions you apply the Newton's laws, sigma fx = mx double dot and all those things you know. And then you will apply these Newton's laws for moment direction also and then you find out the equations of motion. So you can also write down these equations of motion directly in the vector form instead of drawing the free body diagrams you can just consider the vector form and then you can write down directly in the vector form the equations of motion and all the science and all those things will be taken care of in the vector form. Now for the robotic manipulator this 2R manipulator we will essentially follow the same steps. You will first carry out the kinematics analysis as we have seen

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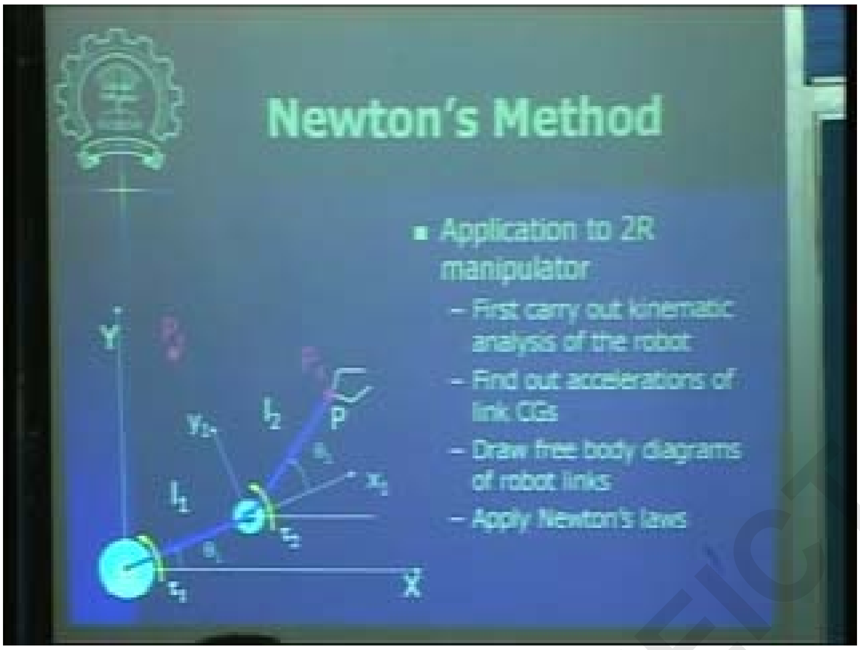

and then you will find out the accelerations of the CGs of the links then you will draw free body diagram in this case it is easy to draw the free body diagram in this case and then you will apply Newton's laws to find out finally the expression of the equations of motions. Now you will find that these equations of motion you obtained from the Newton's formulation and the lagrangian formulation they will be exactly the same.

Now we will use this understanding basically to generate our equations of motion for the robotic manipulator. This is the way how the dynamic equations of motion for the robotic manipulator are obtained.

So we will stop here and in the next class we will continue with this robot dynamics. We will see how to extend this in general for the n link robotic manipulator and then we will have some introduction to the control in the next class ok, thank you.