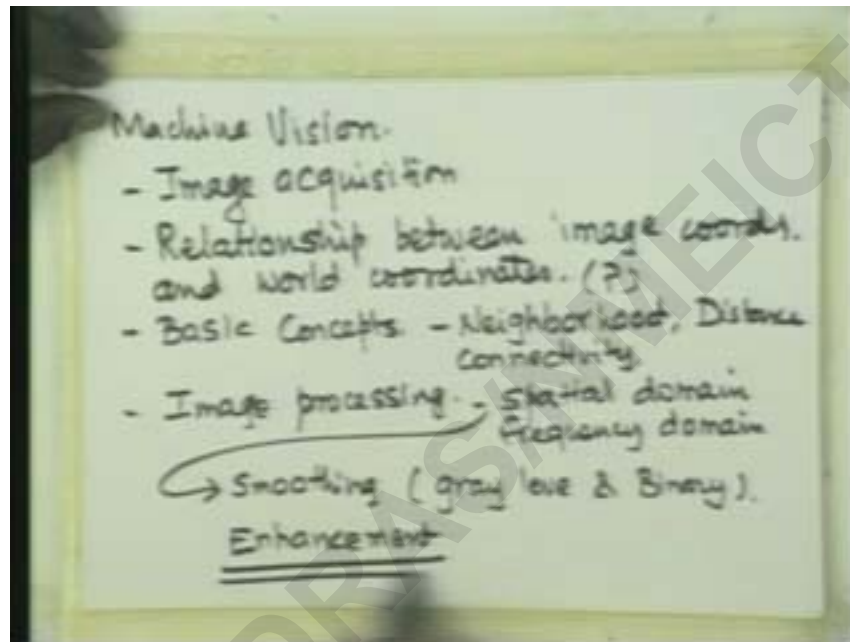
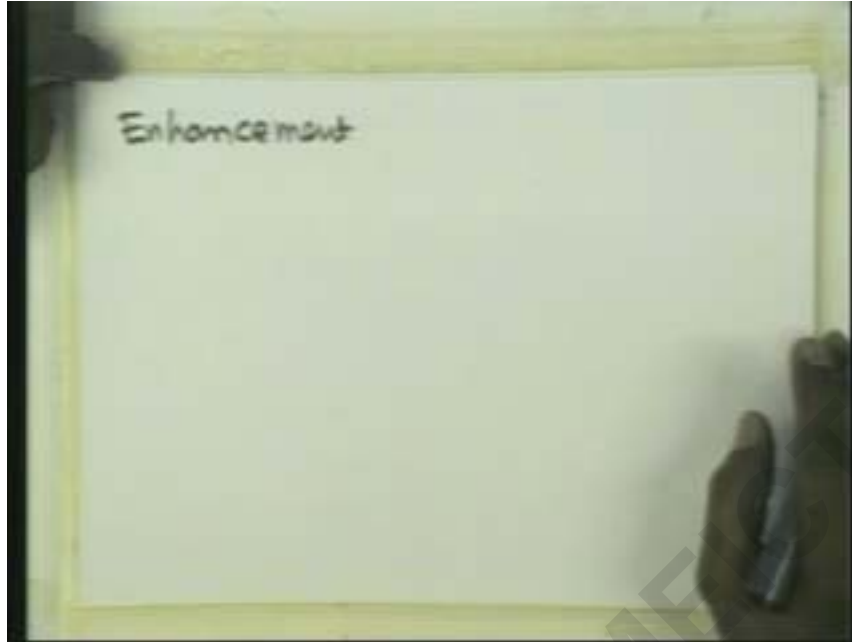


ROBOTICSProf.B.SethDept of Mechanical EngineeringIIT BombayLecture No – 29Image Processing

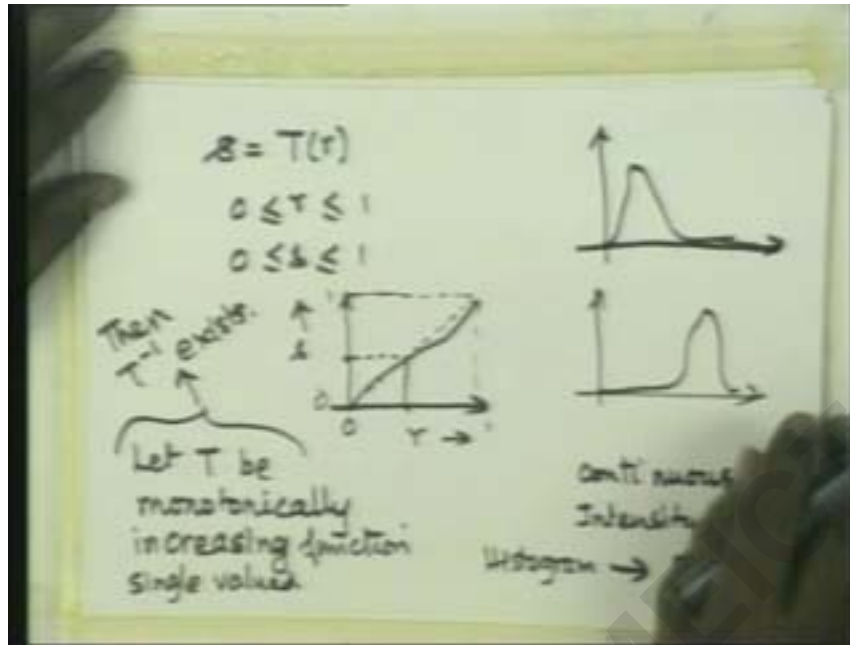
Good morning so for what we have seen is that in machine vision we have different steps(refer slide time4:08)



and starting from image acquisition in which we saw different type of cameras which are used for image acquisition then we went on to look at the relationship between relationship between the image coordinates and the world coordinates and we saw here that using the four by four matrices homogeneous transformation matrices we found actually not a non linear transformation called the perspective or imaging transformation right so we look at p then we moved on to look at some basic concepts and in this we saw the concept of neighborhood distance connectivity etcetera right okay then we started to look at image processing and basically we saw two different ways of doing it which is the space space or spatial domain and frequency domain okay these two types of methods we talked about gave a overview on these and then we started to look at the special domain techniques and then we started out by looking at smoothing right under which we saw different type of averaging and including binary images so smoothing off grey level and binary what we will start looking at today is what is called enhancement okay enhancement(refer slide time4:25)



essentially means that you are going to take care of some errors which are not random in nature okay smoothing or noise reduction is basically for random type of errors where you mean we had some zero mean or regular distribution for some distribution and then we want to get rid of that noise or eliminate reduce it the enhancement we are going to look at techniques which are going to help us to see the picture little better now there is a lot of subjectivity involved when we look at a image in the image one what will appear as a good image to one person is not necessarily going to be agreed upon by everyone when we look at these filters and the result of image processing filters you will find that some of these will have some amount of subjectivity but things like if you have variation of intensity light falling on this paper supposing I look at this particular paper that is in front of you so then if there is non uniformity of light then even if the object is uniform in its properties the effective properties you will have different shades of gray coming from different part of the same surface although the distance from the viewer may be the same right so in that case you will want to make sure that the appears as the same right so you could do some kind of corrective action so one of the basic ways in which we do enhancement is by looking at histogram okay in histogram(refer slide time13:19)



basically we are talking about if I look at some intensity level okay I have some intensity level I take some intensity level how many pixels are there at that particular intensity level so if I if I do the full mapping of that at this instance intensity level we have so many pixels so these are number of pixels and this is the intensity so then I will be able to plot something depending on what the picture contains and I will get some kind of a plot like this or I can show this as a histogram and each of these intensity level I have some number of pixels corresponding to that so that is basically how you will define the image histogram depending on how many levels of gray you have you will have a histogram of something of this one now if this is basically low intensity or darker areas and this is light and then here it is very clear that you have some histogram in which there are more pixels which are dark then there are light okay so this is some kind of dark switch kind of image if you take another image you may have some other histogram for that and then this will be generally a bright image okay so histograms can tell us a lot of things and in particular if you have a histogram which is having intensities your full range is lets say from zero to two fifty five intensity levels but most of your picture data is contained in very narrow band of intensities right some times this is you know it is not easy to decide per details when you have a picture like this so enhancement can help you again

so enhancement essentially deals with the transformation that you talked about earlier that if you have r as the pixels of the original image then we apply some kind of rule to that and we find out what the new pixel intensities up

so these are intensity transformation for example if you have a image like this perhaps it is better to take this frequency this intensity level and take this pixels and show them here and take this intensity level and show it somewhere here and so on so that you can actually spread this out so if I do all that then perhaps my picture will look something like this I have spread out all the intensities so now I have very bright and very dark pixels so the image has better contrast compared to a picture which has got a very few levels of grey which means everything looks the same grey level and the details are hard to see okay now what we will do is we will look at this transformation and we will normalize

this particular transformation so supposing now let me in continuous domain or in continuous variable of you know intensities in the original image then what I have shown you as histograms can also be shown as probability density functions right instead of discrete values one can have you know if you have continuous curve like this that again means that this is basically a dark kind of picture versus a picture like this which is going to be a brightish kind of so this is now in continuous domain continuous intensity okay and then a histogram becomes a probability density function

okay probability density function so what I would like to do is to normalize this and say that my intensity variables are you know $0 \leq r \leq 1$ so if I have zero to two hundred fifty five as the image intensity levels I will say okay let me divide it by two hundred fifty five so that you will start from zero and you will get one as the maximum intensity corresponding to white areas

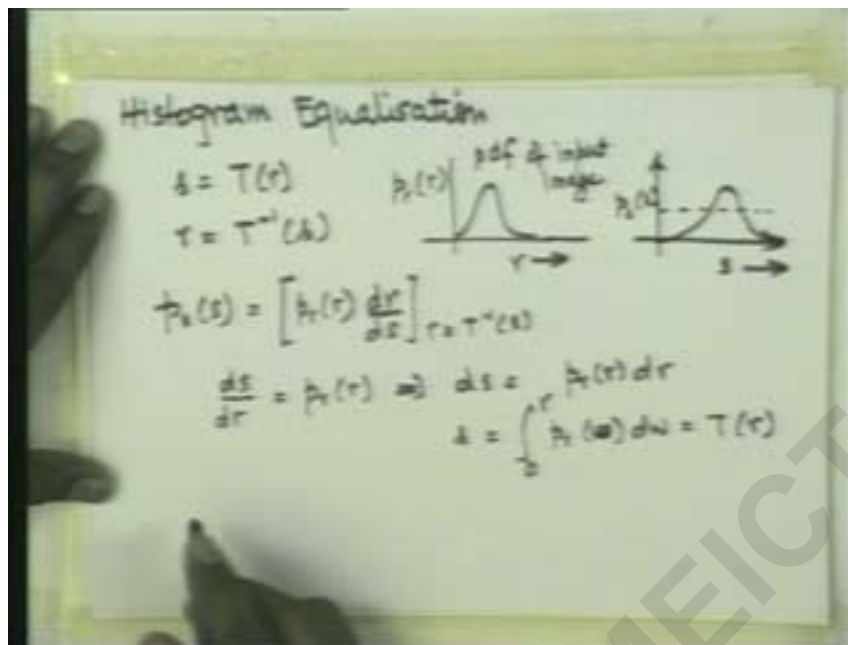
similarly I will say transformed image should also have the same range of intensities moreover if I if the transformation is such that so now let me look at how the transformation may look like so If I have r here and s here then as I mentioned in the last lecture if I have a straight line like this this is zero one zero one then the transformation does not do anything it just takes whatever the input image is output image is the same there is no change however if there is some variation here some other deviation from the straight line then it is going to transform the image in some fashion

okay and so I am going to look at the transformations which are going to be monotonic in nature okay so let t be monotonically increasing function right if I do that then there is a unique value corresponding to unique value will come if I also assume that this is a single valued function okay so monotonically increasing function and also single single valued okay so if I have a single value corresponding to each per each input I have a single output value then it is possible for me to define a inverse transformation right that is so if this property is satisfied then okay if this is so then t^{-1} exists

okay this will be useful for us in in doing some enhancement techniques okay so let me look at the first technique which I will call histogram equalization idea of this I have already given you and I said that if I have a very narrow band picture and if I want to spread it out to cover most of the intensities then I am I am doing some kind of equalization of histogram

so here let us consider this transformation which has the properties that I have already mentioned and therefore I should be able to also write r is equal to $t^{-1}(s)$

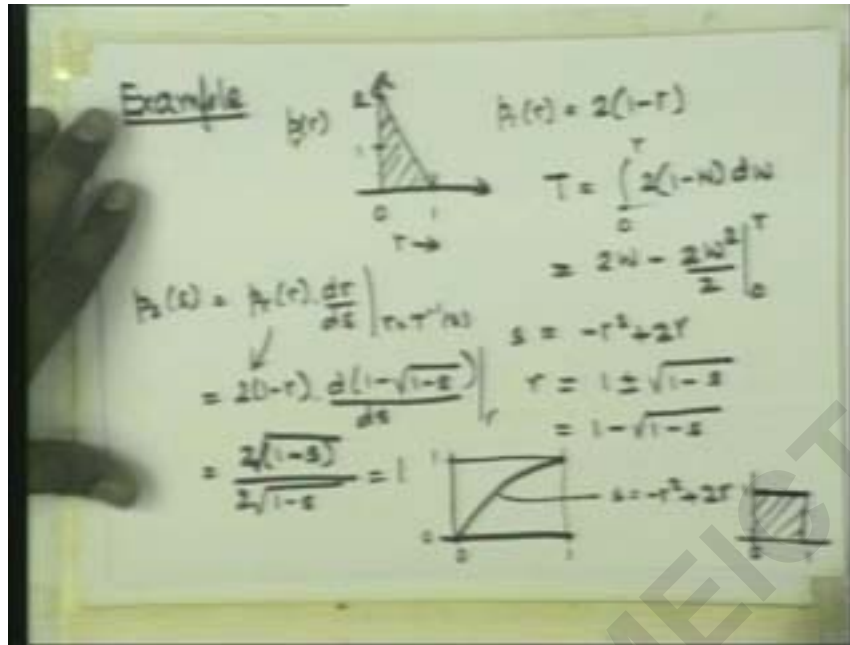
okay now what I want is if we have some probability density function for r okay so let me this is my pdf of input image or original image then I am saying this is some probability as a function of the intensity right the whatever the transformation is it is going to now lead to some probability density function for the output image which I will say $p(s)$ as a function of s where s is the intensity of the output image and it will have some okay whatever the shape is the relationship between these two probability densities(refer slide time 17:13)



can be established and that is given by p_s of s is equal to p_r of r that means the input image times $d r d s$ evaluated at r equals to t inverse s okay so this relates the two probability density functions and the transformation is coming into picture here so if I want equalization what I want is that my probability density function of the output image should actually be a uniform distribution that all intensities are at the same probability of occurring so that can be done if I look at if I consider now supposing I am having this thing here now the basic idea is that I would like to cancel this r so if [noise] for some $d s$ by $d r$

right then this and this will get cancelled off and I will get unity all intensity values from which is what I really want right so this only means that ds is p_r of $r d r$ and I can integrate this to get s is equal to limit from zero to $r d w$ okay and this is nothing but the transformation that I am looking for right if I do this then I can see that I will get a transformation which is going to give me equalized histogram okay

okay so I think what we can do is we can look at an example and let's make the example so in this example (refer slide time 22:09)



lets say we have a input image which has a probability density function like this so I want the area to be unity so I am going to have this from zero to one so this is going to be maximum probability density this thing will be going upto two

so this is $p(r)$ of r this is the property of the input image so that means this is basically a little darkish kind of thing

now so I can describe this in analytical terms so I can say that $p(r)$ of r is nothing but two times one minus r so that at r is equal to zero I have the value two at r is equal to one I have the value zero right so what I am saying is my transformation that I am looking for is actually integral of over r two times one minus w dw right

this is the cumulative probability density function so that I can evaluate because I have the expression for the probability density function here so I get I will get minus w two times okay two times w minus w minus two times w squared about two from zero to r and that is nothing but minus r squared plus two r right this is my transformation that I have defined now

if I take whatever in input intensity r is and I find out the output intensity s which is going to be relating to the original intensity like this then I will be able to equalize the histogram

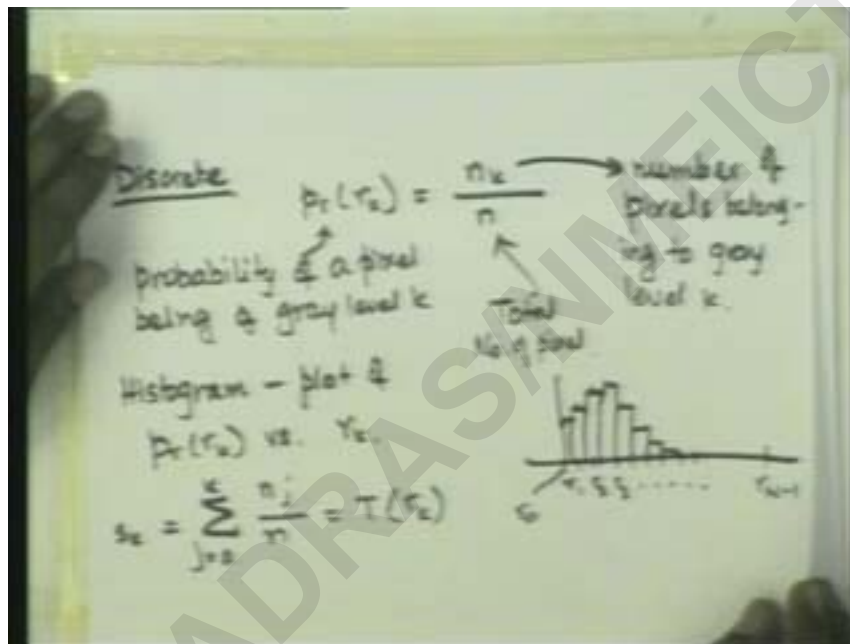
okay this is what I am going to demonstrate now so let us now reverse this equation so that we have the inverse transformation and in the inverse transformation I will have r is given us one plus or minus square root of one minus s now we have to remember that r should vary from zero to one only therefore I have to basically take the minus sign right if I take the plus sign then it is going to become more than one so infact it should be one minus square root of one minus s

and therefore I can calculate the probability density function of the output image as $p(r)$ times dr by ds [noise] okay evaluated as r inverse s which is equal to if I substitute I have two times one minus r times [noise] one minus square root of one minus s ds evaluated at r and that you can see that it comes out to be what I am looking for which is I can substitute for r here also and therefore I will get one minus s square root and this if I take

the derivative I will get the same here one over two one minus s so this indeed will come out to be one

so the transformation that I have got now is basically r squared with the minus sign plus two r so that is going to look like a parabola one zero zero one this is nothing but s is equal to minus r squared plus two r okay and

the output probability density function is going to be simply zero to one one its one everywhere this area obviously must be the same as this area because they are not altering the total number of pixels of course we cant talk about pixels this is uniform continuous domain description of the image which is not going to be so because we have discrete resolution of the camera so one can adapt this problem for for discrete images so we can write discrete form of this(refer slide time25:37)



the probability density function now becomes the histogram and that is nothing but number of pixels of grey level k divided by the total number of pixels

so this is now probability of a pixel being of grey level k okay grey level k is that r_k and this is the number of pixels okay belonging to grey level k and this is nothing but total number of pixels this is very straight forward

this is just saying that whatever number of pixels I have in grey level two or three then divide by the total number of pixels that will tell me the probability of a pixel being of that grey level right

so now if I look at the so histogram is nothing but is a plot of okay is the plot of $p_r(r_k)$ versus r_k discrete intensities r_0 r_1 r_2 r_3 etcetera etcetera r_n or $r_n - 1$ starting from r_0 and then whatever number of pixels I have in each of them is going to give me the pixel the intensity histogram whatever it may look like

okay so what will be the since you will be dealing with discrete numbers we have to be able to see how you will do the equalization here okay so equalization we have to find the transformation which is equivalent of integration right so for integrating the we will have to do is approximate by a summation so if I do $\sum_{j=0}^k n_j$ over n summation from zero to k that

will give me s_k that is the transform intensity values this is nothing my transformation from r_k

okay I think the best thing to do is to look at an example that will illustrate how this technique will be used so we will look at now an example that's right so if you can think of it is a look of table that you have a given intensity level now all pixels of that intensity level we going to transform to some other intensity level in the output image right but what those intensity level should be is what how to figure it out is what we are looking at if you want to equalize the image so that you have good distribution you have roughly equal distribution on pixels in all different intensity levels yeah [noise] correct that you can do and in fact that is one of the techniques after looking at histogram equalization we will look at histogram specification histogram specification precisely deals with that kind of the problem but you will see that it is actually once you understand equalization specification will be just a slight extension of that

okay so let us take an example of a some grey level image and how do we describe the image let's have only limited number of grey level say eight grey levels so k going from zero one two three four five six seven okay now the intensity of this is going to be let's say uniformly between zero and one

so last one is going to be seven over seven or one okay now depending on the image the number of pixels in this (refer slide time 29:40)

k	T_k	N_k	$M(r_k)$	S_k
0	0	789	0.19	0.19
1	1/2	823	0.20	0.39
2	2/2	850	0.21	0.60
3	3/2	876	0.21	0.81
4	.	929	0.23	0.89
5	.	248	0.06	0.95
6	.	122	0.03	0.98
7	1	91	0.02	1.00

4096

64

intensity level will have to be counted from the picture so let's say this is seven hundred ninety this is one thousand twenty three this is eight hundred and fifty this is six hundred and fifty six

okay then we come to three hundred and twenty nine two hundred and forty five one hundred and twenty two and finally eighty one

so this is our image so if we add it all it comes to four zero nine six so I am assuming it in pixels by sixty four will give you four thousand ninety six total pixels okay so now our probable of the histogram is going to be this n_k divided by the total number of pixels

which is four thousand ninty six so one can compute that as “zero point one nine” “zero point two five” “ zero point two one” “zero point one six” “zero point zero eight” “zero point zero six” “zero point zero three” and “zero point zero two” and so we have now basically the histogram and what I will also like to do is to find the cumulative sum and j by n going from j going from zero to k so going from zero to zero is am going to have only one of them which is this

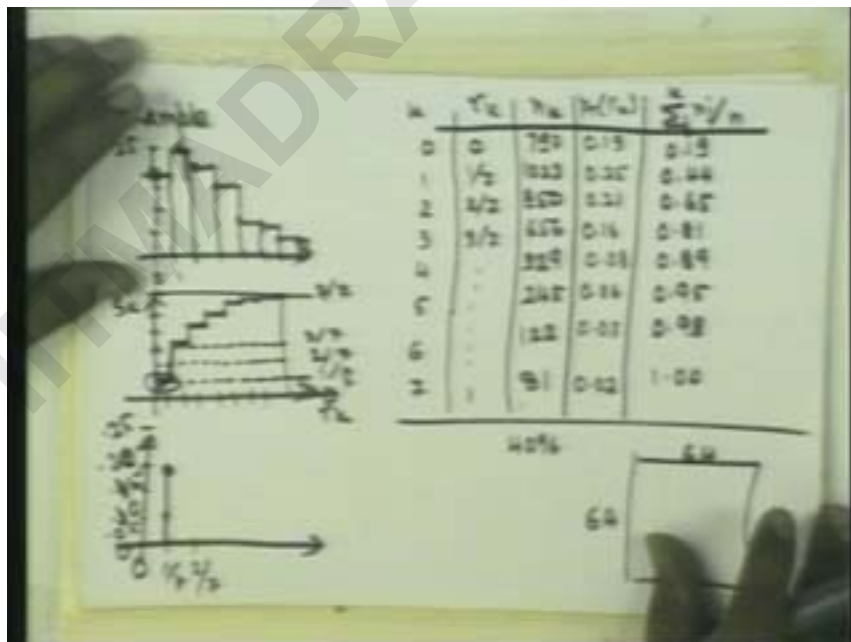
then the cumulative will become “zero point four four” “zero point six five” “zero point eight one” “ zero point eight nine” “zero point nine five” “zero point nine eight” and “one point zero zero” okay so this is defining for me the transformation so now let me see how everything is looking

this is my input image zero one two three four five six seven the value the maximum values is “point two five” so divide that into five equal parts “point one nine” will come somewhere here is the first one “point two five” and we have “point two one” and then we have “point one six” one six will be just above there

and we have “point zero eight” and we have “point zero six” “zero three” and “zero two” so this is for histogram if I show it

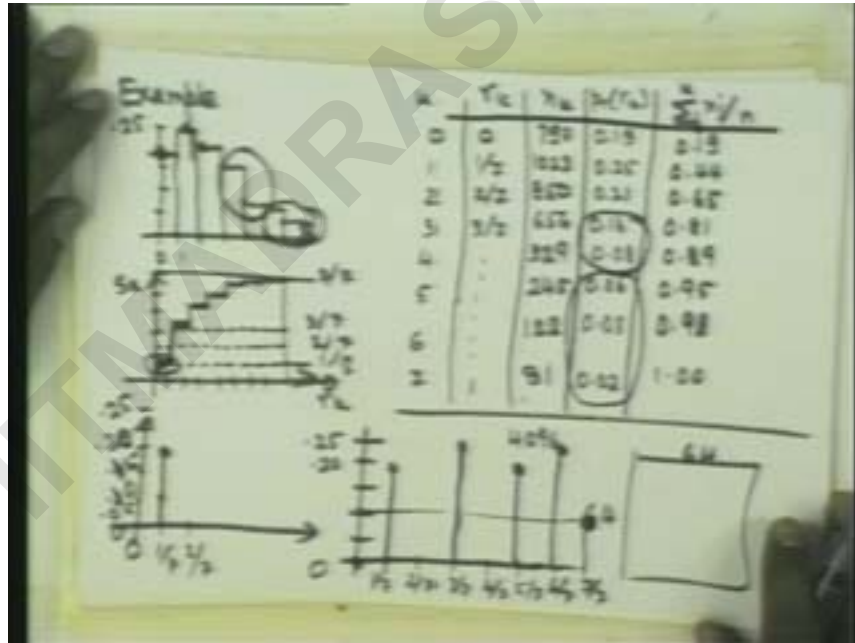
okay this is my intensity histogram [noise] the transformation is going to be defined by this which is going to be simply if I plot r k versus s k transformed image and here I have to go from one to one so each divisions here and so “point one nine” is what I had is here some where then I have “point four” so this will be somewhere here

then I have “point six five” its going to be somewhere here then I have eight somewhere here [noise] then nine five it will be to draw so it will be a somekind of distribution some kind of transformation okay now what I have to do is I have to ensure that my(refer slide time 34:32)



output image is equally distributed right so now if I divide this x is also in seven or eight different categories here I basically have zero one by seven two by seven upto seven by seven seven by seven is here

so now I need lines here which is going to correspond to one by seven okay two by seven and I am going to I mean will be difficult for me to retain accuracy here but what I let us look at the first view once so now look at the output histogram and zero one by seven two by seven lets get those on okay so now at zero I have nothing okay one by seven I have this is the closest this is this level here is closer to one by seven than to two by seven so here because I am dealing with discrete numbers I have to now choose between one or the other okay in the continuous case we did not have this problem so I have these many pixels must belong to one by seven so those are corresponding to seven ninty pixels or "point nine one" of the total so this should have "point one nine" supposing this "point two" this "point four" "point six" and so on so okay actually I have to no I should make my scale the same as this so lets say this is "point two five" "point two zero" "point one five" okay "point one zero" I missed out line here this is this and "point zero five" and zero so now "point one nine" is here I hope it is clear let me otherwise provide a separate one a little more clear okay "point two five" "point two zero" I want too many more this is one by seven this is two by seven (refer slide time 37:08) three by seven four by seven five by seven six by seven and seven by seven okay so what I am saying is that there is no pixels here now I have pixels which are all the first category pixels or zero pixels has been transferred to s one pixels okay now next one is "point four four" which is very close to three by seven okay so two by seven I have no pixels at all okay three by seven I will have all these pixels which will go

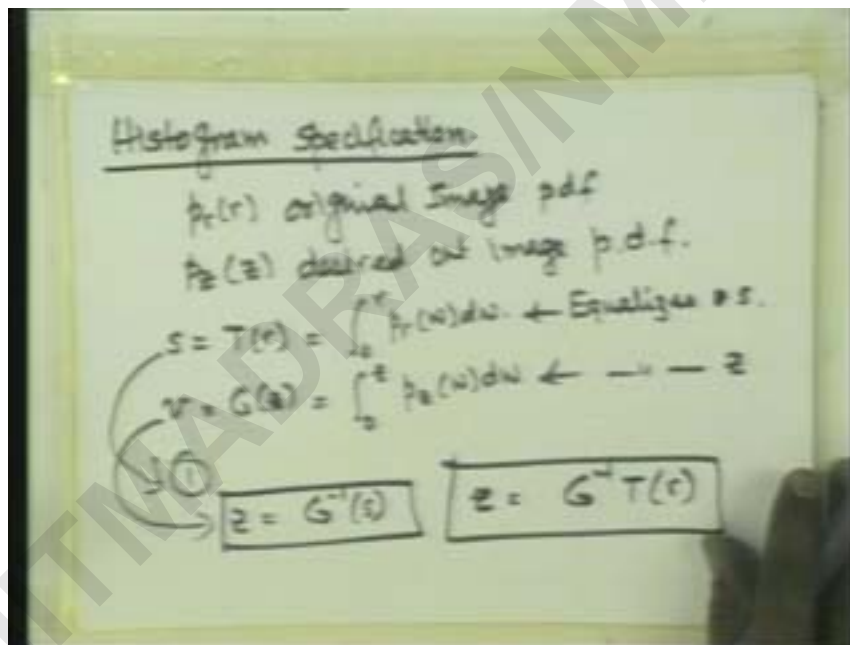


there so these pixels were "point two five" so I have in the output histogram I have all those pixels which were at the intensity one by seven in the original image are now pixels with the intensity three by seven which are in the fourth category of intensity so you can see that it is not changing these pixels the number of pixels in each category so for is the same right of course we are pushing it this way because we had more pixels in the dark region here now we have to put more pixels in the lighter region

so we are going to these shift appropriately and you will see some of these categories will get combined together the last three categories will get combined together the second lies two categories will get combined together will give us a histogram which will have this next category will be five by seven which will have "point two one" so five by seven has "point two one" then the next two categories are [noise] combined and that is "point two four" so this six by seven is going to have "point two four" and

last three categories here will get combined and that is the total of "point nine" and "point one one" so "point one one" will be somewhere here this is going to be the output histogram that i will be given

no not zero at one intensity level one because if you see one seven that is it is close to "point one nine" okay so this is I have we have done the same thing that we did for the continuous case only now there are approximations and there are discrete so the histogram does not look it does not look like a uniform values all all for all the intensities but in a average sense this speak a little higher so in pixels you do not have any intensity level you do not have any pixels and it is basically some kind of a equalization and it will help to view the image better if it is equalization so before we move on lets just quickly look at um histograms specification (refer slide time41:12)



I will not go into details you have already got an idea about histograms and how to deal with them um specification

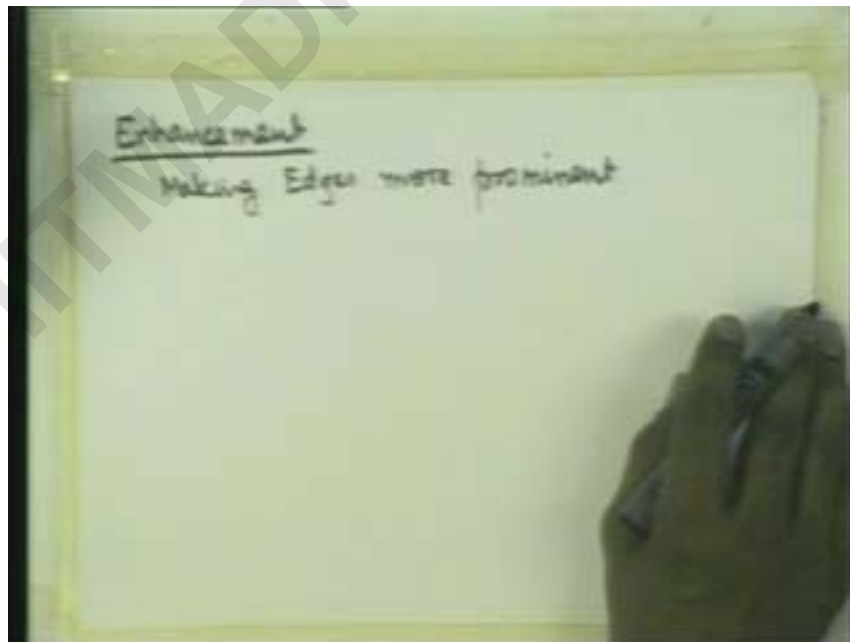
okay what we have seen is supposing I look in continuous domain only so I have one $p_r(r)$ of the original okay image this is the p d f of the original image and I have some desired $p_z(z)$ of the p d f of the desired image of the desired p d f of the output image output image probability density function what I will do is I will just use what I have used so for so i I know that I can find the transformation which will take my input intensities and equalize it so this is the transformation which is given as zero to r p r w d w okay this equalizes okay out equalizes s this is the output image of this particular transformation similarly supposing I took the desired output image and I say okay can I equalize that also so

whatever specified p d f was there I can find a transformation which is going to equalize that so say that is v v is equal to some g of z which is of course going to be defined in the same way zero to z p z w d w okay so this equalizes my other image which is the z image right now all I have to do is if I equalize this this and this image are the same actually right because both are have the same distribution which is the equal distribution of pixels in different intensities so all I need to do is to find the inverse of this so the procedure is going to be like this that I will first use this I will find this and then I will use this in the reverse sense to get z is equal to g inverse of v which is nothing but s because both are equalized so it is the same so this is going to do the job or which is ofcourse so z is equal to g inverse of t of r right you get the logic behind this so I have um some probability density which is of the image that is given to me I I am looking for a particular speci specified distribution of probability density function I can equalize both I know how to do that in one I go from input image to equalized image

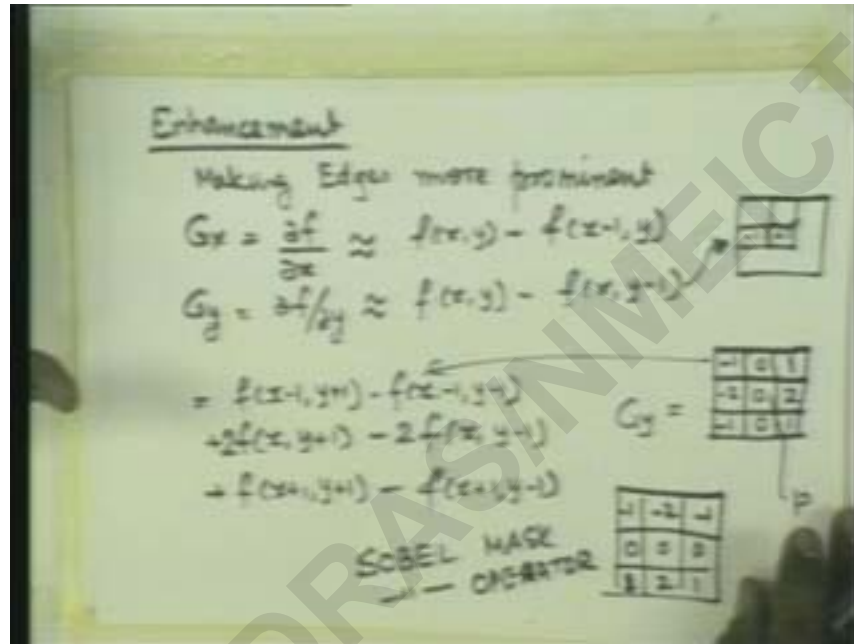
then from equalized image I go by reverse transformation of the second probability density function so that I will now end up with from equalized image to the specified histogram or probability density function

so this in each of these case what I will do is in the next class I will give you some examples then it will become clearer to you what happens take a image apply this type of histogram equalization what happens specify a particular histogram then what do you get and you will see that it makes a remarkable difference in terms of the way we the picture appears to us the information is not any additional information but you will see that from just a grey looking picture if hardly any details that you can discern you will start to see details when you enhance the

Z is the intensity yeah intensity variable of the output image which is also going from zero to one so it is a total intensity range okay now let us look at a few other enhancements(refer slide time43:12)



one important enhancement is making edges little more prominent okay making edges more prominent edges are usually at the boundary of two objects of two different regions of some different properties so therefore if we can enhance them we can clearly see different things in that in the image so edges are places where you have change of intensity that is how you will determine the change of color or change of intensity so if you are talking about grey level only color will make it easier but if you look at only grey level then we are talking about change in intensity that means we need to have some kind of estimation of gradient of the intensity so gradient if I define x gradient (refer slide time 47:34)



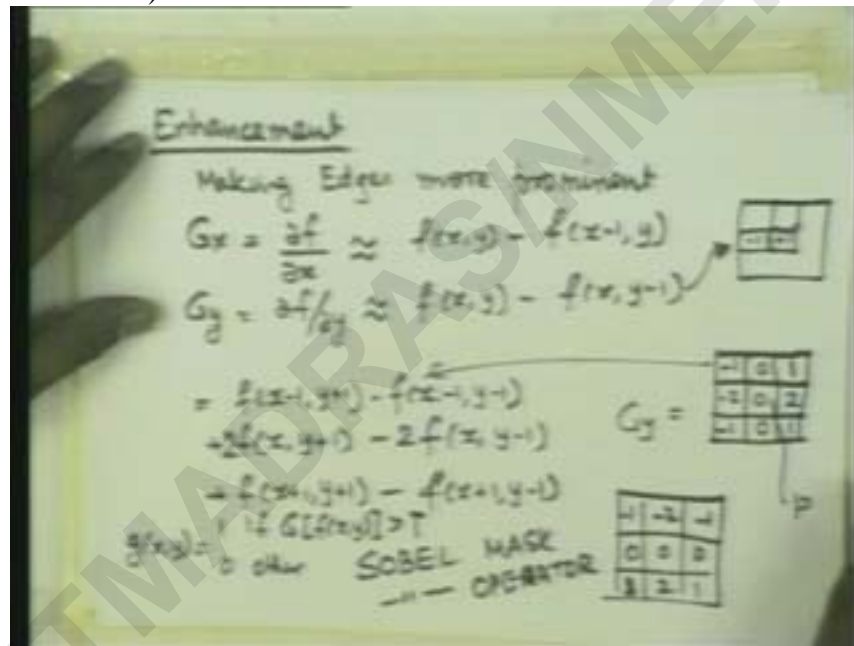
as $\frac{df}{dx}$ okay which can be estimated to be $f(x,y) - f(x-1,y)$ okay this is the gradient in the x direction similarly I can have a gradient in the y direction which will be approximately $f(x,y) - f(x,y-1)$ so we are looking at just different approximation for derivative the better approximation is actually if I try in terms of masks okay so this here of course we were having pixel p I am only looking at x one for the x one I am looking at the previous pixel and for the y direction I am looking at the previous pixel here so I am looking at the difference between the intensities here as a measure of gradient but a better so this is plus one and this is minus one lets say for this a better operator or a template is is more robust supposing for the G_y I want to find so if I take minus two zero two and minus one zero one and minus one zero one so what I am doing is I am finding out the gradients along the x value and one value prior to x and one value after that x with respect to the pixel of interest which is this p right

so then it kind of averages the gradients just above just below and gives more weightage to the gradient at the in the middle

so this gives you a good estimate of of the this is the estimate for G_y

you can write out the full thing if you want but basically it will only be saying that this is nothing but $f(x,y)$ plus one

okay sorry f y plus one minus so this this will correspond to so this approximation now is f um we are talking about x minus one right um y plus one minus f x minus one y minus one right then I am talking about f this is now x level x y plus one two times minus two times f x y minus one plus f x plus one y plus one minus f x plus one y minus one right its similar but its more robust calculation on that similarly for the x gradient I can say this is sorry one two one zero zero zero minus one minus two minus one okay this um is very common mask which is used for finding radiance is called sobel operator sobel mask or sobel operator okay once you find basically what you is going to do is if the image was uniform then this mask will come out to be close to zero if there is change in intensity in the x direction then this mask will give you a high value y mask will not give you a high value if on the other hand you have a oblique line then both of these masks will give reasonably high values so after going through this mask one can then say okay I can threshold and I can say that g of x y in the output image is now going to be one or zero one it will be if my gradient (refer slide time 48:27)



okay of the original image is greater than some threshold value t otherwise its going to be zero so then I will only pick out the pixels which are belonging to the x okay and that will clean out the image a lot because you may have lot of details available in the image but what you are interested in is basically regions boundaries which are separating various regions so if there is a hand and there is a background you perhaps want to look at the hand or you have a part washer and a nut and bolt line on the image and you want to look at the boundaries of this so that you can recognize the washer as a as a disk with a whole okay or some transformed or deformed version of that depending on the angle of vision so those kind of things can be done using enhancing the edge and finally making it binary also so I think we will end here and we will look at higher level vision form in the next lecture