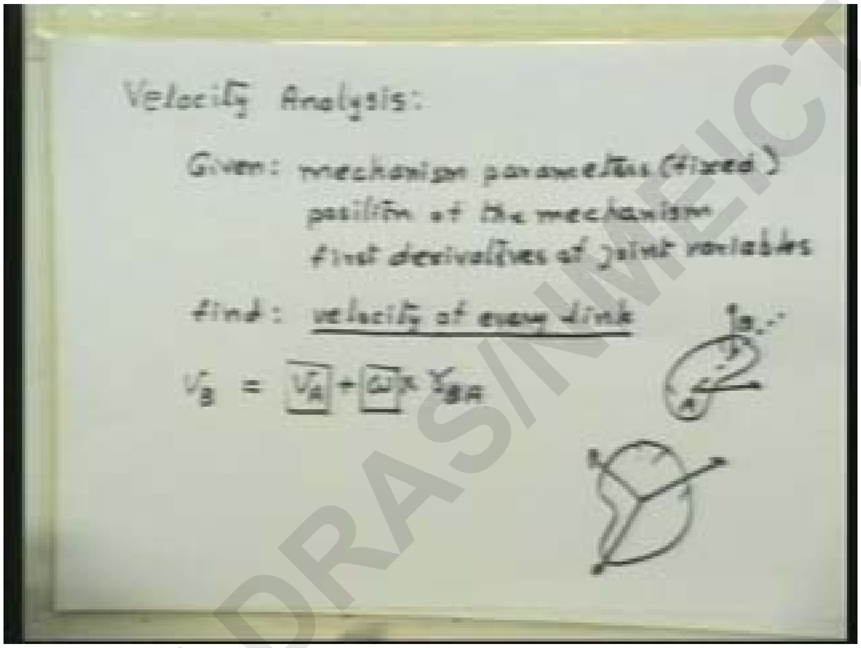
ROBOTICS

Prof. K. Kurien Issac
Dept of Mechanical Engineering
IIT Bombay

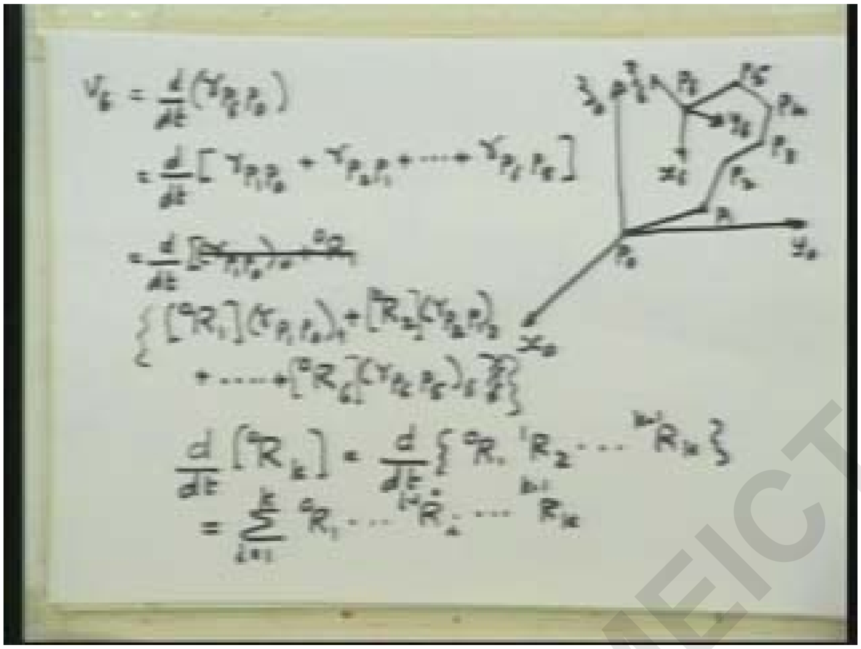
Lecture No – 23

Velocity Analysis (Time: 1:21)

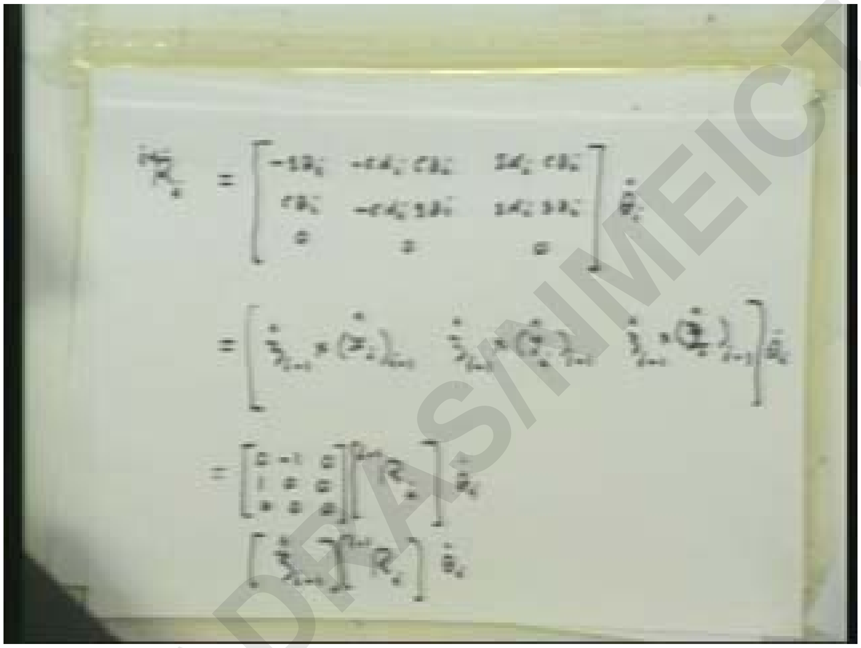
We started with velocity analysis. I will quickly go through and take us to the place where we stopped. So, the problem of velocity analysis is what is laid down here (refer time : 1:31)



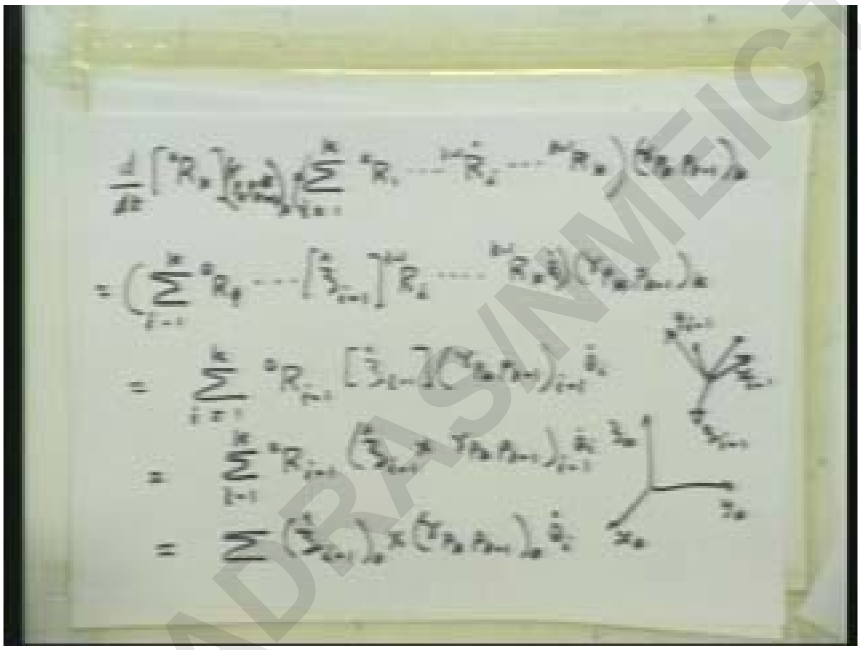
we have given the mechanism parameters, both the fixed and the positions of the mechanism, those are joint parameters, or positions of every link, and we are also told at what velocities the joints are moving, for which essentially amounts to the first derivatives of the joints variables. We need to determine the velocity of every link of the mechanism. Now, velocity of a link in space, of a rigid body in space, can be specified by six parameters. If you take any point on the body then the velocity of that point – the linear velocity of that point – with respect to some reference frame, and then we can defined something called the angular velocity vector for that body which is a three-dimensional vector, and once we have these six parameters we can obtain the velocity (linear velocity) of any other point on the body using this relation, the linear velocity of any point B on the body can be obtained in terms of the velocity of the point A and the angular velocity using this expression where this is the vector cross-product, the vector product, that is the cross-product, fine? What about the angular velocity of any other point on the body? A point doesn't have angular velocity actually, so the statement itself is wrong. Right, the body has angular velocity, the point has linear velocity. Now we started doing this calculation and that is done in the traditional way (refer time : 3:37)



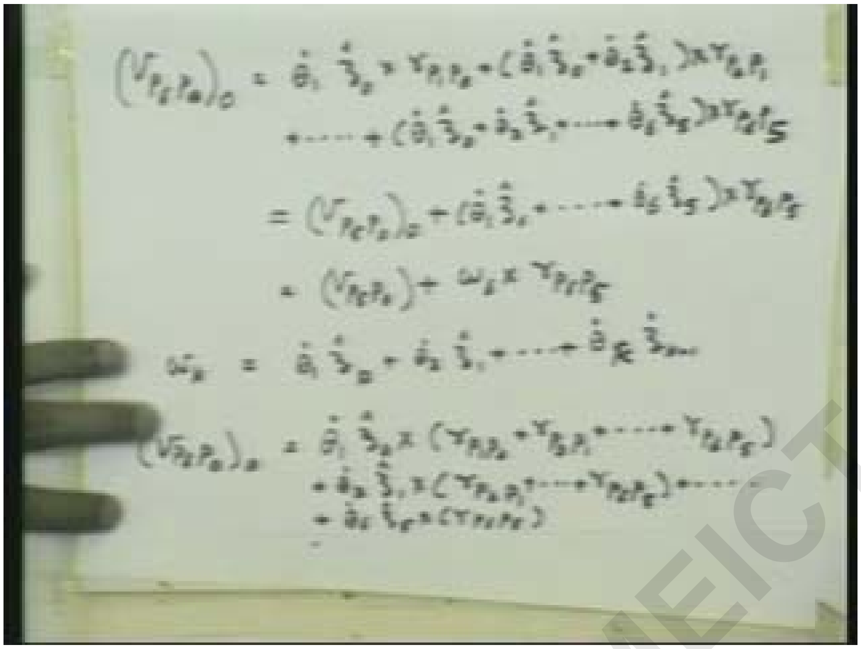
that is, the manipulator is represented by these segments, P nought, P one, P one, P two, P two, P three, up to P five, P six, where P nought, P one, P two, P three, etc. are the origins of the reference frames we fixed on the links, fine? So we get the vector P nought P six. We are trying to now obtain the linear velocity of the point P six which is on the sixth plane, which is on the last plane, ok? So basically the rate at which the vector P nought p six changes with time, that is what we are trying to obtain. So that can be obtained. This particular vector P nought P six is made up of these separate vectors, so add them up. So when we take derivative of this vector r P six with respect to P nought, we take the derivative of all these vectors. Remember, these vectors are represented in the global reference frame, or expressed in the global reference frame, ok? So that is implied here. I do not put an extra suffix to clutter up the equations, so now these vectors, since they are represented in the global reference frame, they can be expressed in this fashion, for example, P five to P six, that is r P six with respect to P five, can be obtained from the local, the sixth reference frame, representation of that vector, premultiplied by the rotation matrix which takes any vector, prevector from the sixth reference frame to the zeroth reference frame, fine? So that is how each of these vectors can be expressed. So now, when we take the derivative of this particular expression we need to take the derivative of each of these rotation matrices. These vectors are now expressed in the local reference frame, and so they have fixed quantities so they don't have time derivatives, so the time derivative is zero. So we don't have to take separately the time derivative of these vectors. So, when we differentiate these with respect to time this will simply be the dot product, I mean the time derivative of this matrix into this vector, there is a time derivative of this matrix into this vector, and so on. So we will have this general term, general operation of taking time derivative of a matrix – rotation matrix – from this point. Rotation matrix which takes vectors from the kth reference frame to the zeroth frame, ok? That itself is a chain of rotation matrices multiplied together, and so when we take derivative we use the product formula, and that is, if you take two scalars, then the time derivative of u multiplying v, where u and v are time, are functions of time, is u dot v plus u v dot. We use that. That applies in this case too, ok? So, if we do that, then this becomes this particular sum of terms, each term the ith term involving the dot product, the time derivative of the rotation matrix I with respect to I minus one, ok? Sum up all that, you get the, this particular time derivative, ok? So, yesterday when we finished the class, I asked you to explicitly take the time derivative. Did anybody get to an answer there? If you wait long enough, you know that I will start answering, so that's not good enough. You should answer yourself. Ok, let this now look at this, the time derivative of rotation matrix i with respect to i minus one. So you know that rotation matrix, we had derived the expression for that, so I will just give the time derivative. We can verify this. (refer time : 24:41)



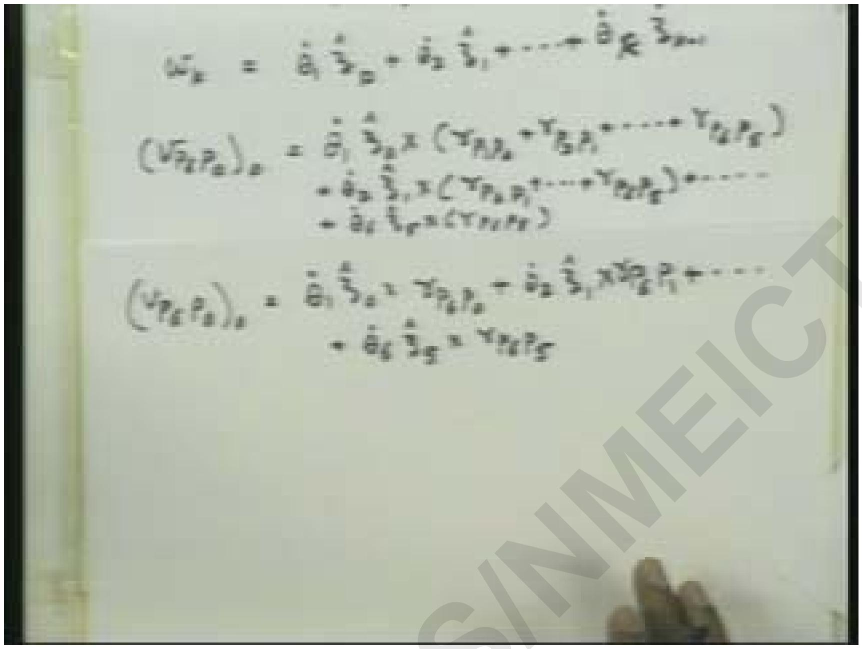
The whole thing multiplied by the time derivative of theta I, if we take the time derivative of rotation matrix turn out to be this expression. So now it turns out that this particular matrix that you have here, if you look at each of these columns, then they turn out to be of this form. Again, this is something you should verify. It is the first column, it is the cross-product of the unit vector along the x y axis, and the unit unit vector along the z i minus one axis, both expressed in the i minus one reference frame, turns out to be that, in the first column turns out to be that. The second column, the same unit vector z i minus one with respect to y i unit vector. Again this, in i minus reference frame, and again, sorry, this is z. Now this is not zero, because these are two different vectors, the angle between them being that alpha, so this matrix turns out to be this. Now, this can be expressed as, this can also be seen to be fine, this particular, the following matrix, this is a skew-symmetric matrix multiplying, yeah, these vectors, a matrix made up of these vectors, so that is R i with respect to i minus one, fine? I will call this particular matrix this is the skew-symmetric matrix. I will call this matrix z i minus one cap, and put a matrix symbol around that, ok? This is useful instead of considering it as a cross-product initially, I consider it to be a matrix by multiplying this rotation matrix, and then I will revert to this cross-product later on, and come back to the cross-product later on. So the important thing to realize is the following, you see. Finally, if you look at it, when we substitute this back in to the equation there is a vector which will multiply, post-multiply this. S,o if you take up to this term it turns out to be a vector, the whole thing turns out to be a three-dimensional vector. That gets premultiplied by this three by three skew-symmetric matrix. It is equivalent to taking the cross-product of this vector with the vector you are going to get later on, fine? Both these things are the same, are equivalent. So, let me now do that substitution. What I mean by substitution is this: we put this particular thing back into this, and where does this go? This goes into this, in all this. So, all of them are post-multiplied by these vectors, ok? So substituting it back. Or before substituting, let me do some simplification. So, we were taking the time derivative of(refer time : 26:30)



this, and one of the terms in that was the time derivative was the following term, this turns out to be, so consisting of terms like this, fine, and this gets post-multiplied by this particular vector r p k with respect to p k minus one in the kth reference frame, so let me just add that. This is in the kth reference frame. Let me read it out if you can't read: this is r p k with respect to p k minus one in the kth reference frame. This whole thing is, we are taking the time derivative of this and multiplying with this, and the time derivative of this is what I have written here, so the whole thing is multiplied with r pk with respect to pk minus one in the kth reference frame. So each of these terms then will get multiplied with this, so if you look at one of those terms, so let me retain this summation, and now I will substitute for what we just now derived for this, if you remember, which has derived a form for this, fine? I will do that substitution: this expressed as this skew-symmetric matrix, multiplying R i with respect to R i minus one. So this is this term, and so on up to R k with respect to k minus one, and this whole thing multiplying. Now let me take this in, and then let me collapse these things, and also these rotation matrices, we get the following, so this rest of the rotation matrices here multiplied turns out to be simply this, and then you have these skew-symmetric matrices now the rest of the matrices here, and then this multiplied, multiplied with this turns out to be r pk with respect to pk minus one that segment yeah, in the i minus one reference frame, fine? Now I shift back to the cross-product because it is easy to see, to do the algebra there, remaining algebra there. Now, this I write, now this is a vector and this is a matrix, instead of that I write it as multiplication of two vectors, cross-product of two vectors. So cross- product of this vector with this vector, both expressed in the i minus one's reference frame. So finally, what we get here is that each term here, or rather, the ith term here, is of this form, right? Cross-product of two vectors expressed in the same reference frame premultiplied by a rotation matrix which takes vectors, three vectors, in that reference frame to the global reference frame. That is, these are free vectors in the i minus one's reference frame, cross-products of two vectors, so lets consider the global reference frame and this is the ith reference frame, i minus one's reference frame. Take two vectors here, take cross-product of these two vectors, and then expres that in this reference frame. That is what is being done here. The answer is exactly the same as taking the cross -roduct of these two vectors expressed in this reference frame, these vectors separately expressed in this reference frame, and then taking cross-product, fine? This is the same, so I can write this as, yeah, I actually did not write one particular term out here when I made the substitution, I missed out one term. Let me bring that back. The time derivative of this at this theta i dot, which I had missed out in the substitution, fine? That is appearing here, but further on it was missed out. So theta i dot is there, ok? So, when we do the substitution we need to retain that. So, there is a theta i dot involved here, but this is R i dot. There we need not worry, but once we get this form, there is a theta i dot, fine? This is a scalar, so you can multiply later on after all the matrix multiplications. So here the theta i dot is here, so then, come back to what I was saying. Multiplication of these two cross-products of these two vectors in this reference frame and transformingg it to the zeroth reference frame later can be obtained by transforming each of them first to the zeroth reference frame and then doing the cross multiplication, ok? So this turns out to be this vector i minus one in this zeroth reference frame, take the cross-product of that with this segment, again this now is expressed in the zeroth reference frame, and theta i dot, fine? So now what we need to do is put this back into the velocity expression for the velocity of the point on the last reference frame that we are trying to obtain, that is what we started with, fine? We need to put it back there, but this has now become fairly simple [???27:14]. So if we go back, we were trying to go back the expression for this particular point, the velocity of this particular point, ok? And while we were doing that, we found that we needed to take derivative of this. Having found an expression for the derivative of this multiplied with the corresponding segment, we now put those expressions back here, fine? So, when we do that, that is, the time derivative of any term like this is of the following form: i is equal to 1 to k, so the kth term inside this bracket after we take derivative, is of the following form, ok? So, if you substitute it back you get the following (refer time : 36:40)



The velocity of the point P six with respect to P nought in the zeroth reference frame, P six with respect to P nought zeroth reference frame then turns out to be the following. So the whole thing has acquired a fairly simple structure now. One moment; P six with respect to P five. Take this term alone. This is the linear velocity of the point P one. This is the linear velocity of the point P one. Take this entire thing. That is the linear velocity of the point P two. So if you take all the terms up to here, what you get is the linear velocity of the point P five, ok? So what we have is the linear velocity of the point P five to P nought in the zeroth reference frame, plus this particular term, fine, which I will retain like that, and take the cross-product of the position vector from P six, P five to P six, that is P six with respect to P five. This is of the form, if you remember, velocity of a point on a rigid body can be obtained from the velocity of another point on the rigid body and take plus the cross product of the angular velocity vector with the position vector from the reference point to the [???32:10] point. So this is nothing but, so basically, velocity of the point P five plus angular velocity of the sixth link, cross-multiplying the vector from P five to P six. So in this serial chain, the angular velocity of the kth link (this is a vector, by the way) is obtained very easily with that k. Remember, z k minus one is the axis along the kth joint, ok? The way we set up a reference frame, z k minus one axis is along the kth joint. That's why it multiplies the kth angular speed, ok? The angular at the derivative of the joint angle of that particular joint. So we get an expression for omega, omega six also can be obtained, and we get an expression for the velocity of the sixth link, and the velocity, i mean the velocity of point P six on the sixth link, so velocity of the point P P k the kth link can also be obtained. This is the partial sum of this, that is the partial sum of that, right? The kth partition. So, only, in understanding that, it is useful to actually club a few things together so, this P six, with respect to velocity of P six in the zeroth reference frame, can be obtained also in the following way. What I am going to do is, in each of these terms theta one dot is involved, right. In five of the terms theta two dot is involved. In four of the terms theta three dot is involved and so on. I club terms in those specific thetas, and then we can write the following. This cross-product of a set of vectors which, so r P one with respect to P nought, all these segments, P two with respect to P one, and so on up to r P six with respect to P five, plus similarly for theta two. Here, this term will not be involved, the remaining terms will be there, and so on up to the last term theta six dot, (refer time : 38:40)



I can write this as, mainly algebra done a little systematically, you will arrive at this. So, I club all this together. This is, when I sum up all these vectors, I get r P six with respect to P nought. If I sum up all this I get P six with respect to P one. This for a physical interpretation which will immediately make you understand the physical importance of how exactly the linear velocity gets generated. Can be interpreted in a few different ways. This is a useful interpretation, and remember, all the vectors involved here, on the right hand side, are expressed in the zeroth reference frame, fine? We have to remember that, which actually takes as to a generalization, but I will come to that. Let me, before that, let me go back to the picture we have, and interpret this particular equation. So if you come back to our original picture, this first term, you can imagine as follows: this is the linear velocity imparted to this point. By holding all these joints as rigid, as fixed, and rotating only this joint, right, with the angular velocity theta one dot, so this term arises like that, right? What about this term? Hold all the joints except the second joint fixed, and move the second joint with the velocity theta two dot, remaining with the rest fixed, fine? The linear velocity imparted to this particular point through that exercise is this term, and so on, so each of the component linear velocities that you get here, vector velocities that you have on the right hand side by clubbing the expressions like this can be physically interpreted like this. They correspond to this physical operations of holding all joints except one fixed and rotating that joint, fine? So, this is an important picture to have in mind, ok? So, finally, the velocity of this turns out to be physically very easy to obtain. All you need to do is, one by one, you move a joint keeping all the other space. The velocity you get out of that is the linear velocity that you will get for the point. Now, this is to, whether the joint is revolute or prismatic. It holds, you can be sure that it holds. Ok now, we have got an expression for angular velocity and linear velocity. Now we will put that together into finally the expression for the velocity of the end effector, so the velocity of the end effector consists of two separate terms, or velocity of the kth link of the serial chain. I will, expression for, so let's put the linear velocity first, and then the angular velocity. Put all this into a single vector. How many elements does this have? Six elements; three for this, three for this, consists of this then; this is the first column, the second column is going to multiply the vector theta one dot, theta two dot, and so on, and then the last. So this matrix multiplying theta one dot, theta two dot, upto theta k dot. This gives both the forward and inverse relationship that we want, that is, this expresses the velocity of the kth link in terms of the joint variable derivatives up to the kth joint, right? Rotation you give to the later joints that doesn't affect the kth link in a serial chain. In a parallel chain it is different. So this is called the Jacobean, different from what you talked about yesterday. So, k could be six. That gives the full Jacobean. Why is it called the Jacobean? That comes from the following. If you have a function of, if you have, say, m functions of n variables, the first order of partial derivatives of this, with respect to the variables is called the Jacobean, right? So, rest of the partial derivative of f is called the Jacobean. This is nothing but that, right? If you have the position of the kth link as functions of these thetas, position is represented by six parameters, right, and they are functions of the thetas, so that is why this is called the Jacobean. This is for a serial chain with w to s. There are some very important conclusions that can be made from this. Remember the elements of the Jacobean; the columns are nothing but this, six-element columns are very simple to obtain, right? They are nothing but these unit vectors along the separate axis, and the cross-product of the unit vector along those axis with some particular vectors, ok? Since all these can be obtained by forward kinematics, we will have thetas, you can immediately obtain all this. So you can write the Jacobean very easily. You don't have go through the derivation which was done right now, all that you have to know is this, ok, and this comes from a very simple physical picture also, right? The angular velocity is simply the sum of the angular velocities you get from these individual rotations, ok? One of the important information that we get out of this is the following. In various positions of the manipulator, remember these are functions of the position, ok, at various positions of the manipulator, what is ability of the joints to impart velocities in all directions that you want? This is a very important issue. It relates to capability of the manipulator in moving in various directions and in applying forces in various directions, right? So all that can be obtained out of this particular Jacobean. And I said this is the forward relationship. Inverse relationship is also like that. If this is a square matrix and if it is invertible, then this can be obtained by premultiplying this with j inverse. That is, if j inverse exists, fine? So, all these can be. So the inverse relationship is also right here, the forward and inverse. That's why I didn't separately define the inverse velocity calculation prompt ok so now in the next class which will be the last class for kinematics um we will see those physical interpretations see how we can get the relationship between static force static forces from the same relationship and mainly see how acceleration relationship can be obtained we won't derive that but we will indicate how that can be obtained.