ROBOTICS

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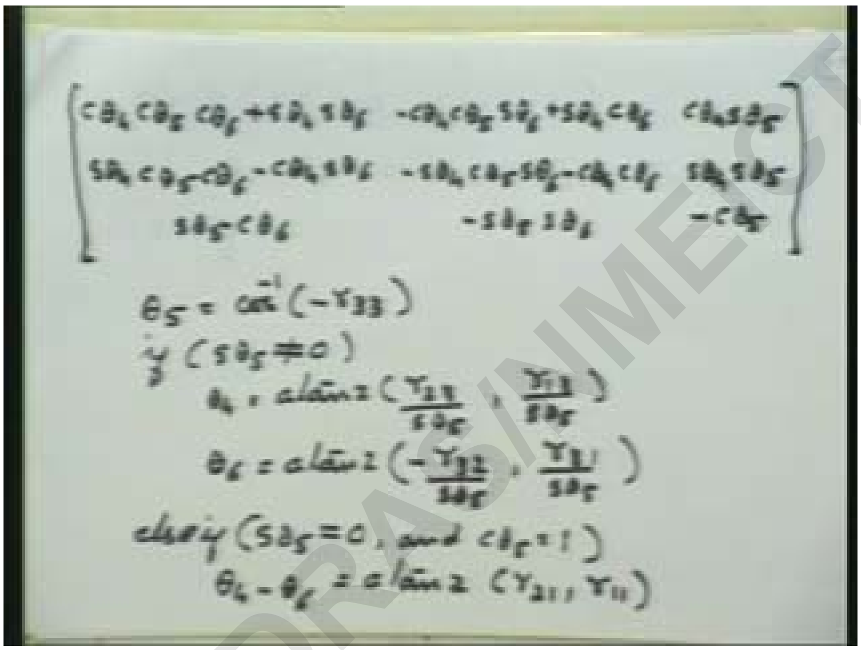
Lecture No – 22

Velocity Analysis (Time: 1:16)

So, we went through the derivation of the joint angles given the end effector position and orientation. At the end of that, we were looking at (refer time : 01:36)

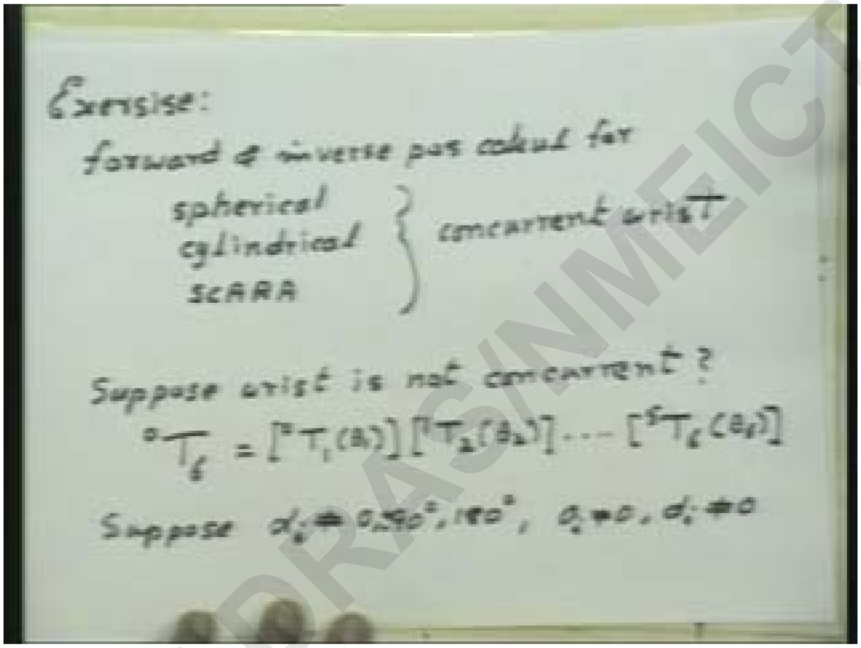


the part of the derivation in which the angles of the wrist theta 4 theta 5 and theta 6 were calculated and the calculation can be reduced to a simple algorithm like this. Now, having come to this stage, for the manipulator that we are considering, given the end effector position and orientation, how many solutions are there for the angles, joint angles? Four. How many solutions are there for the set of angles, theta 1, theta 2, and theta 3? There were four solutions already, right? Two for theta 1, and each of those values of theta 1, least two solutions for theta 2, so altogether four.

So any combination of theta 1 and theta 2 gives a unique value for theta 3. So, four solutions there. Now, how many here? If you look at this calculation there are two solutions, right? Unless r 3 3 is 1 or minus 1 there are two solutions. That is, if it is less than 1 in magnitude. So, there are two solutions for theta 5. What about theta 4 and theta 6 in the case where this is satisfied? That is sine theta 5 is not equal to 0. There is one solution. So, theta 5 has two solutions, and these two have one solution. This is the general case, ok?

So, there are two more solutions are rising from the solution of the wrist. So, altogether now, how many solutions are there? There are eight solutions, because for calculation of these values r 1 1 to r 3 3 there are four possible sets of values. For each of them you get two solutions for theta 5, so altogether, there are eight solutions. Now in this calculation, is there a case where there is no solution? If at all it occurs it has to occur in this calculation, right, and this will not have a solution if the argument, that is minus r 3 3 has, is greater than 1 in magnitude. Well, can this occur? It will not occur if the matrix, the numerical matrix that you have calculated, 3 by 3 or rotation matrix, that is, r 6 with respect to 3 is orthonormal. If that is orthonormal, then r 3 3 cannot be greater than 1 in magnitude. That is guaranteed if the calculation is correct, but when we do calculations, inversions of matrices, multiplications, there could be some small numerical errors, so in a case where r 3 3 is supposed to be 1 or minus 1, it could turn out that numerically you have calculated some slightly larger, fine, but theoretically that cannot happen.

Now, as an exercise, (refer time : 17:25)



you can perhaps do the following, fine? These are simpler than what we have just now seen. It is necessary to identify each of these cases: what are the number of solutions; what are the situations in which the solution doesn't exist; what are the special cases when the number of solutions is less than the maximum number you can have necessary to understand in the calculation; how these things are indicated. In all these, to make the calculation simple, we can consider concurrent wrist. So this naturally leads as to the question, what if the wrist is not concurrent.

How do we proceed with the inverse kinematics? Forward, is no problem. For the serial change that we have seen, doing the forward calculation is very simple, but inverse, for the inverse calculation, we had relied on the fact that the position, global position of the point c, can be obtained, given the end effector position and orientation, so that the first three angles can be calculated immediately. We had used that fact. Now we cannot use that fact any longer. So, what is the option? So, inverse kinematics, remember, is the calculation given transformation of 6 with respect to 0, which is the position and orientation of the end effector, and knowing the expression for this, this is given numerically, and knowing the expression for this, 1 with respect to 0, which is really a function out of variable theta 1 2 with respect to 1, which is the function of the variable theta 2, and so on, upto T 6 with respect to 5. This will be a function of theta 6, fine. We know these matrices, their elements are functions of theta, the appropriate theta,

so how do we get the values of these thetas? How do we solve this when the wrist is not concurrent? Any answer?

So don't think in the algebraic direction. You may not be able to get expressions for these angles the way we got in the case of the concurrent wrist sixth revolute manipulator. We may not get those simple expressions, so what do we have? How do we proceed? Can you get to a numerical solution at least? How do we proceed for that? Iterations, meaning what? Simply keep iterating this equation, or how do we do it? Four? Right. Jacobean of what? yeah, not Jacobean of the T matrix. What does this equation represent?

On the left hand side is a numerical 4 by 4 matrix; on the right hand side is a matrix whose elements are functions of theta 1 to theta 6. So, when I equate two matrices, what does it mean? It means element-by-element equality, which means, the 1 by 1, the ij element here, is equal to the ij element here, right, so 1 1 element is equal to the 1 1 element here.

So, how many equations can you write from that? Sixteen equations? In our special cases of our homogenous 4 by 4 matrices, four of those equations are trivial, right, the last row is 0 0 0 1, this side as well as this side, right? On both sides the last rows are anyway the same, so we cannot, no point in equating them. So, remaining twelve equations are possible, right? So, you have actually, in this equation, matrix equation, twelve scalar equations. In how many unknowns? In six unknowns. If these equations are consistent we should be able to solve that, right? Consistency is guaranteed by the fact that this left hand side 3 by 3 matrix, the sub-matrix, is an orthonormal matrix, and here also, whatever you do here you will get the same corresponding matrix as orthonormal. Because of that, consistency of the twelve equations are guaranteed. So, you will be able to find numerical values of theta 1 to theta 6 which satisfies this, provided the point is reachable, or, this is an achievable configuration. If it is not, then even if the equations are consistent, you will not get the solutions. That is, equations are dependent, but you will not get solutions, ok?

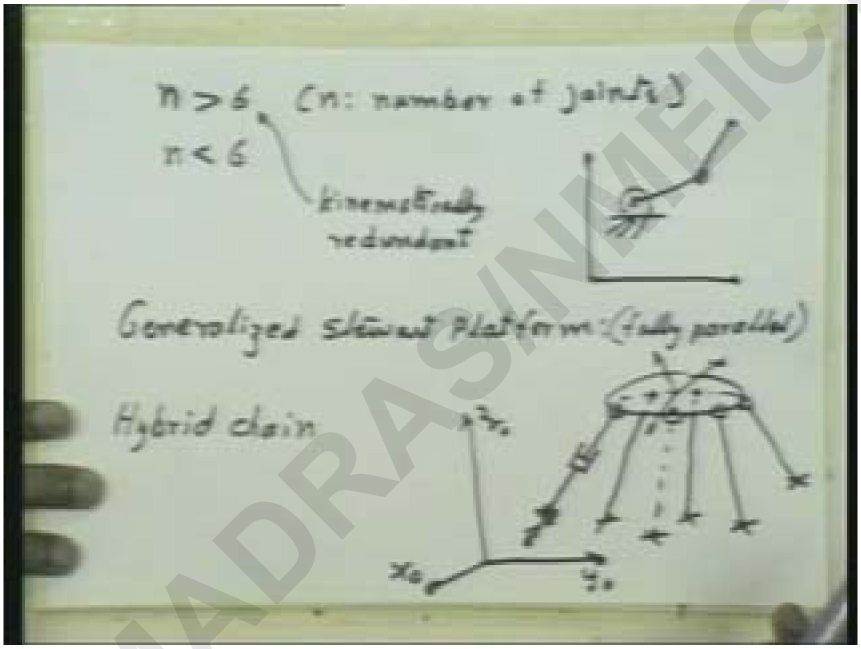
So, this is the brute-force way of solving it; the numerical solution is the brute-force way of solving it. We will not be able to answer questions like how many solutions are there given a set of a particular value for this matrix. Those are questions we will not be able to answer with any level of certainty unless we do very very exhaustive searches in the solutions space.

So, this is the brute-force way of doing it, but what happens is, when the wrist is not concurrent, there probably are slightly simpler ways of solving it, but the most general case is the following. See, this alpha i is, the link is, neither 0 nor 90 degrees plus or minus 90 degrees, or 180 degrees, if it is not these values but some odd values, and if a i's are not equal to 0, none of them, and d i's, none of them is equal to 0. This is the case of a very general manipulator, right, which doesn't have those special properties that one joint is perpendicular to the next, or parallel to the next. Suppose those special properties are not existing; you have a general 6 r manipulator. This is even more general than this, more complex than this. Even this can be solved using the numerical technique, that numerical approach, that we just outlined. Equate those twelve equations; take up those twelve equations and solve them. The method which Vipul was mentioning, using the

Jacobean, is the Newton Rapson technique for solving systems of equations. We use the inverse of the Jacobean in order to do that, so, that is one of the techniques you can use. That is not necessarily the most preferred technique, just one of the techniques.

So even in this case, you have to take recourse to numerical solutions. In this case, many of those, it is that, many of those alphas may be 0's or 90, all of them are either 0 90 or 180 degrees, and the solution may not be as difficult to get, as in this case. We may be able to use some of these relations to simplify things, ok? This particular problem, inverse kinematics of this was called the Mount Everest problem by [???17:02]. It means it was the tough problem. He actually said that the corresponding closed loop kinematic chain position analysis is a tough problem, ok?

Now, there are a few more things which have to be considered. Suppose you have a serial change, (refer time : 17:35)



and suppose n is greater than 6, n is the number of number of joints in the serial chain. Suppose n is greater than 6, and you are doing an inverse kinematic solution where the position and orientation of the end effector is given, you need to find out the values of these n variables, n joint variables. How many solutions do you expect in general? how many solutions do you expect in general? As many as the number of joints? Even in the case of 6 we saw the number of solutions was eight which is two more than the number of joints, so that is immediately incorrect. How many solutions do you think are there? That is correct. If the number of joints are more than six there are infinite number of solutions. The reason is fairly simple. No, the reason is the following. The number of degrees of freedom of the mechanism is more than 6, let's say 7, in case of 7, and what are we giving as input? One end effector position and orientation. How many parameters are involved in that? So, although I said there are twelve equations which can be used for solution only six of them are really independent, really speaking. You need to only six, the others are actually redundant. It may be useful for solution, but they are really redundant. So really speaking, you have only six parameters, six equations to use in order to find out seven variables. So, in general the number of solutions is infinite, right? So, suppose n is less than 6 there are manipulators with five joints; in fact each extra joint, unless it is essential, it shouldn't be used. Each joint may cost something like a lakh of rupees, right, not just the joint, but the motor, the driver, the controller, everything put together, may cost that much. So, suppose n is less than 6, still you have six equations, right, you still have six equations if you have specified the end effector position and orientation, and you have only less than six numbers of variables to determine, unknowns to determine. It is not necessary that the equations are consistent, right, you may not get solutions unless the end effector position and orientation that you have specified is what that particular mechanism can achieve. Unless that is ensured in your specification of the end effector position and orientation you will not get solutions. So remember this. There are manipulators like this and these are manipulators you used for certain special purposes where you don't need that extra degree of freedom and then make sure that what you specify as an end effector position and orientation does not involve the extra degree of freedom, right? You need to be careful about that. To give a trivial case, if the planar manipulator, let's say, of two joints at, I can specify the position of this particular point in a reference plane, right? This is confined to the plane. I cannot, in addition to that, specify the orientation of this link unless it corresponds to that particular position of the point on the end effector, unless it is consistent with that, ok, because the orientation of this link is dependent on the position of this point, ok? So these manipulators are called kinematically redundant, because it is more number of degrees of freedom than you need to position the end effector with 6 degrees of freedom, right?

Then you need you have more degrees of freedom than really required for the sixth reauired to be given to the end effector. Now we had seen the serial manipulator. Let's take the case of the parallel manipulator. So take the case of the Generalized Stewart platform, yeah, used in industrial manipulators. I haven't really seen one. No, I haven't seen a piece of industrial manipulator which is redundant degrees of freedom, but if you go to mobile robots, which are normally used in the industry, snake robot for example, but this is not used for positioning the nose or the head with 6 degrees of freedom, those degrees of freedom are used for slithering along the floor, so they are needed for a specific reason, fine? Right, yeah, that question is a bit relevant; that question was is redundant degrees of freedom not really required? Isn't it essential in certain situations? All of us have a redundant manipulator; our arm has 7 degrees of freedom, including the wrist. Correct? You can count that? Can you count the degrees of freedom of the arm including the wrist? Don't include the hand, which there the degrees of freedom blow up. Total, there are 27, fine? But consider up to the wrist, yeah. What is the shoulder? What sort of joint is the shoulder? ball and socket joint, but does it have, that is the construction, does it have 3 degrees of freedom? Can you show that? Yeah, within a limit that is ok. All of you say that the shoulder has 3 degrees of freedom, right? What about the elbow? The elbow has one. Next, what should we consider? The wrist. How many does the wrist have? Is it a spherical joint? Not really a spherical joint, it is a Hooke's joint. Although Hooke did not design it, it's a Hooke joint. How many degrees of freedom does the wrist have? (1) Like this, (2) This particular rotation is not really the wrist although it happens at the wrist. No, that rotation, really, it is not rotating about the axis, it is moving like a Hooke's joint, right, when you do a flick with the badminton bat, we use that sort of a motion, but, fine? So I said seven. Where is the extra one? Yeah, this motion, fine, is actually starting out, the relative motion starts out from here, there are two bones here which slide, with respect to each other, fine, to provide an axial rotation at the wrist, ok? So we can put it at the wrist if you want to. So, 6, 3 degrees of freedom here, 1 here, and 3 here, 7. So that's why I said we have a redundant manipulator out here, ok? Yeah, the reason for that, we can't ask the Maker, the design, or why that was so, but we can probably speculate. The rotations, the movements of these joints are restricted, right? For example, the elbow, I can move from here up to here. If I try to move further I will break my elbow, right, and similarly the wrist, even the shoulder.

So when you have restricted joint motions, that is one case where you need extra degrees of freedom in order to be able to position the end effector whatever way you want, ok? Yeah, yeah, I haven't really seen that, but what in machine tool industry, the end gripper, you know, the gripper, which has to close to hold objects, or the welding gun, let's say the spot welding gun, the ends have to actually pinch together the two parts of the plate being welded, plates being welded. That degree of freedom is usually called a half-degree of freedom in machine tool industry. How? Because usually, the end positions are what matters. In between, it is not important. So, I don't know whether that is what you meant, yeah, so I am sure there must be, because spot welding robots actually have to reach in to certain very difficult to reach positions in order to do some things, ok? But it is very important how many degrees of freedom should something have, and what is the extra advantage that it offers. Now, you are paying quite a bit extra for that junk, ok? So, reaching around obstacles is definitely one thing,

ok? Let's come back to the Stewart platform. Do you remember this? It was discussed in one of the earlier classes. This is called the fully parallel.

Two of your classmates have worked on the kinematics of this. Then they were in the second year, somewhere, second year B Tech, somewhere. Now this, the fully parallel one, is somewhat like this; you have the end effector on which there are some joints, lets assume these are spherical joints, there are two more on the other side, like this, ok, and these are connected to the frame. Usually, this is a Hooke's joint. What you have here is a Hooke's joint. This disallows axial rotation here, ok? This has a spherical joint. So there are six points at which these get connected. Each of these limbs, these arms consists of a cylinder and piston, so the distance between this point and this point can change, and the actuation is precisely there, that is, the input, ok? So, let's assume that Hooke's joint is outer. You can work out the number degrees of freedom, that will turn out to be 6. Now the question is how do we do the forward and inverse kinematic calculation for this?

So the global frame, somewhere here, so let's say x nought y nought and z nought, not setup according to [???32:20] notation, just put that, to specify the problem, ok? This body is the end effector. How many links are there? If you don't consider the separate links in the Hooke's joint, let's forget that for the moment, no, more than 8. Each of them, each of these limbs, are 2 links, right, so 12 of them here, plus 2, you have 14 links, ok? So we need to set up 14 reference frames, messy thing. Finally, we don't really have to do that, but anyway. So how do we do the inverse and forward and inverse calculations here? The forward problem is: given the positions of these points, given the positions of these points on the link, and given the lengths of these links, these limbs, determine the position and orientation of the end effector as well as each of these limbs. The position and orientation of these limbs are not very important, sometimes they are important for interference calculation. What is important is in what position and orientation you can place the end effector. That is what is primary, right? So this is the forward calculation. hat is the inverse calculation? Given the position and orientation of the end effector and the local positions of these joints in this reference frame, these joints in the world reference frame, find out the lengths of these limbs. So which problem do you think is easier to solve? Inverse is trivial actually, right? Inverse is trivial because we specify this is the end effective reference frame.

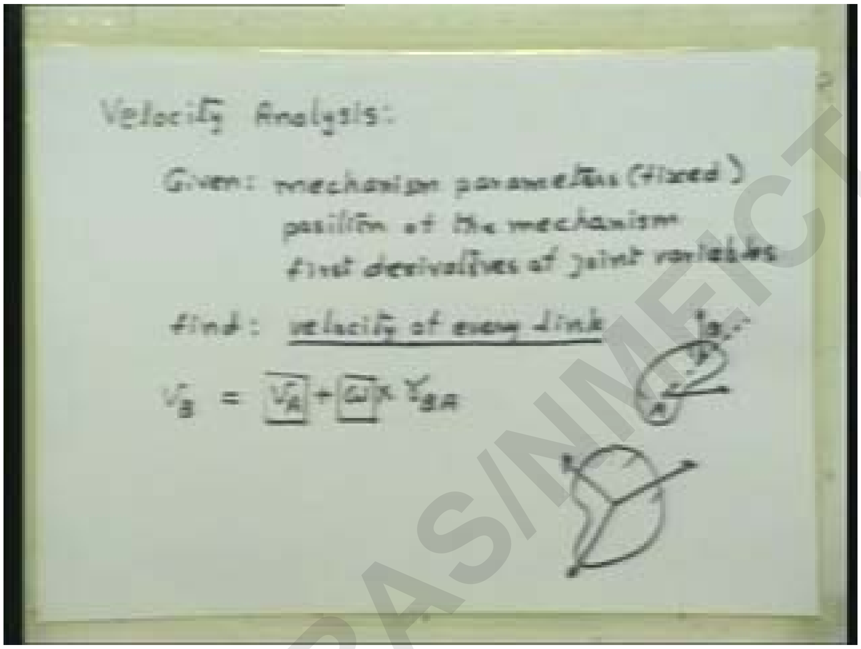
What we know is the position and orientation of that. Since we know the location of these joints with respect to the local frame we can find out the locations of those joints in the global frame immediately, right? We know the locations of the other ends of these limbs in the global frame, then we can find out the distance of each limb, the length of each limb, and that is precisely the inverse calculation. So this is trivial and very simple. The problem is that with this inverse calculation if you keep the lengths at those values this is not necessary that the end effective goes to the position and orientation that you specify. That is the problem here. The forward kinematic calculation position calculation is very tedious here, very involved here, and not only involved, the number of solutions are many. The number of solutions are discrete in this particular case, but it is just not unique. There could be several solutions, ok, and this problem is difficult to do, and this is equivalent to the Mount Everest problem [???35:55] for, specified for, or mentioned for the serial change. It is equivalent to that. As tough as that, but it becomes simple with certain special configurations of the Stewart platform. The way Stewart or somebody else designed it originally, it is actually much simpler than what I had drawn here. This is very general, ok? For the special configuration, or the parameters which are chosen for the actual Stewart platform, this calculation becomes simpler, ok? We won't go into that, but I just wanted to tell you that.

So, that is one thing. So inverse is very simple here, trivial, so however, it could get that complex unless the configuration is simple, and finally, suppose we have a hybrid chain,

what you, yeah yeah; you can. The question is: why do we need these six points on the end effector? The reason for asking the question is that we know that if you specify the position of three points on a body, means position and orientation gets fixed. The answer actually is in that statement. If you are able to fix those points, then the position and orientation gets fixed, but just because, suppose you remove three of the limbs, then by specifying the position the length of the remaining three limbs we are not fixing the end points in space, fine? We are not fixing those end points in space by just specifying the lengths of the limbs. You can see that, if degrees of freedom comes out from a fairly simple calculation. So what is a hybrid chain? A hybrid chain is partially parallel and partially serial, so maybe little easier than this and easier than the complex serial chain. Maybe. Could be as difficult. Problem could be that both the forward and the inverse may not have unique solutions, neither of them, ok? So all these things could happen, but remember that with the machinery that we had just now developed, the important reason why we went through the details of how to specify orientation, position, of rigid bodies in space, how to set up linkage parameters, mechanism parameters, and how to use the transformation in order to set up the equations, the reason for that was to build up precisely that machinery, using which you can systematically analyze mechanisms even if you are not able to get expressions for, let's say, the quantities, unknowns, you want to calculate, you can take recourse to at least numerical calculations finally, because you can set up the equations in a systematic fashion.

So that is the reason why all this machinery for the presentation was really discussed in this. You don't get to see it normally in a kinematics course, because in most kinematics courses [???39:37] kinematics. Robotic is one of the places you get to see spatial kinematics, ok?

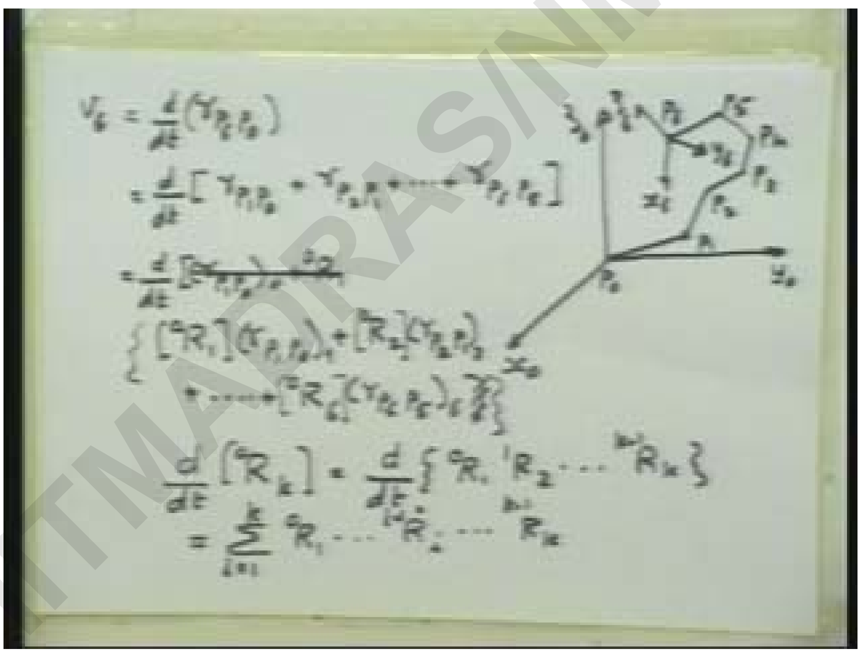
Now let us go on to the next topic which is, do velocity analysis. (refer time : 46:31).



Right. You have done velocity analysis of planar mechanisms in your [???40:07] machines course. We can do the same thing here. In general, it turns out that velocity analysis is simpler to do than position analysis. That will be the case here also but the algebra involved may turn out to be a little formidable. Let us pose the problem. As in the case of the position analysis problem, there is a forward and inverse problem, is not really, they are solved simultaneously together, more or less, ok? We basically set up the equations for that, so now let's define the problem given mechanism parameters, so basically, the fixed parameters, position of the mechanism, so this is supposed to be known. What is meant by position of the mechanism? Position and orientation of every link of the mechanism. That is what is meant by that, ok? We know that. So, the forward problem is, in addition to these first derivatives, the first-time derivatives. So in the case of the 6 r manipulator that we saw, this theta 1 dot theta 2 dot upto theta 6 dot. If there is a prismatic joint in between, there will be a corresponding d i dot, and because offset is a variable, there will be a corresponding d i dot. What is to be determined? We need to find out velocity of every link. So, what does this term mean, velocity of a link? A link is a rigid body. Velocity of a link is defined by six parameters, the linear velocity of a point and the angular velocity of the body, fine? A body, a rigid body, has a velocity field which is given by the following equation: take any point b; the velocity of that point b can be obtained from the velocity of another point which you assume to be known, plus the angular velocity of the body is the three-dimensional vector, is cross-product with the vector B with respect to A, that is, the vector from A to B. So, if you have a body like this, if you know the velocity of the point A, you can find out the velocity of any other point, let's say, B if you know the angle of velocity of the body, fine? The cross-product ensures that the velocity of the point B, when you calculate, it will turn out that the projection of this vector on to this line joining this two points A and B is the same as the projection of this vector on to the same line. That is the rigid body constraint, right? Whatever way the body moves the points A and B should remain at the same distance, ok? So, the relative velocity can only be perpendicular to this line which is A and B that is ensured by this. In addition to this, there is a magnitude of rotation which is given by this. So, the velocity field of the body is defined by this, and you can regard this parameter the velocity of some point, and the angle of velocity of vector as the six parameters defining the velocity field of the rigid body, ok?

So, if you have a body, and if you have fixed the reference frame to that, fine, you can regard the linear velocity of the origin of the reference frame as this the linear part of the velocity of the rigid body, and then the angular velocity of the rigid body, fine? Ok, this is what we need to find out for every body: the velocity of a point, and the angular velocity, [??46:31].

So, let's proceed to do that. You have written down? (refer time :46:40)



So, again, we use the serial manipulator, that is easier to deal with. So, this is the origin of the global reference frame. So let this particular point be called be P nought; this is the origin of the reference frame on link 1. Let's call that point P 1. So, the other serial manipulator is made up of a series of points like this: P 2, P 3, P 4, P 5, and finally P 6. You remember P 6 is the origin of the reference frame we had fixed to the sixth line, ok? P 6 is the origin of that, so if I draw the reference frame there, x 6, y 6, z 6, like this there are several reference frames involved. I now try to express the velocity of the point P 6 with respect to the global reference frame, so for that what I do is I take the vector P nought to P 6, and take derivative of that, right? So, what I want is that velocity of the link 6 which I define as the velocity of this particular point P 6 is the time derivative of the radius vector from P nought to P 6, which the notation for that is P 6 with respect to P nought. This is the vector, fine? This consists of the following vectors: P 1 with respect to P 6, P 2 with respect to P 1, and so on, right? So, P 1 with respect to P nought plus, and so on up to [???50:12], and remember something important is that all these vectors are expressed in the global frame. They are vectors expressed in the global frame. So, because of that, I can use the local position in the local reference frame, and premultiply it with the appropriate rotation matrices to express it in the global reference frame. So, this can be written as reference frame 1, plus R 2, with respect to 0.

This will involve a little bit of algebra. The reason why this was written like this was these vectors in these reference frames are fixed vectors, but there is a slight confusion here, and the confusion is this. Look at this vector, 6 with respect to 5, so 6 with respect to 5 is the vector from here to here, fine? From the origin of the fifth reference frame to the origin of the sixth reference frame, and that is expressed in the sixth reference frame. It is a fixed vector in the sixth reference frame. So that is why when we later take derivative (one moment, let me make the notation like this, and putting this, ok), so remember that.

Now, when we take derivative here take any typical matrix like this, this transforms some kth um reference frame to the 0th reference frame we we will encounter um we will encounter when we take derivative, the derivative of this particular matrix which is the derivative of a chain of matrices multiplied, and so on, to R k, k minus 1, right, and when we take derivative of that it is element by element derivative of each matrix and the product so if you use matrix product definition it turns out to be the following,

right? This is the product rule. So it will involve taking derivative of this rotation matrix of one link with respect to the previous link, fine?

So take that particular derivative you have written down. If you need any clarification you can ask me.

Yeah, so I won't proceed beyond this today because it is already 12:30. What you need to do is you have the form for this matrix, right? Take derivative of that, look at that and see if you see anything special in that. We'll have to make use of special property of that in the further derivation, ok?