ROBOTICS

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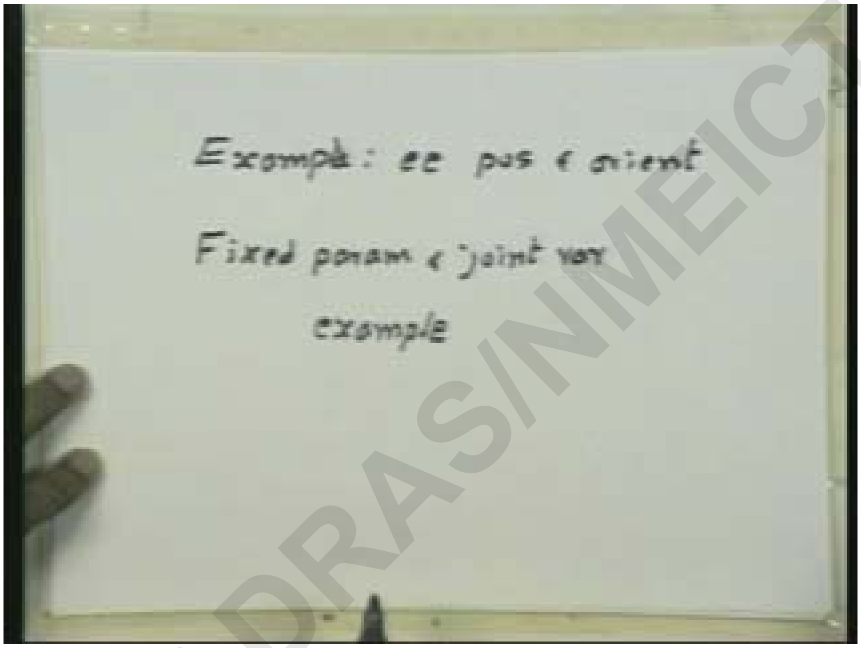
Lecture No. 19
Trajectory Planning (1:20)

We were seeing an example of how to specify the end effector position and orientation for a specific task. So today what we will do is complete that example. (refer slide time 02:22)



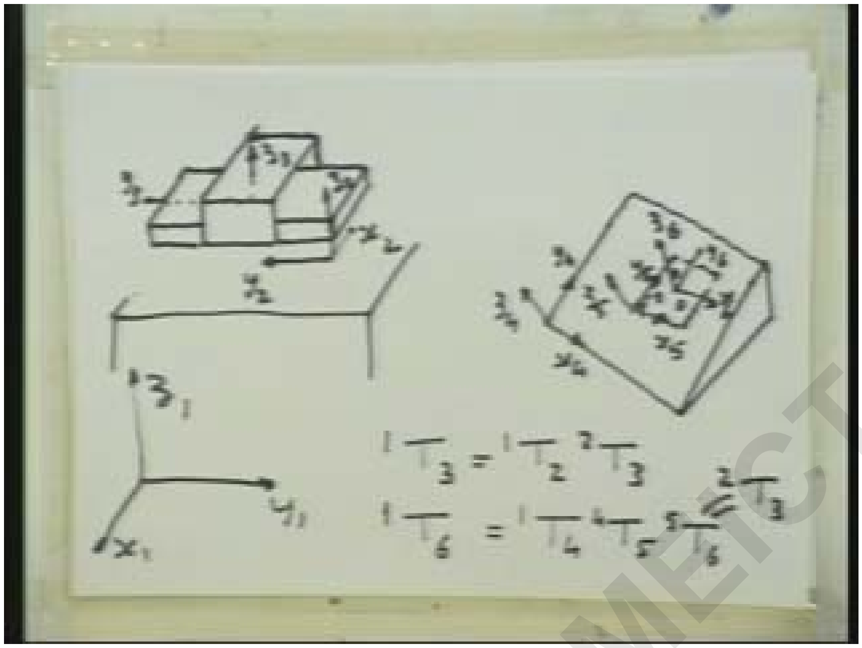
Then we will look at the following, very important concept of how to define parameters of a mechanism. We will see an example in this particular case and there is a particular transformation, which is used when we do the inverse and forward kinematics and which uses the way we set up the reference frames on the links. So we will see that transformation.

So let us start with the example we saw. The problem was the following: there is a part placed on this table, which has to be picked up by an end effector, a gripper, and transferred to this particular position, where the pegs of this part would go into the holes which are there in this part and the part has to be brought to this position and assembled.

So if you remember, we had set up some reference frames in order to specify this particular position. One was that we fixed the global reference frame x_1 y_1 z_1; with respect to this, we need to get the end effector position and orientation. The location of the part itself, the particular part on the table, is given by the reference frame attached to it at this x_2 y_2 z_2.

It was attached in a particular fashion in order to make the locations of the pegs on that part easily definable. I am not showing the pegs here.

Now the end effector has to finally come and hold this part. (refer slide time 14:16)



So when the end effector is in a particular position with regard to the part, with respect to the part, the position and orientation of the reference frame attached to the end effector will be as follows: x_3 y_3 z_3; I had given the details in the last class.
So to start with, we need to get the transformation between three and one. Where exactly is the end effector with respect to the global reference frame x_1 y_1 z_1? Now to specify where it has to finally go, we have this inclined plane and we have fixed the reference frame to the inclined plane x_4 y_4 z_4 starting from this particular corner of the inclined plane. Since the orientation of the inclined plane is fully known and this particular position is given, we can get the position and orientation of the fourth reference frame with respect to one.
When finally this part is transferred here, this particular location of the corner, which we call a, is given here; so it can be specified with respect to this reference frame and the orientation is along this axis.
When this reference frame two is in the position five, the end effector reference frame is in the position six.
So we need to also get finally the end effector position six with respect to the global frame. And remember these transformation matrices are the four by four homogeneous transformation matrices, which include the orientation information as well as the offset of the origin.
Now what would this be? We know two with respect to one, where the part is placed with respect to the global frame; and we know three with respect to two, where the end effector is with respect to the part, in the first position of the end effector
and actually all positions of the end effector, when the end effector grips the object.
This is the relationship between the end effector reference frame and the part reference frame. So the position and orientation of the end effector is now given by multiplying these two matrices to get T_3 with respect to one. That is fairly straightforward.

What about the final position of the end effector? How would you do the transformation? What is it that you know with respect to the final position? I have from one to three, so I know the end effector position, in the first position with respect to the global frame. Now I want the final position, which is six. But do you know three to four? This is in a particular position; and this is in a particular position. Now how do we get three to four? I haven't actually given four with respect to three or three with respect to four; four with respect to one has been different right there actually we need to use that.

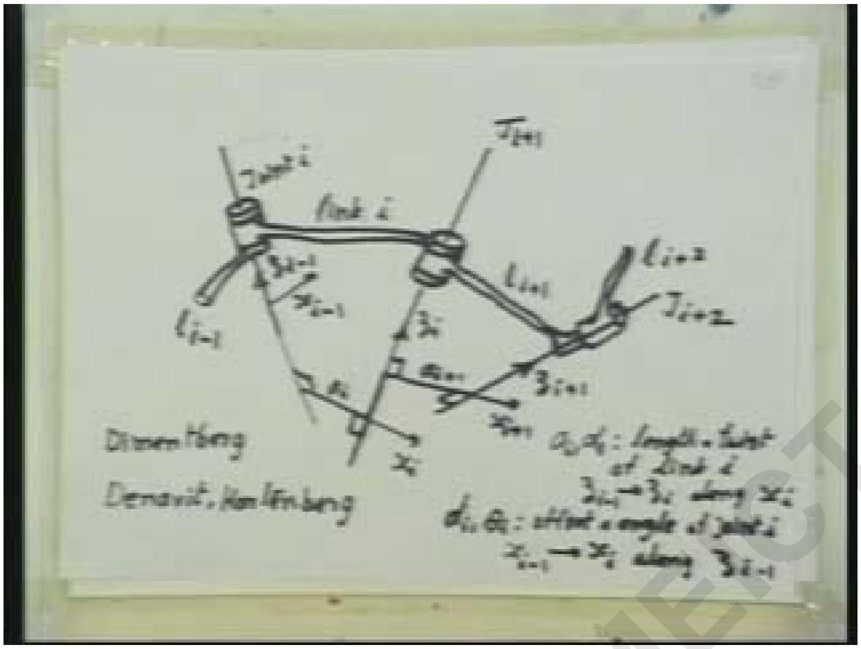
We have already got three with respect to one; we know four with respect to one, so we can get three with respect to four. That is possible; but directly we know four with respect to one. So we have four with respect to one; five with respect to four; six with respect to five; so this is known, and this is known. What about this? This is the exactly the same as three with respect to two.

So because you have gripped the part and you haven't changed the gripping position when it is transferred from this position to this position, so the grippers reference frame with respect to the part reference position hasn't changed. That is why these two are the same. So it is as simple as that in notation, but the key thing is that you need to find out the elements of these matrices. In this particular case they are fairly easily derivable. Now if you remember the motions of manipulators that give us specifying, using joint interpolation,

what you would use is not the initial and final positions of the end effector alone. For example, suppose I want to have the end effector move from this position to this position, these are only the final positions, the end positions.

We need to now obtain all the intermediate positions. How do we solve that problem? We specify some intermediate positions and orientations also. Now these are specified taking care of certain things; for example, if the initial position is here and the next position is here, and the one before the final position is here and the final position is here, what would happen? It will take a dig in to the table, which you don't want; you don't want a collision between the objects. In order to take care of that, the intermediate positions have to be specified appropriately.

What do you want the end effector to do when it starts off? You want to make it lift off the table. That has to be ensured. So the next position should be this x_3 y_3 z_3 translated upwards; somewhere above that. That can be obtained by the position of x_3 y_3 z_3 and the new position would simply be a translation along the z_3 axis with nothing else changing. You can simply translate it. (refer slide time 35:46)



So transformation matrix for the next position between three and that would be something very simple, and so you can get the transformation between that position and the part reference frame. So the transformation, which gives you the new position with respect to the global frame, can also be obtained very easily.

How about the position of the end effector just before you place it at this position? It also has to be above, ideally above this particular plane, so that can also be obtained by translating x_6 y_6 z_6 along z_6 has axis. That is all that is required by some appropriate distance. Again you can very easily get another position and then if you move the end effector from the first position to the slightly raised position to the slightly raised position here and finally to this position, you expect to get a reasonable motion of the part and the robot, so that collision wouldn't occur between the tables and the part.

So this is something you can do yourself. Let us get to the crucial question, how do we specify position and orientation? How do we specify parameters of a mechanism when the mechanism is a complex spatial mechanism? You had done something like this for planar mechanisms and it is fairly easy; you had defined something called link lengths in planar mechanisms, and if there is a translating, prismatic joint, then some angle is required. So let us now draw a fairly complex mechanism and see how this can be done. You can start drawing after I finish.

What I have shown here are four links connected by three joints. Is that clear? So there are two revolute joints; there are two links connected by them. There is another link connected to this link, through this revolute joint. This is a prismatic joint here, connecting this link and this link.

So there are four links; the issue now is to identify the parameters of this particular mechanism. Let me call this link i and let this joint be joint i. So this is joint i plus one; this is link i plus one; this is link i plus two; and this is joint i plus two; and this is i minus one.

So what we are going to discuss today are useful for serial chains; when it comes to parallel chains or linkages with loops, the connections that we are going to set up would have to be modified at little bit. So these are the joints and these are the links. You can see that these joints are not parallel and may not be perpendicular or parallel it may be shown if I draw the axes of the joints—this is the joint axis for this joint, for this joint we have an axis like this, and for this joint you may have an axis like this.

Now what would you call link lengths? There are some lengths involved, so look for fixed parameters of this particular mechanism. What would you consider as fixed parameters so that you will be able to use them to then determine how one link moves with respect to the other.

The person who first set up a nice convention for defining link lengths was this person called Dimentberg; sometime in 1940s, he had a few publications where these things were defined very well. So what he proposed was the following: fairly simple, you have the physical link here, no need to consider that as the link. Instead of that, what can be regarded as a parameter between these two joints or this link and what is the role of the link? Remember the link is rigid; its role is to keep the two joints on the link in fixed relation to each other.

So we are seeking this fixed relationship, or parameters, which define this fixed relationship.

If you now look at the following thing consider the common normal between the two joints. This is the common normal between these two joints. Will that common normal shift as the link moves? Remember the two joints are fixed on the link so relationship between the two joints is going to remain fixed wherever the link is. So as the mechanism moves, the link moves; and as the link moves does this distance length of the common normal change? It cannot. That is number one; number two is suppose you look along this particular common normal, what do you see? You see these two joint axes with a certain angle between them. These two joint axes in general are two skewed lines in space. So when you look along the common normal, you see two lines and there will be an angle between them. Would this angle change as the link moves? That also wouldn't change.

These two can be two of the fixed parameters of the link. For this link we can identify two parameters—one is the length of this normal and other is the angle between the two joints when you see along the normal; and these two parameters are called the link length and the link twist.

Let us assume that two joints are together or parallel and then you twist them in order to get the angle between them and you keep it there after that. So what we have are a_i and alpha$_I$: the length and twist of link i. If I were to do that what we have is a_i here and the angle between these two; I won't clutter up the figure by putting that angle there. We will see it in more detail later.

This is link length a_i; similarly, the common normal between this axis and this axis is the link representation of this link, i plus one. So let me draw that common normal; so this is a_i plus one.

What about the angle between the links? We have now drawn simple representation of two links, i and i plus one. What about angle between them? How do we define that? Because this being a revolute pair, what changes as the mechanism moves, is the angle between the links. Look at these two segments, a_i and a_i plus one. If you look along the joint, there is an angle between the two. That can be regarded as the joint angle; in a revolute pair that changes.

But the way we have drawn is such that ithas also a distance between these two links along this joint axis. Will that change for the revolute joint as a mechanism moves? That doesn't change. So this particular distance is called the joint offset and this angle is called the joint angle and these are called d_i and $theta_i$: these are the offset and angle of joint i. So for joint i, we have to look at the two quantities on this joint. What we have shown here is for joint i plus one but d i is the suffixes suffix i goes with joint angle. ok
So this is what Dimentberg did; let me just ask one more thing here. There may another joint on this particular link at some other point and there will be a joint axis over there, and the common normal between that joint axis and this axis will be the representation of that link. Now along this joint also now you can talk about two parameters: the joint offset and the joint angle.
As the prismatic joint moves, which of these two parameters is going to change? The offset and not the angle. In a prismatic joint, the offset is a variable, and the angle theta is a fixed parameter; whereas for a revolute joint, the angle is the variable and the offset is the fixed parameter. Both the parameters are associated with a link; link length and link twist are fixed. This is what is done by Dimentberg and later on Denovit and Hortonberg set up reference frames on this mechanism on the links with a particular convention and then defined these link lengths and joint offsets. By the way this is d, and not alpha. They defined how to measure these parameters, set up a convention for that, so that signs and all those things are used consistently.
So it is useful to learn that; it is not necessary, essential, that you do that. The basic concept is that; you can set up reference frames the way you want to, but this is a nice and convenient way of doing that. It is called the Denovit and Hortonberg notation, which is commonly used. So the way they set up reference frames is like this: look at this link I; the reference frame attached to link i will have its origin on this joint. The x axis will be simply an extension of this link so draw it further and you get x_i.
Remember x_i y_i z_i is fixed to link i; the z axis is along the joint z_i and y_i is the cross product of z_i and x_i. The convention is very simple: attach the reference frame on the link at the second joint of the link. This is joint i plus one, joint i, the link is between link i is between joint i and joint i plus one. It could have been between joint i minus one and joint i; it really doesn't make much difference. It is useful to stick to a convention so that you are consistent.
So x_i y_i is linked like this and similarly we can regard that there was this particular link, which had this x_i minus one along that and z_i minus one along the joint. So z_i minus one is along joint i. x_i minus one is perpendicular to that, and is coincident with the link i minus one.
So now the measurement of a_i and $alpha_i$ is done like this: look at z_i minus one and look at z_i.
You can translate z_i minus one along this x_i and rotate it to make z_i minus one coincide with z_i. The translation you have to do is a_i and the rotation you have to do is $alpha_i$—that is the convention.
Translation and rotation of z_i minus one is to make it coincident with z_i by translating and rotating about zx_i—that is the direction, and rotation is in the right-hand screw direction. That is the positive direction. The reason is, these two, z_i minus one and z_i are perpendicular to x_i. So look along x_i, you see the angle fully, full projection.

Did you get the question? I have to repeat the question: the question was, can you rotate z_i minus one about x_i to make it coincident with or align with z_i? It can be done because both these are perpendicular to x_i.

a_i and $alpha_i$ are obtained in this following fashion: take z_i minus one to z_i along x_i. Similarly d_i and $theta_i$ are defined in the following way: translate and rotate x_i minus one; we are talking about d_i and $theta_i$, so associated with joint i. Translate and rotate x_i minus one about z_i minus one to make it coincident with x_i.

The convention is the right-hand screw direction, so we take x_i minus one to x_i along; so this is the convention that is used and we set up the reference frames like this.

There will be the reference frame attached to link i plus one so it will have its origin here and x_i plus one axis is the extension of this and so what is the z_i plus one axis along joint i plus two?

This is the convention that we are going to use; we are going to set up reference frames like this and define the mechanism parameters like this for a robot, and then after that, we can do the inverse and forward kinematics in a systematic fashion. Let me put some points associated with what we now did. I had directed z_i along this direction—is that the only possibility? It is not. It could have been in the opposite direction. To start with, a joint doesn't have a direction, unless it is a helical joint. Then you can define some direction; otherwise a revolute joint doesn't have a direction. It has a sense, I mean there is a direction in space associated with that; without it, it could be in one of the two directions.

Similarly the prismatic joint—but the prismatic joint has even one more thing which is not really well determined with regard to this. Why did i put the prismatic joint through this particular position in space?

What is special about it? In a revolute joint there is a specific axis about which one link rotates with respect to the other.

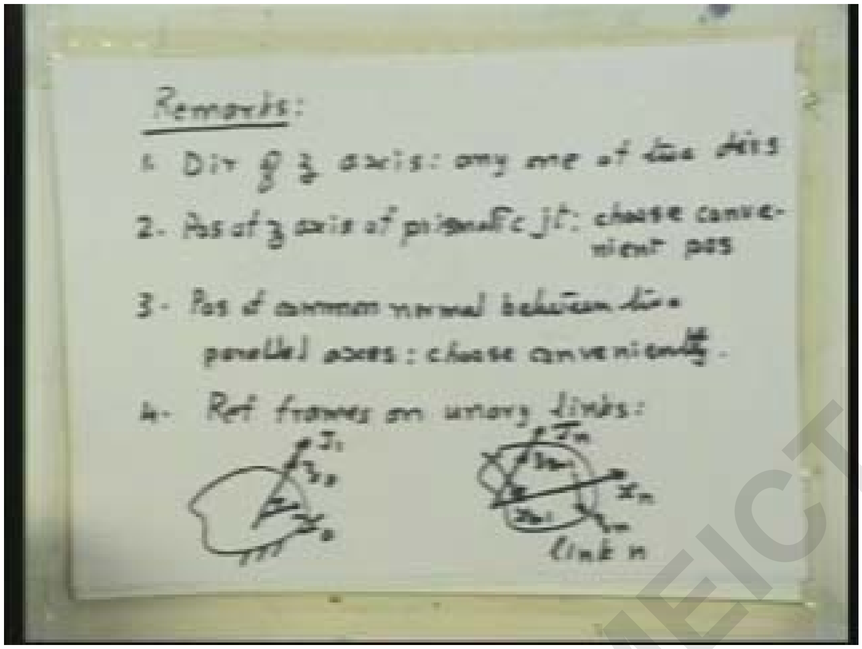
In a prismatic joint there is only a direction; there is no particular axis along which a prismatic joint translates. I could have put it as well out here,

provided I made the direction properly; so the axis of a prismatic joint can actually be located anywhere provided the direction is correct. So we locate it conveniently; so let me note down these points for you.

Any one of two directions would do, so choose anything convenient, anything would do.

It doesn't have to be associated with that physical piston and cylinder that is out there or a slide and the guide; this is out.

Suppose on a link—it is a binary link, which means there are two joints—the two joints are parallel, then where would you put the common normal? There are an infinity of common normals. (refer slide time 44:12)



Again choose something convenient. This is one more thing: in a robot are all the links binary links? A robot, an industrial manipulator, is typically a serial chain; so most of the links are binary. There are two links which are not binary—the base and the end effector. What type of links are they? They are just one joint each, whereas in our convention for setting up reference frames, we use two joints on a link. How do you set up reference frames on links when there is just one axis?

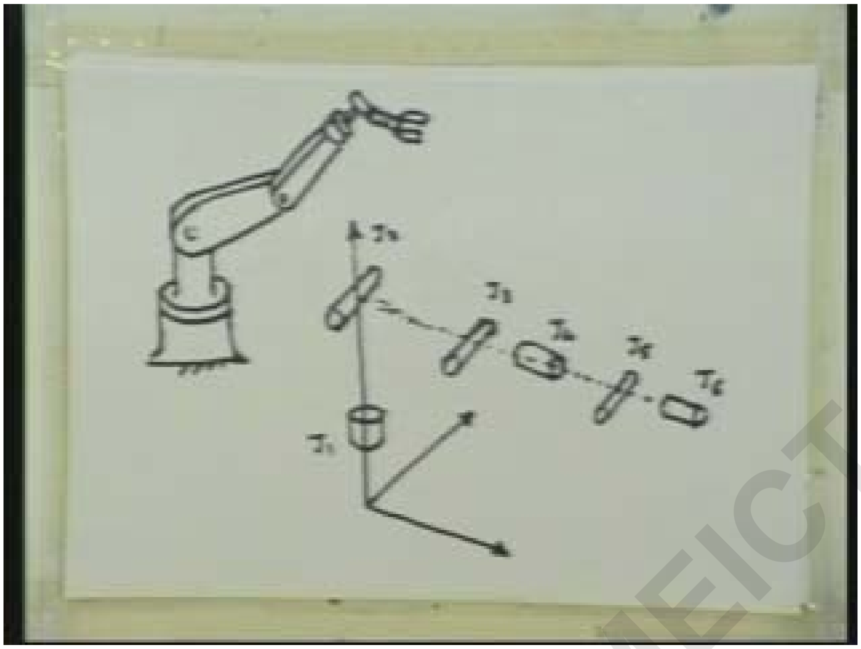
Take a base link, so this is base. Suppose there is an axis on this. This is joint one, so z_0 is along this. What about x_0—x_0 z_0 y_0—this is the twist to link the first link.

So this will be link 0 by our convention; it could be anywhere along this. Take something perpendicular to this; that is all. That is all that is required.

Similarly, consider the last link: there is the last joint associated with that; and already we have set up z_n minus one; let's say x_n minus one. We have to set up the reference frame x_n y_n z_n to this link, the last link. How do we do that? Choose arbitrarily any point for the origin of the reference frame.

Drop a perpendicular to the z_n minus one axis from that point and then choose this as x_n; anything perpendicular to that can be z_n. It may not be chosen like this; it may be more convenient, but we could align it with this if we want to. So this is all that we have to take care of; more freedom is available to us in how to fix these reference frames. So these are some of the points we need to remember.

Now let us take up the sixth revolute puma; that is, in a sense one of the more complicated manipulators, and then set up these reference frames, define these linkage parameters, and then after that, do the forward and inverse kinematics. Let me start this off: if I will first draw a puma—I am drawing it roughly; <u>(refer slide time 53:14)</u>



it is not important that you are able to draw this particular figure. So if you remember what the puma is, there is a vertical pose. Have you gone to the robotics lab and seen this, the puma? Have you seen rino? There is this vertical pose, so this link is the second link, the base link rotates about a vertical axis.

Then there is a horizontal axis here, about which this link rotates. It is like our shoulder. Then there is another axis, which remains always horizontal, about which this rotates with respect to this. So this is like the shoulder; and this like the elbow; this is like our upper arm and this like the fore arm. So that makes what is called the regional structure; the orientational part of the mechanism is this. There is this particular wrist, which is something like this. About this axis, this whole thing can rotate; and then there is another axis here, about which this can rotate; and finally the gripper: the end effector can rotate about an axis like this.

These are the three axes; I will show them in more detail later. There are three axes about which the end effector can rotate.

This is somewhat like the puma. Now let us see how to set up the reference frames for this. So something which is useful to conceive, to do, when you set up reference frames is to imagine the robot in a special position, in a special configuration. It will be fairly easy to figure out how to fix these axes if you actually stretch out puma.

Then all this axes, their relative positions and all that can be fairly easily represented on the paper; otherwise the problem is if you put it in some odd position, it is very difficult to represent in it space. So ideally, with Demontberg's convention of using common normals between axes in order to represent the link, you will see that the serial chain is made up of a sequence of segments in space, which are adjacent ones and are perpendicular to each other.

So you have the link, you have the joint offset, which is perpendicular to that,, you have the next link which is perpendicular to that joint offset, you have the next offset which is perpendicular to that particular link, and it goes on like that in a sequence. It's possible in many cases especially in the case of puma, to align these offsets and joint axes along some global x y and z directions. So if you do that, drawing and consuming it becomes simple.

So, we will do that. I represent the first revolute joint by the cylinder; its axis is vertical. The next one is parallel to this, so another cylinder is parallel to that. I stretch out the whole thing along this axis direction, so the next one is out here, further along this. So I have actually the three axes already drawn; one is the vertical axis, then the horizontal axis, third one is the rotation about this longitudinal direction.

So I have an axis like this, and then further along that, there is this particular axis, which, stretched out like this, turns out to be parallel to this and then further along this, there is a third axis for the wrist, which is like this. These are the six joint axes in a particular stretched out position of the puma. So I will call this joint one joint two joint three joint four joint five and joint six. So imagine that these joints are now frozen in space like this. Now set up the reference frames; it will turn out that many of the link lengths are zero. But any way, set up the reference frames and then we will list out what the various parameters are. We don't have time for that today; we will do it tomorrow.