ROBOTICS

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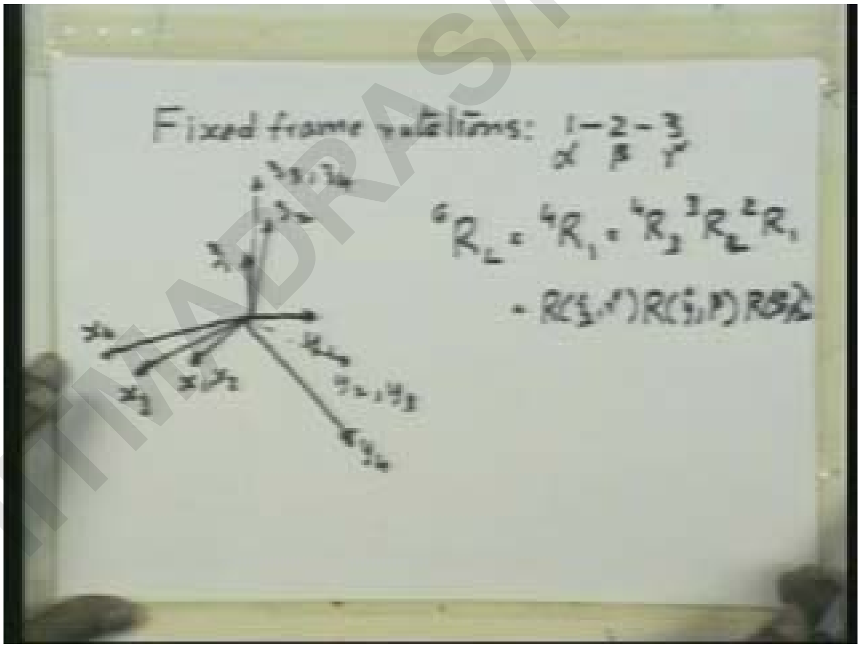
Lecture No. 18

Trajectory Planning (1:20)

In the last class what we saw was how we can specify orientation using just three parameters. In the way we were specifying orientation and precision earlier, we were using twelve parameters and we know that a rigid body requires only six parameters to specify precision and orientation. So out of twelve, there are six parameters which are dependent on the other six; so it turns out that what we use for orientation is that orientation matrix, which itself has nine parameters, out of which six are dependent; six are redundant, in a sense. So we saw how Euler angles can be used in order to specify orientation with just three parameters. Now we started looking at fixed rotations about fixed reference frame axis in order to specify orientation parameters. I'll just sketch out that again and go on to the next topic.

So the fixed frame is x y and z axis by angles, let's say, alpha, beta, and gamma. So let us call this x_1 y_1 and z_1. I will be doing the rotations and sketching it in a slightly different way than I did yesterday so that it becomes little more clear. (refer slide time 09:09)



So we assume that the body reference frame is coincident with the global reference frame to start with, and we rotate about the x axis by angle alpha plus. So if we do that, the z axis rotates in this direction, y axis in this direction by alpha, in order to give you the new position of the body.

Now instead of doing that, I am going to rotate the fixed reference frame with respect to the body reference frame by angle minus alpha, about x axis. If I do that, z axis will go here, y axis will go here.

So the new z and y are here; and x remains where it is. So I call this y_2, z_2 and this is the same as x_2.

So now here which is the global reference frame and which is the body reference frame? x_1 y_1 z_1 is the local reference frame now; x_2 y_2 z_2 is the global reference frame. So this way I need to use just one figure to show all the axes. So now I look at the next rotation, which is about the global y by angle beta. So global y is now y_2; I rotate that by angle beta, angle minus beta, instead of beta.

So this goes here, z_3; don't worry about the sense of rotation; basically remember that this x_1 axis or other x_2 axis will go somewhere.

So we need to only see them separate so this is x_3 now, this is z_3; so y_2 and y_3 are the same; y_1 is this.

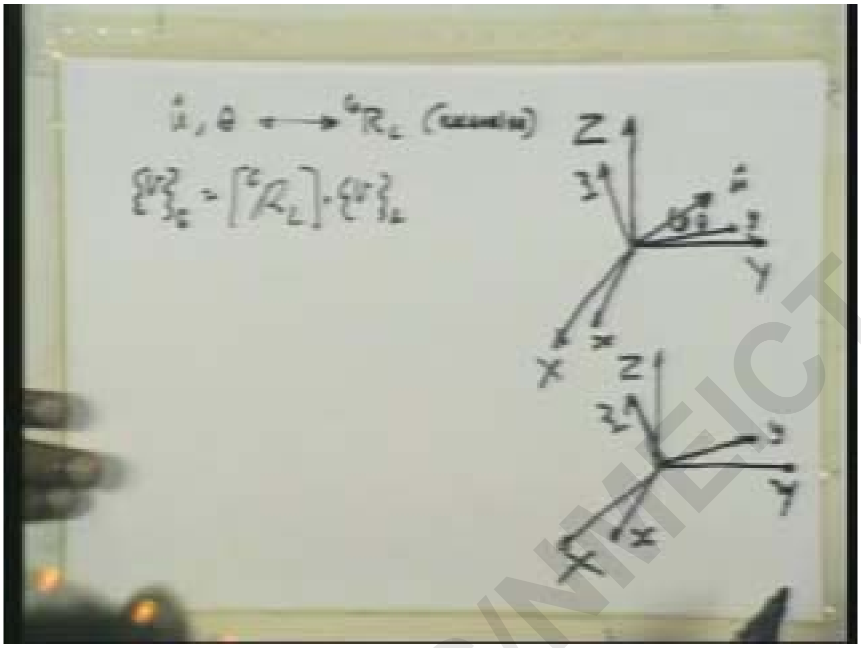
Now, which is the global reference frame? x_3, y_3, z_3. Which is the local reference frame? x_1, y_1, z_1. Now, for the third rotation, which is gamma, with respect to the z axis of the global reference frame z_3, you rotate the rest of them by angle gamma. Instead of that, I rotate about z_3 by minus gamma. If I do that, z_4 remains a result and x_3 will go to x_4, y_3 will go to y_4. And now the rotation matrix, which corresponds to the local to global—which is the local and which is the global now? The local is x_1 y_1 z_1 and the global now is x_4 y_4 z_4. The rotation matrix that we are talking about from local to global is from one to four. So, based on the notation we have given, this is three to four, two to three, and one to two. So what is three with respect to four? That is the rotation gamma about the z axis by angle gamma, so R about z by angle gamma. And two with respect to three is about y by angle beta and one with respect two is about x by angle alpha. So if you have this notation, if you have this way of specifying orientation, the corresponding rotation matrix of local with respect to global will turn out to be simply this.

The local frame here is one. Which is the local frame? One; and the global frame is four. So the rotation matrix of local with respect to global is reference frame one with respect to reference frame four because four is global and one is local. This is alpha beta gamma; in the last one we did, which is about the z frame, that is z_3, we rotated to shift y_3 to y_4 and x_3 to x_4, that is, by rotating with minus gamma.

But we are now talking about three with respect to four, and not four with respect to three. If it is four with respect to three, it is minus gamma; if it is three with respect to four it is gamma. So fixed frame rotations can also be done in whatever sequence you want and we can give some notations for these angles and you will get a valid representation of rotation. This final thing will turn out to be an orthonormal matrix. Now a third way or a different way you can specify a rotation is to do the following: the rotation of the body about a point, whatever be the set of rotations that you give, the final position with respect to the first can be obtained by a single rotation about some axis.

So if you have a global reference frame and you if you have some axis, let's say, u, about which you rotate by angle theta, you will get the new position of the reference frames;

let's say x y z. You can imagine the xyz reference frame to be fixed to the body and so the body has displaced from coincidence with the global frame to the new position. (refer slide time15:28)



So now the entire rotation is specified by two parameters u and theta. Does this seem to be an even shorter description of rotation? No. Because there are three elements in that u vector of which only two are independent that is normal; it's a unit vector.

So imagine u to be unit vector and then the angle theta is the rotation by which you give about this unit vector. So there are actually four parameters here; out of these, three are independent. So it should be possible to determine this if we are given the local to global rotation matrix. And it should be possible to do the reverse also: given these two, you should be able to find this. So this is an exercise.

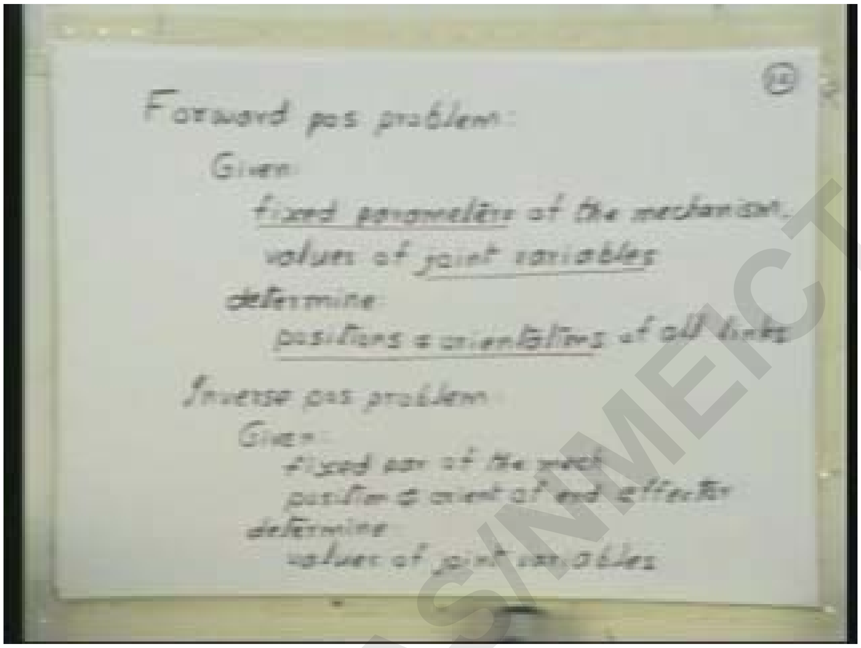
Try to work this out. It is quite useful to do; I will give you just one hint. Look at two positions of the rigid body. Imagine that the rigid body is initially coincident with the global axis and now it is rotated and the reference frame has gone to small xyz. Now if you will look at the two positions of the rigid body, most points would shift from their original positions to new positions.

What about the origin? It will remain where it is. In addition to origin there are a set of points which remain where they are— along the u vector so that is what we call the axis of rotation.

So due to this transformation, local to global, multiplying let's say a vector in the local frame, you get it's vector or the position itself because the origins are coinciding in the global frame. So this can represent positions of points themselves. There are points which remain where they are; so if we use that fact, we can immediately get u. So you try to formulate that and try to determine.

So try to do this yourself. Now I would like to go back to our original definition of the inverse and forward kinematics problem. If you look at your notes, we have defined it in a particular way.

The forward position problem is given the fixed parameters of the mechanism and the joint variables determine positions and orientations of all links. so we have been defining position and orientation; and the inverse problem given the fixed parameters of the mechanism position and orientation of the end effector determine values of the joint variables. (refer slide time 17:04)

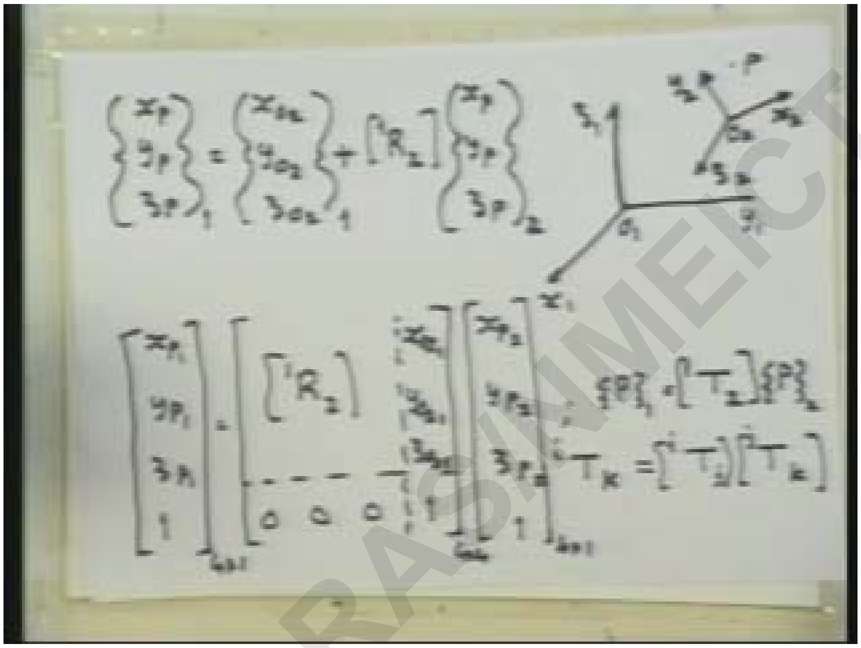


So let me take a case where we need to specify, find out the position of the end effector, in a specific problem, where we need to manipulate objects. We need to take an object from some place one another and we need to specify, we need to find out how to specify the position and orientation of the end effector for that, so that what we have in covering would be illustrated by a very clear example. In order to make it a little simple, I will introduce a concept which will make the notation of finding out new positions, and if you have a sequence of displacements the notation for representing position and orientation will be a little more compact. This is something we will be using fairly frequently. If you again let me draw the axis, let's say it is x_1 y_1 z_1, you have the local frame somewhere here. So y_2 z_2 x_2: this is the point o_2, the origin, and this is the point o_1. The transformation of a point, let's say p, in this reference frame to its global position is given by x of p; y of p in reference frame one equals the location of the origin of this reference frame in this frame; let's call it x_{o2}, y_{o2}, z_{o2} again in reference frame one plus this rotation matrix, which takes vectors in two to one, multiplying the local coordinates of the point p.

So this is how the transformation of positions of points takes place. I can express this in a slightly different way using what are called homogeneous transformations. I represent each vector, a position of points, by a four-dimensional vector.

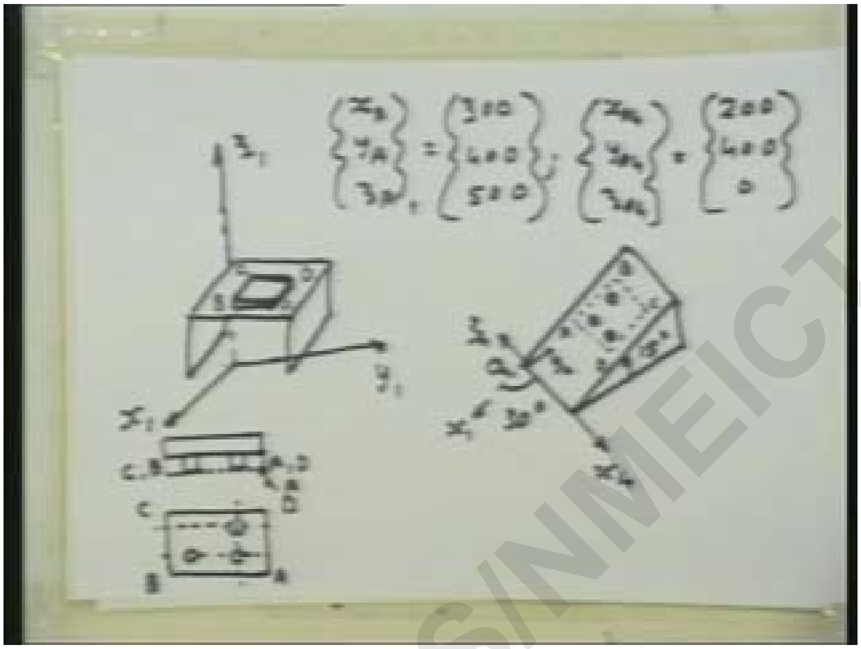
So instead of this, I write it as x_{p1}, which represents not p1 but the coordinate of p in one, exponent of p in one. Similarly y_{p1} z_{p1} and I add the number 1 here, which is necessary for making the transformation compact. One has to be kept there always.

So this is the extended four-dimensional vector of this three-dimensional vector in this transformation, then this multiplies the local coordinates.
Now here, the elements of this matrix of the rotation matrix and this offset; so here you have the three by three rotation matrix; this portion is the three by three submatrix of this four by four matrix. This is four by one and this is four by one. These elements are zero and this is one. This column is this particular vector, x_{o2} in one.
Now remember this transforms the locations of points, so this has the local coordinates of the point and this has the global coordinates of the point. (refer slide time 24:18)



So let me call it p in reference frame one, which is equal to this transformation matrix, which is four by four, taking two to one and p in two. This has a property that you can express some k with respect to i. Remember this transformation matrix contains information about position and orientation because it has the rotation matrix and offset of the origin; that could be specification for position and orientation. So position and orientation of the kth body with respect to the ith body can be obtained from an intermediate transformation of the jth body with respect to the ith and then the kth body with respect to the jth.
This actually makes the chain multiplication very compact if we didn't have that and we have used this non homogeneous form for the transformation then what would happen is that this particular translation term will actually build up into very large set of frames, which is cumbersome to deal with. So this becomes more compact this build-up is going to occur right there in the columns here but you don't see it when you write it
when you when you don't expand it you don't see it. so there is some amount of economy in notation. Just try to see what this would be if you have given k with respect to i from that how do you find out i with respect to k? You know the elements of this; from that what you get the elements of this? Again, do this as an exercise. This is the inverse of this—one is the inverse of the other. So this notational convenience is necessary for us.

Now let us apply what we learnt to a specific problem. So there is a table on which there is some part placed. This has to be picked up and put on and assembled on to this part. So I will show this particular part in detail later. It has three pegs; this is something we had seen earlier; it has three pegs and those have to go in to holes here (refer slide time)



Now with regard to dimensions, positions and dimensions—let me give the edges or the corners some names, so let this be the corner A, B, C, and D. Let me show this part in more detail. If you look at it from side, in this direction, what you will see is the following. This is corner A, and behind that D, and this is the corner B and C.

So there are these pegs, which are there below the body. This is the corner A, B, C, D: so there is one peg here, another here from the same line parallel to these sides and there is a third peg here.

So this is placed on the table with the pegs down. This has to be picked up and brought to this position and assembled.

Here let me draw the outline of the part when it is assembled. This corner is A; this corner is B, C, and D.

This is essentially the operation. Let me use the reference frame to locate things; if we use the global reference frame like this, along this frame, we will call it x_1 y_1 z_1. If I consider this point to be A so let me draw a line to this and through this and call this particular point as A. Is that clear in the figure?

I call that point A; x_{A1} is the specification of the problem. x_A y_A z_A in reference frame one, which is the global reference frame, is are given as 300, 400 at an elevation of 500 from the base of that—all these are in millimeters.

So I am talking about the point in contact with the table. The point which is in contact with the table there is actually no material point. Imagine that it is in contact with the table.

I also give the following information. This is an inclined plane; this is horizontal, these two edges are horizontal; the angle that this makes with the x axis, is, let's say, 30 degrees and the angle that the inclined plane makes with the horizontal is 15 degrees.

Now if you look at A in this particular reference frame, I will give numbers based on the way I worked out so we will see that a little later. I will call this x_4 axis and the y_4 axis is horizontal in this direction and z_4 is perpendicular to both of them. So the origin of this, that is, this corner of the inclined plane that is o_4 with respect to the global frame is given in addition to these angles. So x_{o4} y_{o4} z_{o4} is on the x_1 y_1 plane. So that's given; these two angles are given; the angle made by this edge and this particular inclination are also given.

Now in addition to that, you are given few more things or rather let me ask what else needs to be known? Origin of what? Of the object; so B A B A edge is along y_1, the AD edge is along x_1. That is the way it has been oriented in the first position; second position is for you to work out. A few most specifications have to be there you have to work it out you will work it out. what needs to be done is on is the gripper has to come, pick up the part, right and take it to this position—to the position where you can push it in to assemble. If you remember the joint interpolation discussion in this course, the input for that, to determine how joints should move or certain locations of the end effector from what we used in the inverse kinematics problem to work out the corresponding joint angles of the joint variables, form the inputs for calculating this interpolation.

So what we are trying to do here is specify the position and orientation of the end effector at whatever convenient points which are necessary to pick up this object and place it here. There are a set of points or set of positions and orientations the end effector has to go through. So we have to take certain decisions; for example how would the end effector pick up this object?

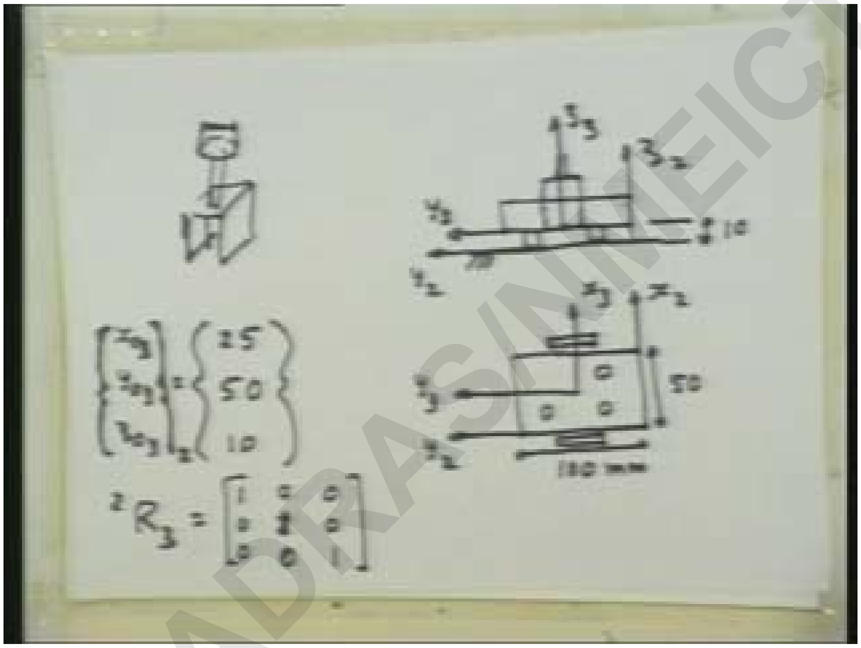
So let us look at the object a little more. This is on a table; so a gripper will come—let's assume the gripper has two flat faces, which can be closed, which can be brought together. So we can take it to the top of the object and close the gripper to hold the object tightly.

So how will you place this? On top of that, the robot hand is there, the robot arm is there; the rest of the robot will be there. How do you pick it up? Let me show it from above also. The width and length of the piece are known; let's assume 50 millimeters and 100 millimeters. Remember what we need to do is locate these things, these pegs, on top of the holes; you need to do that finally.

So how do you place, how do you pick it up? There are many ways of doing that, let us think of some simple way. We can pick it up with one of these sides of the gripper touching this and other on the other side, or touching these two faces. If we try to hold it by its edges, it will not be firmly held. So I can bring the gripper on top of this. How far down? It shouldn't be below the lower edge; that is, assuming that when you push it in, the lower face will be flushed with the top face of the part which you want to assume—that may not be the case; the peg may be longer than the hole. So you may be able to bring down, further down; but it is safe to keep it up to here. This is the place you can locate; so the gripper could be like this, the robot further up like this. So if you look at it here you will see the gripper coming down like this. In order to specify the end effector position and the orientation, after the object is gripped, we need to know the relationship between the object and the gripper. Because what we know are the initial and final locations of the object and not the end effector so after the gripping is done we need to know the relationship between the object and the gripper, the end effector, in order to be able to say where the end effector should be in order that the objects are at these two positions. So that is why we need to decide on how to grip. Let me fix the reference frame to the object.

If you look at it from above, let this be x_2 and let this be y_2; the zy location of the corner is here. So this is one way you can fix the reference frame; you can do it whatever way you want, it is not really very important provided you define it very clearly.

Now this particular gripper, once it grips, also has a reference frame attached to the gripper—let me consider it to be at the center of the gripper. So the axis fixes to the gripper x_3 y_3 (refer slide time 47:28)



and here, perhaps, it is a good idea to put the origin of that reference frame at the edge of the gripper. So if you do that, then this is the z_3 axis and this is the y_3 axis.

So now we can decide what should be the location in this direction to grip the object. What would be that direction? The location of this origin this particular origin, which is the reference frame fixed to the gripper, this direction is fixed because if we put it at the center of the gripper, that will be the center of the object where the gripping is done. Since this is 50 millimeters from this edge, this particular point will be 25 millimeters from this edge.

If I center it in this direction also, this would be 50 millimeters from this edge. Now I can specify the position and orientation of the gripper reference frame, that is, the end effector reference frame with respect to the object reference frame as follows. What are the coordinates in the x direction? This is with respect to two. So let me use curly brackets. What are these? We have centered it, right so what is the x direction coordinate? 25. y direction? z direction? No. The height of this hasn't been specified.

So we will say that this is i plus 10 millimeters. What about the orientation of three with respect to two? Would it be the identity matrix?

All you need to do is look at the x_3 vector in the second reference frame. What are its coordinates—the y_3 vector in the second reference frame and then z_3 in second reference frame? It is one, it is aligned; it is aligned with the second reference frame. So the identity matrix is the rotation matrix

So position and orientation of the fourth reference frame with respect to the third reference frame and third reference frame with respect to the second is given by these parameters.

Now my question is, what all do we need to know in order to do this operation? Let's go back to what we did earlier. The object is placed here. Its A, the coordinates of the point A in the global reference frame one, is given. The other thing we know is the coordinates of this particular point, which is one edge of the inclined plane in the global reference frame, and we know the directions of two of the edges. So we haven't worked out the coordinates of this reference frame: the position and orientation of four with respect to one. That is something we can do given this information. It is required. That is one thing; the other thing is where will this reference frame be, reference frame two, which is attached to the object B, when the object is in the first position?

That is something we can work out. We have attached reference frame to the object and we know the object in the first reference, first position. So the orientation matrix of two with respect to one—

two is the reference frame attached to the object with respect to one in the first position of the object—can be obtained.

That is, R_2 with respect to one, position one of object; two will go somewhere else when the object goes here. That is something we will have to work out later; so what is this? I will have to bring back and show this partially.

So there is the x_1 y_1 z_1 reference frame; and when we look at it from above in the x y plane, we see this.

In which direction is x_2 derived with respect to the global reference frame? It is parallel but in the opposite direction; it is in the negative x y direction. So this particular first column would be minus 1 0 0. Now similarly, if you look at y_2 in this direction, y_1 is in this direction; they are parallel, that is, in opposite directions. So you have zero minus one zero. What about z_2 and z_1? They are in the same direction; so 0 0 1;that is, R_2 with respect to one is the first position of the body. Now we can get the position of this particular point A in the global reference frame and the coordinates and the directions of the reference frame. When it is assembled, our job is done.

We have all reference frames at appropriate positions and transformations; now we can simplify do a transformation to get the positions that we want. Let us construct that; these are exercises you have to go through if you want to really specify even the position of the end effector.

We have this inclined plane; we have put some reference frame already on that: x_4 in this direction, y_4 along this edge but origin being here; and z_4. I had fixed y_4 along this edge actually; so let us not consider this; this is y—along one of the edges of the inclined surface, not the horizontal surface. And then perpendicular to both of them, you have z_4.

And now when the body is placed here, your x_2 y_2 z_2, which is attached to the body, will be aligned along this. So I will call that new position x_5 and this is y_5 and this is z_5.

In this frame this corresponds to A and the two holes that you have are here; the three holes that you have are here. So finally when the object is placed here, this x_5 y_5 z_5 have to coincide with the body reference frame x_2 y_2 z_2. If that is done, you will see that the holes get aligned. So we have to find out the position and orientation of this reference frame in the origin, in the global reference frame, and for that we can actually do it through this particular reference frame, x_4 y_4 z_4.

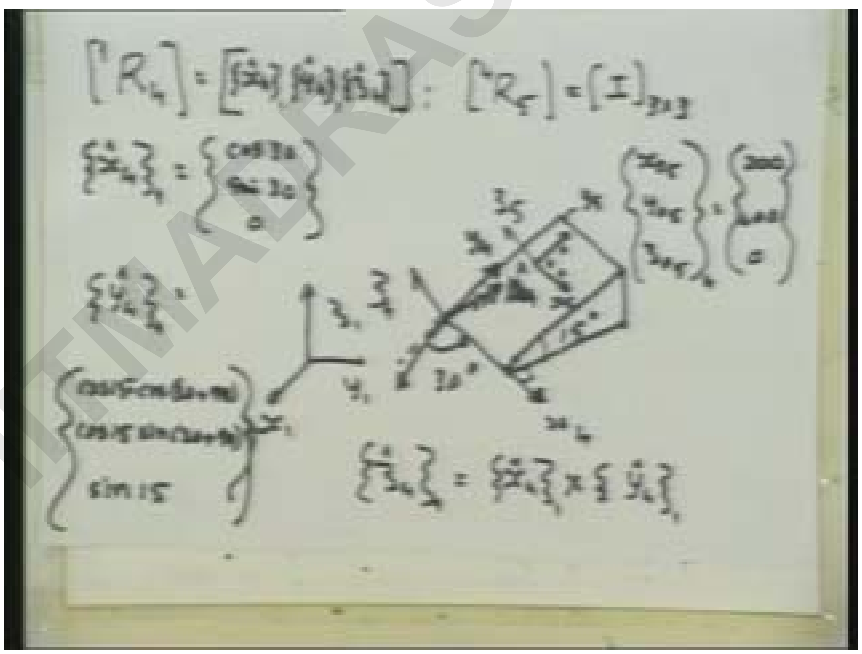
We have to know the origin of this particular point, which we have already specified, if you remember. This is o_4, which was specified as the global reference, the global coordinates of the point o_4, which is the origin of the fourth reference frame, as 200 400 0.

Let us now work out the orientation matrix R_4 with respect to one. So if you look at this, you will see that it consists of three columns: the unit vector along x_4; the unit vector along y_4; and the one along z_4. These are the three columns that each of them expressed in the global reference frame. So what is x_4— the unit vector along x_4 in the global reference frame? I am talking about this reference frame, this particular vector. If you remember, our x axis is like this and this angle was given 30 degrees; although it doesn't look like that, it is 30 degrees.

Now what are the coordinates of this? Cos 30 sin 30 0.

What about y_4 unit vector? y_4 is along this. What about its z coordinate? This is 15 degrees; z coordinate is sine 15. The the z axis is along this; z_1 axis is along that; so what about the z_1 coordinate of this y_4? It is sine 15.

What about its horizontal component? The vertical component is sine 15; horizontal component is cos 15. So we have two cos 15 components here, which further get projected on to the x_1 and y_1 in those directions. (refer slide time 01:02:43)



So this is 30 degrees with respect to x_1; this is 30 degrees plus 90 degrees. The projection, the cos projection of this unit vector onto this is simply along this. So, that has cos 30 plus 90, and cos 15 into sine 30 plus 90, with respect to the y_1 projection; this x_1 projection. You can easily see that this is the unit vector; and this is the y_4.

So now what about z_4? We can do the same thing; project and find out. But we can also get it through both in one.
So you need to really find only two; we can get the third from this cross product. I said it can be specified by three; I didn't say we need only three because in order to do the transformation we need all this. It can be specified by three or twelve; depends on what we used to specify.
So if you look at it carefully, you will see that we have used really only three. Why do I say that? Look at the way I specified this particular reference frame, the orientation—the way I specified really, I have used that; try to figure that out.
We have four with respect to one so we should be able to get this.
What about five? Finally, we need five with respect to one. So we can get five with respect to four and then we can get five with respect to one. What is this? This is an identity matrix; so i_3 by three is the identity matrix.
But is that enough? I need to also locate it so I have to know with respect to this edge and with respect to this edge what is the location of the point A? So this being reference frame five, we have o_5; so x_{o5} y_{o5} z_{o5} in this reference frame four.
That is the easiest way to specify that. That is the one easy way of doing that; I had given some values to them. There is a slight change in values, so I will give values here. This is 200—so this is something you have to work out numerically—400 0.
I had actually given it earlier as four with respect to one, which is not correct. I have some other numbers for four with respect to one. I will modify that now; position of five with respect to four is 200 400 0.
So four with respect to one is 500 800 0. So we will work out inverse kinematics of this later; we'll work this out numerically.
I suppose we have given everything that is required. I could have done it in a slightly different way; worked it out instead of this. These points are these particular locations. Working out is a little different from this. Instead of that, I gave it very conveniently, saying that when placed, the point A will be at this particular point, which is easier to specify. Now the exercise is, find out the location of the end effector just before gripping, at the first position, and find out the position of the end effector when just before pushing the pegs into the inclined plane. I think we have specified enough to get to that.
We can use the homogeneous transformation matrices for calculating it; multiplying four by four matrices takes time so you can do it on a computer.
So complete this exercise; in the next class let's see what the answers are.