ROBOTICS

Prof. K.Kurien Issac
Mechanical Engineering
IIT Bombay

Lecture No :15
Trajectory planning (Time : 01:20)

First off, what has been covered, and also telling about the total course, an overview of what is robotics was discussed first, then robot anatomy was discussed in some detail. So, which included mechanisms, actuators which corresponded to the muscles, transmission for transmitting the motions from the motors to the joints, and then, sensors which are required for feedback to tell the robots what it's fate is, and controllers which are used to control the robot.

So these are things which have been covered under anatomy, and after that, we started on this last topic called robot control [noise].

We started with specification of task, and how motions can be specified, or calculated, at the joint level, ok?

So, there are seven calculations which need to be done for control apart from motion calculation [noise], that is what I will concentrate on, and the two important topics there are position and velocity calculations.

So we will have two weeks on that. I will get into the details of that soon. After that, we discussed what can be called as intelligence robot, so, essentially part planning, and after that only we get through the lowest level of control where the robot motors are controlled at the lowest level in order to carry out the motions which are specified. Then a very important sensor of robotics will be discussed, that is, vision, camera, and how image processing takes place in typical robot problems, and that will be the details of robotics. Then we will deal with some advanced topics and also some space studies.

So, what I will be covering in the next two modules next two weeks, are these calculations, simply these calculations for position and velocity, fine?

Let me motivate why such calculations have to be done. While we face the problem, consider that some block has to be picked from some level and placed at some other level, fine? This has to be picked up by a robot with a gripper. So let's assume we have specified the initial, some positions of the robot, or rather, the gripper, so naturally the gripper has to close onto the object at this position, and finally, when it takes it here, it has to release it, fine?

Now, in order to ensure that this is taken up, and not into the table, into the surface, we may say that immediately after this, this has to go up. So, maybe to a position like this, and before it is put down, to ensure that it doesn't come inside the corners, it has to be taken up like this, and so we will specify some points here, positions here,

so that this could be the first one, first position, second, third, and finally we put down the block and release it, right? When we give these specifications, in order to make the robot move from the position 1 to 2, to 3, to 4, we need to calculate what would be the joint angles, or joint positions, so that we can control motors in order to make the join joints follow those variations of joint angles with respect to time. This is what we learned as joint interpolation, right?

So, in joint interpolation, with respect to time, let's say you specify time starting from t 1, t 2, t 3, t 4, which correspond to these four positions, fine? And you have calculated at each of them what are the joint angles.

So this is some theta 1, some joint angle of the robot varying with respect to time. Now the question is, how do you get this theta? How do you get these thetas which will make the robot take the end effector, the gripper, take it to position 1, to 2, to 3, to 4?

So [???7:30]. Imagine that this is the robot, fine? [noise] How do you calculate the these joint angles? Yeah, this particular problem is called the inverse position calculation problem in robotics. So given the end effector position, determine the positions of the joint angles, values of the joint angles, or joint, it may be a prismatic joint, in which case the extension of the joint says that the gripper is placed at the position you specify, right?

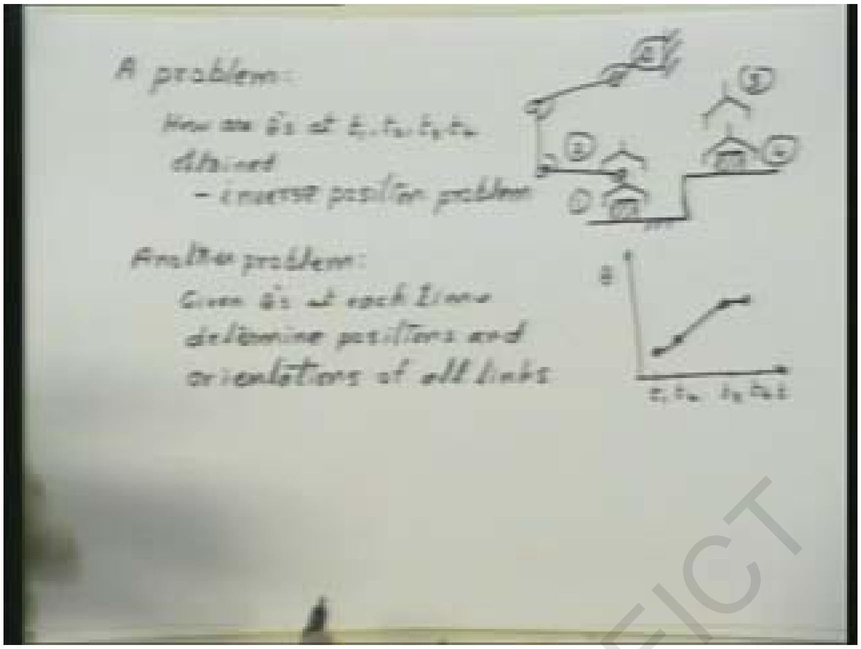
So one problem is you will realize that in order to calculate the entire part, or the entire trajectory of each of the thetas you need the thetas at least at these points, only after that can you do this joint interpolation, fine?

So this particular problem is called inverse kinematics, or inverse position problem, [noise]. Now, consider another problem. If you remember, we have taken you to position 1 to 2 to ensure that it doesn't collide with the table, or the surface on which the block is kept, right? And immediately before putting it at 4, we had specified a position 3 to ensure that it doesn't dig into the table when it places [???10:20].But how are you sure that these joint angles which are calculated will ensure that there is no collision in between? How do you ensure that? So, imagine that this computer which is controlling the robot is there, how do you program it to check this? That the constraint is that no part of the robot, or the end effector, or the objects, should fit, go into this region, fine? So, how do you [???11:12].

You are really talking about how to find this particular trajectory of each of the thetas such that it doesn't begin to, or collide with the other object, right? The problem I am posing is not that. Assuming that you have done a joint interpolation the way it was discussed in the previous module, where we have got now the thetas at every point of time from t 1 to t 4, fine? How do you check whether any collision happened? By doing simulation. What do you mean by that?

Yeah, so basically we have to check for every theta every time every point of time from t 1 to t 4 for the corresponding thetas what are the positions of all the links, and whether the material parts of those links are colliding with either each other or any part of the environment, right?

We need to check. What you mentioned would be corrections. If we find that there is a collision, we need to then correct this particular trajectory which we have got and take care of that, right? And that is actually a higher-level problem. We are now looking only at that. So this requires the ability to calculate the positions of the links given the joint angles. This is called the forward problem in robotics. So these two problems are very crucial to controlling the robot, and calculations which these two problems have to be solved for the manipulator for which the controller is designed. You have to have algorithm to calculate this, fine? (refer slide time 14:10).

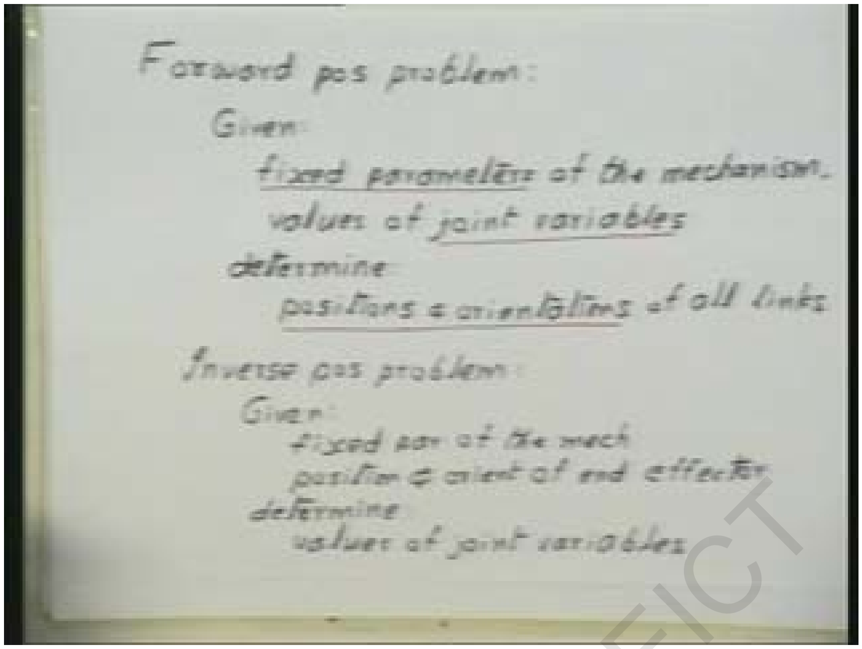


Now, let me shows the problem in full detail, and let us formally state this problems so that we can solve them. So the forward position problem is the following,

fine? So there are [noise] certain terms here which are not very clear at this stage. One is, what is, what are the fixed parameters? What do you mean by joint variables? Even more fundamentally, what is meant by position and orientation of a link these are terms which are not very clear. Maybe in a simple planar setting it is easier to understand that, but in a spatial mechanism [???16:46], or in a little more [???], and we need to also finally determine the solution we need to do these calculations, so, that is what we are going to discuss in detail.

Let me just write down the inverse position problem. So, same thing as this,

fine? It is clearly one is the inverse of the other, that is, what is given in one, and what is to be determined [???18:34]] here, right, with one difference, that is, here positions and orientations of all links have to be determined from the joint variable values, whereas here, given the end effector position and orientation and you are not given the positions and orientations of all other links, only the end effector, you need to determine the joint variables. So, there is some subtle differentce here, fine? So, same things are not really defined, these things are not defined yet. (refer slide time 19:14).



So, let us first have to do that, but before we do that, let us actually pose the problem in a very simple setting and solve it.

So we will look at some simple planar manipulators and solve this. So, a two-revolute planar manipulator with three links. So what are given are the link length, right, and the joint angles, theta 1 and theta 2.

You need to determine the positions of the two links, right? So when you determine the positions of b and point p, you know where the links are, right? Let's call this as global reference frame. So in this global reference frame the coordinates, x B y B, that is the position of B, and the coordinates, x P y P, position of P. Is that [???22.13], the orientations of the links, so let me call them as alpha 1, alpha 2. I mentioned there are three links. Which is the third link? The ground, fine? So what is the calculation? It is very straightforward and simple. x B is it is l 1, sin theta 1. What about x P plus l 2? Yeah, this is fine? Very straightforward and simple. So, similarly, y P.

Now, let us look at the inverse problem. So the inverse problem in some reference frame at which the [??23:47] of the first link is [???}, we give the position of p. In addition to that, we also know the link lengths. We need to calculate theta 1 and theta 2. Before we go on to that let me just pick out one thing. We said we need to calculate in the forward problem (this is from 2). In the forward problem, apart from the positions of the points B and P, also the orientation of the links, right? So, what is alpha 1?

So I have not defined alpha 1, so we will say the angle made by the directed segment ab with the x axis is alpha 1, so that will be the same as theta 1. Angle made by the directed segment BP, with respect to x axis, is alpha 2. So what is alpha 2? So, that is fairly [???25:20], fine? So now (noise), the inverse problem is given the position of the point on the end effector, that is, find out and l 1 and l 2 the link lengths, find out the theta 1 and theta 2.

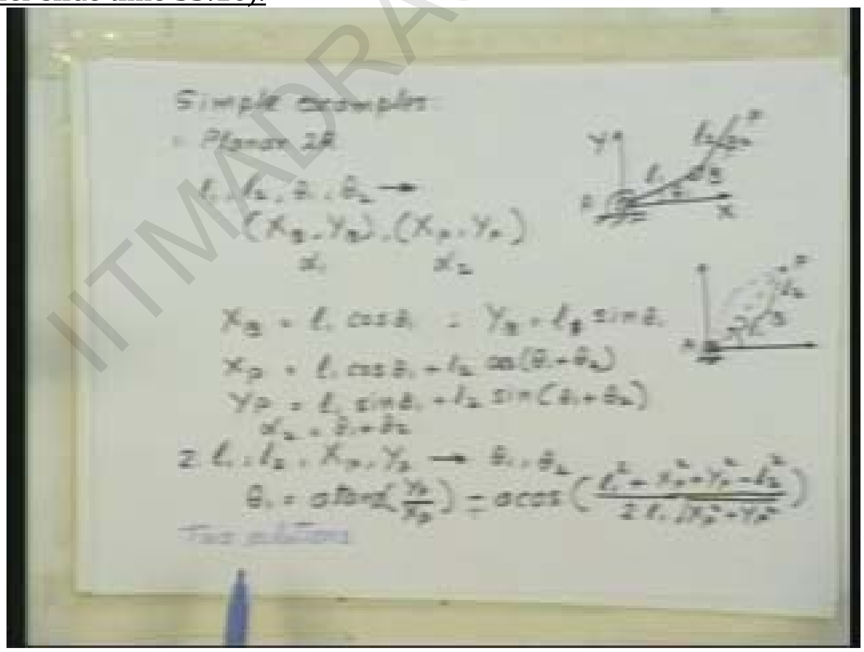
So, I will do it here. It is fairly simple, that calculation. If you draw the mechanism in some position and also connect B to A (this is B), then this angle can be easily calculated because we know the position of p, correct? And now, look at this angle ABP, all sides are known. So this angle also can be calculated, right?

So this angle minus this angle will give me theta 1, fine? Once I have theta 1 I have the position of B and I have the directed segment BP in this coordinate system. So I can find out alpha 2. Once I find out alpha 2, I subtract theta 1 from it to get theta 2. So let me write it down.

So, that is this angle, fine? When I use the cosine rule as follows, this is the side l 1, and this is the side l 2, and this side is the distance from here to here, fine? So, if I use the, say, l 1 square, you agree with it? This is the cosine rule.

Yeah, a tan is the cotan function for inverse of tan inverse, and a cos is the cotan function, cotan most of the languages have that. That is one thing. The other thing which should be noted is that this angle a tan, tan inverse is normally known to within two quadrants, you don't know exactly which quadrant it is if we use a tan, but we know exactly where yp and xp are, what yp and xp are, and so, where this particular point p is, in which quadrant it is, so we can actually precisely calculate this angle including the quadrant in which it is and there is the function which gives you that, which takes two arguments, it takes, it is called a tan 2 in most of the languages, and you give two arguments, y p first and x p next. So we use a tan 2, you get this uniquely, otherwise, you get two solutions for that. This is one point. The other point is about this particular angle [noise].

How many solutions are there for this angle, inverse cos? If you get some 30 degrees, minus 30 degrees also will be valid. There are two solutions, right, and both the solutions are valid, actually. If you look at the other solution it is different. So, this could be minus or plus. (refer slide time 33:16).



So the inverse kinematic problem here has two solutions. This is very important to note. Is it that in every case there are two solutions? It can be more than two.

I am talking about this particular problem, yeah. Well, there is a particular case where it can be more than two. It can be infinite, but let me first ask the question in the other direction: can there be less than two? One can be there. When it is stretched out, for example, if the point p is given as some point on the boundary of the work space, right, then there can be only one solution,

yeah, it can be. But suppose it is outside the work space there cannot be any solution. It should show up in our calculations, right? When we say there is no solution, one solution, two solutions, infinite solutions, it should show up in this particular equation that we have written. Where does it show up? Right. cos inverse argument has to be between minus 1 and 1. If it is outside that, then there is no solution, and the function is not defined outside that domain. And if it is 1 or minus 1, it has only one solution, right? If it is 1, you can have only 90 degrees as the solution. If it is 1, you can have only 0 as the solution. If it is minus 1, only 180 degrees as the solution, right? In this case there is only one solution, and if it is less than minus 1 and greater than minus 1, and less than plus 1, there are two solutions, ok? These are the ways it occurs, and you can recognize it from this calculation directly. Another point that one is to notice here is the following. I said that in the definition of the problem of inverse kinematics if you look at this the position and orientation of the end effector was given, right, whereas if you look at the way we pose the problem here, what is given here, only the position of the point on the end effector is given, the orientation is not given.

Can you figure out why that is done? Right, that is correct, but more generally the answer is there is a 2 degree of freedom mechanism. You cannot specify three position variables independently. The third one will be dependent on the other two, so you cannot give the arbitrary specification for the position of the point on the end effector as well as the angle made for the end effector, ok? That is why only this was done.

So now, let us go back to some of the things which were not fully defined when we stated the problem. We said that something called fixed parameters of the mechanism has to be given. What are the fixed parameters here? l 1 and l 2, right? Those are fixed. Any other fixed parameter? Ground is fixed, but what about the parameter? I could have actually positioned this anywhere here, right? The location of the point a in the global reference frame really is a parameter, can be treated as a parameter. Here, I actually simplified it by putting the origin of the reference frame right at the point a [???35:02],

ok? If I had put it here, x a and y a would also be some fixed parameters, ok? Now, what about joint variables here? Theta 1 and theta 2, the way it is given here, why did I define theta 1 and theta 2 this way? Why not give alpha 2 instead of theta 2? Is a 2 fine? But, I suppose, the calculations would be as simple; alpha 2, yeah, right, the actuator is mounted on the link and if you think about the control problem it is the motors that we have to rotate in order to make the joints rotate, right? And joints are going to rotate. The angle the joint rotates by is essentially the angle by which one link rotates with respect to the other. That is why the angle is defined as joint variable, is defined here as the angles made by one link with respect to other rather than with respect to base link

So, the angle between the two links which are at the joint, ok? These are things which we will do in a more systematic way when we go to specifying this type of spatitial mechanism. These are fairly simple planar mechanism fits.

So, let me come to the 3 r manipulator which we mentioned, and pose this problem. Anything else to be determined here? Theta 1 has to be determined. What about theta 2? That also has to be calculated. How do you do that? Yeah, we know alpha 2. No, do we know alpha 2 right now? We don't know. We are to calculate it. How do we get that? We know theta 1 now. We can find out what b is, where b is. Once we know where b is we know we can find out the angle bp. That is alpha 2.

Once we get alpha 2, subtract theta 1 from that, we get theta 2. That's clear? Everyone write it here. So let's look at the 3 r manipulator and pose this question. I will pose only the inverse kinematics here.

So given the link lengths, so let me show it first and then set up the reference frame. I put the end effector at the end, so that the orientation become relevant, and let me consider the point on the end effector, always it appears to be not on the end effector. It is not on the end effector; this is some point fixed with respect to the end effector, fine?

The link lengths are l 1, l 2, and l 3. The joint angles defined the way we had defined earlier,

so what is given here are these link lengths and the fact that the end effector, this special point p, so this is a, b, c. This particular point, p, on the end effector which is typically called a reference point which is used for defining work space, if you remember, reachable work space was defined using a point on the end effector, fine? So, a particular point p is here and the direction of the link has to be this, which is essentially given by this angle, so position of p and the angle alpha are what I have given. So l 1, l 2, l 3, x p, y p, let us callit alpha 3, because it is the third moving link, alpha 3, right?

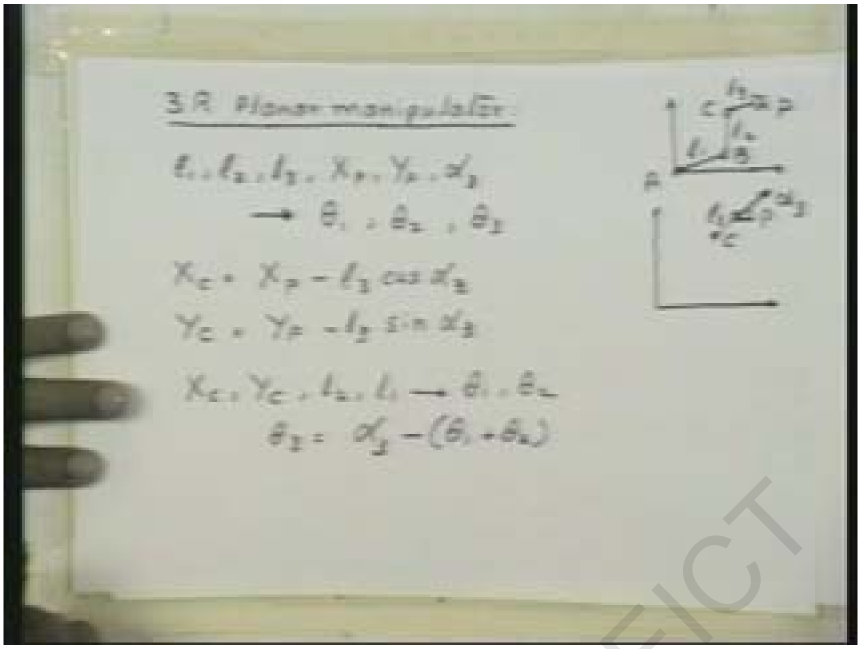
We need to calculate theta 1, theta 2, theta 3. So this is the point, p, and this is the direction made by the vector, cp, right? Alpha 3 is the direction made by the vector cp. how do you now calculate the joint angles?

Right, this is the same as before. Did you here that? The suggestion is that we know the position of p and the direction, cp, right? With this information, we can calculate the position of c. How do we do that? Geometrically draw this, extend this line backwards by distance l 3. No, no need to get in all; you measure this out so you get the point c, so that this distance cp is l 3. The remaining part is the same as what we just did for the 2 r manipulator. It is as simple as that, fine? Though simple, this is actually one of the crucial steps in even the more complicated manipulator. It is something we will do even in the more complex manipulator because of the particular property of the kind of risk which are used in this manipulator, ok?

Is the procedure clear to everybody? I will just write it down. Fine. So, from x c, y c, l 2 and l 1, we can calculate theta 1, theta 2, as we just now did, right,

and theta 3 is nothing but alpha 3 minus alpha 2, right, and alpha 2 is theta 1 plus theta 2, fine. It is as simple as that. So again, the question of how many solutions. This is the crucial question in most of the, in all the problems.

So, one, two, three, four, five, four, two, what? Yeah, why two, why not four? No, alpha 3 is given as the angle for the direction of cp. That is the unique angle. (refer slide time 43:17).



so you can only be, cp can only be in that direction, then only two, right, because this has only two solutions. This is unique. So, we have already found that this is two solutions. Can there be a case where there is one solution? There could be. And no solutions? That also is possible, right, but that will show up in this calculation, fine? But, the case of no solution, it is important to realize, may not be a point which is not reachable. There could be a point which is reachable by the position, there could be a location which is reachable by the point p, but still, there may not be a solution. The reason is that we would have specified an angle, alpha 3, which the link should have when the point reaches that. So you imagine that the whole thing is stretched out, let's say, in this direction, then cp can only be zero, right?

The position is xp equals l 1 plus l 2 plus l 3, right? y p is 0 somewhere about here, right? y p is 0, and angle, if anything other than 0 degrees is given, it is not possible, right? A little inside you will find that only a range of alpha 3 is possible. That, you would have studied, that that's the workspace is one in which this particular point, p, can reach with all orientations possible for the end effector, fine?

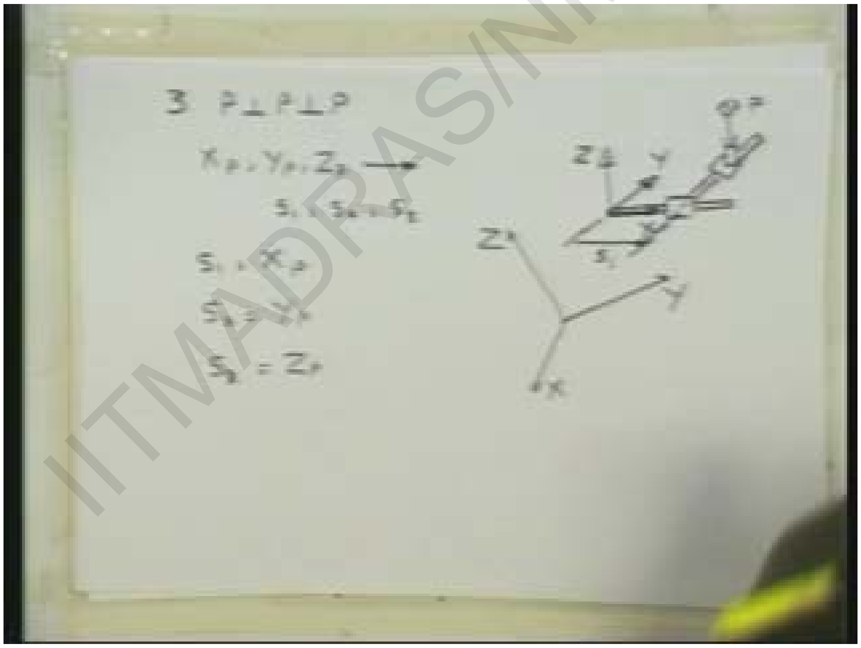
So, there are points, the reachable work space, which can be reached but you can not orient the end effector any way you want. That is restricted. This

shows the case like that, but I showed, I told you about a case where this particular manipulator's inverse problem has infinite solutions, and I didn't really describe what that is, right.

Assume that l 1 is equal to l 2, and x p and y p are 0, right? So, this gets folded in and p coincides with a. We are demanding that p should be at a, so all that is required is that the angle theta 2 should be 180 degrees. Theta 1 can be anything. Correct? So, this is a case where there are infinite solutions. So, such funny things can occur,

ok? So, these problems are fairly simple. We should progress to more demanding problems which involve spatial kinematics, and let me introduce one problem which is fairly simple even if the mechanism is spatial. This is a Cartesian manipulator with three prismatic joints. They are orthogonal, mutually orthogonal directions for the prismatic

joints, so let me locate it in a very simple fashion, so one link moves along this, that caries another.
So x, y and z. You have understood what the manipulator is? This is the base joint; this link can slide along that taking this entire structure along with that, making [???]. In this there is a member which is perpendicular to that, slight perpendicular to that, horizontal again, and this particular slide can slide on that, and on that there is another link on which this link can slide up and down, fine?
Let's assume this particular point, p, is x p, y p, and z p. The location of this point on this, in this reference frame, is given. What do we need to find out? Any fixed parameters on the mechanism? Well, there are some fixed parameters, but let us not worry too much about that. Let's see how to solve this, fine? Those will be defined later.
We need to find out what the joint motions are, positions are, so let me call it, let's say s 1, s 2, s 3, where s 1 is the distance by which this particular link has moved from the origins, s 2 is the distance by which this link has moved from this point, and s 3 is the distance by which the point p has moved above this particular point, fine?
So the question is, what other values has s 1? Fairly straightforward; this is x p itself. Correct? If I define the joint positions like that, s 2 is y p, and s 3 is z p, ok? Why did the simplification occur? Because I oriented the joint at the global reference frame axis along this convenient direction. (refer slide time51:47).



So, the whole problem was very simple to solve, so we should make it little more complex. What we should do, is orient the global reference frame in an arbitrary fashion, or in some other way. So, let this be xy. The joint variables are still s 1 which is the motion of this position of this with respect to some initial position; position of this with respect to some initial position; and position of this with respect to some initial position. So, those remain the same. So we can actually solve it a little later when we define orientations and things like that, fine?

But, this appears so simple and trivial, can be complicated by a small change, ok? So, you try to solve it later; yes, that is very good, yeah.

So now, in order to, we had defined the problem of forward and inverse kinematics, and we have now seen two planar mechanisms which I have fairly easily solved, and one spatial mechanism which is easy if we set up the reference frame in a particular fashion, fine?

Now, we need to look at spatial manipulators in its generality, and in order to do that, we need to define some of these things which were left undefined; we have to define them.

So, what we will first take up is the idea of position and orientation. How do we define position and orientation of rigid body, fine?

We have done a bit of this when you did engineering mechanics, or a corresponding physics course. That we will do it formally. Any questions here? Which means you have understood everything. Can I ask questions? Yeah.

Let us look at this concept. So we will do it only for rigid links. So, what is meant by position of an extended body like this book, for example? We can talk about positions of points, right? That is easy to talk about. We said, as the Cartesian reference frame, rectangular coordinate system, and specify positions using that. Position of a point is easy to define, but a body has infinite number of points, right? Do you have to specify the position of all those points in order to … so

[noise] so what exactly is the concept of position? We can define very clearly position of points. It is fairly easy to define.

So, if there is some body, take a point on that, plot the position of that point in a global reference frame [???]. This is a fairly simple concept, right, [noise] and as the body moves, the position of that particular point, let's call it p, that changes, ok?

Similarly, what about orientation of a body, an extended body? We can talk about directions of vectors. That is something are familiar with, right?

We need to, what I'll try to do is try to relate position of bodies and position of the body and, or rather, position of points on a body, and directions of directed lines on a body, to the idea of position of points and directions of vectors. These are simple concepts, for example, if there is a vector on this body, we have drawn a line with , direction, fine?

That particular vector can be transposed here, translated here, with this scale coming out here. The coordinates of the tip can represent the direction, fine? It could be any vector or any size in this direction that can represent the [???58:05]. Typically, we take unit vectors, ok?

Now actually, instead of asking what is position and what is orientation of rigid body, we should really try to figure out why are they useful, fine? What is it that we need to specify? How do they become useful?

So, let us look at the typical situation where a robot works, and see whether something like this is useful. So, one case is take a pressure vessel which welded out of sheets and some dish ends. You need to form this well, weld the two parts, two edges of the sheet together, and weld the ends to that, and we need do it with a robot.

How do we specify how the welding guns should move? We have a welding gun; this is attached to a robot; this will move along this, right? How do we specify that? Yeah, the path that should be taken by the point at the tip of the weld gun, we can specify that. That should be along this particular line. What else? You should specify, in order to make the robot do this work, the speed with which it should move along that, so in addition to path,

I can say, trajectory, the trajectory of the tip of the welding gun along this should be specified, which includes velocity.
So the time information also is known. What else? What about the orientation? Maybe we should keep the tip a little away from that hot zone, not the tip, but the rest of the gun away from that hot zone, right?
So we can probably orient it slightly in one direction. So, we may specify that this particular vector which specifies the axial, the vector which is axial to the gun, should be in a particular direction, right? These are things we should physically specify, but the gun can still rotate keeping the point right there, and keeping this orientation for the final path, the gun can rotate about that particular axis and still maintain the orientation which is important to us, and also maintain the position of the tip.
Correct, and talking about, let's assume the gun is like this, and I am saying that this tip should be at this particular point, and the orientation of this particular axis of this particular thing, or this part of the gun, should be in the particular direction. But I can still rotate the gun about this. I haven't said that that should be locked in a particular position. Maybe it is not important to lock that.
So, it depends on this specific situation. Suppose we want a robot to hire a gun, right? What is important is that the gun barrel points towards the target, and that can be done without actually specifying everything, ok? So, this is one example. Another case, (with this, I will stop) is consider an assembly problem in which there are certain resources here, into which small projections on some part have to fit. The path, let's assume, is like this, so, I am showing it from one side. So there are some small projections here which have to go into these holes, right? This is an assembly problem where we pick up this part, and we bring it here, and we insert it - a snap fit.
Now, in order to specify where the path has be brought, on top of this, before we start the insertion operation, what all do we need to specify? Position of one hole, ok, if we specify the position of one hole, let's say, one peg, let's say this hole, position is specified, so this particular point on the peg, or on the projection will be brought here, ok. Then what? One vector direction so now you are saying two points, so let us take two points. So I take this particular point, so this sender of this particular peg will have to be poised on top of that hole, correct? Is it sufficient, not sufficient?
Right, because once you position these two points in space like this, I can rotate the whole body about this line, right? So, we need to figure this out properly, right, and this the way the operation has be specified for bringing this part on top of this and insert it, can perhaps be specified in many ways.
So, all this has do with positioning of points on the body and directing vectors on the body. That is why I said these are the two important things, and using this we can arrive at a specification of position and orientation of extended rigid bodies which has infinite number of points and infinite number of vectors in the, on the body, ok?
So, we will do it in the next class, but before that let me recapitulate, What we are tackling in the next two modules are calculation of positions and velocities of rigid bodies, especially, interconnected rigid bodies, manipulators are made up of interconnected rigid bodies, so we will develop a formalism, or a set of tools with which we can do this calculation. This will be useful not only for the manipulators but also for mechanisms in general and spatial mechanism, fine? We will meet in the next class.