ROBOTICS
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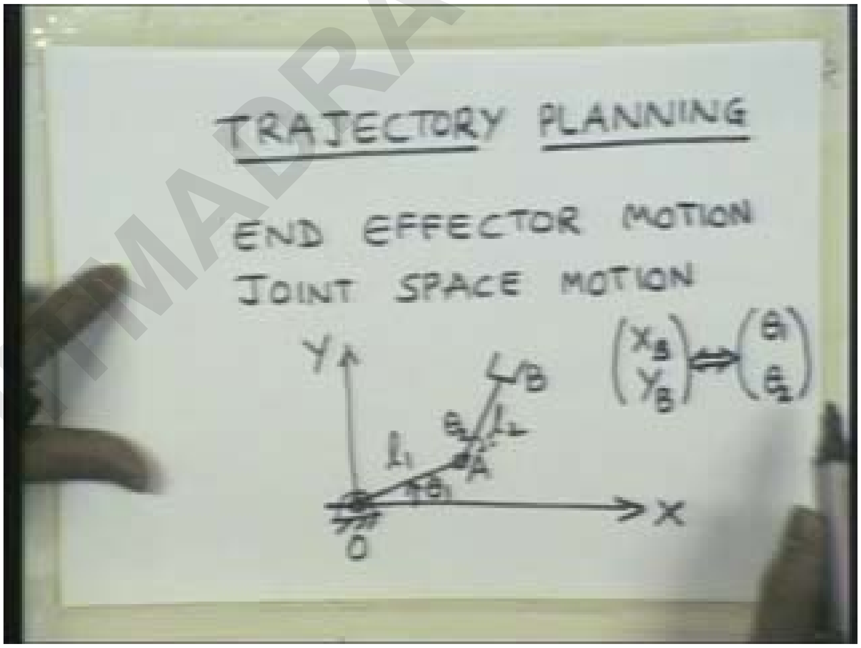
Lecture No-14
Trajectory Planning (01:18)

We have been talking about what is known as trajectory planning. We have looked at various types of tasks a typical robotic manipulator is required to do, and the requirement needs to be translated into a certain desired motion characteristics with respect to time either in terms of cartesian space in which the end effector operates, or in terms of the individual joint motions.

So we looked at the end effector space, or joint space motion. We looked at the specific example of a planar 2 R manipulator which we will also have occasion to discuss in today's class, where I have indicated how these two are related, because through the kinematics of the linkage with the link length l1 and l2, if this is o a b, you will be able to relate the individual joints motions, so X B Y B will be the end effector coordinates in the global XY frame, and the individual joints will have motions, theta 1 and theta 2. These can be related through the kinematics of the change, and therefore, we said we could convert this specified task to the individual joint requirements, or vice versa, one is easier than the other, that we discussed in one of previous classes,

and so, we looked at the simpler task of actually planning the trajectory within the joint space. (refer slide time 03:15).



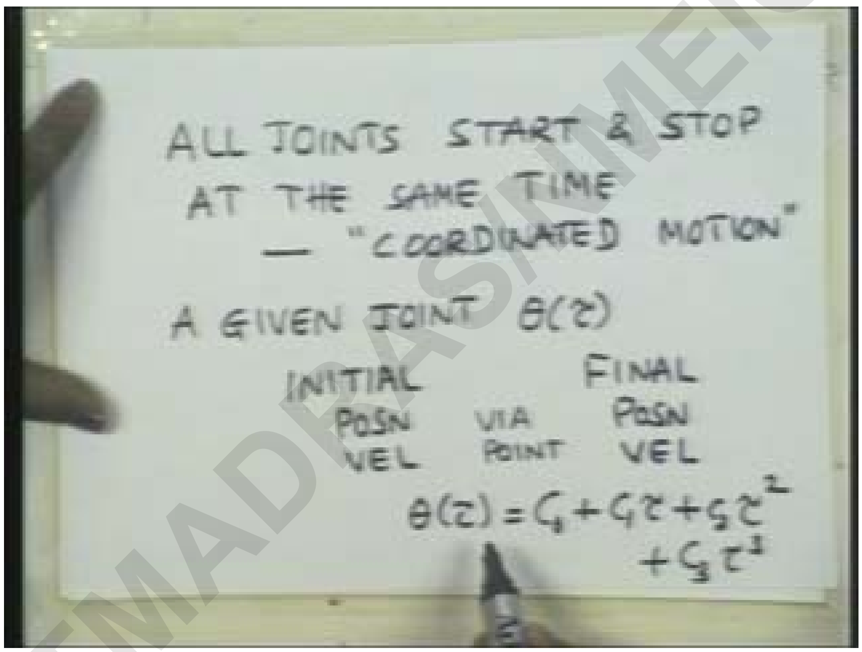
We called that the joint interpolated motion. One of the things that we said there was that we expect all the joints to move together. That means, all joints start and stop at the same time because that gives us the coordinated motion that we are looking at.

So, this is what is known as the coordinator motion. This is the case that we looked at, and we focused our attention on one single joint, and I showed how its trajectory can be planned given the specific desired motion characteristics. We looked at, for example, a given single joint;

we considered a revolute joint or a general variables theta of tau and we said initial position is given, final position is given to begin with, so the position is given, velocity could also be given, and so we fit a cubic polynomial through these.

We have four conditions: position; velocity at initial position; position; and velocity at the final state. Therefore we can determine the four coefficients. We looked at how exactly I will go about doing these calculations .

We also said in between we could have a via point being specified, and then I showed how this is modified and drawn. We looked at various strategies; how this type of joint interpolated motion can be achieved, how the individual joints motion in terms of position, velocity, acceleration can be planned. (refer slide time 15:09)

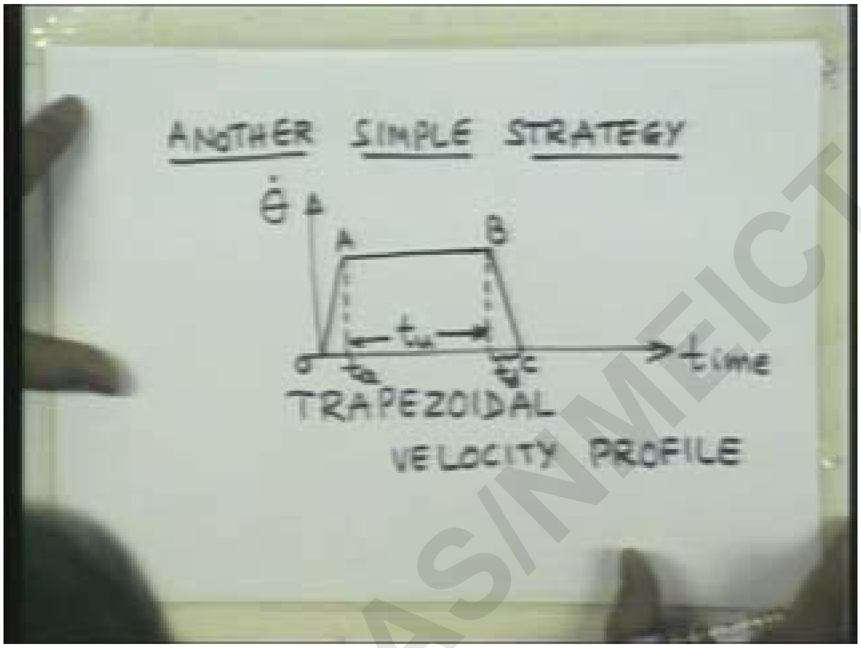


We look at yet another simple strategy of trajectory planning in today's lecture and show how these things actually reflect in a particular application example.

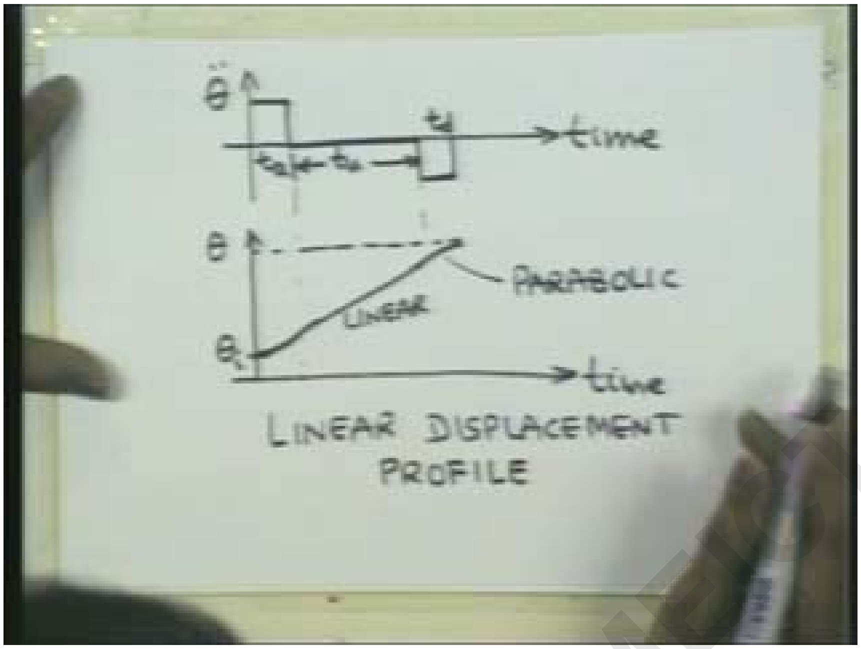
Let's look at, for example, another simple strategy. This strategy is based on the following idea that if I want to move from one position to another position please note that on the y axis I am trying to draw the velocities of the joint.

What it tries to do is do go through a uniform acceleration phase for some period of time, maintain that uniform velocity that is achieved after some time, so that you will have linear displacement profile, and then I come back to 0 velocity. So this is what is known us the trapezoidal velocity profile. This is one of the common strategies that is used, because this is a very simple strategy. We are talking in terms of the uniform acceleration; so let me call this a phase point o, let it be A B and C, let this time for uniform acceleration, be t a, uniform acceleration phase, and correspondingly, this period

of time I call it t d, the time I spend in uniform deceleration to come back to 0 velocity, and in between time I call it a uniform velocity.
So, this motion planning trajectory planning strategy has very simple characteristics, that I rev up to a particular speed and maintain that speed, tune it at that the speed for a certain amount of time, and then when I decide that it is time enough to come to rest at the final position, then I just decelerate it to that point of time. (refer slide time 7:20)

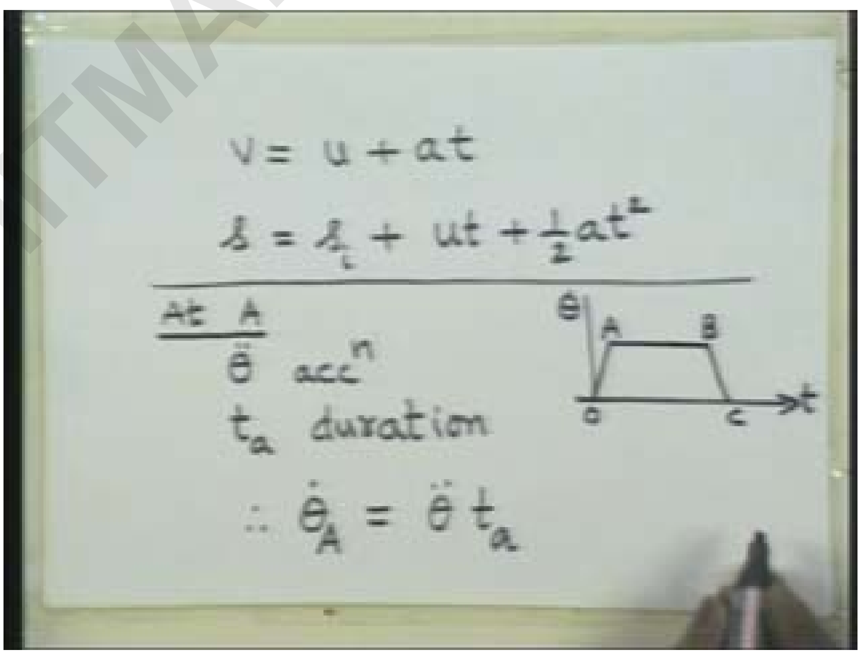


So correspondingly, I could have the velocity and acceleration profiles drawn, the acceleration profiles correspondent to this simple strategy will therefore be theta double dot first initially uniform acceleration phase, and then 0 acceleration because you are cruising at a velocity that you achieve, and then you go through a uniform deceleration.
Let this be the t u, as we discussed, this is t a, this is deceleration, and this period is t u. So intermediate case of uniform acceleration.
Correspondingly the velocity profile as you can imagine, the displacement profile, as you can imagine, in the uniform acceleration phase, it will start from the initial position theta i to the final position that I am trying to go through at the final time. It will start with some kind of a profile corresponding to uniform acceleration so it will give you a parabolic curve, and then it will take off on a straight line, and then it will have another parabolic blend corresponding, so this is linear, and these are actually parabolic blends,
so that it may smoothly match the next segment of motion. It is essentially a linear displacement profile. So in other words, the trajectory planning strategy that we have used using the trapezoidal velocity profile can be referred to as the essentially linear displacement profile. Towards the ends you have these parabolic segments which will blend with the next segment and point. So we can make some simple calculations for this profile to see how the velocity, acceleration, and displacement vary with respect to time, so that you move from the initial position to the final position. (refer slide time 10:04).



All the relations that I will use in this strategy are based on the high-school physics relationship that you must have worked out. Final velocity is initial velocity plus acceleration time, and the distance traveled is initial distance plus velocity plus half at squared.

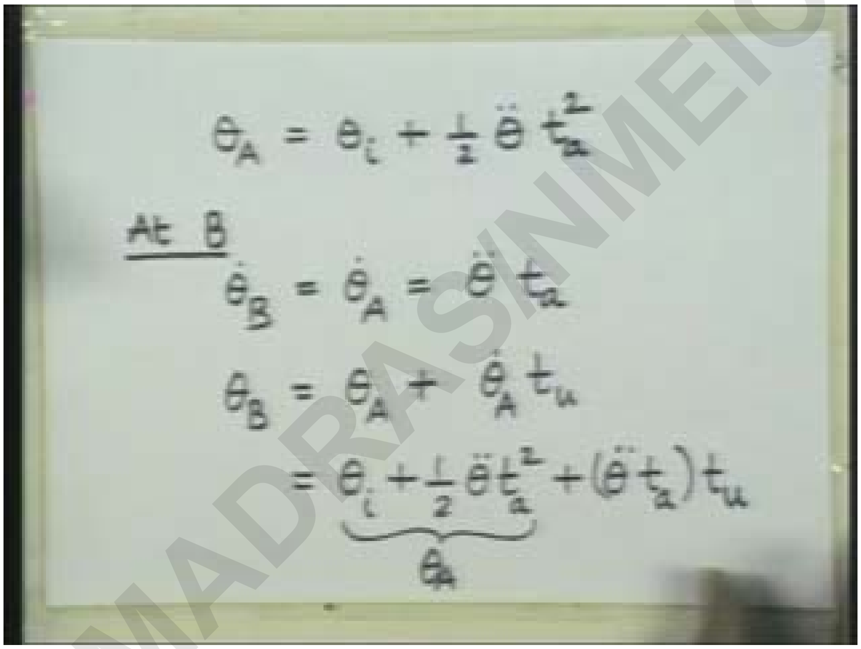
These are the only two relationships that I will need to use to plan this kind of a trajectory and therefore this is a very simple strategy. So let's look at A, the position A, where we are talking about theta dot, this is time, so this is A B C o, at A, because of the uniform acceleration for the period of time t a, so theta double dot is your acceleration, t a is the duration, therefore, theta A dot is because it is going with 0 velocity, initial velocity, it is simply theta double dot time t_a. (refer slide time 11:50)



And theta A is theta i whatever is an initial position there at the point o, plus half theta double dot t_a squared.

Now correspondingly at B, when you reach the position B, you have cruising at a uniform velocity that you have attained at the point A, and therefore, theta dot B is same as theta dot A, that is, the acceleration you had and the time period for which you are accelerating, so that velocity remains the same is a uniform velocity phase, and the position is therefore theta A plus theta dot A time t_u for the time period that you have been moving through. So therefore, this will be theta i plus half theta double dot t a squared plus theta dot A, theta dot A is theta double dot a so theta double dot t_a times t_u.

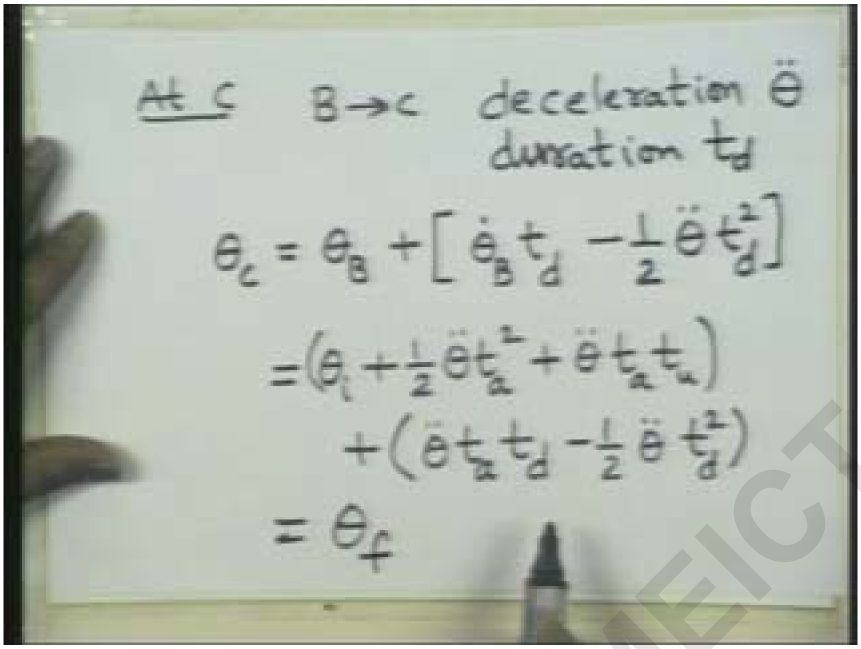
So I just substitute the theta A into this equation so this part is essentially theta a and this is the part due to the uniform velocity t_u. So I get the position at the point B. (refer slide time 13:32).



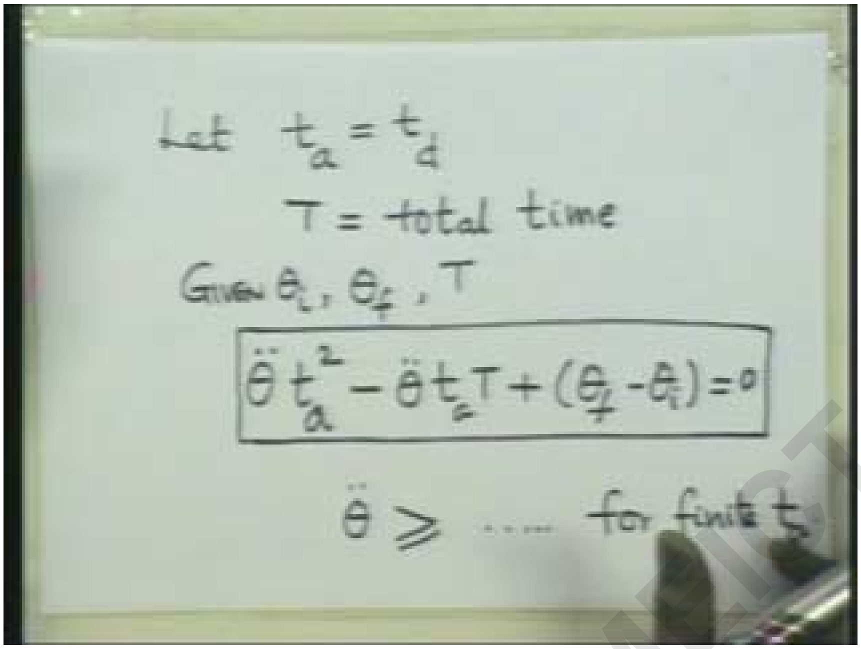
Now similarly, for the point C, I am decelerating from B to C. Again magnitude-wise I use the same, for simplicity I use the same level of acceleration that I used earlier as the deceleration now and t_d is the deceleration duration.

So, theta C is theta B, whatever is the position at that point, plus theta initial velocity for theta B dot t_d ut plus halt u_t squared, so a being deceleration minus theta double dot t d squared—

the same simple relation for the displacement and uniform deceleration phase. So, using the previous relation that we have obtained, the theta B is theta i plus half theta double dot t_a squared plus theta double dot t_a t_u plus theta B dot it is theta double dot t_a t_d minus half theta double dot t d squared. This looks a rather formidable expression but it is based on the two simple relations of high-school physics for uniform motion, a uniform acceleration or uniform velocity. That is all that we used in this. (refer slide time 16:05).



What I am going to do now is again for the sake of simplicity, I will say that let t acceleration be same as t_d, ok?
So the time period for which I accelerate I also spent in the deceleration phase.
So, let T be total time, and remember that theta c has to be finally equal to the theta f that is prescribed; that is the final position that has been prescribed here, so that is the condition that you are going to satisfy that at total time T you are going to be at point C, which is corresponding to the theta final.
So, we have this total time given theta i, theta f and total time T we can get an equation which relates all these three. I will skip the simplification procedures, I will give you the final equation, it comes out to the following.
This is the equation that relates the various parameters in this trajectory planning strategy. That means you have a certain initial position, you have a final position and you have given the duration that you can take for reaching this final position from the initial position and how much acceleration I can take could depend on the joint actuators or you could work with the specific time for which you want to be in the parabolic blend region and how much time you want to cruise at the uniform velocity. So, choosing one of these two, the acceleration, or the time for acceleration, you will be able to determine the other.
So this is the simple quadratic equation in t_a. You can easily work out that for finite t_a.
Theta double dot has to be greater than for finite t_a. I will leave it to you to work out this simple condition if you want to get a finite t_a, that means, nonnegative time, real time. Then you need to have sufficient acceleration. So your acceleration should be chosen appropriately for this strategy, following this simple equation and then all these very simple set of calculations. So
how you plan this strategy is, given the initial position and final position and the time duration, you choose either the acceleration that you want to use or the time period for acceleration. The rest of it is calculated from this equation and then you will be able to plan the entire strategy. (refer slide time 18:34).

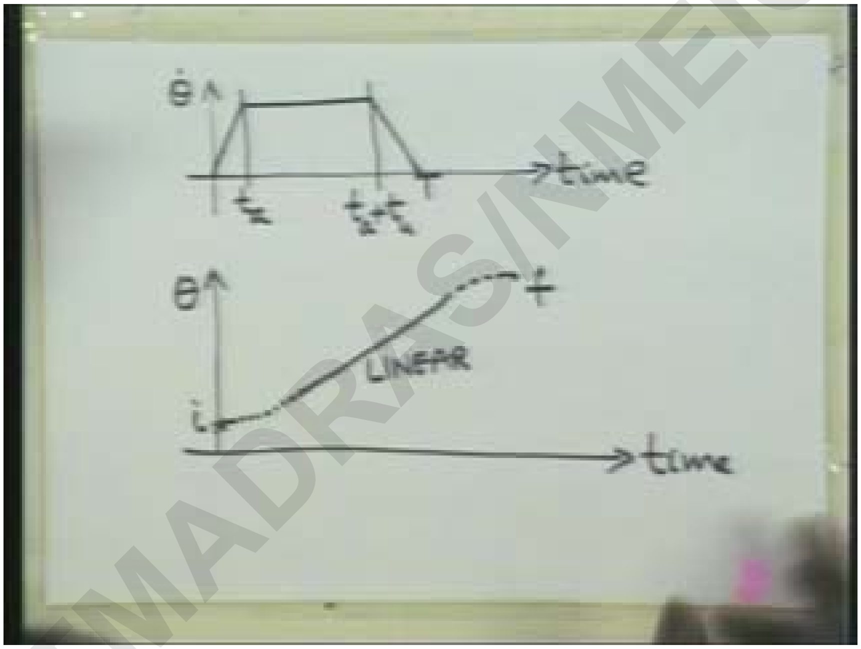


So for the duration zero to t a, we have, this is t a plus t u and this is the total time t.
So, for this duration you use the straight-line velocity increasing uniform acceleration phase, you have the corresponding theta of t, you have, this is cruising at a uniform speed, you have the corresponding theta of t, and here, this final segment, there is uniform deceleration you will have a corresponding theta of t.
 So, this strategy is again correct. You may have a certain limit on the maximum acceleration that you can achieve depending on the joint actuator characteristics, so you could choose the acceleration, but at the same time, the acceleration has to be sufficiently high, otherwise, if you look at this equation, you would not have real tu, ok? So, acceleration, that we see should be sufficiently high but it is limited how high you can take the acceleration, is limited based on the joint actuator characteristics, and therefore, you could start with theta double dot and then find t a. But the complication here is that because you would like to have coordinated motion you look at what happens in this equation,
 for example, you look at what happens if you are looking at a robot manipulator with multiple joints, each joint corresponding to the end effector motion from initial position to the final position, could have different theta i and theta f.
 So the range of motion required would be different, whereas you would want all the joints to start and stop at the same time, so the time duration is the same, but the range of displacement that we need to achieve would be, in general, different, and therefore, your maximum accelerations that we need to choose will also be different.
So this is the only point you need to keep in mind when you actually think in terms of implementing this strategy for the real manipulator with multiple joints.
So, one strategy that is done is that you look at the slowest fellow and reduce other fellows to that, so that everything goes as per the same time sequence, so you have a coordinated motion, ok? In that way, you are not taxing others, it is only that you are not able to fully utilize the potential of all the things.

But you look at the slowest joints in your scheme of things, and then follow the time scale based on that reduce to the accelerations and ultimate speeds, terminal speeds of other joints based on that.

That is one point you need to keep in mind. So this is governing each particular joint, the kind of equation that we have just now derived. So the range of motion in general could be different, so you need to keep that point in mind when you plan this strategy.

So this strategy is essentially simple, and the corresponding profile, once again, will be that you have an initial position, you have a straight-line motion going through, and then you have a final position. So this is essentially the linear position that I am talking about. Now, this is the initial position, this is final position. The other point we mentioned was that between the initial and final positions; it is common that you prescribe certain via points where you want the manipulator to pass by, may be to avoid certain obstacles in the path and so on because as the end effector moves through the Cartesian space you know what are the obstacles and so on and so forth. (refer slide time 22:28)

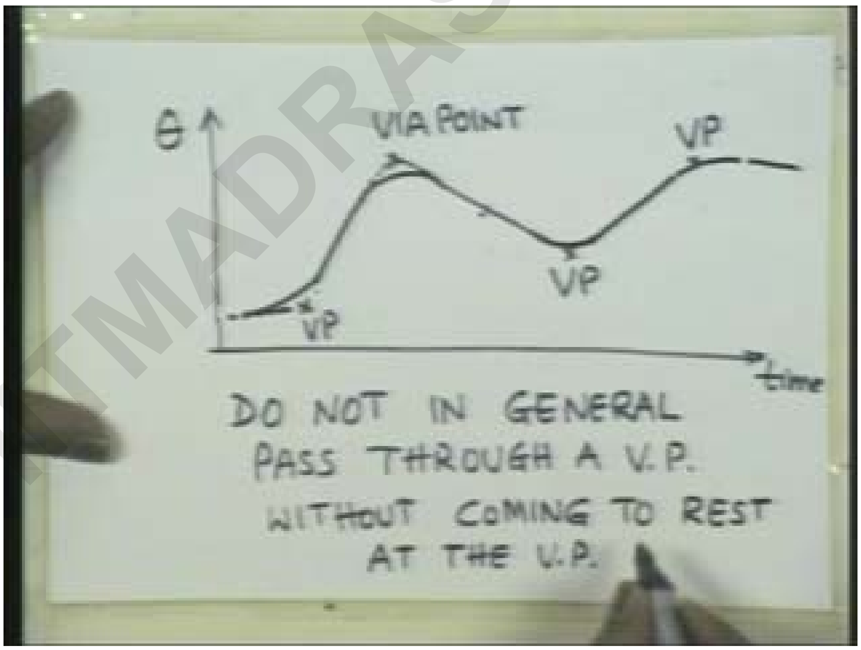


So how do plan this strategy when I am going to extend it to the case of via points? We would avoid the detailed mathematical calculations but I would indicate to you how actually it can be done. So you have the time and theta: one point here, and another point here, the third point here and fourth point here and points across in general. This is not the initial point this is not the final point. These are the set of points, via points we are just trying to see how the motions can be planned using this simple trapezoidal velocity profile that is the linear displacement with parabolic blends.

It is coming from the previous segment so I should have some kind of smooth transmission here, so this is the linear portion. I need to go in a straight line from here to here, so I could have some kind of smooth transmission here with the parabolic curve.

Similarly I need to have a straight line here so I can have a smooth parabolic blend here like this and so on to the surface. What you would observe here is that I am not actually passing through the via point. This is another via point; this is another point and so on.

We don't pass through via points in this strategy; in general, with this type of curve fit, I do not pass through via points.
Now if you insist that it should pass through a via point because, in general, when we are talking in terms of avoiding a certain obstacle or so, you say that you should go close by to this, you are not really particular because you are not specifying a certain velocity or certain acceleration to be achieved at the via points. You are requiring that it pass close by that with smooth velocity and acceleration but if you insist that this is not enough, I should go through the via point, then you can actually put two pseudo via points such that this is the real via point that you want and then it goes through that. That is the simple strategy you can have.
So this can be the actual via points through which you want this fellow to move through but you are actually putting two via points also on the other side close by so that it will be lying close by on the linear profile. Otherwise, in general, without coming to rest at the via point, this strategy can not pass through a via point.
So we just note that do not in general pass through a via point without coming to rest. If you say that it can come to rest at the via point then it can pass through you can work it out but at the same time I said you can use pseudo, these via points and then make this go through a real via point that you decide.
So you can kind of beat the system with this strategy by describing the pseudo via points. So this is very simple strategy based on a simple trapezoidal velocity profile of uniform acceleration cruising at a uniform velocity and then uniform deceleration and we can plan this sequence of segments and fit the entire motion. (refer slide time 26:37)



So this is yet another strategy; like this you can come up with several strategies for actually trajectory planning. Between any two via points I am actually using a trapezoidal rule but in the trapezoidal rule if we see this at time t equal to 0, my velocity starts from 0 and then comes uniform acceleration. That is why if I am saying that it must pass through a via point without coming to rest, then it is not possible. But I can come to rest, start from rest there, then I can pick the simple trapezoidal velocity profile of uniform acceleration from 0 velocity like we just now worked out. Otherwise, I have to define two pseudo via points and then come to the actual via point through the linear profile.

But you can always modify this scheme and come up with refined strategies and so on. But the point is you keep it as simple as possible because these calculations—I have emphasized in one of the previous classes also—are offline trajectory planning strategies. I do this computation before hand and these remain my set points for that trajectory. But real trajectory generation, motion generation, has to be done in real time when the robot operates as a closed feedback system in real world.

All the computations in your strategy should be possible to be done in real time when you are talking in terms of a task to be done or executed in real world. So the time consumed in general for the trajectory planning will also be a very important point for all these computations. So we need to keep that in mind. This strategy is essentially to keep a simple scheme, which does your job.

So let's look at one simple example; we have looked at various kinds strategies and I have indicated to you how these strategies can be planned and so on and so forth.

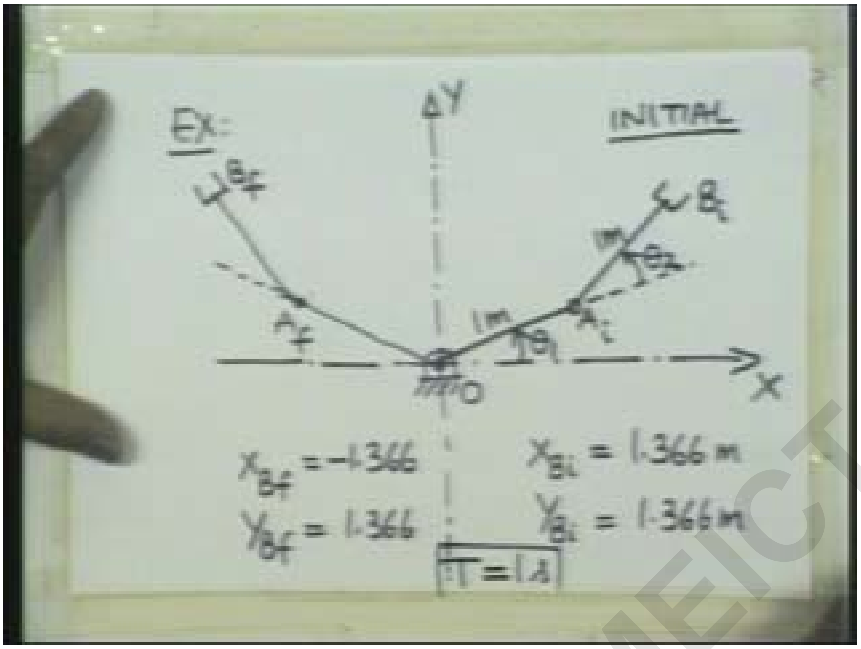
Now we perhaps is the time when we actually look at a particular example and see how these strategies perform. Let me think in terms of simple example problem, continuing with our two R manipulator that we talked about. So let me say that I have a planar or two R manipulator, which is in an initial position like this. I wish to move the point B to a final position on this; for simplicity I have taken it to be mirror symmetry above the y axis to illustrate the point. So I choose some point here and this is some point here, this becomes a point here, and so we are talking in terms of this, the final configuration.

So this is A; we can call it A_i, B_i, A_f, B_f. So now we want to move it—the problem statement is that I want to move the end effector point B from its initial position given by this. We can work out the specific numbers correspondent to this position from B_i to B_f, use any of these strategies that we have just now discussed. For simplicity I choose the link length to be 1 meter, 1 meter. We could choose any arbitrary length; as an example I have taken length to be simply 1 meter each.

And I define this as my theta 1; I define this as may theta 2. There are two possibilities for defining what theta 2 could be: one is that I measure it always with respect to the global Cartesian space xy, the orientation of this particular link in this coordinate time, but more convenient is to refer to this in this manner. That is the reason I say that it is more convenient if there an actuator that is driving this link. It is likely that it will be sitting on this positive line when you look at the multilink robot manipulator. So if it is sitting on this then the actuator's motion is actually relative to this, the motion of this length, and therefore it is convenient to look at theta 2 being defined in this manner. We could also always refer to this with respect to the global extrapolation; it doesn't matter which way you follow but just you have to do it consistently throughout, that is all that is important. In this particular example, I have taken theta 2 to be in this manner.

So now x_{Bf}—I will write here x_{Bi}. I have chosen the desired position to be 1.366; 1.366; x_{Bf} to be the mirror reflected position.

So the final position will be on other side of the y axis, it is the exactly reflected position. So 1.366 remains the same and this becomes minus of this. So I need to move here to here; let me say that the time taken be 1 second. (refer slide time32:52)



Again for the sake of simplicity of calculation I have taken 1 second. So let this motion from the initial position to the final position be done in 1 second.

Now let's see how we will carry out this particular task.

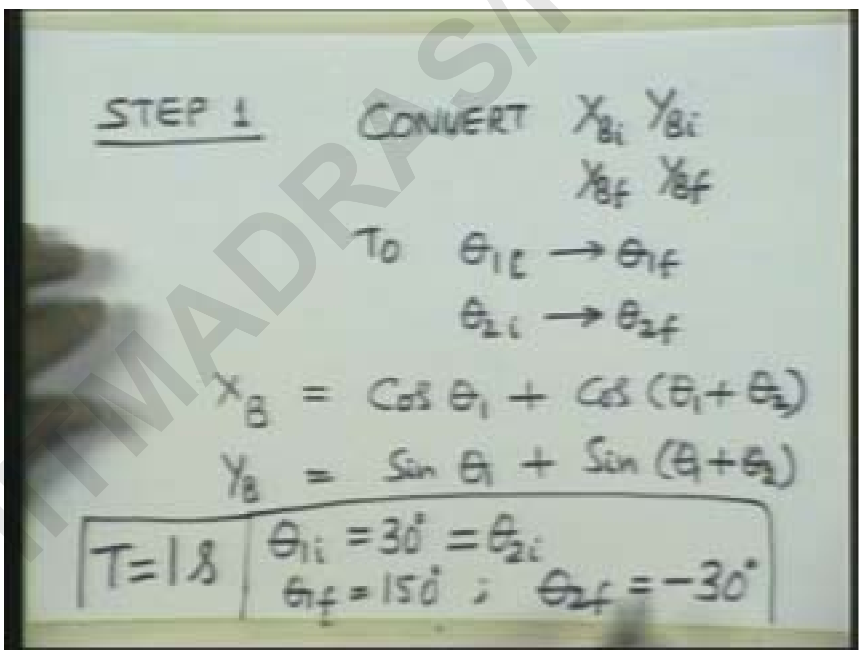
So the first step I need to do in our scheme of things is that for the step (1), I need to convert X_{Bi} Y_{Bi} X_{Bf} Y_{Bf} specifications on to theta_{1i}, theta_{1f}, theta_{2i}, theta_{2f}. So I know the range of motion that I need to get. I do that through a simple calculation. So l 1 and l 2 being 1 meter, it is simply cos of theta 1 plus cos of theta 1 plus theta 2 and Y_B is the sine of theta 1 plus sine of theta 1 plus theta 2. So I just give you the numbers; it is possible for you to sit down and calculate these things. I have chosen the numbers in such a way that these turn out to be very simple: theta 1 initial is 30 degrees, so also theta 2 initial; theta 1 final turns out to be 150 degrees because it's a mirror reflected position and I have taken, theta final turns out to be −30 degrees.

When you are looking at all these inverse calculations given the Cartesian and effector positions to calculate the joint space coordinate positions, there are multiple solutions possible. It refers to the particular branch in which the mechanism is operating and therefore we choose one particular scheme of solutions and then you continue with that throughout your calculations. Because the manual mechanism is likely to be assembled in one particular branch and it continues in that branch throughout the motion, we choose here in this manner.

Here I am asking it to move in that manner; but the particular velocities that I require at that singular position are such that I don't get into trouble, because we will see the solution.

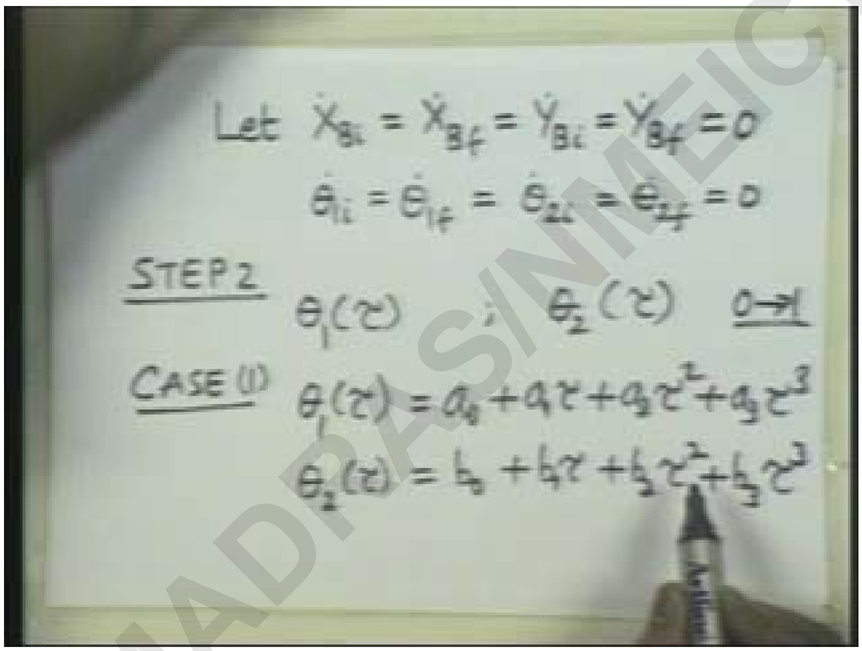
I have taken a simple example of a planar two R manipulator. In general for a robot manipulator operating in 3D space we need to have a strategy to represent the position and orientation of a rigid body in 3D space. Certain coordinate frames that are linked through these keep moving within the global partition frame and I need to keep track of the relationships between the various coordinate frames, that is, the link attached coordinate frame with reference to the global partition frame of place.

So the formal ways of doing this, the notations that are required for this, and the way in which I define the various angles defining the orientation in 3D space, all these will be discussed separately in a set of lectures under the topic of robot kinematics.
There you would worry about the solution, multiple solutions, problems of solving the forward kinematics, that is, given the joint coordinates finding the end effector's position. This is typically known as the forward kinematics, the easier of the two problems given the end effector positions to calculate the joint space motion; that is, the inverse kinematics, which is slightly more difficult. The same thing extends to the velocities and acceleration.
So given the end effector velocities to find the joint motion velocity, you may or may not get a solution at all; because the end effector velocity that we desire may not be achievable at a particular position for any finite velocities for the joints as we mentioned in another previous class also. All these issues will be discussed extensively in a separate series of lectures that would come under the topic of robot kinematics
So that is why I have taken a simple example here to illustrate the point, so that these can be discussed at length in a separate lecture. So to recapitulate, my problem specification has now become that I need to move from an initial position of 30 degrees to a final position of 150 degrees in joint 1; an initial position of 30 degrees to −30 degrees in joint 2 and of course, our problem specification is incomplete if I don't specify the velocities. The time taken is the same, so this will be 1 second. (refer slide time 38:03)



So that is the same, now how do I specify the velocities? Now let's say that, for simplicity, let x_{Bi} dot be equal to x_{Bf} dot. I am taking all velocities to be 0. This simplifies the problem for us. Given any finite velocities you will be able to also calculate exactly in the same manner the time variations described, using the equations we have already obtained.
So correspondingly theta 1 i dot is theta 1 final dot, theta 2 i dot, theta 2 final dot, all velocities are 0.

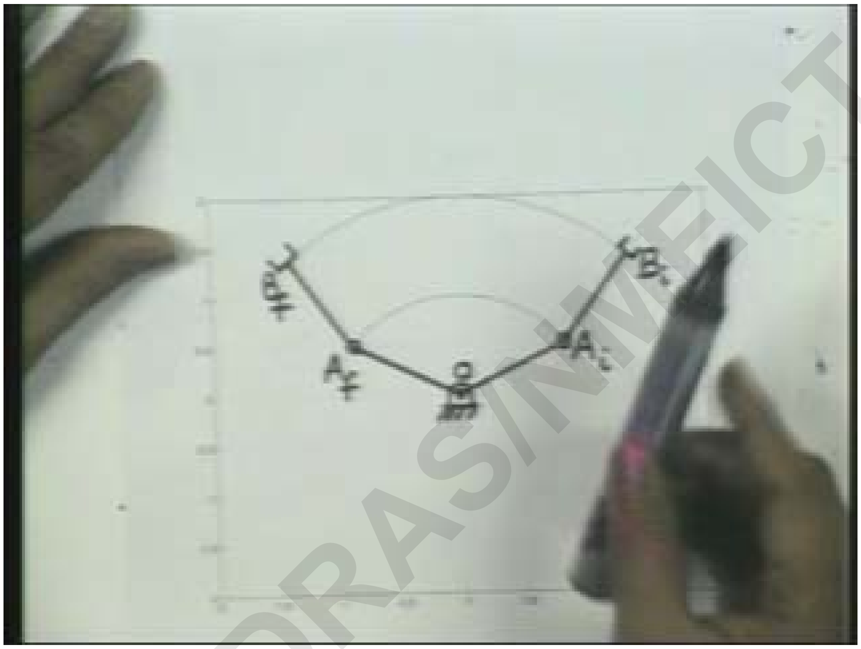
So I move on now to step 2 of my calculations, where I say that I need to split a particular trajectory theta 1 of tau, theta 2 of tau: 0 to one segment. Here come the various strategies that we have so far discussed. So to begin with I choose a simple strategy that we discussed in one of the beginning lectures, where I talked about the given theta and initial position, final position, initial velocity, and final velocity: I can fit a cubic polynomial.
So theta 1 of tau is a_0 plus $a_1$tau plus $a_2$tau squared plus $a_3$tau cube, theta 2 of tau is similarly b_0 plus $b_1$tau, $b_2$tau squared, $b_3$tau cube; the tau varying from 0 to 1: one single cubic polynomial. It's possible to find all these coefficients without much difficulty because these conditions are known, we have derived equations that govern these coefficients and so you will use these to obtain the solution.
I will give you the final solution for this particular problem using this strategy of using a single cubic polynomial. (refer slide time 40:00)



So theta 1 of tau turns out to be, in radians; theta 2 of tau turns out to be pi by 6 minus pi tau square plus this. Tau varies from 0 to 1.
These are simple cubic, polynomial strategies with respect to time for the individual joint motions. But my problem doesn't end here. This is the joint space coordinate. Correspondingly, I go to step 3, which is to estimate x_B of tau, y_B of tau. From the forward kinematics that is given these two, how these vary with respect to time. I need to now look back finally at how the end effector is actually moving from this position I wanted it to go to before. So I need to find that. So let's look at how the actual solution will behave for this strategy.
So this source, what we show here, is the initial configuration and the final configuration and how actually it moves from the initial position to the final position. So that is what we are trying to achieve. Each of this joints theta 1 and theta 2 will vary in a cubic manner and the end effector finally moves the tip of, the end of the robot manipulator. The end effector moves from point B_i to point B_f in this manner. This is symmetric at

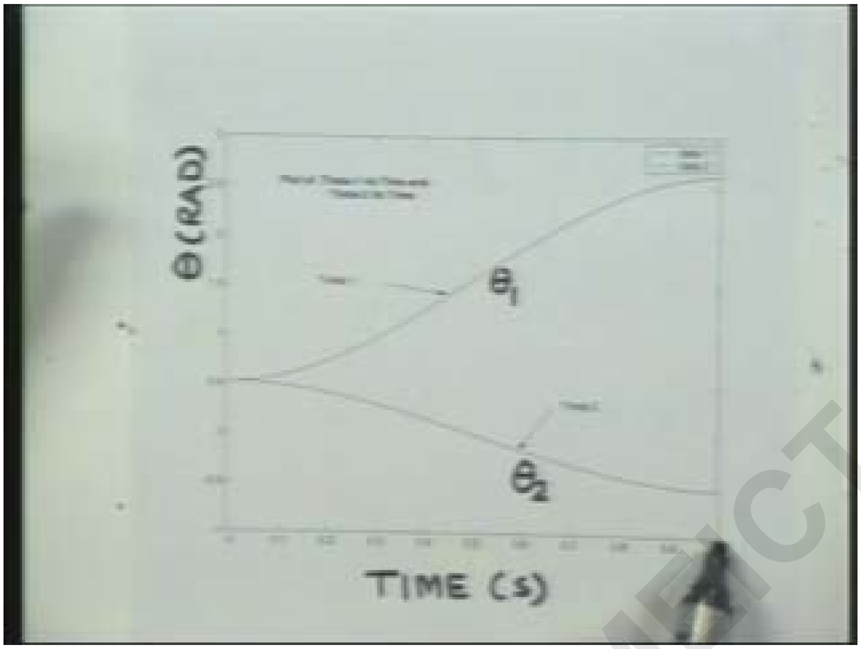
about this particular configuration because of the various things that I have specifically shown.

Now this therefore is some complex function of the joint motions theta 1 and theta 2; in general, the end effector motions. When you do the trajectory planning in the joint space, you could fit certain known trajectory profiles in the joint space, which could be an essentially linear trapezoidal velocity profile or it could be a cubic curve or something like that in the joint space. But the final end effector's motion with respect to time could be dependent on the kinematics of the robot manipulator. (refer slide time 42:28)



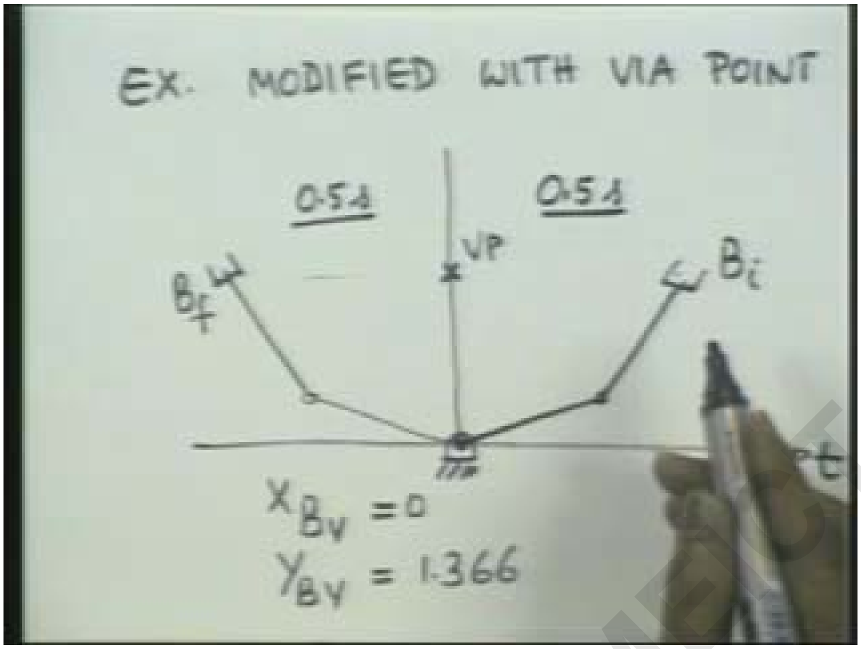
It will have something like this. So how do the theta 1 and theta 2 themselves vary with respect to time? For this case, it is easy to see. The theta 1 has to go from pi by 6 or 130 degrees, I mean 30 degrees to 180 degrees pi by 6. So, it can smoothly go in one cubic curve in this manner.

Theta 2 has to go from pi by 6 to − pi by 6; that also goes smoothly in one cubic trajectory in this manner. So this is theta 1 and theta 2 cubic trajectories with respect to time 0 to 1 second; one single cubic curve. (refer slide time 43:50)

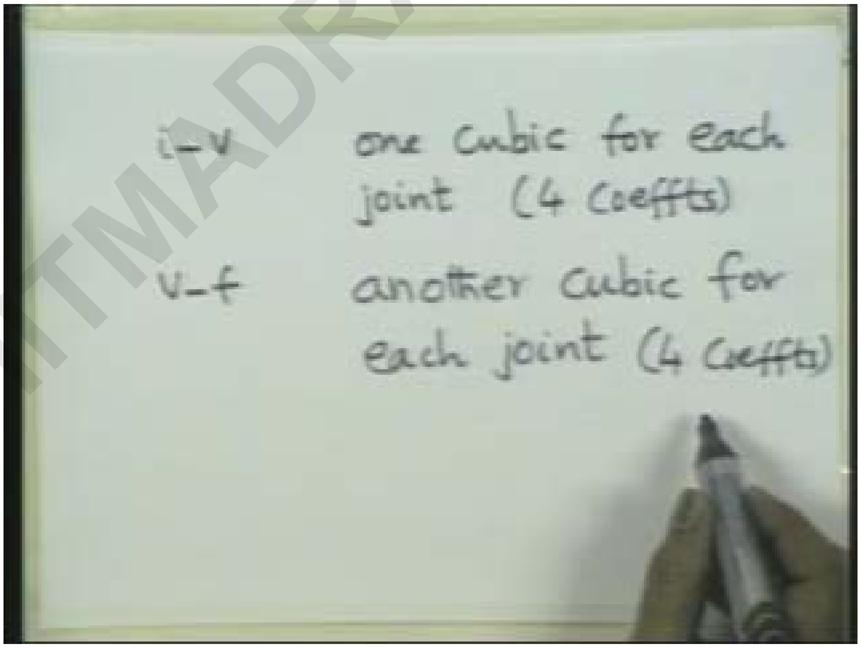


Now let's look at how I can achieve this kind of motion when I also specify a via point. I am trying to kind of make you think that when I am actually looking at the motion of this end effector B from the initial position to the final position, supposing I want to actually move in this straight line or somewhere close to it, at this stage, it was the first step when we solved the problem, we just ignored any constraint on that. We had an initial position and a final position. Now let us say that let it be close to the straight line joining these two; that means I actually specify a via point in this step. That's the next statement in the problem.

So I would actually say let it move through the same initial position to the final position for example, modified with via point. That means, time—I have initial position, I have a final position, but in between I specify a via point, which is this: B initial, B final, but specified as a via point. I am saying that let this take 0.5 seconds, let this take 0.5 seconds, and let this x_B via point be 0, y_B via point be 1.366. That means, it is the straight line that joins these two points: initial and final configuration. So I am putting x axis at position 0 and taking a via point on this, in the duration of 0.5 seconds on either side. This segment should be accomplished in 0.5 seconds, this segment should be accomplished in 0.5 seconds. So the total time duration is the same, initial position and final position are same, so everything is same in this problem, except that I have introduced the via points. Now if I am introducing a via point, what do I do? I need to put two cubic segments of motion: one from this to this and another from here to here. (refer slide time 46:35)



So let's look at how I do that. so initial position i to via point v, 1 cubic ok one cubic for each joint; remember there are two joints. so i am talking in terms of for each joint i need to do this. ok v to f another cubic for each joint again Now this has four coefficients, this has four coefficients, each cubic will have four coefficients so we need to determine eight coefficients to be able to get the entire trajectory from 0 to 1 second. So if you recall the discussion that we had earlier, (refer slide time 47:31)



what we are going to do is to say that initial position and velocity are specified here for two conditions; final position and velocity are specified for two more conditions. I am going to specify that this position should be retained when I come from this side or I come from this side, when I start off from here. So this segment or this segment should have this position: so two more conditions and I am going to finally obtain two conditions from the fact that velocity and acceleration should be moved continuously when I move from here, through this to this.

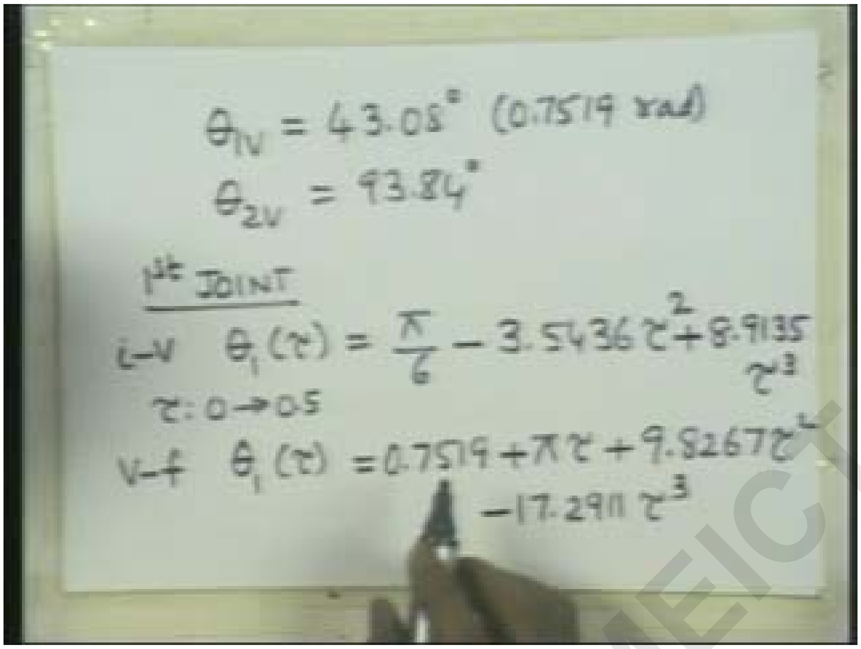
So totally I will have eight conditions: two here, two here, and four here. so I will able to solve for the eight coefficients I am skipping the calculation I will just show you the results but you should be aware of the issues that are involved, so we will use all the equations that we gave in the earlier schemes and then find out all the eight coefficients, four coefficients for this segment and four coefficients for the other segment: initial position to the via point, via point to the final position.

Let's look at how this would be; but before I do that, please remember that the via point has been specified in terms of the X and Y coordinates of the point B. So I need to convert it into theta 1 via point, theta 2 via point. Because I am doing trajectory planning in the joint space, I need to always convert it into the joint space so that I need to do an extra calculation.

When the via point is conveniently specified, it is convenient for the user/operator to specify it in terms of the motion of the end effector in a desired straight line. I specify a via point for initial position and final position but for the trajectory planning, I need to convert it back into the joint space; so I need to calculate theta 1 v and theta 2 v. So I do that and then I fit the cubic polynomial. Let's look at how it will appear.
Just for take of completeness I give the theta 1 v turns out to be 43.08 degrees and theta 2 v turns out to be 93.84 degrees.

For the sake of completeness of calculation, you can verify this. Then correspondingly, you will get the joint motion results, final results, first joint theta 1 of tau is pi by 6 minus 3.5436 tau squared plus 8.9135 tau cube. Similarly, theta 1 of tau is this, 0.7519 radian. So 0.7519 plus pi tau plus 9.8267 tau squared minus 17.2911 tau cube. So this is from initial to the via point, via point to the final point; each time tau—please remember that tau is local coordinate, time coordinate—is varying from 0 at the segment's beginning and whatever is the corresponding time at the end of the segment from 0 to 0.5 in each of these segments.

So the expressions, cubic expressions, are given; similarly you can get the expressions for the second joint. Let's look at how actually that motion itself looks like. (refer slide time 51:40)

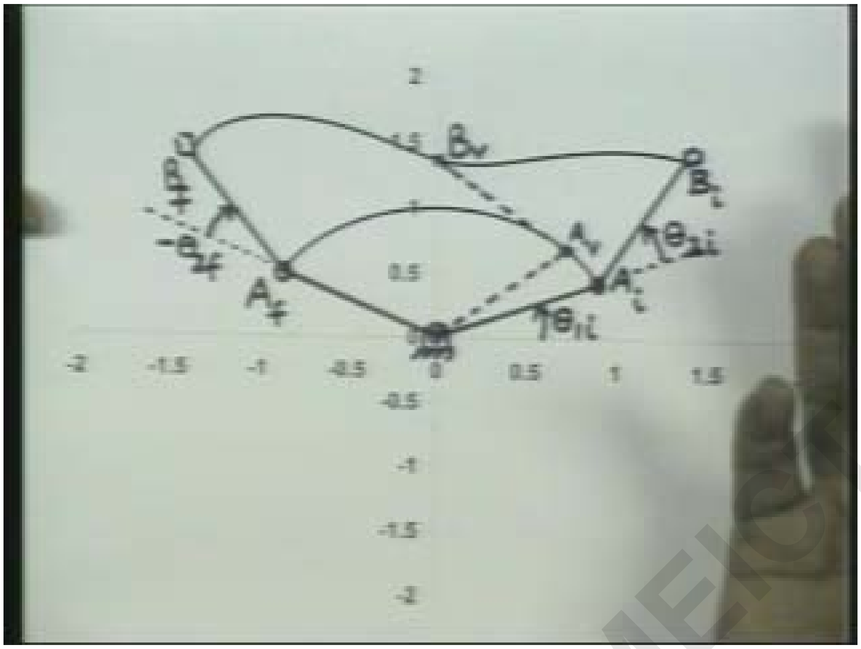


So I have tried to capture here how the motion of the end effector is from B_i to B_f through a via point that has been specified. When you look at the via point, it is illustrative that you look at how the configuration of the manipulator is in that via point for this via point, the end effector, to be here.

Then you are talking in terms of the manipulative segment, this kind of configuration, for the kind of numbers that I have just now given.
So you are expecting that theta 1 should go from its initial position to the final position through a via point configuration base. Theta 2 should go from here to here through a via point configuration ok the actual trajectory that you get for the point A and the point B is shown here it is closer to a straight line then rear one but definitely not a straight line that was required. ok

so if you insist having a straight line then you have option to specify many more via points and make sure that it is as close to the straight line as you wish or you could specify, if you say that there is a particular type of weld profile that I am trying to do, then i specify actually the motion of B exactly as an equation, as the function of X and Y, the trajectory that I want to get.

That becomes a continuous path problem.
So we are talking in terms of the continuous path being approximated with sufficient number of via points, so that I come as close to it as possible; or the other way is to actually specify this continuous path as a function, an exquisite function of X and Y and so on. Therefore you have to work out the joint motion to achieve this, exact straight line, which you wish. But remember that once you specify these positions, X and Y coordinates of this, (refer slide time 52:51)

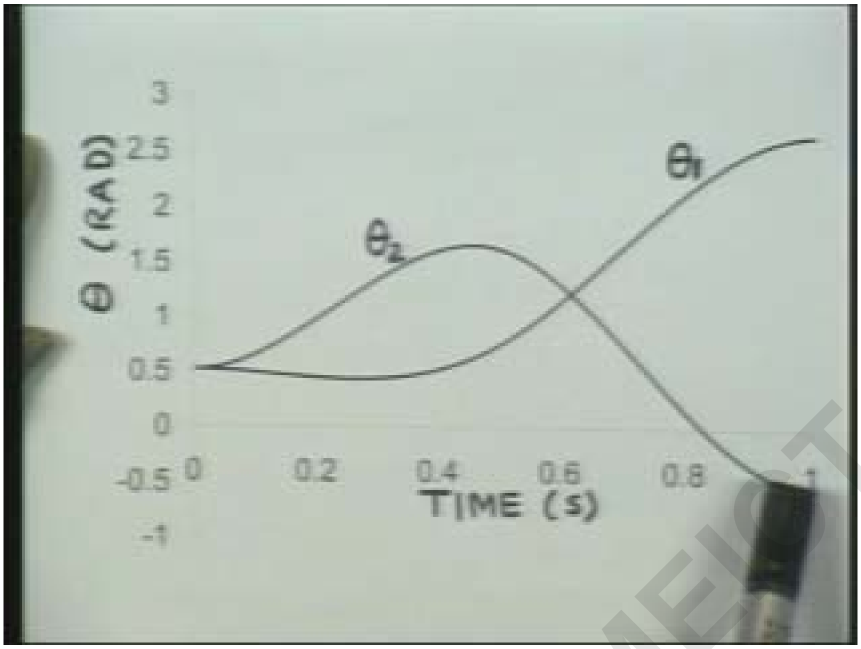


in this particular case, and the time that you can achieve, the time duration that you can take for all these intermediate positions, as a continuous path, then your complete motions is prescribed. You have no freedom to choose theta 1 and theta 2 in any manner that you want. This can only be directly completed from the specified trajectory for this particular example.

The moment somebody specifies how the end effector should be as the function of X and Y, the trajectory is given there and the time is specified too; so the positions, joint positions, joint velocities, everything gets fixed.

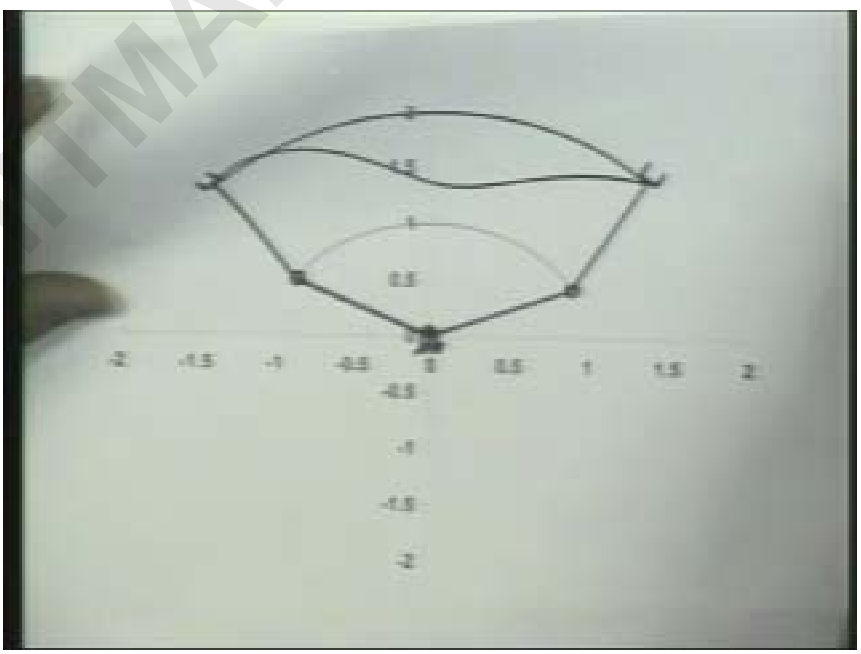
So this is an approximation to that, just to illustrate how a via point can be used to make it move through some profile.

Now let's look at the two joints motions: theta 1 continues roughly the way required from pi by 6 to 5 pi by 6; it was smoothly going through one cubic segment earlier. (refer slide time 55:02)



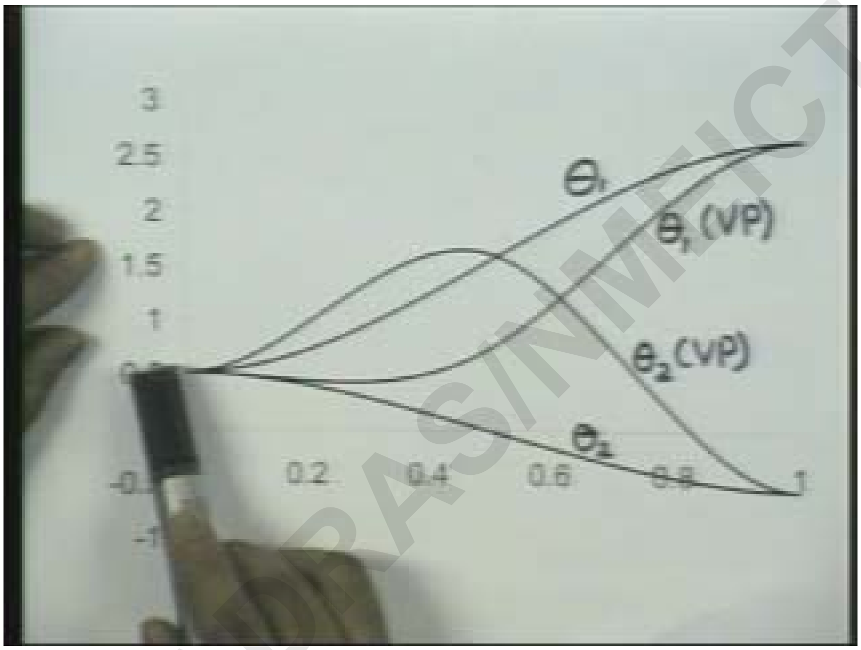
But now we have two cubic segments at about this point: 0.5 seconds, so theta 2 has a major upheaval.

We have one cubic segment here and another cubic segment here. I tried to capture these motions together for your benefit of understanding. So if you look at this to compare these two strategies, in the first strategy where I had no via point, I am using the single cubic curve fit for the entire trajectory motion. It goes in this motion. For the second case, where I tried to bring it closer to being a straight line from this to this, I have specified this point and therefore I have got some curve like this. But in this case, it is indistinguishable in either case on the scale of this plot. It is indistinguishable, so you will see a single curve here, the motion of this. (refer slide time 55:42)



The joints motions themselves are compared in this curve where, in the duration of 0 to 1 second, you have theta 2 going from plus pi by 6 to minus pi by 6 with a smooth, single, continuous cubic curve in the first case but it goes through a via point in this manner in the second case. This is theta 1 in the first case without via points and theta 1 with the via point.

So the joint motion trajectories that we get when you specify these via points following this strategy should be very different, and you can use all the equations that we have discussed in all these strategies. You can vary this strategy, use the simple trapezoidal profile also, or use 434 strategies (refer slide time 56:14)



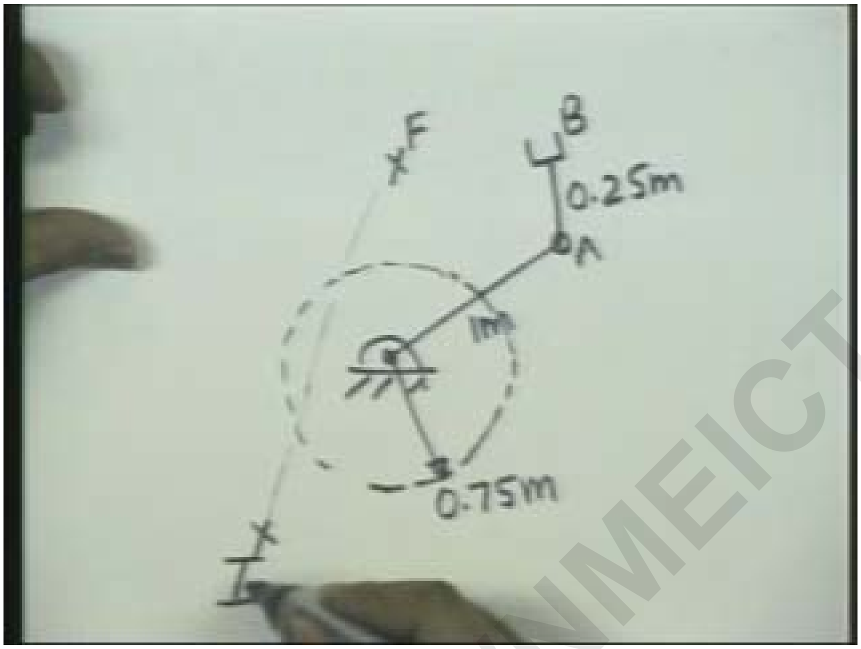
that we discussed in an earloer class and so on and so forth. So essentially you can do this but if you do insist that I need to able to specify the Cartesian space motion of the end effector I insist that this has to be straight line path or any such complex curve. Because if the end effector is holding a welding gun, and you are actually welding two parts together on, say for the example, an automobile body frame,

you have a particular x y relation that the end effector point to move through; you cannot have an approximate relation here.

If we specify that, then your joint motion will have to be computed accordingly; but there is just one danger I want you to be aware of when you look at that kind of trajectory planning in the Cartesian space.

If I modify the same example problem, and I say that I have a two R manipulator that the first link is 1 meter, as we have taken, but the second link is only say 0.25 meter and just to illustrate the point, I am taking some numbers.

Then you can easily visualize that there is a center circle of points, of radius 0.75 meter when this skip B collapses with this, then you trace the circle. These set of points cannot be reached by the end effector at all. So if the user wants to go from point A or initial position I to a final position F, he unintentionally specifies the straight line path; all the points from the entire straight line path must be reachable by the end effector. (refer slide time 58:35)



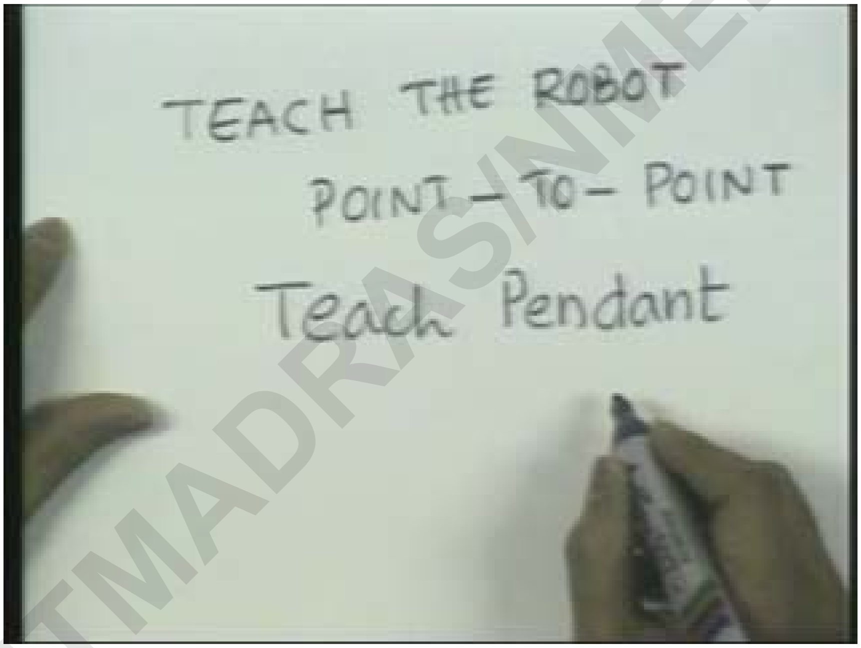
So we are talking in terms of complex kinematics relationships for the three dimensional motion of a robot manipulator end effector team to be kept in mind by the user, by the operator, before he specifies a particular trajectory in the Cartesian space. So it is usually difficult to ensure that these types of kinematic constraints are not violated.

So it's convenient to specify the initial and final positions or certain via points to avoid the obstacles and so on and so forth, but it is actually easy to implement it in the joint space and work backwards. Another point we need to keep in mind is that this is the trajectory that is given, continuous path that needs to be followed. The corresponding joint motions required will have to be completed through the inverse kinematics at each position, inverse kinematics for a multi-degree of freedom robot. Manipulation is going to be definitely more complex than the forward kinematics problems. So those computations have to be done repeatedly for each of the positions from the initial to final position in the Cartesian space and sometimes in real time.

So this is another point you need to keep in mind. And some of these positions, even if they are reachable, could be close to singular configurations when you will require joint velocities to be significantly large, to achieve the kind of motion that the operator has specified for the end effector skip when he is trying to plan the trajectory in the Cartesian space.

So when we looked at the joint interpolated motion we considered that yes, it is possible to do the Cartesian space trajectory planning but now we are in a position to appreciate the nuances involved in the planning of the trajectory in the Cartesian space. Yes it is possible to do it, but there are certain issues that you have to keep in your mind.

So overall if you look at the trajectory planning problem that we looked at, we have kind of decomposed it into two parts. I first said the Cartesian space trajectory that somebody desires, is split into joint space; then in the joint space, for each joint, we split it further into two problems that is, offline trajectory planning and the online motion control, motion generation and control. We looked at the problem of motion planning, trajectory planning, of plan. How various strategies can be used to get these motions; then these remain as set points when the motion is actually generated in real time, lower level controlled algorithms based on the joint position feedback with encoders or whatever, sense that you have a reach of the joints, try to achieve these type of joints. That is the way we have neatly decomposed the problem. But one issue that we have not really addressed is how does the operator specify these points? I have all the time said that this is the via point, this is the initial position, and that is the final position or joint effort; so the operator for the given robot manipulator should be able to specify this. That is where you talk in terms of a teaching the robot manipulator these executable tasks—the task that needs to be executed. This is a fairly complex problem in itself depending on the level of intelligence a robot manipulator has. If you are not assuming any level of intelligence in this, you can actually teach that through a skilled worker. So we can teach robot manipulator through specific points: point to point. (refer slide time 01:02:48)



We are talking in terms of teaching a robot those points. So I have actually what is known as a teach pendant, which is like a remote control. When I take it, move it to various points, I reach the point that I want the robot manipulator or the end effector to reach. I say that yes, we reached the point now. Then I instruct the control computer to capture the joint position at those points,

and move on to another point and so on. If it is a continuous path that I need to specify then I have to actually capture the path as the individual joints as a road and then this end effector is moved through. I need to capture these individual joint motions with respect to time, but even though we say it is continuous, we need to remember that there is a digital computer that is storing these and there is certain trampling rate at which these points are taken continuously as input points and then any of your strategies can be worked out.

So with this, we come to an end of the discussion on the trajectory planning topic. The concomitant issues of a mobile robot moving around in an obstacle-strewn environment, dynamically trying to figure out the strategy of how it should actually move around, is more complex issue that we have not discussed in this. Those of you who are interested can pursue this particular topic for further studies.