Heat and Mass Transfer
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Lecture No. 09
Heat Conduction-6

We were looking last time at problems of two and three dimensional unsteady state heat conduction; before that we had looked at 1 dimensional unsteady state processes in an infinite cylinder for which I had given you a solution, a series solution and a solution in the form of charts and how to use them and there I had menti1d that the same type of solution is available for a long solid cylinder as well as for a solid sphere and then we had started discussing two and three dimensional unsteady state heat conduction and I said suppose we have a long rectangular bar like this and of dimensions 2 a and 2 b and initially this bar is at a temperature

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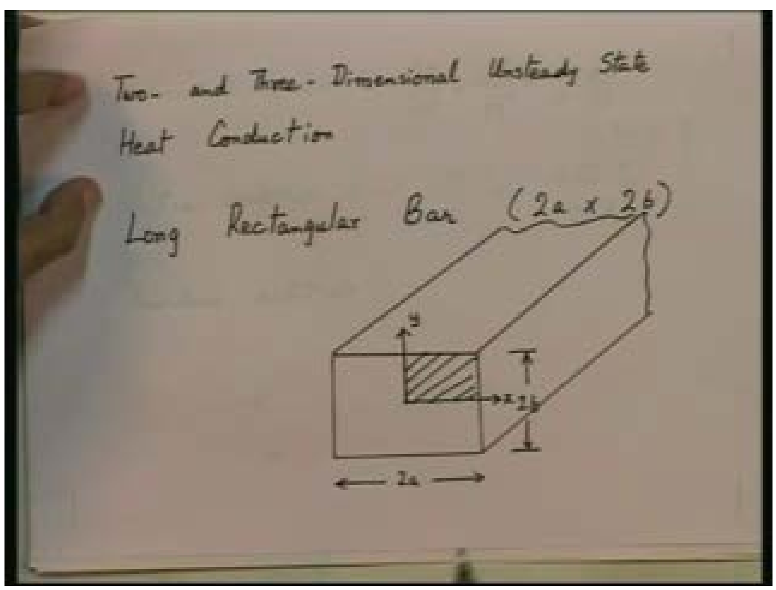

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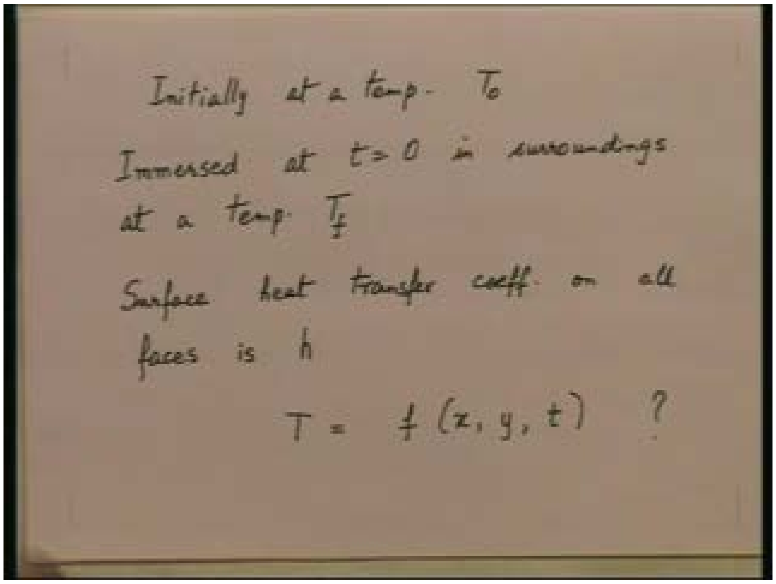

Initially, it is at a temperature T 0; it is immersed at time t equal to 0 in surroundings at a temperature T_f. There is a surface heat temperature coefficient on all faces h, then obviously the temperature is going to be a function of x, y and time. We would like to solve for this temperature distribution so we will formulate the problem like we did for the 1 dimensional unsteady state. Let us formulate the problem; that means let us put down the differential equation, the boundary conditions and the initial condition for this problem.

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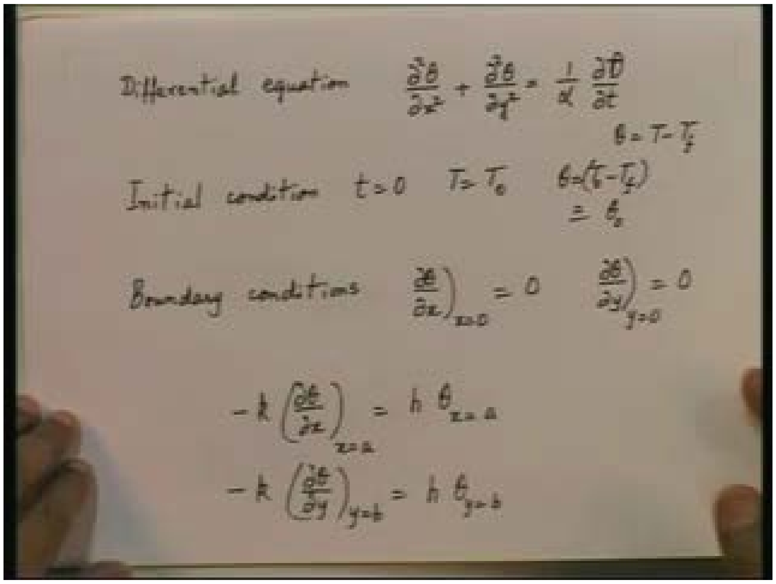

First of all, the differential equation for this problem, go back to the general differential equation and take T as a function of x, y and time eliminate the heat generation term. Take k equal to constant and you will get differential equation straight away as d 2 theta dx square plus d 2 theta dy squared is equal to 1 upon alpha dT dt; that is the differential equation you will get d theta dt not dt dt d theta dt.

Now, I am putting the differential equation straight away in terms of theta because I am defining theta in the same way. theta is equal to T minus T_f like we did earlier, T_f being the surrounding temperature. So what is the initial condition we have to solve this differential equation subject to the initial condition? Subject to the initial condition at time t equal to 0, T is equal to T 0 or theta is equal to T 0 minus T_f which is nothing but theta 0. And the boundary conditions will be as follows; now while putting down boundary conditions let us make use of the symmetry of the problem. That means let me go back to the sketch which we had of the bar. We have a choice we can either put down boundary condition that x equal to y equal to x equal to minus a, y equal to b, y equal to minus b or recognizing that there is symmetry in this problem.

We say let us solve the problem only in the positive quadrant in which case we will put down the boundary conditions at x equal to a x equal to 0 y equal to b and y equal to 0; that is what we will do. It should recognize symmetry; you need to solve only in the positive quadrant. Let, so we will put down boundary condition at the four edges of this positive quadrant and what will they be? The boundary conditions would be at x equal to 0, d theta dx at x equal to 0 has to be 0. Symmetry requires that d theta dy at y equal to 0 also must be equal to 0; symmetry requires that and then at the 2 faces x equal to a we will get minus k d theta dx at x is equal to a is equal to h times theta at x is equal to a and at y equal to b, we will get minus k d theta dy at y equal b is equal to h in to theta at y is equal to b.

This is the full formulation of the problem - the differential equation, the initial condition and the boundary conditions in the positive quadrant making use of symmetry at x equal to 0 and y equal to 0. Now this problem has been solved; also this differential equation subject to this initial condition and the given boundary conditions has also been solved and the solution is very neatly enough comes out to be as the following form. Now we are not solving the problem ourselves; I am just going to give you the solution which has been obtained. The solution which has been obtained is as follows.

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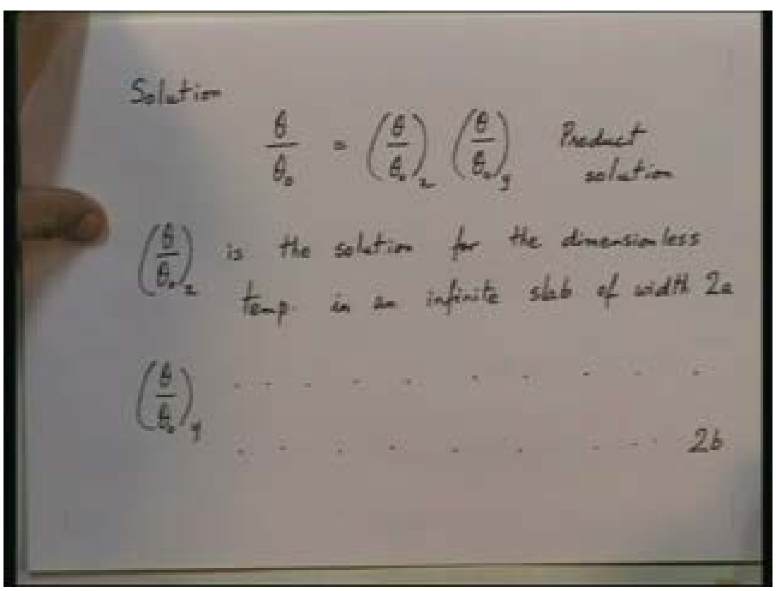

It has been shown that the solution to the problem is given by theta by theta naught the same way that we had for the one dimensional situation is equal to theta by theta naught x multiplied by theta by theta naught y. This is what we call as a product solution, a product type solution and the quantity is theta by theta naught. x means theta by theta naught is a subscript; x stands for it is the solution. theta by theta naught x is the solution for the dimension less temperature; for the dimension less temperature in an infinite slab of width 2 a. That is what it is which we have already with us; this solution we have from the infinite slab solution and theta by theta 0 y is the same thing, is the same solution is the solution for the dimension less temperature in an infinite slab of width 2 b.

So, for a rectangular bar all you have to do is to treat it as if you have got 2 infinite slabs - one of width 2 a, one of width 2 b. Obtain the solution theta by theta 0 x for a slab of a width 2 a, obtain the solution theta by theta 0 y for as infinite slab of width 2 b, multiply the 2 to get theta by theta naught for the rectangular bar 2 a by 2 b; that is all you have to do. So it is very very neat solution which has been obtained and as you know theta by theta 0 x and theta by theta 0 y will come. These values will come from the charts which we had earlier or from the series solution for the one dimensional case which I have given you earlier. Now the same idea can also be extended to a 3 dimensional situation.

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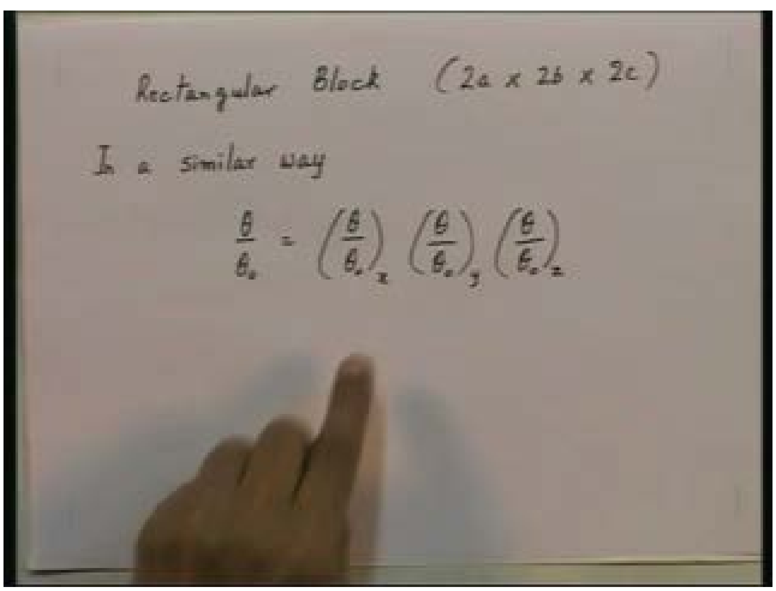

The 3 dimensional situation is the following: suppose I have a rectangular block. Let us say I have a rectangular brick or rectangular block of dimensions 2 a by 2 b by 2 c, a rectangular brick or block of this size. So initially at a uniform temperature t naught I put it in surroundings at a temperature T_f; the heat transfer coefficient and all the six phases of this block is h. Find the temperature at any point in this block as a function of time. Now t is going to be a function of x, y, z and time and interestingly enough in the same way in a similar way, the solution has been obtained for this case, has been obtained to be theta by theta naught is equal to theta by theta naught x multiplied by theta by theta naught y multiplied by theta by theta naught z where the 3 quantities on the right hand side theta by theta 0, x theta by theta 0, y theta by theta 0 z are the solution for infinite slabs of width 2 a 2 b and 2 c which we can obtain from the series solution I have given you earlier. So it is a very very straight forward job to use these product solutions for one - the rectangular bar a long rectangular bar and secondly for a rectangular block of 2 a by 2 b by 2 c.

Now we have come to the end of what we wanted to do with unsteady state conduction. We have done one dimensional unsteady state in an infinite slab, we have done one dimensional unsteady state and an infinite solid cylinder. We have also done something for a, I have also indicated that similar solution has been obtained for a sphere and I have indicated how these solutions can be applied to problems and can be extended to two and three dimensional unsteady state problems for rectangular bar - a long rectangular bar as well as for rectangular brick. Now we want to go on to a new topic and that new topic is the topic of fins or extended surfaces

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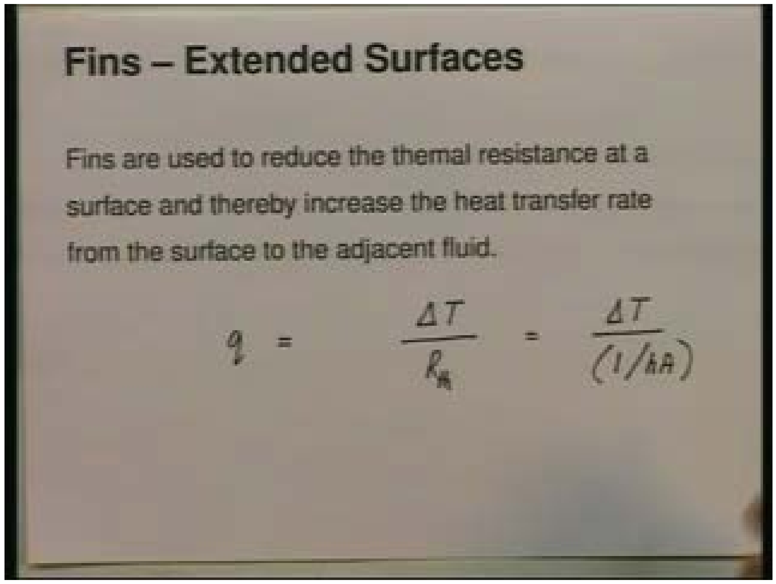

The first thing we want to ask ourselves is when are fins used? We want to go to the topic of fins or extended surfaces. Fins are used to reduce the thermal resistance at a surface and thereby increase the heat transfer rate from the surface to the adjacent fluid; that is when we use fins or as you call them extended surfaces. I repeat - fins are used to reduce the thermal resistance at a surface and thereby increase the heat transfer rate from the surface to the adjacent fluid.

Now, what is the heat transfer rate given by heat transfer rate q? For any given surface, it is given by the thermal temperature difference between the wall and the fluid or the fluid and the wall delta T divided the thermal resistance and if there is a heat transfer coefficient h at the surface and the area of the surface is a, then we would say it is delta T divided by 1 upon hA. delta T is the difference of temperature between the wall and the surrounding fluid; so q is given by this.

Now suppose for a given situation this thermal resistance 1 upon hA, this thermal resistance 1 upon hA is too high. I want to lower it in order to increase q; I can’t change delta T, I want to lower 1 upon hA is a the thermal resistance in order to get higher value of q. Introduction of fins or extended surfaces helps p in lowering this value of the thermal resistance. Now let us look at some geometries that are used; let me show you some instances. A fin is being used here is the figure.

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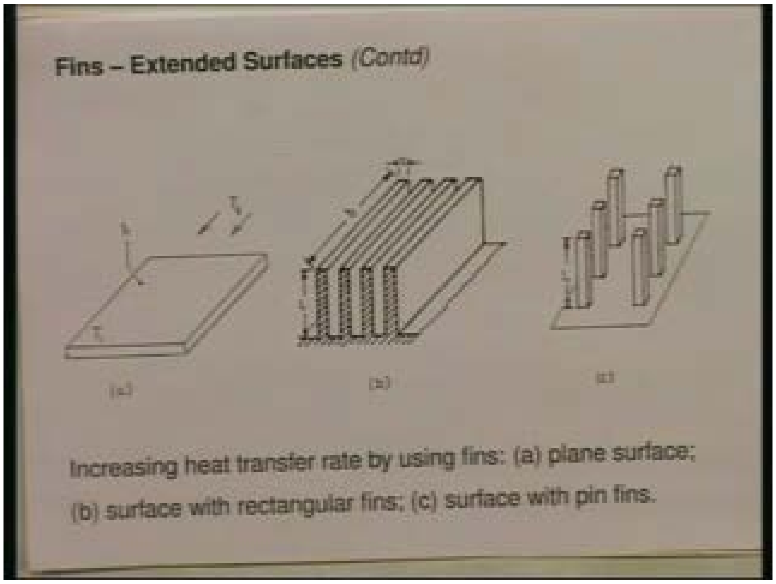

this is the plane surface area a, h is the heat transfer coefficient, T_f is the fluid temperature over it; we get a certain heat transfer rate from this surface. I want to increase that heat transfer rate so what do I do? I put fins on this surface; the fins in this case as shown are nothing but thin plates which are stuck on the surface at right angles to the surface. The fins are height L, width b and thickness T; usually thin plate side by side would be stuck and they would help to reduce the thermal resistance or the fins could be in the form of just rod sticking out of the surface. These would if they are thin rods, thin relative to the length L, with here would be like pins; so if we call them is pin fins typically.

They could be square in cross section, they could be circular in cross section, they could have any cross section that we like; the point is we are adding some surface area on the main base area which is the horizontal part here and the addition of this area through the use of extended surfaces helps to reduce the thermal resistance; that is the main point about fins or extended surfaces. Let us look at some more examples; I now go to the case of a cylinder. Let us say this is a hollow cylinder.

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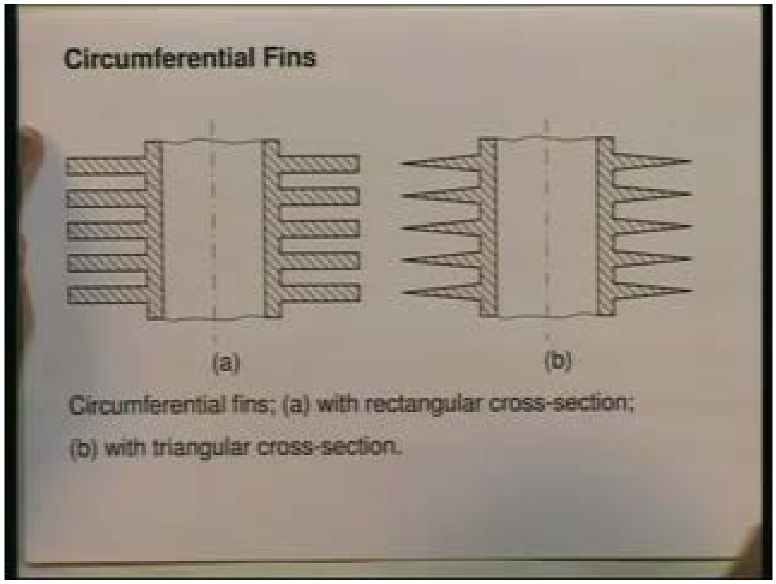

It could be say the cylinder block of an engine, a internal combustion engine loosing heat to the surroundings. In order to reduce the thermal resistance at the outer surface and therefore increase the heat transfer rate, we put circumferential fins and the ones that are shown here are circumferential fins which are rectangular cross section around this cylinder. In b here, these are also circumferential fins around the cylinder but they are circumferential fins with triangular cross section, we could have a variety of cross section.

I am just showing 2 as an example; typically we use circumferential fins with rectangular cross sections or with the slight taper or with triangular cross sections. These are typically used and as I said the purposes again the same as for the plane surface, the idea being to reduce the thermal resistance on the outer surface. Now what we are going to do in this class is the following. We are going to look at one particular fin namely the pin fin.

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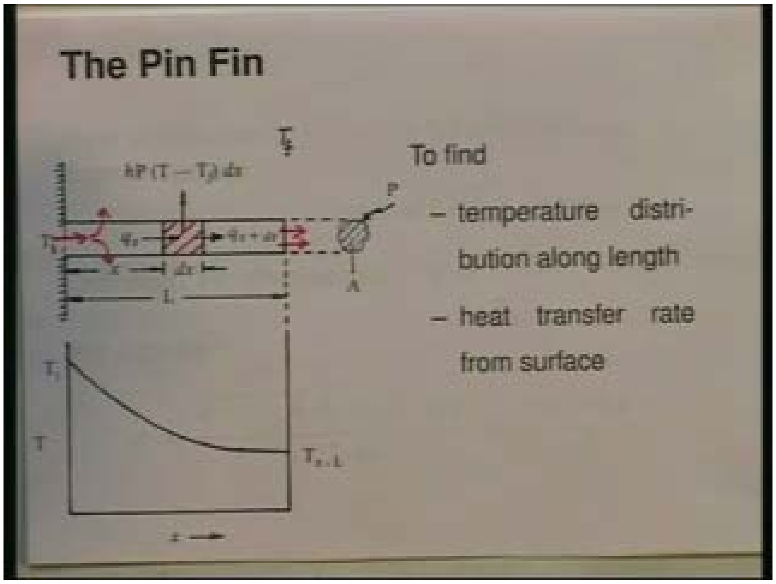

We are going to analyze it in some detail. What is the pin fin? A pin fin is nothing but a rod sticking out of a plane surface; one could have series of that. Let us analyze one; what we do for one is valid for all. So, pin fin is nothing but a rod sticking out of a plane surface and we use the word pin to signify that it is a cylinder rod, that is why we use the word pin and the moment cylinder, what follows is that we don't have to worry about the temperature variation across the cross section of that pin fin. What we need to worry about is only the temperature distribution along the length L of the pin fin, that is the idea.

A pin fin, a cylinder fin because of its slenderness, there is negligible variation of temperature across the cross section. The only worth while variation which is worth analyzing is the variation along the length L; so let us look at such a pin fin. This is a pin fin here; now in the previous sketch, I had shown it vertical. Now I am showing it horizontal that is the only difference.

Here is the base surface which I have hatched here and the pin fin is of length L. I have drawn some arbitrary cross section and said - assume that the perimeter of the pin fin is p and its cross sectional area is A. It could be circular, it could be square, it could be rectangular cross section; it doesn't matter. The point is it is a slender cross section compared to the length L; that is the point. Now I have already said that the moment because it is slender variations of temperature along the cross section are negligible like this there are negligible. Therefore, temperature will vary only in the x direction, the x direction being the direction around the length and since we are dealing with the steady state situation, temperature will therefore only be a function of x.

For such a pin fin, let us say the temperature at the base is T_1; let us say the temperature at the base is T_1. Now you know very well if the surrounding temperature is T_f and T_1 is greater than T_f,, then it follows that along the fin, as you move along the length outwards away from the base plate base surface, the temperature is going to drop and is gradually going to approach the value of T_f as you go further and further way; that is what you are going to see.

We want to solve for this temperature distribution. We want; this is the function of x we would like to get T as a function of x, we will like to solve for it and once we have solved for it, we will be able to calculate the rate at which heat flows from the surface of this fin. So our objective with this simplification that is a pin fin; therefore T is only a function of x, our objective is to find the temperature distribution along the length that is T as a function of x and to find the heat transfer rate from the surface of the fin; all along it is surface that is our objective.

What we will do is the following - we will say consider in order to derive the differential equation for this case, we will say consider a closed system like this shown shaded by me here. That means at a distance x, take a cross section at a distance x plus dx. Take a cross section; you will cut out an alimentary volume of width dx and cross section equal to the cross section A of the fin. A times dx, you will cutout an elementary slice A times dx; consider the heat balance on this elementary slice of width dx; what do we get let us look at that now. Let us write that down; let us apply the first law of thermodynamics to A times Tx.

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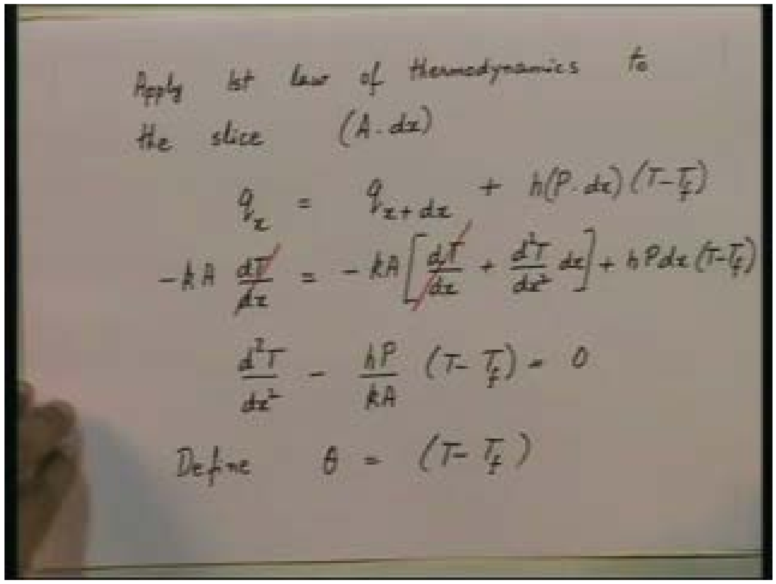

Apply first law of thermodynamics to the slice A time dx; now let me go back to the sketch here again for a moment. Notice I have drawn 3 arrows here; qx which I am showing here is the rate at which heat has been conducted into the element by heat conduction that is qx. qx plus dx is the rate at which heat is being conducted out of the element by heat conduction and we also need to consider the heat flowing by convection from the surface of the slice and that is h the heat transfer coefficient into the surface area p times dx where p is the parameter in to the temperature difference T minus Tf. So this is the heat being lost by convection from the surface of the elementary slice. So what do we have now? Let us put down expressions for these quantities.

The first law says qx is the heat flowing in by conduction that must be equal to the heat flowing out by conduction plus the heat being lost by convection from the surface and that is h pdx which is the area into T minus T_f as the convective heat loss from the surface. Now from Fourier’s law, qx is nothing but minus kA dT dx and that is equal to qx plus dx is minus kA dT dx. If I move a distance dx in the positive x direction, I will get d 2 T dx squared multiplied by the distance I move dx; that is the increment on dT dx plus hPdx T minus T_f and on simplification you can see the dT dx term is going to cancel, this going cancel with this.

So on simplification, I will get d 2 T dx square. If minus hp by kA is equal into T minus T_f is equal to 0, that is the differential equation I am going to get for this case. As usual, I will again define theta as being the access temperature above the ambient temperature theta equal to T minus T_f; let us define that. And if we you do that, we will get one simplification.

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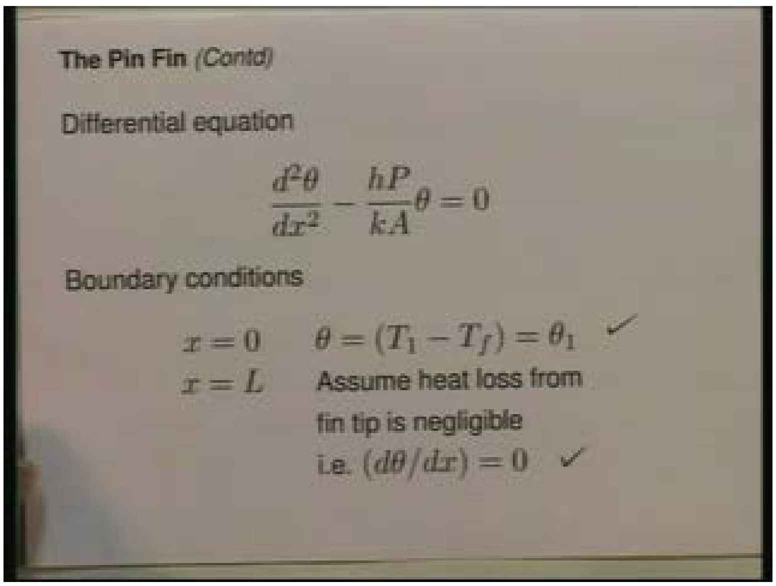

We are going to get the differential equation in terms of theta as we get the differential equation in terms of theta as d 2 dx squared minus hP by kA theta equal to 0 - that is our differential equation. It is the second order linear differential equation, very simple to solve. What are the boundary condition under which we will solve it? We will need to boundary conditions so the 2 boundary conditions would be 1 at x equal to 0 1 at x equal to L. At x equal to 0; T is equal to T_1, so theta is equal to theta 1 straight forward. At x equal to L, we have a variety of boundary conditions to choose from but let us make, let us choose boundary condition which is usually adopted and that is the following. We will say, I have already said the pin fin is fairly cylinder and long relative to it is cross sectional dimensions. Therefore, by the time you reach almost the fin tip the heat flowing out at the edge, the heat flowing out at the edge.

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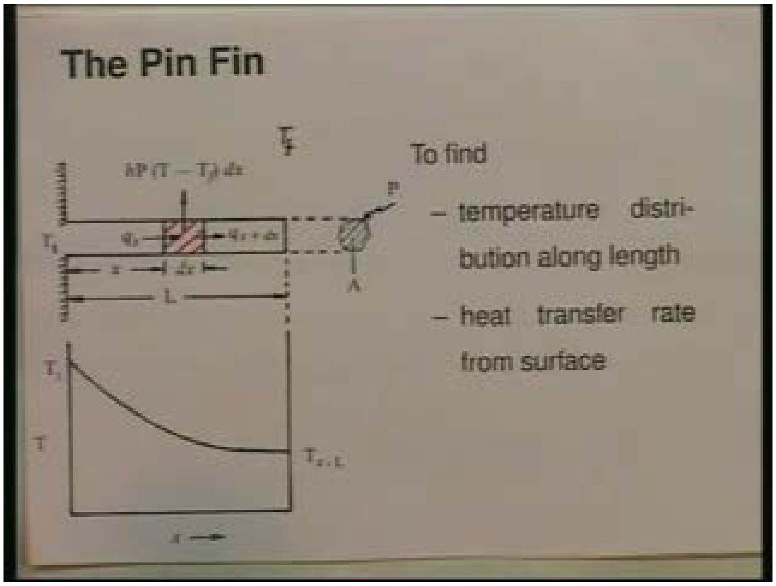

Right here which I am showing again, I am showing the sketch of the fin again the heat flowing out at the edge right here from this face. This is likely to be extremely small; what I have shown by the red arrow just now and if it is small let me assume that it is 0. If it is 0, it follows from Fourier's law of heat conduction that the d theta dx at x equal to L must be equal to 0. So, we make the reasonable assumption that the heat lost from fin tip is negligible and therefore at x equal to L, we will take d theta dx equal to 0. So that is our second boundary condition that we take. Solve these differential equation subject to these two boundary conditions; as I said it is a very simple differential equation

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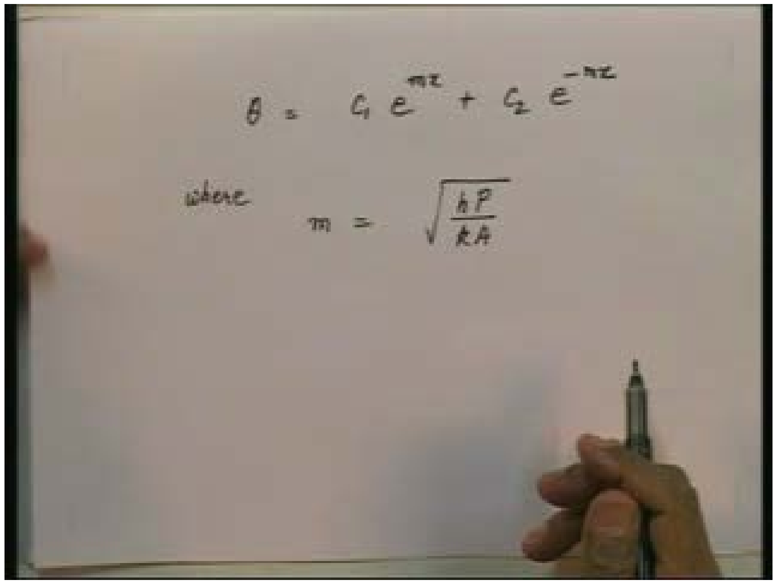

the general solution of this differential equation is equal to; theta is a is equal to c_1 e to the power of mx plus c_2 e to the power of mx minus mx which all of you should know where m is equal to square root of hP by kA so straight forward solution. Now use the 2 boundary conditions to get the values of c_1, c_2 and if you do that we will get the solution to the problem to be for the temperature distribution we will get the following temperature distribution.

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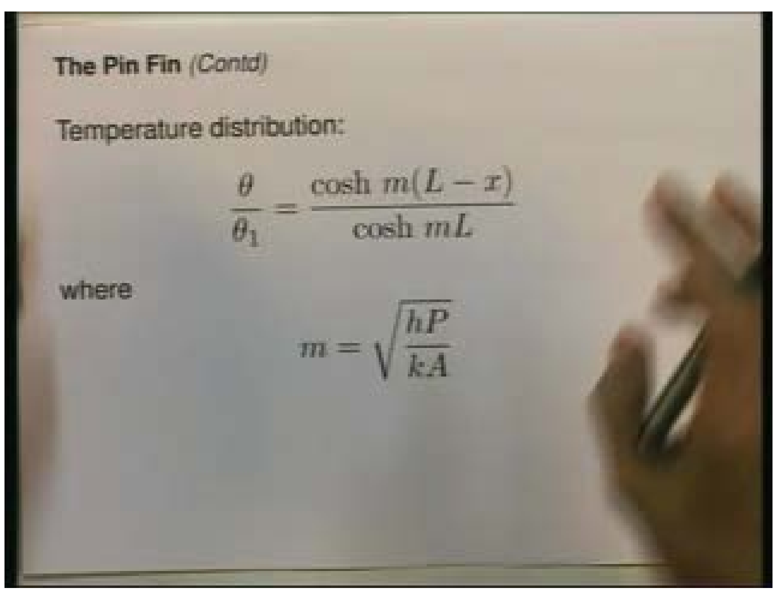

We get theta by theta 1 instead of putting the solution in the form e to the power mx and e to the power minus mx, we will use the hyperbolic cosine and will get theta by theta 1 is equal to the hyperbolic cosine m L minus x divided by the hyperbolic cosine moulding that is what we will get for the temperature distribution, very simple solution that we get because a very simple differential equation where as the defined earlier m is nothing but the square root of hP by kA. So this is the temperature distribution, the equation that we got for the temperature distribution which I sketched for you earlier. Once we got the temperature distribution, we can solve for the rate of heat flow from the fin surface; that is what we are interested in the second thing we are interested.

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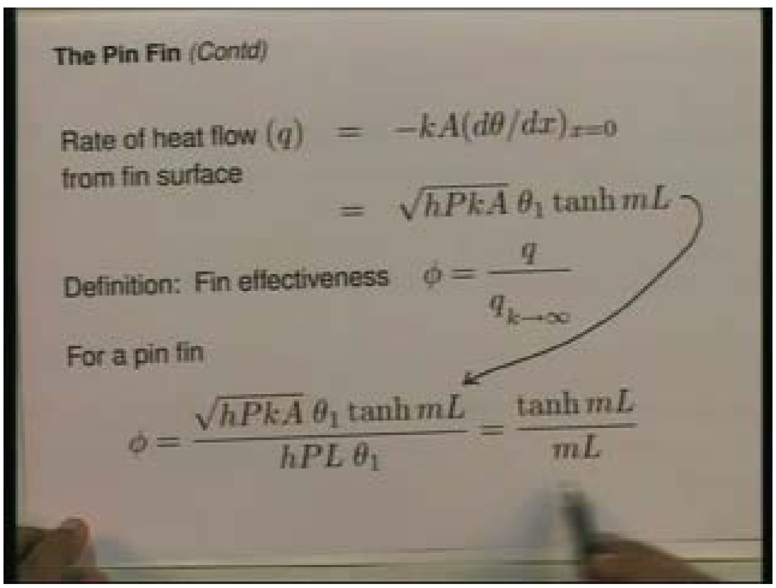

What is the rate of heat flow from the fin surface that we will call as q so when you, what is, what will it be? If I go back to the fin again for a moment, let me show the fin again. We want the rate at which heat is flowing out through the fin surface like this all over to the surroundings. All this heat must have originated at this point; it has to come from the base surface here at x equal to 0, enters the fin and then go out. So, all the heat that is flowing out must ultimately originate here and then flows out like this; so we can also say the heat flowing out all over the surface is nothing but by Fourier’s law is nothing but minus k d theta dx at x equal to 0 that is at this cross section. So let us put that, we will see rate of heat flow from the fin surface is nothing but minus kA d theta dx at x equal to 0 and if you use the temperature distribution equation which I gave to find out d theta d theta dx from it. Put x equal to 0 which is a liner 2 of calculation for you to do.

You can show that this will come out to be the square root of hPkA multiplied by theta 1 multiplied by the hyperbolic tangent mL. So that is the rate of heat flow from this fin surface; so we have got the answers to the 2 thing we are looking for. We have got an equation for the temperature distribution in the fin pin; we have an equation for the rate of heat flow q from the fin surface, so many what is the rate at which heat is flowing? Now with fins, it is usual to define a term called the fin effectiveness for all fins, not just pin fins for all fins it is usual to define a term called as the fin effectiveness which we will denote by the symbol phi and is defined as follows.

We say the fin effectiveness phi is equal to the actual heat flow rate q from the fin surface divided by the q you would get if thermal conductivity of the fin material is infinity. q upon q which would be obtained if the thermal conductivity of the fin material is infinite. Suppose the thermal conductivity of the fin material is infinite then again to go back to the sketch of the fin here; what would be the temperature distribution? If the thermal conductivity of this fin is infinite, then T is equal to T_1 at the base; then an infinite thermal conductivity requires that T is equal to T_1 everywhere along the fin's length. There will be no drop in temperature like I am showing here; you will get T equal to T_1 everywhere. So, k equal to infinity means a temperature distribution T equal to T_1 everywhere along the length of the fin. So, this is how we define a term called as fin effectiveness.

For a pin fin, what we will get for a pin fin phi is equal to the fin effectiveness is equal to in the numerator, I put square root of hpk theta 1 hyperbolic tangent of mL which is nothing this expression. In the denominator, recognizing that the temperature is T equal to T_1 along the whole length if the temperature is T equal to T_1. The temperature difference with the surroundings will be T_1 minus T_f all along the length that means theta

1 and therefore the rate at which heat is lost will be h into PL which is the surface area of the fin p, is the parameter L, is the Length into theta 1 which is the temperature difference T_1 minus T_f. So hPL theta 1 is the heat that would be lost from the fin surface if the conductivity of the fin material is infinite.

If you simplify this whole expression, you will get hyperbolic tangent mL upon mL. So that is the expression for the fin effectiveness of a pin fin. Now how do we use?

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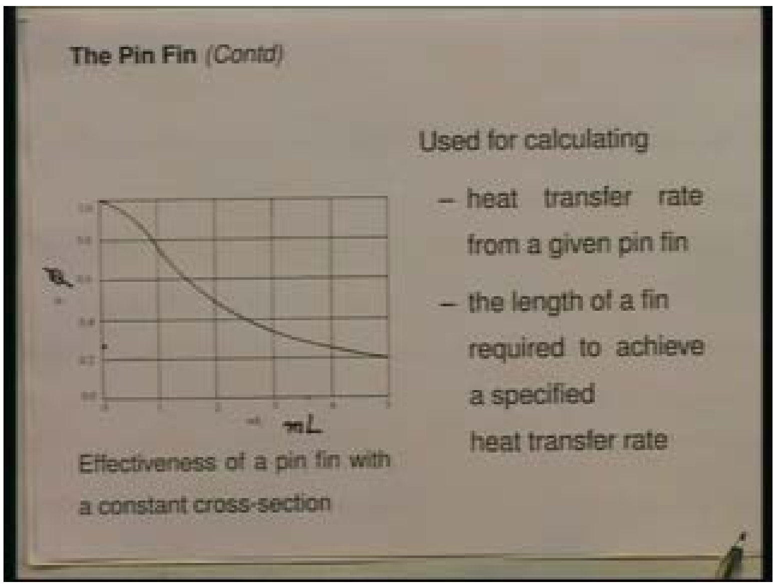

Let me just show you graph and which I have plotted the effectiveness of pin fin, the same equation that I showed you phi equal to hyperbolic tangent mL upon mL is plotted here. This is phi and this is mL; just writing it in big letters again. We are plotting that equation hyperbolic tangent mL divided by mL here. Effectiveness, this is the effectiveness of a pin fin with the constant cross section - phi plotted against mL. How do we use this solution? Now we use it as follows; we say we, suppose I have a pin fin of a given length and a given cross section and ceratin given conditions. For that pin fin, you can calculate mL; the moment you get mL go, this graph here for that value of mL go up. Then go horizontally and get the value of phi for that pin fin; so for a given a fin calculate mL, go up intersect this graph and then go horizontally till you get the corresponding value of phi.

Once you have got the value of phi; multiply by q when you, the value of q you would get if k is equal to infinity and you will get the actual heat transfer rate from the given pin fin. So for a given pin fin proceed first for a given value of mL upwards, then horizontally to get value of phi. You may have to do the reverse problem; reverse problem is find the length of a fin required to achieve a certain heat transfer rate q. In which case, you would go in the reverse direction you would know a value of phi. You would go horizontally and then you would come down in order to find a value of mL. Try doing both types of problems yourself; I would like you to do both types of problems. I will do one type; I want you to do the other type yourself. So let me illustrate now this whole idea; let us take a problem, we will say let us do a problem now of pin fin.

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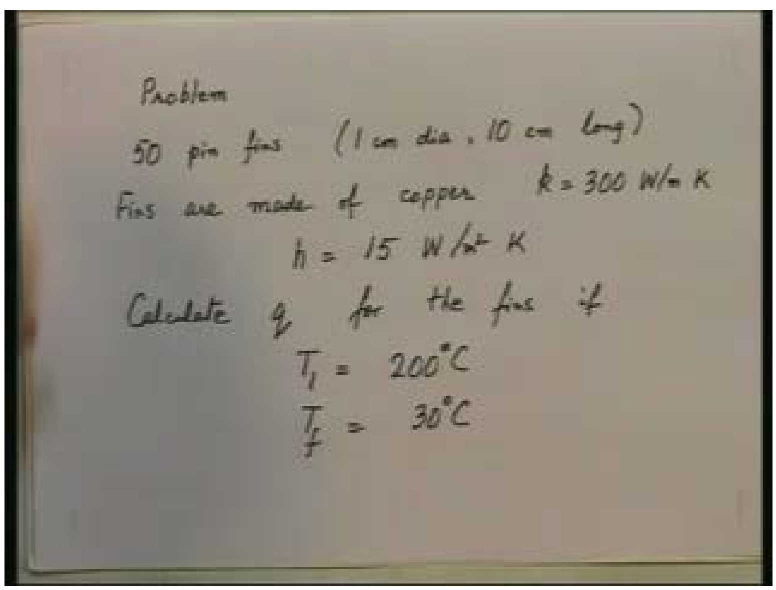

I am going to take up the following problem; let us say on a surface certain base surface, I have 50 pin fins 50 pin fins. Each pin fin 1 centimeter diameter 10 centimeters long; I am just taking some dimensions simple dimensions. Circular pin fins 1 centimeter diameter 10 centimeters long attached to a wall. Fins are made of copper, fins are made of copper k equal to 300 watts per meter Kelvin, high value of k because some metal and that too a fairly pure metal. Value of h at the surface given to be 15 watts per meter square Kelvin. Calculate q for the fins if the temperature at the base T_1 is equal to 200 centigrade and the surrounding temperature T_f is equal to 30 degrees centigrade. This is just a straight forward substitution into what we have done. So let us to that, calculate q for these 50 pin fins. Let us substitute the numbers now, first let us get mL

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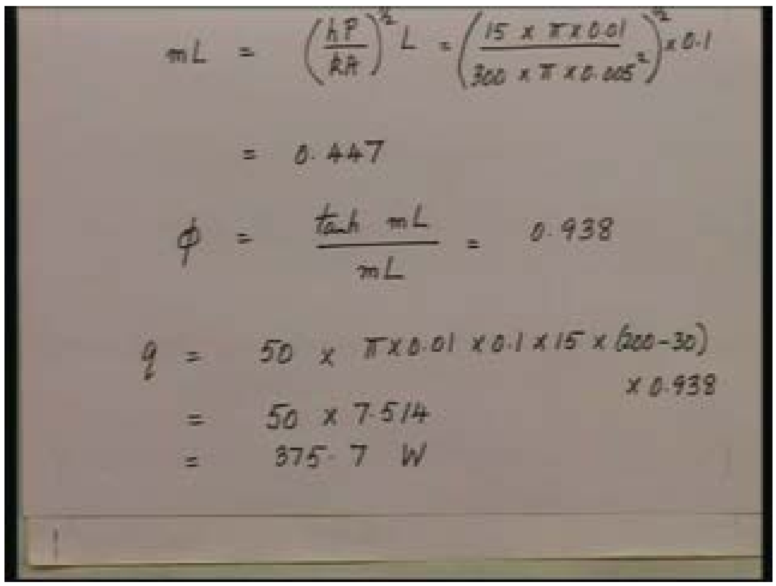

mL in this case mL is equal to square root of hP by kA to the power of half multiplied by L. So that is equal to h is 15 L, P is pi into .01 divided by k is 300. A is the cross sectional area which is pi into .005 squared pi r squared; take the square root of this and multiply by the length L which is .1 in meters - this comes out to be .447. So the effectiveness phi is equal to the hyperbolic tangent mL divided by mL which comes out to be if you calculate it .938. Therefore q is equal to 50, the number of fins into the heat transfer rate from 1 fin which will be pi into .01 into the length pi Dl. This is the surface area of 1 fin into the heat transfer coefficient 15 multiplied by the temperature difference 200 minus 30.

pi d l into 15 into 200 minus 30 this would be the heat transfer rate if k is equal to infinity. If I multiply this by the fin effectiveness, I get the actual heat transfer rate. So this comes out to be 50 into 7.514 which is equal to which is equal to 300 and 75.7 watts, so that is the answer. It is a straight forward substitution problem - given 50 pin fins with the certain data, find the value q which you would get for a given values of a temperature and the surrounding temperature and the temperature at the base.

Now we want to look at another problem; the problem which we want to consider next is the following. Consider that I have the, now doing 1 more problem of a pin fin.

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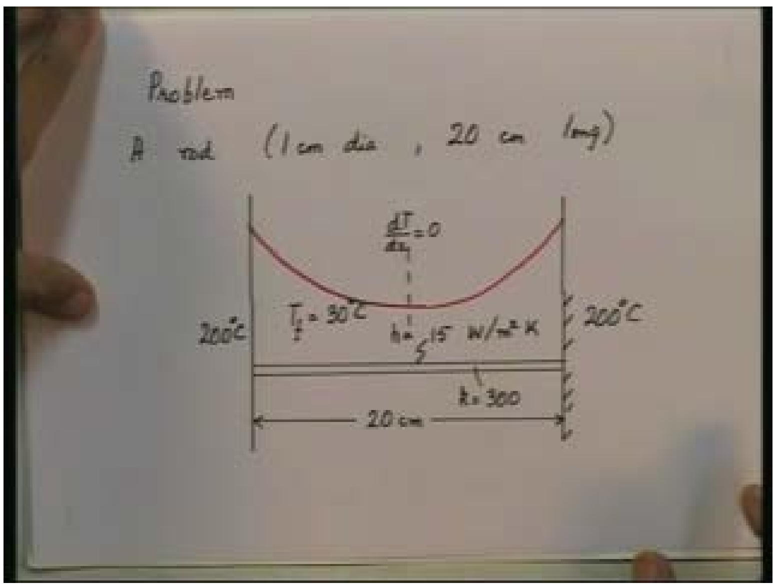

Suppose I have rod which is 1 centimeter in diameter, consider a rod 1 centimeter diameter 20 centimeters long which is fixed between 2 walls each of which is at a temperature of 200 degrees centigrade. This is the situation I am visualizing now. These are 2 walls between which I have a rod struck, thin rod 1 centimeter in diameter 20 centimeters long. This is 20 centimeters long and it is struck, it is fixed between 2 walls and the walls are at temperature, both of them are at a temperature of 200 degree centigrade. We are told that the k, the thermal conductivity k of the material of the rod is 300 so many watts per meter Kelvin. The heat transfer coefficient at the surface is 15, h is

15 watts per meter squared Kelvin and the surrounding temperature T_f is 30 degrees centigrade, T_f equal to 30 degree centigrade.

So, rod fixed between 2 walls at 200, surrounding temperature is 30 degree centigrade, h is 15, k of the rod material is 300 watts per centimeter Kelvin, watts per meter Kelvin. Now what kind of temperature distribution will you get in this rod if you were to sketch it? I think you will agree with me that you have got a symmetrical situation so you are going to get some kind of a temperature distribution which is going to be like this - going to go down and then it is going to go up; that is the kind of temperature distribution you are going to get in this rod. The line drawn in red temperature is going to go down and then going to go up again to 200; starts at 200, goes down and then goes up an at the center that is as a x equal to 10 at the center here, symmetry requires at the center, obviously since everything is symmetrical on both sides, symmetry requires that d theta or dT dx be equal to 0.

So once you recognize that this situation is one with the symmetry what you have in effect - the situation that you have with this rod is in effect equivalent to 2 pin fins which are attached to each other. I would like to find out; by the way I should say that what are we looking for we want to know what is the heat transfer rat from this rod.

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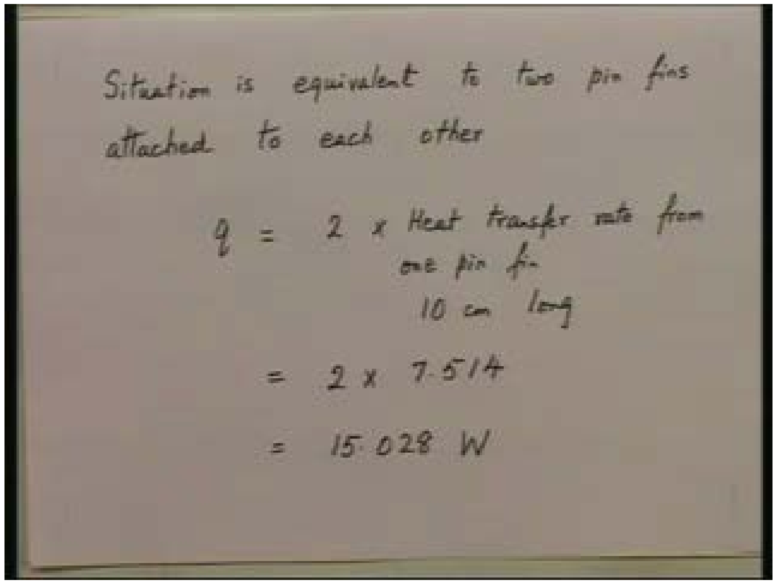

So, recognize now straight away that in this case the situation is equivalent, situation is equivalent to 2 pin fins, 2 pin fins attached to each other at x equal to L. That is what we have got and therefore if I ask now what is the q for this, you say straight away the q for this situation is equal to 2 multiplied by the heat transfer rate from 1 pin fin 10 centimeters long and that is nothing but 2 times 7.514; that is the answer of the previous problem because the data is identical and that is equal to 15.028 watts. So the case of the thin rod in this case is equivalent to 2 pin fins that is what we are really saying. Now we have analyzed only the pin fin here but keep in mind that there are solutions for a variety of pin fin configurations available in the literature and in every case, almost in every case after obtaining the temperature distribution, the solution finally is in the form of fin effectiveness plotted again, some geometric parameter of the fin.

For instance, you recall when I started talking of pin fins, I referred to circumferential fins with the rectangular cross section; you recall I showed this geometry and I said around cylinders we put circumferential fins like this.

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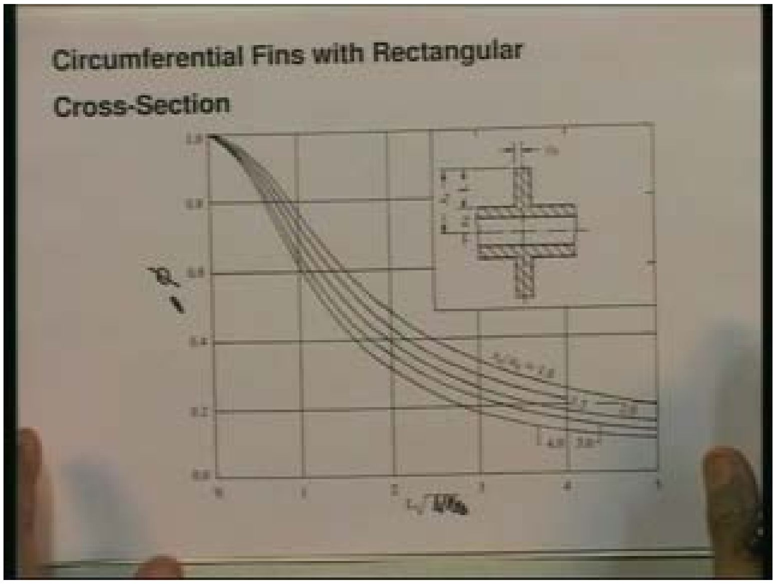

Now for this case, this is a well analyzed case of a circumferential fin around the circular cylinder. For this case, the fin effectiveness phi - let me write this in bigger letters the fin effectiveness phi is plotted against a parameter L square root of h divided by kyb where yb is half the width of half the thickness of the circumferential fin. This is the chart which is available in almost all the standard books; it is not something new but keep in mind this is very similar to the chart which we got for the pin fin.

And what, how would we use it? In the same way given a certain circumferential fin with given dimensions xe, xb, yb, etcetera, go to this chart calculate the x axis parameter L in to the square root of h upon kyb. Then from this for the given value of xe by xb, go up vertically in to the given value of xe by xb, go horizontally, get the value of phi, multiply this phi by the heat transfer rate you would get from the circumferential fin if it has an infinite thermal conductivity. So in exactly the same way as a pin fin, you would use this chart in order to find the heat transfer rate from circumferential fins with the rectangular cross sections. Such solutions are available for a variety of fin shapes.

We have done here in the class only the pin fin and I have shown you the chart for fins are the rectangular cross section but keep in mind such charts or equations are available for variety of fin shapes. Now we have come to the end of what we wanted to do in heat conduction; let me sum up what we did quickly.

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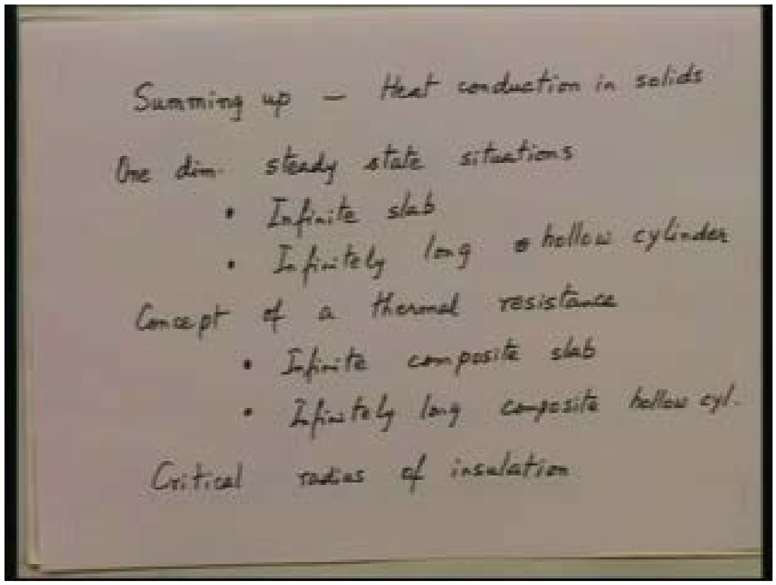

Let us sum up what we have done in heat conduction - first of all we started with one dimensional steady state situations, one dimensional steady state situations and we looked at the infinite slab. We looked at the infinitely long hollow cylinder, got simple solutions for temperature distribution and heat flow rate for these cases. I told you how to solve on your own, to solve the problem for a hollow sphere. Then we introduced the concept of thermal resistance and we got a expression for the thermal resistance again of an infinite composite slab and infinite, infinitely long composite hollow cylinder.

And our reason for introducing the concept of a thermal resistance was - thermal resistance is in series are additive like thermal, electrical resistances in series and then following in this, bases we looked or we introduced the idea of critical radius of insulation for a pipe. Saying that there are situations in which if the outer radius r_2 of a pipe is below a certain value, then the addition of a insulation may not help until we a put a certain minimum thickness. We got an expression for the critical radius of insulation then in order to extend ideas to more general situations, we derived the general differential equation.

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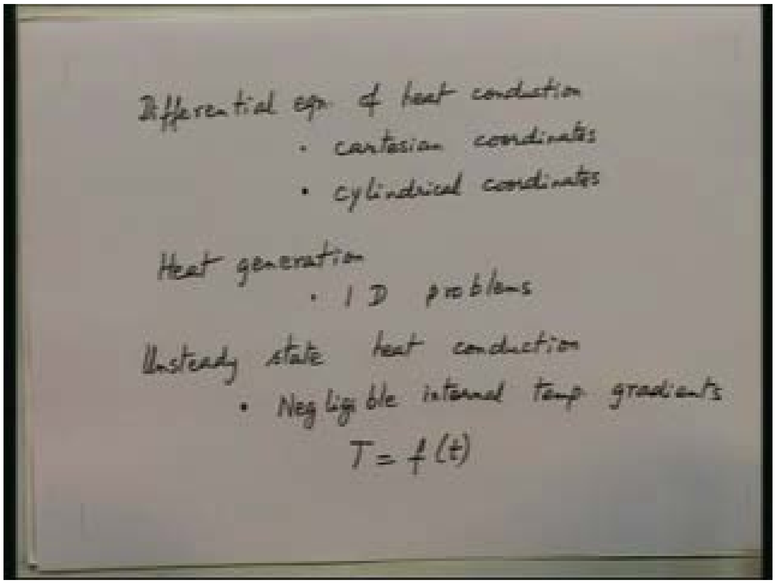

we derived the general differential equation both in cartesian coordinates and in cylindrical coordinate and in cylindrical coordinates. Using this general approach we solved some problems of heat generation some one dimensional problems, 1 D problems of heat generation in a slab as well as in a cylinder. Then we looked at unsteady state, unsteady state heat conduction and in this first of all we said, suppose we neglect internal temperature gradients, negligible internal temperature gradients. Then we have a situation in which T is only a function of time; a very simple which can be solved regardless of the shape of the object but the moment you can not neglect internal temperature gradients.

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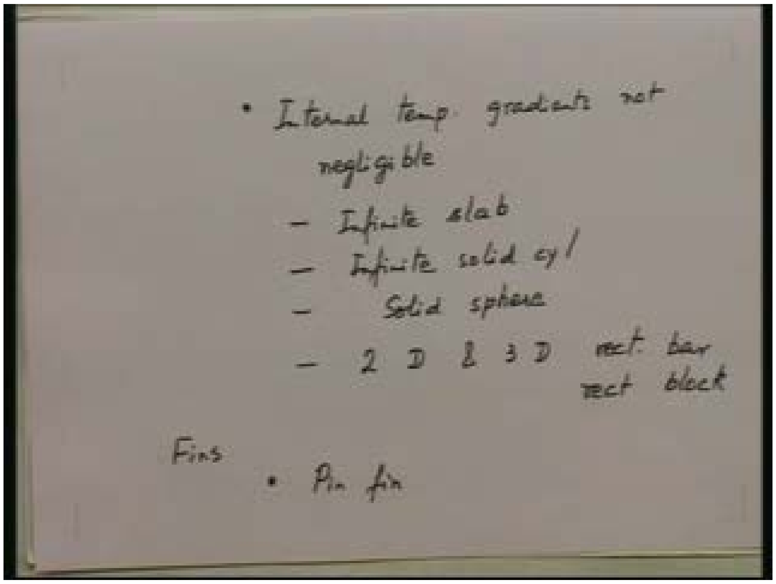

Then we have more complicated situations, internal temperature gradients not negligible and for this case we looked at the infinite slab. I set up the problem; indicated how the solution is available in the form of charts. Then we also indicated how a solution is available in the form of an infinite solid cylinder and a solid sphere. Then we looked today at 2 D and 3 D problems, rectangular bar and rectangular block and indicated how a product type solution is available for them. And finally we have looked at fins and in particular we have looked in detail at the pin fin, derived an expression for the temperature distribution in the pin fin and the effectiveness of a pin fin. I have also indicated that there are charts available for the effectiveness of so many other geometries of fins. So we have come to the end of what we want to do with conduction; next time we will start with thermal radiation.