Heat and Mass Transfer
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Lecture No. 7
Heat Conduction 4

Today we are going to look at some one dimensional steady state heat conduction problems involving heat generation. We will consider one dimensional steady state conduction with heat generation first in an infinite slab and then in an infinitely long solid cylinder. So, let us look at the first situation.

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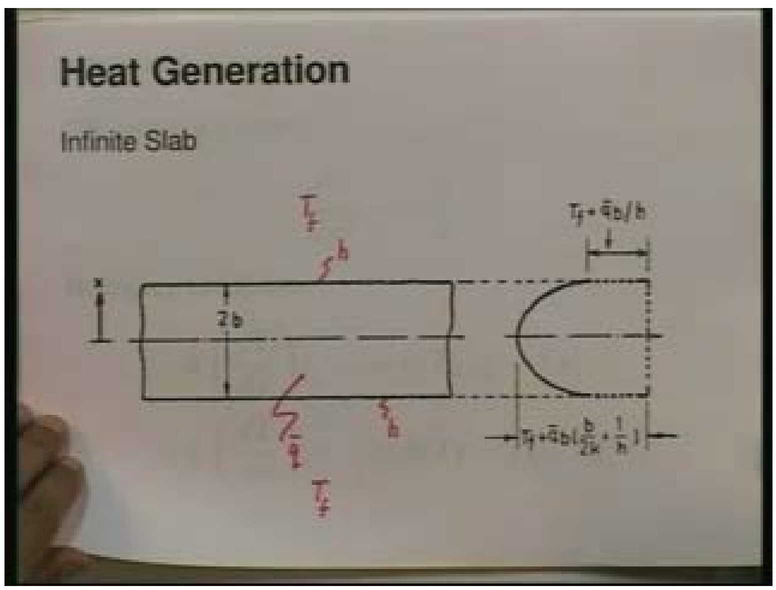

In this sketch here, I am showing an infinite slab of width 2 b and I specify to you that in this slab heat is being generated at a uniform rate q bar; everywhere in this slab, heat is being generated at the uniform rate q bar. We would like to know the steady state temperature distribution in this slab. The distribution obviously is going to be such that I will get a larger temperature in the center, a lower temperature at the edges and we want to find an equation to this temperature distribution which is shown here.

Now first of all, let us formulate the problem; that means let us put down the differential equation, let us put down the boundary conditions of the problem. In order to put down the differential equation, let us look at our general differential equation.

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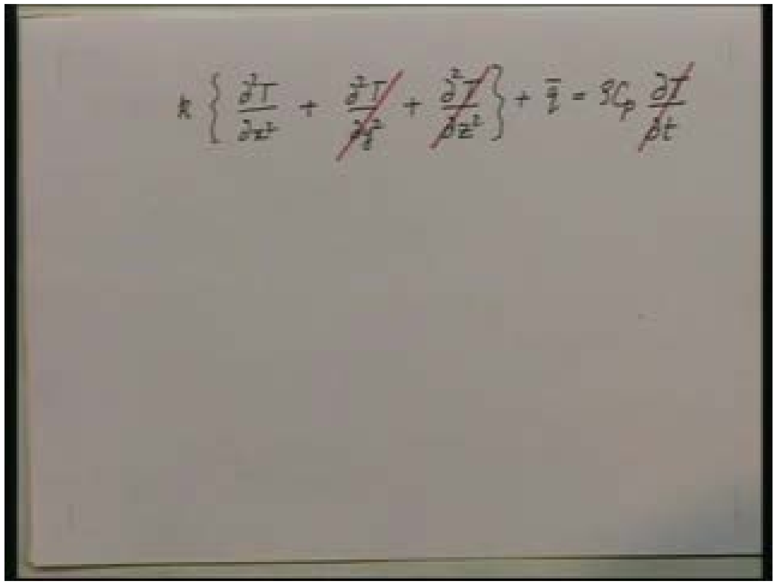

Our general differential equation if k is a constant is K for a general situation is, k multiplied by d 2 t dx squared plus d 2 t dy squared plus d 2 t d z squared plus q bar is equal to rho C_p dT dt – that is our general differential equation which we have derived earlier, where in the general case being one where variation may occur in x y z and there may be an unsteady state also. Now in this particular case, we know that because it is an infinite slab there is variation only along the width v and therefore which we call the direction x. Therefore, there will be no variation in the y direction; there will be no variation in the z direction. We also know that there is a steady state therefore the term dT dt is equal to 0.

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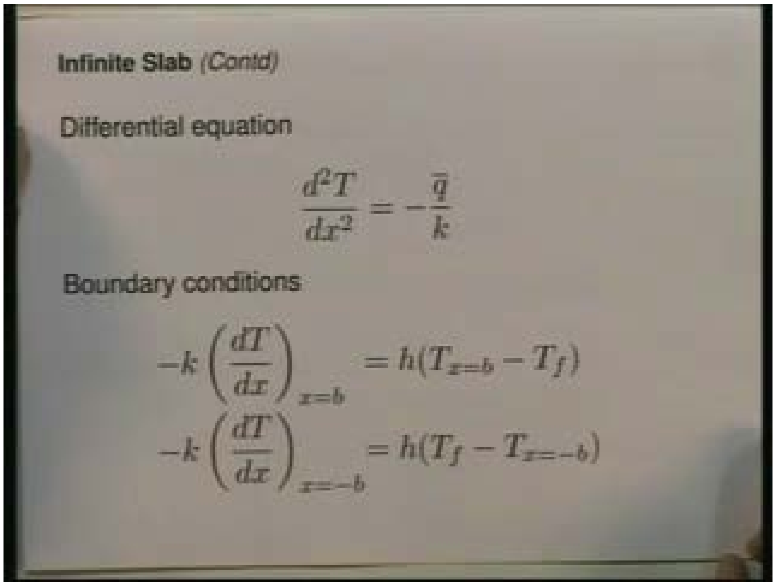

So, in this case the differential equation for which we have to solve reduces to the form d 2 T dx squared is equal to minus q bar by k. Notice now I have gone from the partial differential to the ordinary differential because we have only variation in one direction - the x direction - and there is no variation with time. So we have to solve this second order differential equation subject to the boundary conditions which we have on the 2 faces. What are the 2 faces of the slab? Let us go back and look at the slab; 2 faces of the slab are - x is equal to b, x is equal to b and x is equal to minus b. At both these faces we specify that there is a heat transfer coefficient h; at both these faces we specify that there is a heat transfer coefficient h and that the fluid outside is at a temperature T_f; the fluid outside to which heat is being lost is at a temperature T_f.

So, the boundary conditions of the 2 faces; we have a boundary condition of a heat transfer coefficient and in mathematical terms the boundary conditions will become: on the face x is equal to b, we will get on the face x is equal to b; we will get minus k dT dx at x equal to b is equal to h into T at x equal to b minus T_f. The heat being transported by conduction inside the slab at x equal to b is equal to the heat being lost by convection at the face x equal to b to the surroundings and similarly at x is equal to minus b minus k dT dx at x equal is to minus b is equal to h into Tf minus T at x is equal to minus b.

Notice the interchange; this is now T_f minus T at x is equal to minus b because the heat flow by conduction as given by Fourier's law is always in the positive direction. So this we call the formulation of the problem; we have stated the differential equation and we have stated the 2 boundary conditions at x is equal to b and at x is equal to minus b. Now we have to solve this and solving it is a straight forward thing, just a second order differential equation; I integrate it twice.

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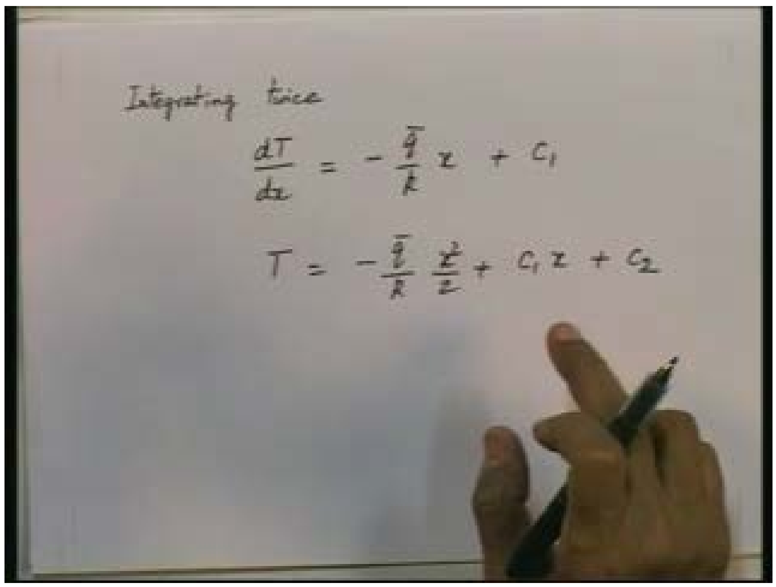

If I integrate it twice I will get, integrating the differential equation twice, first I will get dT dx; the first integration will give me dT dx is equal to minus q bar by k x plus c_1 and the next integration will give me T is equal to minus q bar by k x squared by 2 plus c_1 x plus c_2, that is the general solution. Now I use my boundary conditions which are - I know I have 2 constants c_1, c_2; 2 boundary conditions that x equal to b and x equal to minus b. Put those 2 boundary conditions in, solve for c, for c_1 and c_2 which I will not do on the board here. I leave it for you to do but if you do that you will get the solution for the temperature distribution and one gets the temperature distribution to be the following.

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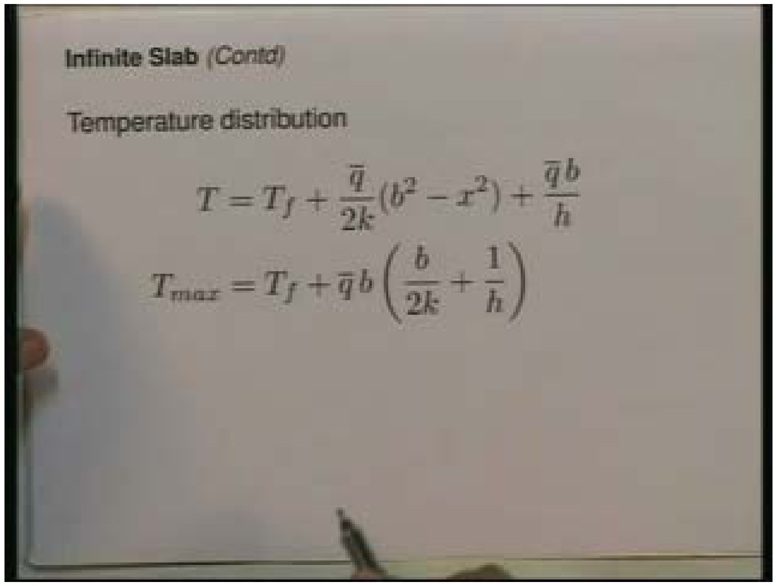

You get T is equal to T_f plus q bar by 2 k b squared minus x squared plus q bar b by h, that is what you get for the temperature distribution in the solid and that is the equation for the temperature distribution which I showed right at the beginning. That is the equation for the temperature distribution which I showed right at the beginning; that is this temperature distribution, it is a parabolic temperature distribution that we get. Obviously, the maximum occurs at the center and if you solve for in order to obtain the maximum value T_{max} at the center, you put x equal to 0, so you will get T_{max} is equal to T_f plus q bar b into b by 2 k plus 1 by h.

So, that is the maximum temperature that occurs within the solid, within the infinite slab at the centered line; so this is the solution to the problem that we have got now. We will solve the same problem for an infinitely long solid cylinder. Suppose I have an infinitely long solid cylinder of radius R and heat is being generated in it uniformly at the rate q bar. I ask myself find the temperature distribution in this solid cylinder and find the expression for the maximum temperature which will obviously again occur at the center line that is at R equal to 0.

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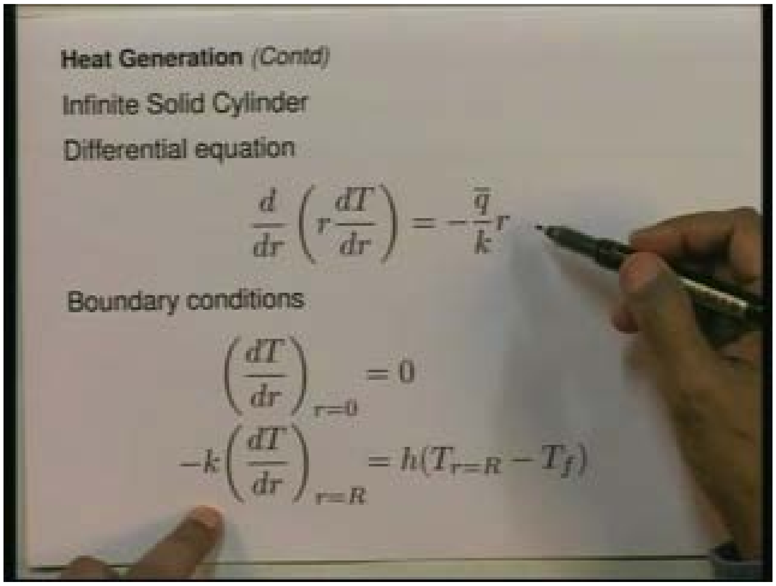

So in this case if you go to the differential equation in cylindrical coordinates; I am not going to put that down but go back to the differential equation which we had for constant k in cylindrical coordinates. In that differential equation, note that T is only a function of r; T is not a function of theta, T is not a function of z. Note also that it is a steady state problem so there is no dT dt term; if you do all that then you will end up with this second order ordinary differential equation - T varying with r and the boundary conditions now are one boundary condition is at the center line.

This is the symmetry condition because we have symmetry at r equal to 0; dT dr must be equal to 0. The other condition is at the face; the round face r equal to R - circular face. minus k dT dr at r equal to R is equal to h T at r equal to R minus T_f. Solve this, that means again like we did for this slab; integrate this twice, put in the 2 boundary conditions and obtain the 2 constants of integration which I will skip this time because it is very similar to what we did for the slab.

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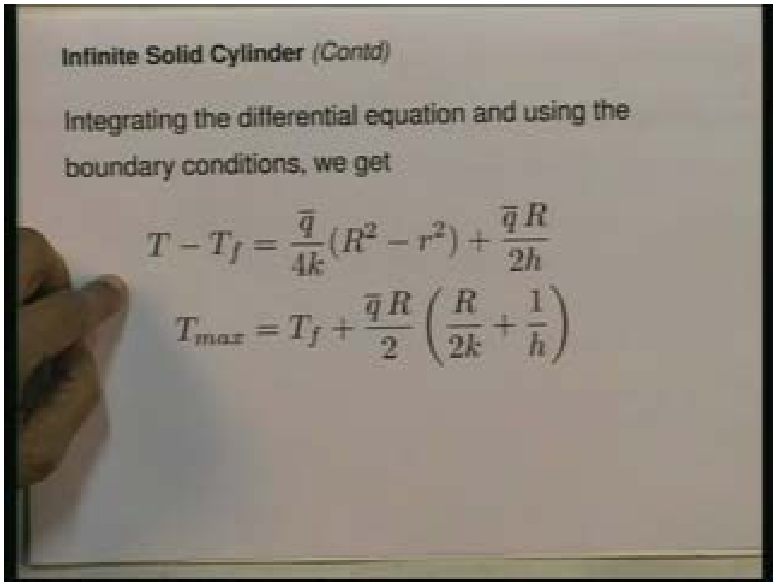

Now if you do that, if you integrate the differential equation and use the boundary conditions, we get the solution to be T minus T_f is equal to q bar by 4 k into R squared minus r squared, close the bracket, plus q bar R by 2 h; this is the equation for the temperature distribution inside the solid cylinder. And the maximum temperature again if you wish to find that out - put r equal to 0 in which case this term goes to 0 and we get T_{max} is equal to T_f plus q bar R by 2 multiplied by R by 2 k plus 1 by h. That is the maximum temperature which occurs at the center line R equal to 0; so these are 2 simple one dimensional problems in which uniform heat generation, which we have solved for the case of a slab - an infinite slab of width 2 b - and for an infinitely long solid cylinder.

We could solve similar problems - one dimension problems also; for instance on your own, I suggest try to solve the problem for a long hollow solid cylinder. A hollow cylinder, inner radius r_i, outer radius r_o, heat being generated uniformly in it. Assume that there is a heat transfer coefficient h on both the surfaces at r equal to r_i, r equal to r_o and the surrounding fluid temperature is T_f both inside and outside. Solve the problem.

You will again setup the same differential equation; you will integrate it twice but because of the change in boundary conditions, you will get different values for the constants of the integration. So that is a problem you can do on your own; a problem of a long hollow cylinder or take the problem on the sphere. I suggest you do that on your own take a solid sphere, assume of radius capital R, assume that heat being generated uniformly in it the rate q bar. Find the temperature distribution in that solid sphere; do that on your own. There should be no difficulty doing that problem as well.

Now let us do a numerical example; just to illustrate ideas and to substitute into the formula that we have just derived.

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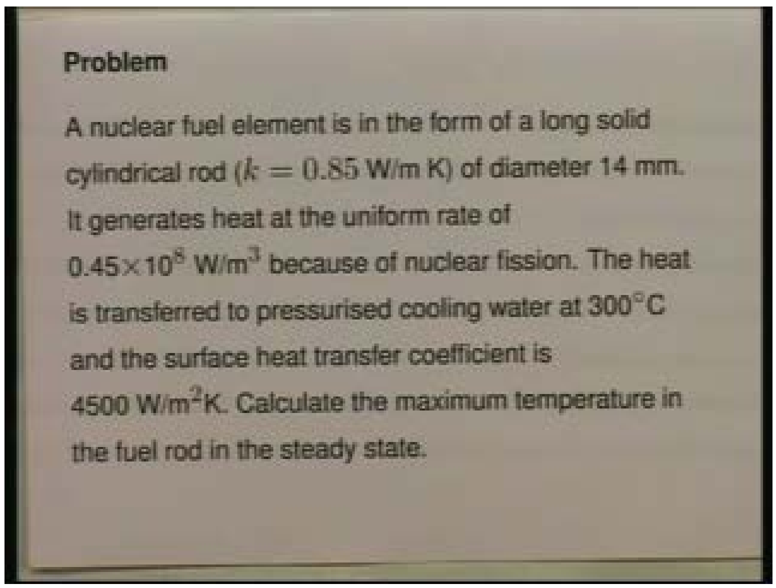

Here is the problem that I would like you to do and let me just dictate it so that you can take it down; that problem is concerned with nuclear fission - nuclear fuel rod in which fission is taking place. And it is as follows: it says a nuclear fuel element is in the form of a long solid cylindrical rod parenthesis k, thermal conductivity equal to .85 watts per meter Kelvin; close the parenthesis, of diameter 14 millimeters. It generates heat at the uniform rate of .45 five multiplied by 10 to the power of 8 watts per meter cubed because of nuclear fission. The heat is transferred to pressurized cooling water at 300 degrees centigrade and the surface heat transfer coefficient is 4500 watts per meter squared Kelvin. Calculate the maximum temperature in the fuel rod in the steady state; I repeat calculate the maximum temperature in the fuel rod in the steady state.

This is the problem; it is a straight forward substitution into what we have done. Notice in a pressurized heavy water reactor we have uranium, natural uranium form of uranium oxide is the fuel. The thermal conductivity is about .85; these are values which are close to real values and heat is being generated at this rate because of the fission. Neutrons are striking and breaking up the uranium atom so we have generating heat by fission at the rate given. Obviously because of heat is being generated, temperatures rise, the heat is transferred but at we get a temperature distribution inside the fuel rod; find that temperature distribution and particularly find the maximum temperature in the fuel rod, that is the problem. Now let us do it; so this is, let me just sketch things a little.

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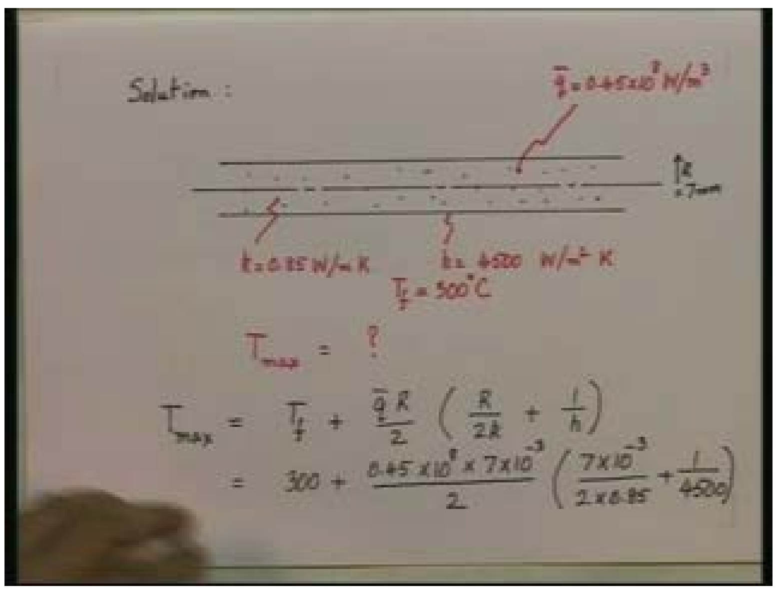

Let us say this is our fuel rod; let me just draw a rough sketch. This is the fuel rod, some uranium oxide, some form of uranium oxide; the conductivity of this fuel rod is given to be .85 so let us put that down. k is equal to .85 watts per meter Kelvin and the heat is being generated uniformly in this bar. q bar is equal to .45 multiplied by 10 to the power of 8 watts per meter cubed. We are given that the radius of the rod R is equal to 7 millimeters, that is the radius of the rod. The heat transfer coefficient at the surface h is equal to 4500 watts per meter squared kelvin and the temperature T_f of the pressurized cooling water flowing on the outside is 300 degree centigrade. Find T_{max}; that is the problem. It is a straight forward problem; it is really just a matter of substitution.

So, T_{max} is equal to, going back to the solution which we had a moment ago I am just substituting into it; Tmax is equal to T_f plus q bar R by 2 into R by 2 k plus 1 by h. That is what we got from the solution for the solid cylinder; so we get, substituting we will get 300 plus .45 multiplied by 10 to the power of 8 multiplied by the radius that is 7 into 10 to the minus 3 meters divided by 2, the whole thing multiplied by 7 into 10 to the minus 3 divided by 2 multiplied by .85 plus 1 upon 4500 which is the heat transfer coefficient and if we do the calculation, we will get this is equal to 983.5 degrees centigrade; T_{max} is equal to 983.5

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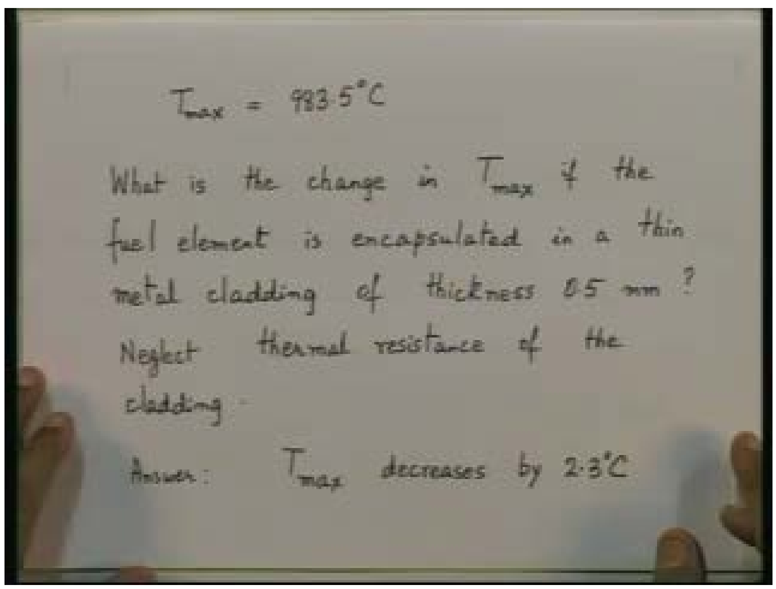

This is the maximum temperature occurring on the center line of the rod; this is the maximum temperature at the center line of the rod. This is what we are interested in finding; this temperature must not exceed the safe value specified for that fuel rod. If it does, then we must either cool more effectively or we should not generate so much heat by fission; whatever it is either you will have to lower the value q bar or increase the value of h or decrease the value of T_f. In someway you have got to do something; if you don’t, if this 983.5 is above a safe value for the fuel rod. Because you don’t want that fuel rod to melt, you want it to be safe but just keep on giving out heat by fission.

Now, let us take up a small extension of this problem. The extension is as follows - I ask you what will, what is the change in the maximum temperature; this is what I want you to do further. Say what is the change in T_{max}, what will be the change in T_{max} if the fuel element, if the fuel element is encapsulated in a thin metal cladding; in a thin metal cladding of thickness of thickness .5 mm. The uranium oxide is in inside a thin metal cladding of thickness .5, what will be the change in T_{max} if I give you if this is the additional information? Neglect, in order to simplify the calculation; I will say neglect thermal resistance of the cladding. In order to simplify the calculation, neglect thermal resistance of the cladding; I want you to do this on your own.

You should find, if you should do the problem on your own; you should get the answer - T_{max} decreases by 2.3 degrees centigrade; that is the answer you should get if you do the calculation on your own. So we have done the calculation for a fuel rod alone and now I am saying suppose the fuel rod is surrounded by an encapsulation which is a metal cladding half a millimeter thick, neglect the thermal resistance of their metal cladding. How much would T_{max} change? The answer is T_{max} should decrease by 2.3 degrees centigrade. I want you to do this on your own.

Now let us now move on to some other situations. We have considered some situations of heat generation. Now I want to move on to situations in which temperature is no more just a steady state that is we want to move on to more general situations in which temperature varies with time. So we want to now consider some problems of unsteady, of the unsteady state.

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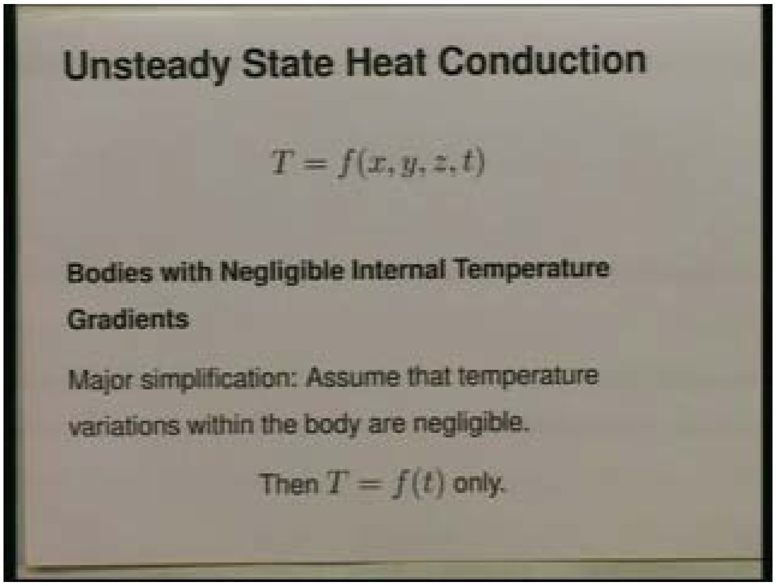

We want to move on to unsteady state heat conduction. In general if I have a general problem in unsteady state conduction, T will be a function of x, y, z and time or T will be a function of R, theta, z and time if I am working in cylindrical coordinate; that is the general situation in unsteady state conduction. And if that were the case; if it is a general situation and I want it to solve for it then you have already seen the general differential equation which I have put down a moment ago but I will just show it again.

You recall we had put down the general differential equation for heat conduction when we did the heat generation problem. This is our general differential equation so if T is a function of x, y, z and time, this will be our differential equation with the d 2 T dy square d 2 T dz square dT dt term all being retained. This will be our general differential equation. So obviously, life is going to be more complicated the moment we go the unsteady state because not only will T vary with space, T will also vary with time. So straight away you are going to have a situation in which T is going to be a function of at least one space variable and the time variable. So straight away you will have a partial differential equation.

Now a major simplification is obtained in conduction problems if one assumes that temperature variations within the body are negligible. It is possible to conceive of situations where these temperature variations within the body are negligible and temperature is only a function of time. For instance, suppose the body - solid body - that we are talking about is undergoing an unsteady state. Suppose that solid body has a very high thermal conductivity; obviously temperature variations within that body are always going to be a very small.

So, a major simplification is possible if we assume that temperature variations within the body are negligible; that is we first take up for consideration bodies with negligible internal temperature gradients and what is the mathematical simplification as a result? The mathematical simplification is that the temperature T is then only a function of time. So first now, in considering unsteady state situations of heat conduction, let us first consider those situations in which we can neglect internal temperature gradients so temperature is only going to be a function of time. Let me illustrate now; let us derive the differential equation for this situation; body is with negligible internal temperature gradients.

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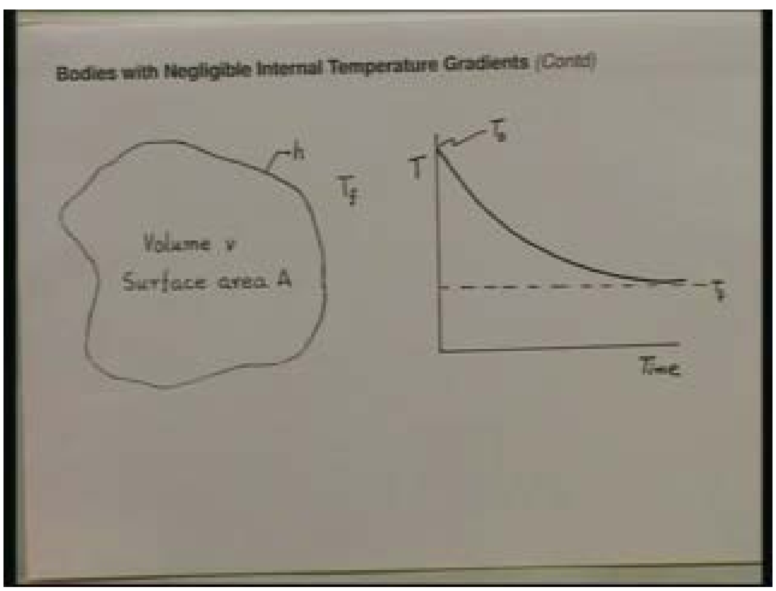

Let us say I have some solid body; could be any shape and let us say its volume is v, its surface area is A. At the surface, this body in during the unsteady state loses heat to a surrounding fluid which is at a temperature T_f and the heat transfer coefficient is h. Let us say the body is initially at a temperature T_0, some uniform temperature T_0. I suddenly put it in surroundings at a temperature T_f and there is a heat transfer coefficient h at the surface. If T_0 is greater than T_f, then the body is going to cool; if T_0 is less then T_f, the body is going to heat up; if we neglect internal temperature gradients, the temperature will only be a function of time. We would like to find that functional dependence.

How does temperature vary with time for such a general situation? Let me first sketch the temperature distribution before we solve for it. For this case suppose that I have a; let me just sketch it, this is the type of graph we are likely to get if we were to plot temperature of the body against time. And starting at time T equal to 0 then, if we were to sketch the temperature, temperature of the body is going to change like this. It is going to go on decreasing and it is going to asymptomatically, asymptotically, heat will approach the value of T_f as it keeps on cooling down. This is T_f; this is the initial temperature T_0. So if the body is initially at a temperature T_0 when it is immersed in surroundings at a temperature T_f, then it cools down and it is going to cool in this fashion.

We would like to find an equation for this line, this cooling curve; we would like to find that equation; that is the problem. Now let us formulate it; first let us formulate the problem. If I apply the first law to the solid body, what will I get at any instant of time?

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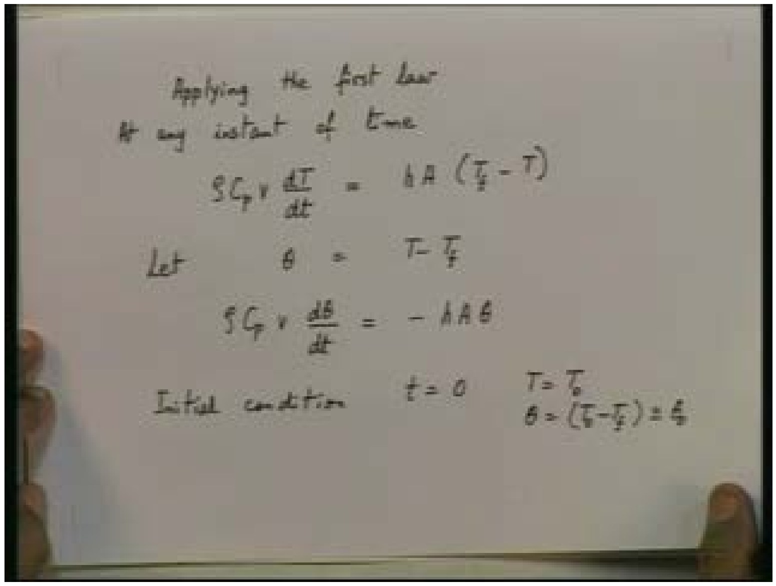

Applying the first law of thermodynamics to the solid body - it is a closed system; at any instant of time during the cooling process, at any instant of time during the unsteady state process, at any instant of time we will get rate of change of energy of the solid body rho C_p v dT dt. rho into v is the mass, C_p is the specific heat, tt dT is the rate of change of temperature with respect to time. This is the rate of change of energy; this must be equal to the rate at which heat is transferred to the body which is h into A into T_f minus T. Mind you the body is cooling, then this will be a negative number and this will also be, dT dt will also be negative because the body is cooling. So the gradient will always be negative; so this will be the applicable differential equation which we have to solve.

Applying the first law, let us make the substitution theta is equal to T minus T_f. I make the substitution theta is equal to T minus T_f; what do I get? If I make the substitution, I make the substitution theta equal to T minus T_f I will get; the differential equation becomes rho C_p v d theta dt is equal to minus hA theta. That is my differential equation; if I integrate this differential equation once, integrating to get the solution and using the initial condition; what is my initial condition? So solving this differential equation and using the initial condition time t equal to 0, capital T is equal to T_0 or theta is equal to T_0 minus T_f which I will call as theta 0. Using the initial condition at time t equal to 0, theta equal to theta 0, we will get the solution to the problem to be - it is a straight forward first order differential equation. We will get the solution to the problem to be theta by theta 0

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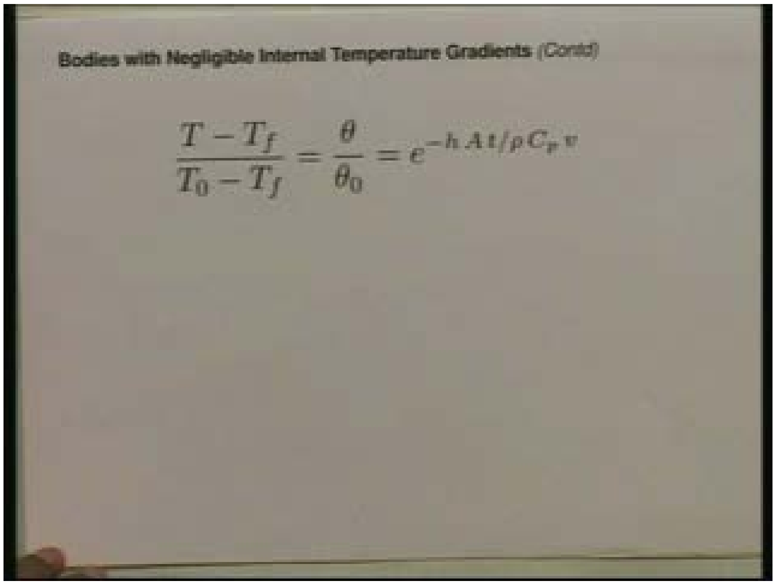

is equal e to the power of minus hAt divided by rho C_p v; that is the answer. It is an exponential decay; theta by theta 0 goes on exponentially decay e to the minus term or instead of theta I can write T minus T_f instead of theta 0. I will get T_0 minus T_f and this gives me the temperature distribution in terms of T. So as I said, the moment we introduce the notion of negligible internal temperature gradients, we don’t have to worry about the temperature as a function of x, y, z or r theta z inside the body. We just say temperature is a function of time; get a first order differential equation and the solution is there.

Now whatever be the shape; suppose the shape is a cylinder then for A and v, I will put in the values for a cylinder. Suppose the shape under consideration is a slab; I will put in the values of A and v for a slab whatever they are. So that I will get appropriate solutions for the particular shapes that I am dealing with. But here I have a general solution for any shape because I have made a very major simplification by assuming that there are no, that the temperature gradients inside the body are negligible. But this we cannot always do.

We can do it in some situations; then if we do, well we got a solution, the problem is over but suppose we cannot neglect internal temperature gradients which is very often the case.

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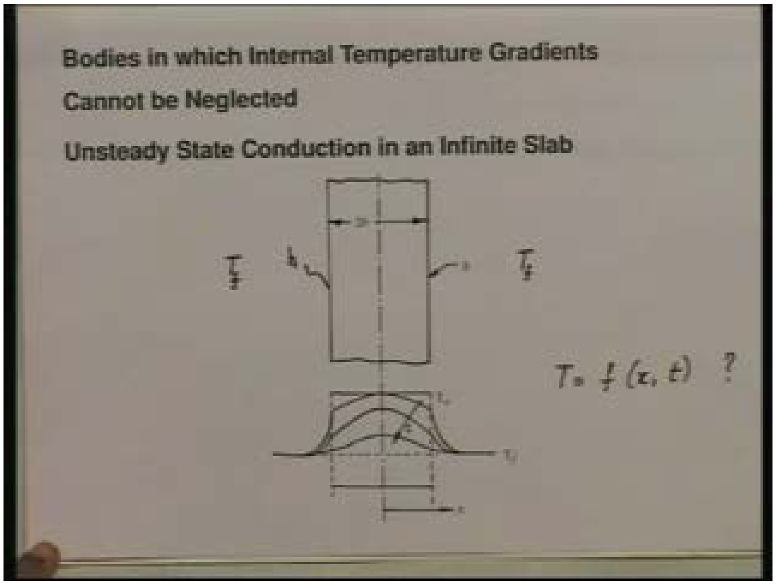

So, now we go on to consider bodies in which internal temperature gradients cannot be neglected; we go onto such situations and first of all we will look at unsteady state heat conduction in an infinite slab. Again, we will again as usual look at in infinite slab; so the problem now which we wish to solve is the following - we have an infinite slab of width 2 b, an infinite slab of width 2 b. Initially at a temperature T_0, it is initially at a temperature T_0. This slab is suddenly put into surroundings which are at a temperature T_f and the heat transfer coefficient at the surface, on both the surfaces - this face x equal to b and this face x equal to minus b; the heat transfer coefficient on both the surfaces is h, on this face as well as this face, h on both surfaces, T_f the temperature of the fluid on both sides, T_f the temperature of the surfaces, of the fluid on both sides.

So, the slab is going to cool and we would like to find equations for the temperature distribution in the slab as it is shown cooling. It could be heating up also if I showed T_0 less than T_f here, the slab will heat up it doesn't matter. We can solve the solution; we obtained will be general enough for both cases. It could be a case of heating, cooling or heating but it is the same physical situation. Initial temperature T_0, surrounding fluid temperature T_f, heat transfer coefficient on both faces h; find the temperature distribution as a function of x and time. Find T as a function of x and time inside the slab; that is the problem. T is a function of x and time inside the slab.

Now just to sketch it, this is how the temperature is going to cool. Initially, it is uniform at T_0; then the temperature is going to look something like this. Then after little while, it is going to look something like this, then something like this and if you wait long enough as time T tends to infinity, finally you are going to get a uniform temperature in the slab by time T equal to infinity which is equal to T_f; the slab will attain the temperature of the surroundings. So we want equations for these temperature distributions as a function of x and time.

Now again like earlier, let us first put down the differential equation and let us put down the boundary conditions and in this case also the initial condition of the problem. What is our differential equation?

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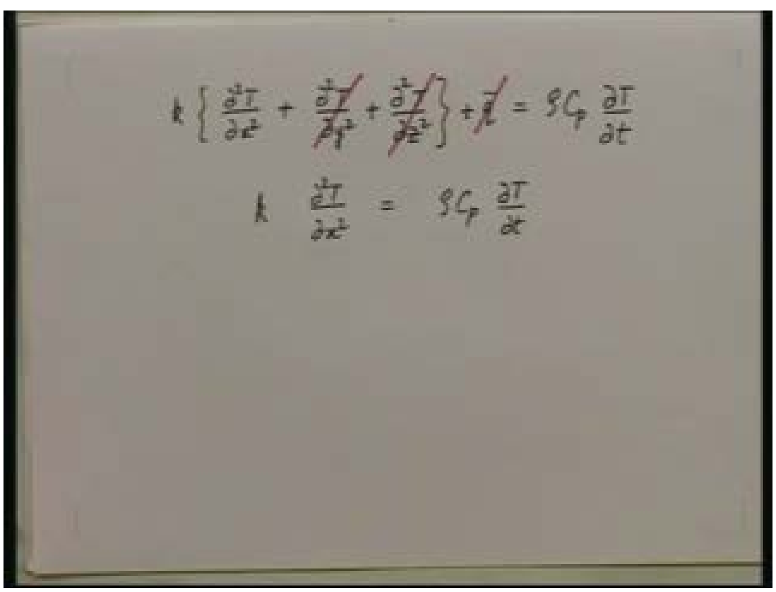

Just to repeat again, the differential equation which we have which is to be solved is, the general differential equation to be solved is d 2 T dx squared plus d 2 T dy square plus d 2 T dz square plus q bar is equal to rho C_p dT dt - that is our general differential equation. In this case, temperature is only a function of x; temperature is only a function of x and time so d 2 T dy square will be 0, d 2 T dz square will be 0 and there is no heat generation in this case therefore q bar will be 0. So in this particular case, the differential equation will reduce to the form k d 2 T dx square is equal to rho C_p dT dt; that is our differential equation in this case, rho k into d 2 T dx square is equal to rho C_p dT dt.

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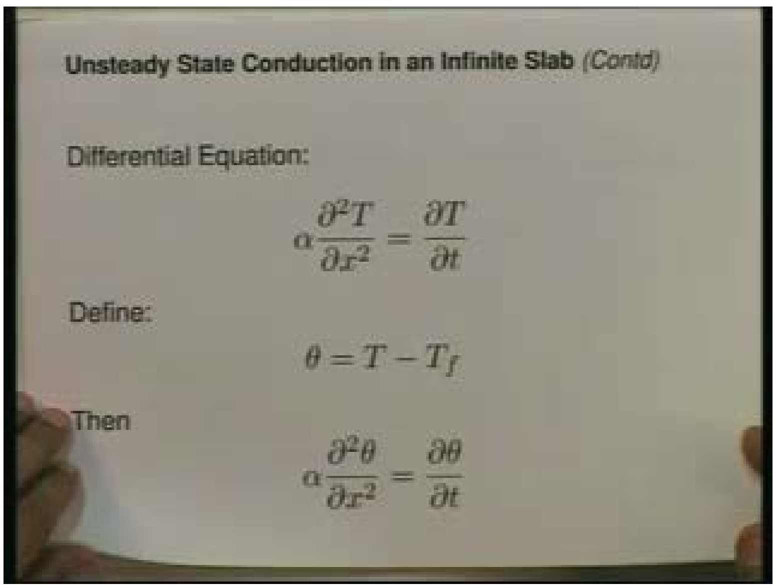

Now, the differential equation which we have got, we can also express it in the form alfa into d 2 T dx square is equal to dT dt, alfa being nothing but k by rho C_p; I think I have mentioned it you last time alfa equal to k by rho C_p is alfa, is the thermal diffusivity of the material. Like the previous case of negligible internal temperature gradients, let us again define theta equal to T minus T_f same way as earlier, theta is the temperature above the ambient temperature. The ambient is the benchmark in which case if I make this substitution, the differential equation will become alfa into d 2 theta dx squared is equal to d theta dt.

So, this is our differential equation in terms of, theta is the variable rather than T_f the variable. Now if this is our differential equation, then we have to solve it subject to the conditions of the problem. This is an unsteady state situation so we must have an initial condition and we must have boundary condition.

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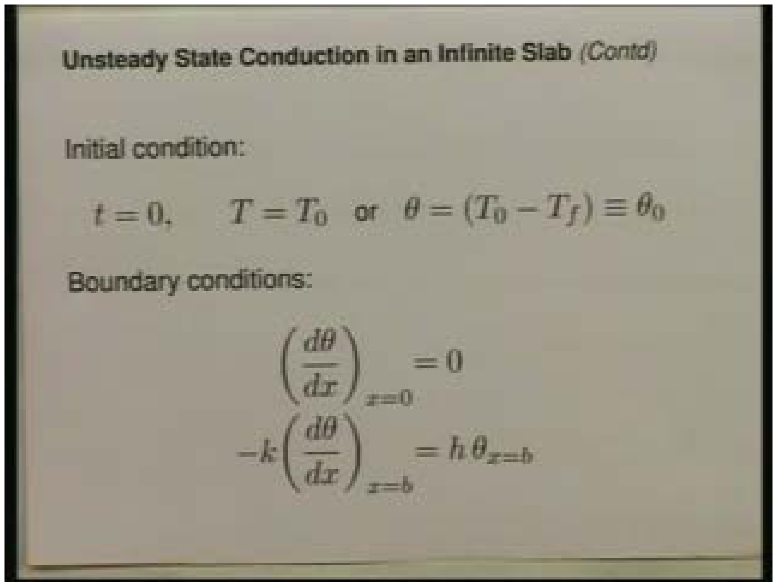

The initial condition that we have is at time t equal to 0, capital T equal to 0, capital T is equal to T_0, I have stated that or in terms of theta, theta is equal to T_0 minus T_f which is nothing but theta 0 which we will call as theta 0. Instead of writing T_0 minus T_f, we will write this as theta 0; that is our initial condition. Now boundary conditions: we have 2 faces for the slab - x equal to b x equal to minus b. Now one thing we can recognize in this case is we have a symmetrical situation about x is equal to 0. So we can also solve the problem in the positive domain that is from x equal to 0 to x equal to b and take account of symmetry and use the appropriate boundary condition which we will get due to symmetry at x is equal to 0.

So, that is what we will do in this case; we will say d theta dx at x equal to 0 is equal to 0 by symmetry because symmetry requires that the temperature distribution in the positive half and the negative half - one must be the mirror image of the other; that is a requirement of symmetry at x equal to 0. If you want one to be the mirror image of the other, then obviously d theta dx at x equal to 0 must be equal 0.

The other boundary conditions is at x equal to b and there we will get our usual condition; by Fourier's law minus k d theta dx at x equal to b is equal to h into theta at x equal to b.

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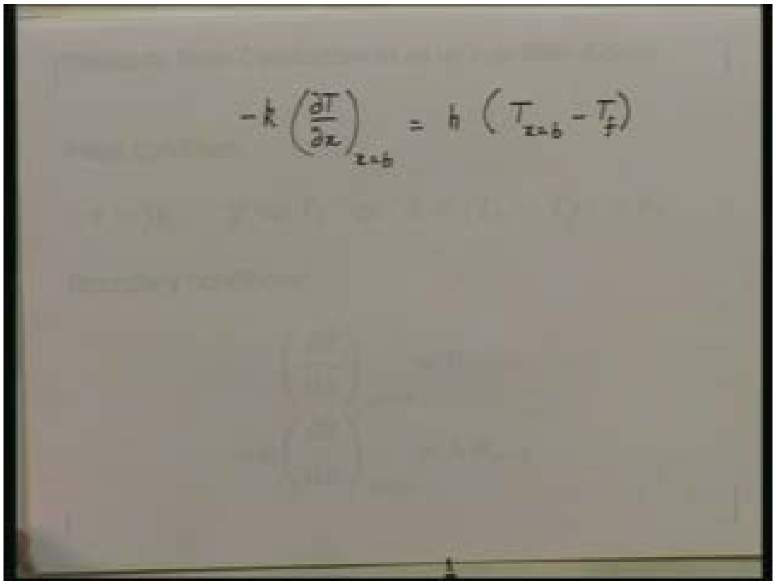

Now instead of writing of the full condition like we wrote earlier, if we write it in terms of t, we would get minus k; let me put that down in terms of T. It would have been minus k, the partial of T with respect to x at x equal to b is equal to h at T, x equal to b minus T_f - that is the boundary condition in terms of T. If I put the substitution theta equal to T minus T_f, this boundary condition will reduce to the form minus k d theta dx at x equal to b is equal to h into theta at x equal to b. So this is my boundary condition for the problem; so we will solve the problem only in the positive half that is from x equal to 0 to x equal to b and we will use the symmetric condition at x equal to 0. So this is the full statement now of the problem; you have got the differential equation, you have got the initial condition, we have got the boundary condition.

Now in this class, we will not solve this differential equation; it is the partial differential equation so obviously it calls for a more sophisticated technique to solve it. We will not do that in the class but I am just going to give you the solution directly as has been obtained.

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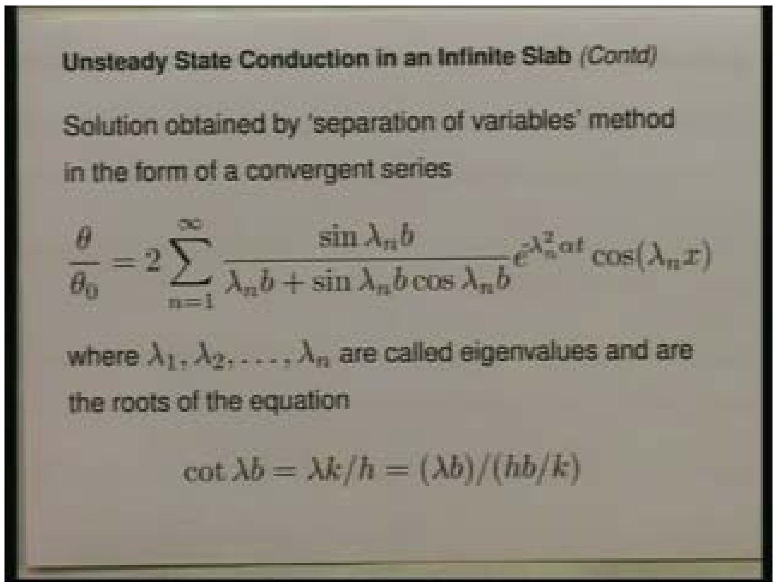

The solution for this partial differential equation is obtained by what is called the separation of variables method. This is the method used for obtaining the solution to the problem and the solution is obtained in the form of a convergent series finally; we get the solution in the form of a convergent series so I am skipping the solution method here because that is not required in this class - how we get the solution, how we solve the partial differential equation. I am simply saying that the partial differential equation which we have stated subject to the initial and boundary conditions which we have stated after solution, when it is solved we get this as the solution.

This is the solution theta by theta naught is equal to 2 into the summation; this is an infinite series n equal to 1 to infinity sine lambda n b upon lambda n b plus sine lambda n b cosine lambda n b, e to the power minus lambda m squared alfa t cosine lambda n x where lambda 1, lambda 2, lambda n up to lambda n, etcetera are called the Eigen values and are the roots of the equation - cotangent lambda b is equal to lambda k by h. This is the solution to the problem; it is a convergent series of this form with the lambdas which are called as Eigen values being given by the equation cotangent lambda b equal to lambda k by h which in turn is equal to lambda b divided by hb by k. Now, let me just write this solution out a little so that you get a feel for it.

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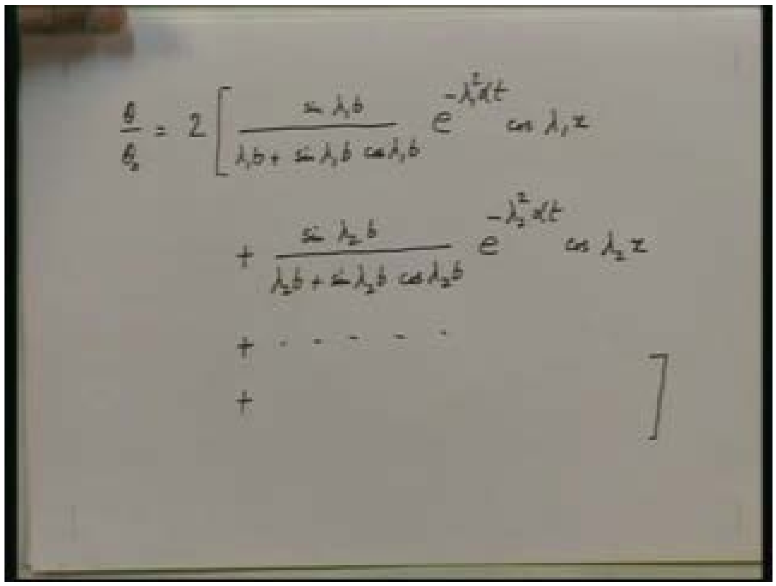

Let us say we write it out, a few terms of the solution, we get theta by theta naught, let me write the solution - put lambda equal to 1, lambda equal to 2 etcetera so that you see the solution. So you will get: 2 multiplied by the first term is obtained by putting lambda 1 b so we will get sine of lambda 1 b divided by lambda 1 b plus sine of lambda 1 b cosine of lambda 1 b into e to the power minus lambda 1 squared alfa t cosine of lambda 1 x - that is the first term of the series. The next term of the series will be plus sine lambda 2 b divided by lambda 2 b plus sine of lambda 2 b cosine of lambda 2 b e to the power minus lambda 2 squared alfa t cosine of lambda 2 x plus so on. Then finally we will get an n^{th} term and we can go on if we actually, to do the numerical work we would have to put in values of lambda 1, lambda 2, lambda 3, etcetera and we will find that slowly it will go on converging and we will stop after certain number of terms if we are doing some numerical solution for a given problem.

The lambda 1, lambda 2, etcetera as I said are given, are called the Eigen values of the problem and are obtained by, are also available in the literature. But let me just graphically illustrate what I mean by them.

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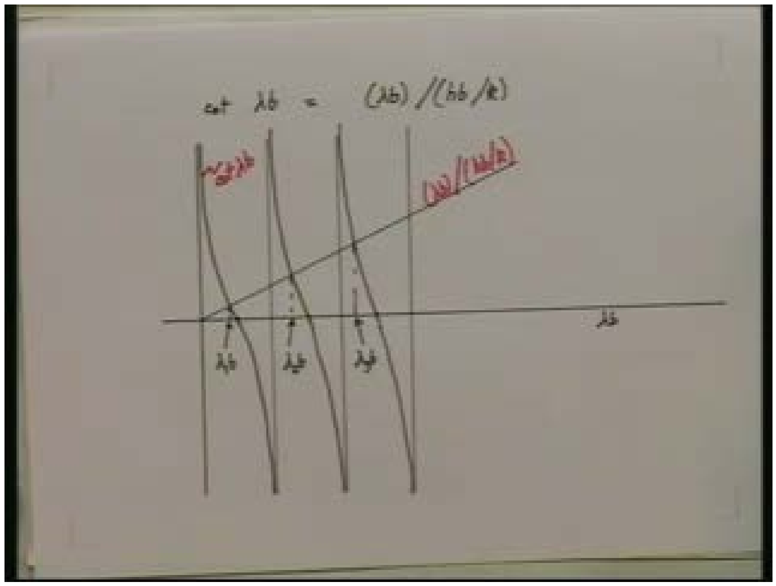

The solution to the lambda 1 is co-tangent lambda b is equal to lambda b divided by hb by k; that is what we are solving for. In order to visualize what we are solving here, it is like this: let us say this is the, let us say this is the x axis, y axis, this is the x axis. We are effectively saying the following - let me draw the cotangent, if I plot the left hand side which is nothing but the cotangent I will get something like this.

The cotangent starts from plus infinity and goes to minus infinity, periodically like this. This is the cotangent term and the right hand side is lambda b so on the x axis I have lambda b and on the, plotting the function, this is the cotangent lambda b which I have plotted. This is cotangent lambda b that I have plotted and lambda b divided by hb by k hb by k is some constant. So if I plot that, I will get a straight line with a certain slope like this and these are the values of lambda 1 b, lambda 2 b, lambda 3 b, etcetera so this is lambda 1 b, this is lambda 2 b, this is lambda 3 b.

The intersection of the cotangent curves with the curve lambda b divided by hb by k gives me the values of lambda 1 b, lambda 2 b, lambda 3 b, etcetera. So this is the solution that we have got for the one dimensional unsteady state temperature distribution in the slab. Now next time; this is the temperature distribution and these are the Eigen values; now next time we will talk a little bit more about the solution and how it is presented in graphical form.