Heat and Mass Transfer
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Lecture No. 04
Heat Conduction-1

Today, we will start with the second major topic in the syllabus namely heat conduction.

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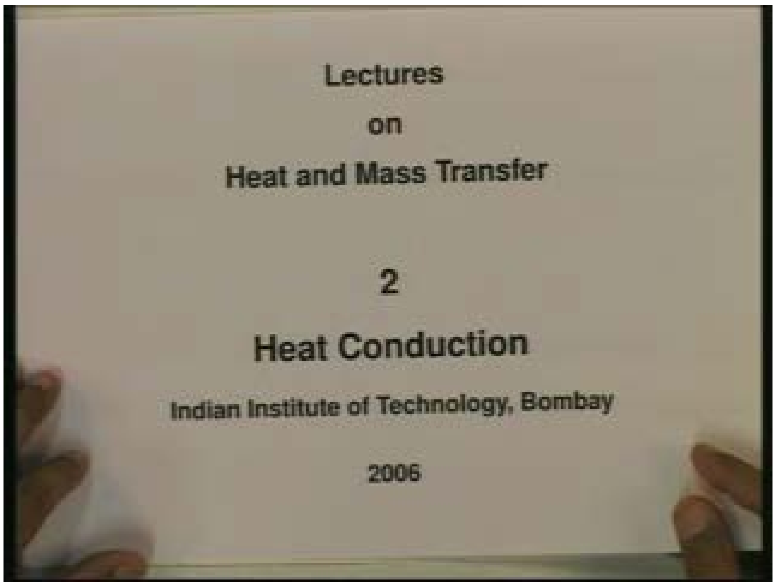

Now, more specifically I should say, we will be talking about heat conduction in solids. We will be concerned with trying to find the temperature distribution in a solid for certain situations, trying to find the rate at which heat flows in and out of solids given certain specified conditions either in the steady state or in the unsteady state.

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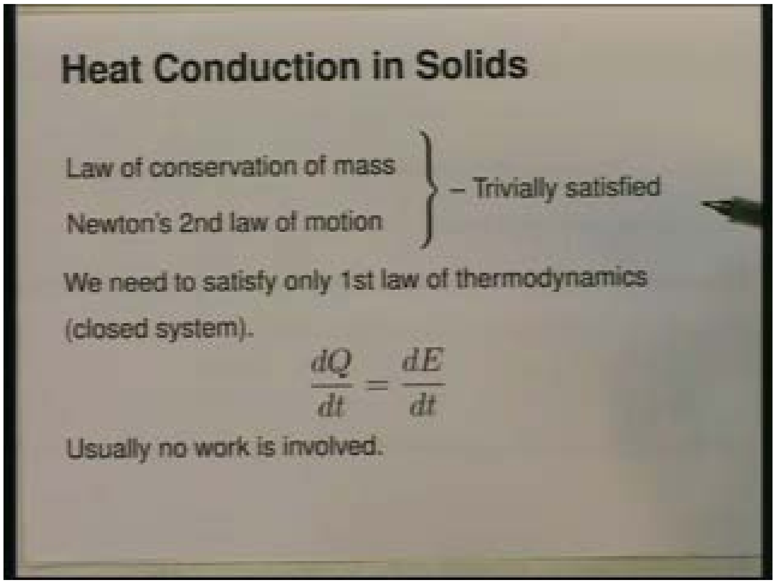

One reason why we study heat conduction in solids first is that of the three fundamental laws the law of conservation of mass and Newton's second law of motion are trivially satisfied when we talk of a closed system within a solid. We don't yet worry about mass being conserved obviously; it is trivially satisfied and Newton's second law of motion is concerned with force and momentum that too inside a solid is not of concern to us. It is like saying almost zero is equal to zero or something like that. What we need to satisfy is only the first law of thermodynamics and that too applied to a closed system. Now, in the first law, we say rate at which heat enters the system minus rate at which work is done by the system is equal to rate of change of energy of the system. Inside a solid, the work term does not take exist; so the first law simply reduces to the form dQ dt equal to dE dt because there is no work involved.

So, this is one good reason why we study heat conduction in solids first. It is mathematically, shall we say a little simpler in the sense that of the fundamental laws, we have to satisfy only one and that too, the first law for a closed system. Now what we are going to do is first of all in order to get a little familiar with the situation of heat conduction, we are going to consider a number of one dimensional steady state situations.

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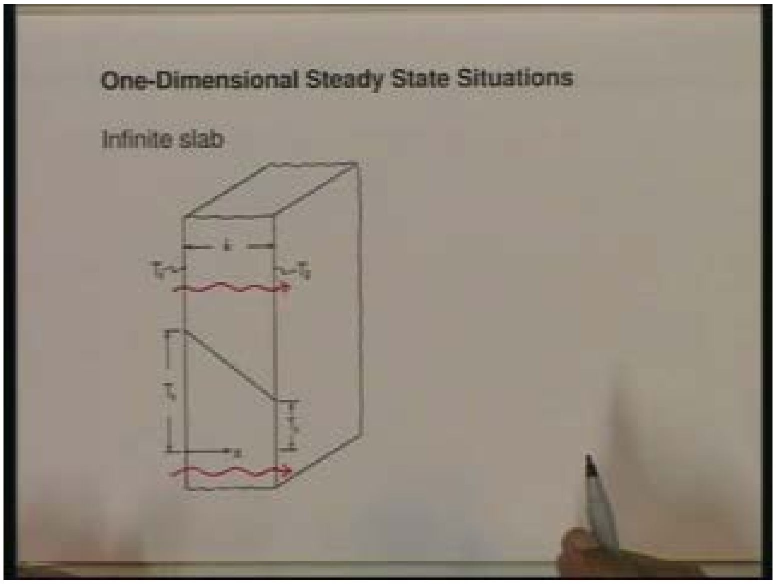

Steady state means the temperature does not vary with time and one dimensional means the temperature varies only in one direction. The first situation which we are going to consider is that of an infinite slab. Let us say I have a slab of width b, its two faces - one on the left is at a temperature $t_{1,}$ the one on the right is at a temperature $t_{2,}$ is maintained at that temperature $t_{2.}$ The thermal conductivity of this slab, let us say is k, small k. We would like to find out - by conduction - what is the rate at which heat flows across this solid and what is the temperature distribution in the solid.

By conduction, obviously heat has to flow in the direction of decreasing temperature. So, if I assume t_1 to be greater than t_2, the heat is going to flow from left to right something like this across the slab. And I would like to derive an expression for the rate at which heat flows across the slab. I would like to derive an expression for the temperature distribution in this slab; that is what I am trying to do. The way I will proceed is of course to use Fourier's law of heat conduction and the first law; these are our basic tools for solving the problem. So, let us first of all now see that - suppose we want to use these. So, I have already made a statement that the flow of heat is one dimensional - let me write that down – one dimensional and therefore and it is steady state.

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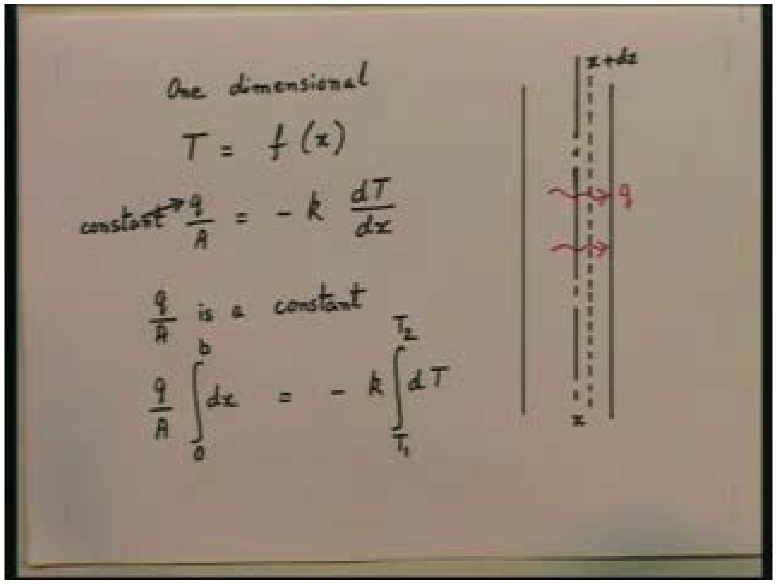

Therefore, the temperature in the slab will only be a function of one direction and we will call that the x direction. Therefore, since temperature is only a function of the x direction, it follows that Fourier's law can be stated as q by A is equal to minus k dt dr, dT dx; notice I am using an ordinary differential not a partial differential. Why? Because T is only a function of x.

Now the first thing I am going to do is I am going to use the first law of thermodynamics first of all, to show that the quantity q - the rate at which heat is flowing across the slab so many watts - is a constant and I do that as follows. Suppose I take a slab. Let me just draw a slab here; suppose I draw a slab. This is a slab and let us say at some distance x from the left face - some distance x - I consider a plane x at a distance x. Now, also consider another plane which I will show in a slightly different manner at a distance x plus dx. Let us say a second plane at a distance x plus dx like this. This is the plane x and this is the plane x plus dx.

Now, consider heat flowing across this slab through the plane x and then through the plane x plus dx. The heat is flowing at the rate q. Now, if the quantity q flowing across x - some area of x - is not the same as the quantity q flowing across x plus dx; if it is not the same, then the first law of thermodynamic says that the temperature of the slab, the energy of the material within these two planes x and x plus dx, will have to increase, either increase or decrease; will have to change, if q is not the same and if that happens the temperature will have to change but we have already said that the temperature is not changing with respect to time. Therefore, it follows that q across any plane x, x plus dx or x plus two dx, whatever, any plane in the slab will have to be the same everywhere. So q is a constant; that’s what the first law - from the first law, we have shown; q is a constant. It follows if q is a constant that q by A is also a constant because A - the area through which heat is flowing across the slab - is the same. It is a one dimensional situation; the heat is continuously flowing across the same area. So if q is a constant, it follows q by A is a constant. So using the first law of thermodynamics, we have shown q by A is a constant. Now, with that information let me rewrite Fourier's law in the form; I will say q by A - which is a constant - into dx is equal to minus k into dT and let us integrate this expression. Let us integrate this expression between the limits; from one end of the slab zero to b - x equal to zero to x equal to b - and the other one at x equal to zero the temperature of the face is t_1, at x equal to b the temperature is t_2. So, integrate it and once we integrate it, we will get the expression; we will get the following expression.

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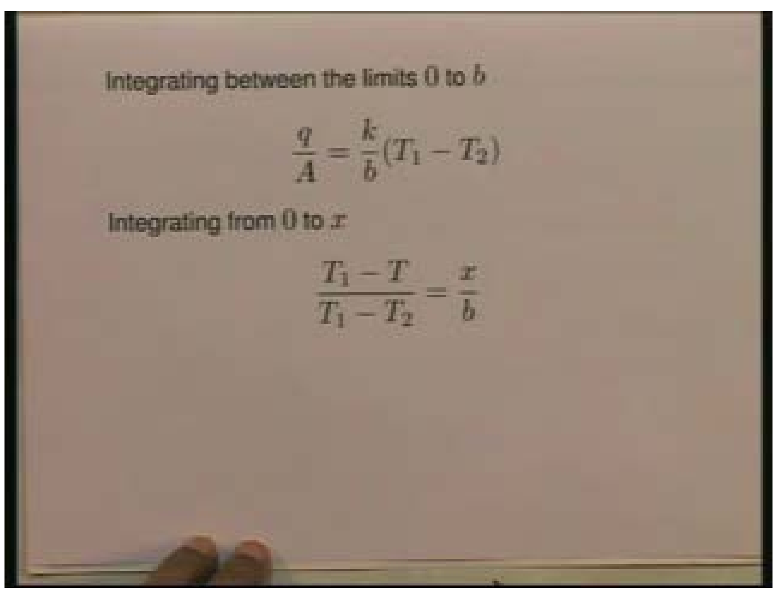

If we integrate it, we get the expression q by A is equal to k by b into T_1 minus T_2 - this is the expression for the rate at which heat flows across the slab. If I want the temperature distribution, we integrate the same earlier expression but from zero to x for x and from T_1 to T for the variable T and we will get the second expression which I have shown here that for the temperature distribution namely T_1 minus T upon T_1 minus T_2 is equal to x by b which is just a straight line; simply saying the temperature will vary in a straight line across the slab. This is derived under the assumption that k is a constant at all points within the slab. Keep that in mind.

If k is not a constant, then I will have to put that variation of k with temperature into account and leave that, put that variation into the integration. So, I will get a different answer in that case. So, this is with the assumption let k is the constant. So, these are our two results for the temperature distribution in the slab which is a linear distribution and heat flow rate across the slab.

Now, we will solve the same problem for the cylindrical situation. Let us say, I have an infinitely long hollow cylinder and it is of inner radius r_i,

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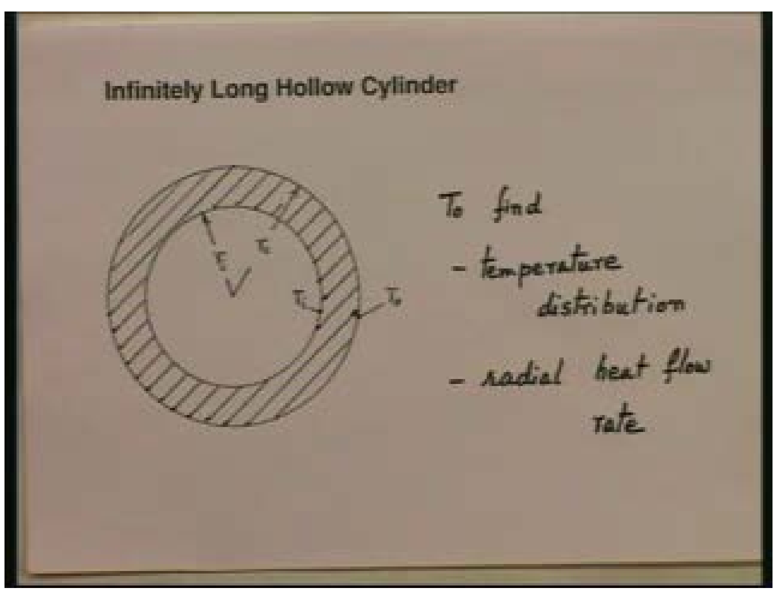

Its outer radius is r_o; the inner circular face is maintained at a temperature T_i, the outer is maintained at a temperature T_o. Its thermal conductivity is again a constant namely k. I would like to find again like in the earlier case; to find first of all the temperature distribution in the solid and secondly to find the radial heat flow rate just like in the earlier case; the only change now is we have switched from a slab to a long hollow cylinder. So, we would like to find expressions for these and we will proceed in more or less the same manner that we proceeded earlier for the slab, excepting that now we will be working with a one dimensional situation in cylindrical coordinates. So what will we have? We will have again if you argue out; you can say it is a one dimensional situation.

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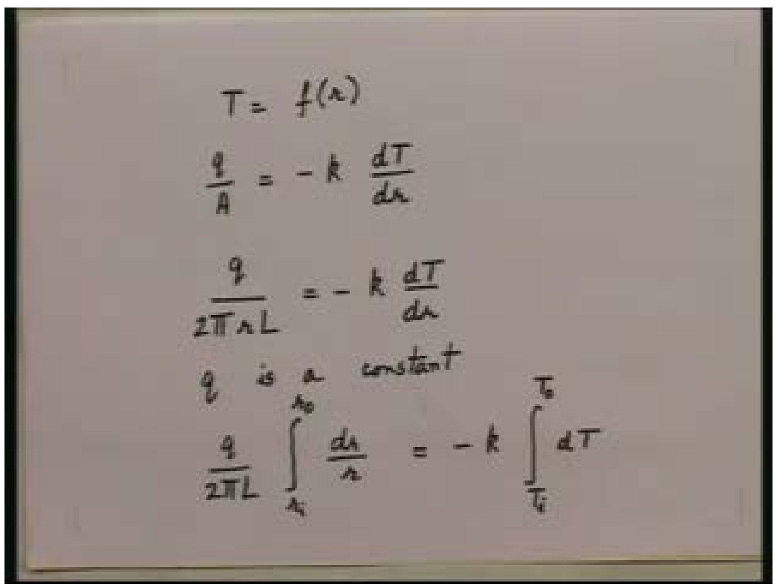

So, the temperature will only be a function of r - the radial direction. It will not be a function of theta or z or of time; time because it is not a - it is a steady state situation. Therefore Fourier's law we dealed - q by A is equal to minus k dT dr where the dT dr is an ordinary differential and now in a ray, in a cylindrical situation, the area A across which heat is flowing is continuously increasing as we move outwards in the radial direction unlike the infinite slab case. Therefore for A, I will have to put q divided by two pi rL where r is any radius within the cylinder; L is some length of the cylinder. q divided by two pi rL is equal to minus k dT dr and q is a constant; just like we had shown in the previous case, we can argue that q is constant by applying - by using the first law of thermodynamics. q has to be a constant. So again, integrate this expression. We write this as q upon 2 pi L; integrate it from r_i to r_o, dr by r and that is equal to minus k the integral T_i to T_o - the inner face to the outer face - dT; that is how we will integrate. So, rewrite the Fourier's law in this fashion; integrate from the inner face to the outer face and after doing that, we will get the result for the temperature distribution and the heat flow rate.

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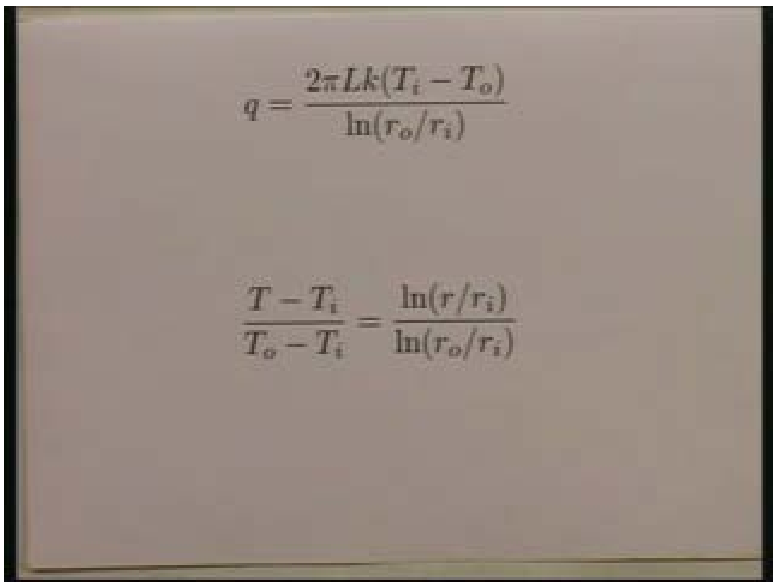

We will get the following - we will get for the heat flow rate q is equal to 2 pi kL into T_i minus T_o upon the logarithm r_o by r_i - logarithm to the base e - and the temperature distribution would be T minus T_i upon T_o minus T_i is equal to the logarithm r by r_i upon the logarithm r_o by r_i; this would be temperature distribution inside the cylinder. So, these are the corresponding results for a long hollow cylinder. Now, let us do a problem. We are going to do the following problem - numerical problem. I am going to say let us do the following

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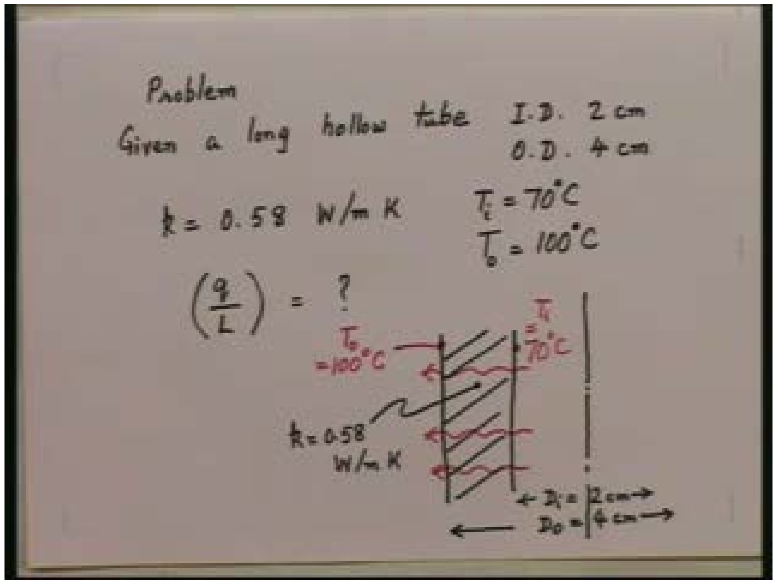

just to illustrate the use of the equation which we have just derived for heat flow rate. Let us say we have given a long hollow tube. Let us say ID inner diameter 2 centimeter outer diameter 4 centimeter. Let us say its thermal conductivity k is equal to .58 watts per meter Kelvin. Looking at this value .58, I hope you immediately recognize it is not a metal tube; it is made of some non metallic material say a plastic or something like that. And let us say the inner face - the inner circular face - is maintained at a temperature 70 centigrade and the outer is maintained at a temperature of 100 degree centigrade. This is the given data. Find the value of q by L - the heat flow rate per meter length of the tube. It is a direct substitution into the equation that we have just derived. Now, one habit - one good habit - which I think you should get into is: always draw a sketch of the physical situation that you are dealing with. Very often, one can visualize it; it is a simple enough sketch, simple enough situation that you can visualize it but still it is a good habit to always draw a sketch.

So, in this case what we will get now? If I draw a sketch, it is going to look something like this. Let us say this is the outer surface, this is the inner surface and let us say this is the center line. So, the inner diameter is D_i so I will have D_i is equal to 2 centimeter. D_o is the outer diameter is equal to 4 centimeter and this is my solid tube whose k is given to be k is equal to .58 watts per meter Kelvin. So, this is the solid tube. Now, the inner face is at a temperature of 70 degree centigrade. This is T_i and the outer face is at a temperature of T_o - is at a temperature of 100 degree centigrade. We would like to find the value of q upon L - heat is going to flow - we would like to find the value q by L; the rate at which heat flows in the radial direction. You can see I am drawing the arrow always in the positive r direction. We would like to find q by L for this situation. It is a straight forward substitution; there is no complication in this problem. I just want you to get into the habit of drawing a sketch, putting down the data and solving the problem. So, substitute into the equation we have; what was our expression?

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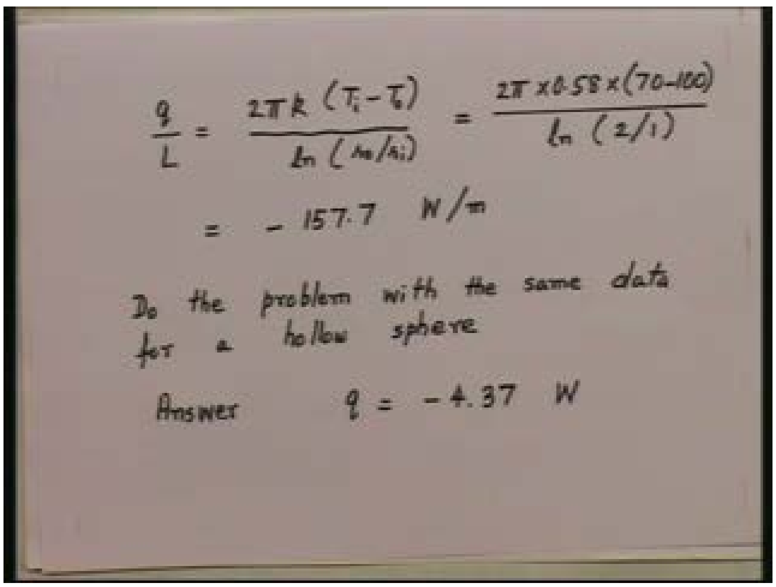

The expression we had was q by L is equal to 2 pi k into T_i minus T_o; that is our expression divided by the log to the base e r_o by r_i. So in this case substituting the data we are going to get 2 pi .58 into 70 minus 100 divided by, this will be divided by the log to the base e 2 divided by 1 - 2 centimeters is the outer radius 1 centimeter is the inner radius. I don't have to put it in meters because I am diving 1 by the other and it becomes dimensionless automatically. And this if you calculate it; if you put in the numbers and calculate, you will get minus 157.7 watts per meter for a meter length of the tube. So, that is the answer to the problem. It is a direct substitution into the formula which we just derived and what is the meaning of this negative sign? Please note it - the negative sign is because in this case the heat is flowing in the negative r direction; the positive r direction is outwards. Since the outer face is at a higher temperature compared to the inner face, the heat flow is actually inwards; therefore, we are getting a negative sign. That is the meaning of the negative sign.

So, now we have expressions for 2 one dimensional situations - a slab and a cylinder. What I would like you to do on your own is the following. I would like you to do the problem that I just solved; do the problem with the same data for a hollow sphere. Repeat this problem; that means assume we have got a hollow sphere, inner diameter 2 centimeters, outer diameter 4 centimeters; inner face maintained at the temperature of 100, inner face maintained at temperature 70 degree centigrade; outer face at a temperature 100 degree centigrade, k equal to .58. Find q; what is q? And the answer you should get - I want you to do this on your own - if you do this problem on your own, the answer you should get is q is equal to minus 4.37 watts. So please do this problem which is an extension of what I just did on your own for a spherical situation. You should get this answer; mind you again the negative sign is because heat is flowing radially inwards. That is the reason for the negative sign.

Now, we want to introduce the idea of what is called as a thermal resistance. You have got solutions for a few one dimensional steady state situations; now we want to introduce the concept what is called as a thermal resistance. A thermal resistance is defined in a manner very similar to an electrical resistance defined by ohm's law. Suppose I have, let us look at the right side of this figure.

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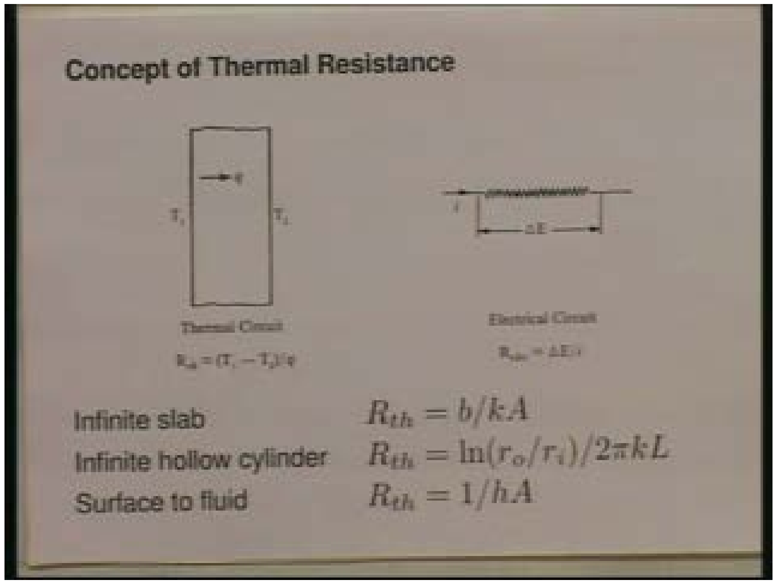

suppose I have an electrical resistance through which a current I is flowing and across which I have a potential difference delta e. Ohm's law states that the electrical resistance R electrical is equal to delta e divided by I right? That is ohm's law. Now in a analogous manner, I define a quantity called a thermal resistance and define it as follows. I say suppose I have a solid and across the faces of the solid I have a temperature difference T_1 minus T_2; this is similar to the potential difference delta E and there is a heat flow rate q flowing across this solid; the q is analogous to the current I. Then, the thermal resistance of the solid is given by the temperature difference T_1 minus T_2 divided by q; that is how we define it. We define thermal resistance as equal to a temperature difference divided by the rate of heat flow; this is the definition of a thermal resistance. Now, immediately I can go back and see the case of the infinite slab. For the infinite slab, what did we have for the, look at the expression for heat flow rate.

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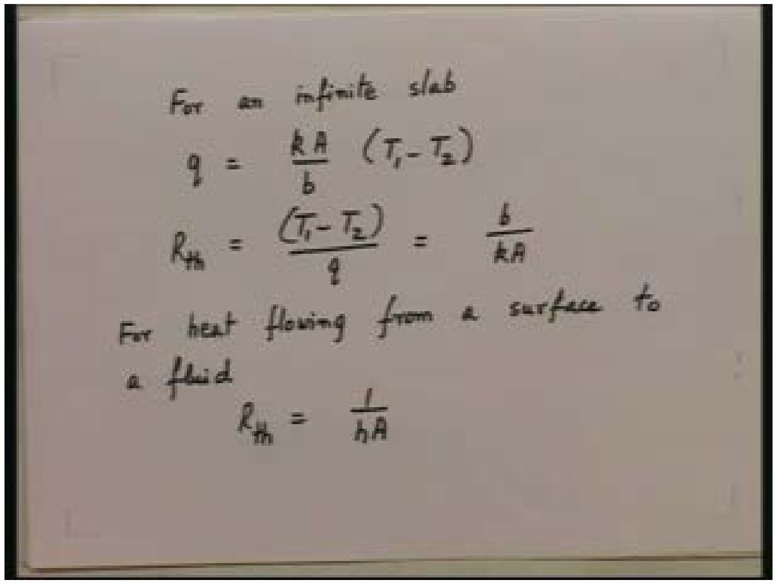

For an infinite slab, what will be the thermal resistance; with this definition that I just gave you for an infinite slab. We just derived the expression q is equal to kA by b; if you go back in your notes you will see kA by b into T_1 minus T_2; we just derived that expression. Compare it now; compare this expression with our definition. How do you define thermal resistance? We say thermal resistance is equal to T_1 minus T_2 divided by q. Therefore, for an infinite slab, thermal resistance would be given by b by kA. That is how we will get the expression for the thermal resistance of an infinite slab. For heat flowing from a surface to a fluid - that is Newton's law of cooling - the thermal resistance, if I again use this definition of T_1 minus T_2 by q; the thermal resistance would be equal to 1 upon hA by comparing with Newton's law of cooling this definition.

For an infinite hollow cylinder - if I go back to the previous transparency now - for an infinite hollow cylinder, R_{th} would be given by the logarithm to the base e r_o by r_i divided by 2 pi kL. So, once I have an expression for heat flow rate and I have a definition for thermal resistance, I can write down what would be the thermal resistance for a given situation. So we have put it down for the 3 cases which we have be so for studied - an infinite slab, an infinite hollow cylinder and when heat transfer takes place from a surface to a fluid. These are our expressions for thermal resistance. Now, why do we introduce the concept of a thermal resistance you will say? Well, what do I get out of it? The main reason is the following - the main reason is you know that when there are many electrical resistances in series $r_{1,}$ $r_{2,}$ $r_{3,}$ r_4, we know very well the total electrical resistance of all those in series is the sum of r_1 plus r_2 plus $r_{3.}$ So, if I have many electrical resistances in series, the total electrical resistance is the sum of the individual electrical resistances.

Similarly, when we have a number of thermal resistances in series, the total thermal resistance is also the sum of the individual thermal resistances; that is why the concept of a thermal resistance is a useful one and we use it. So, let me repeat that sentence because that is a useful sentence.

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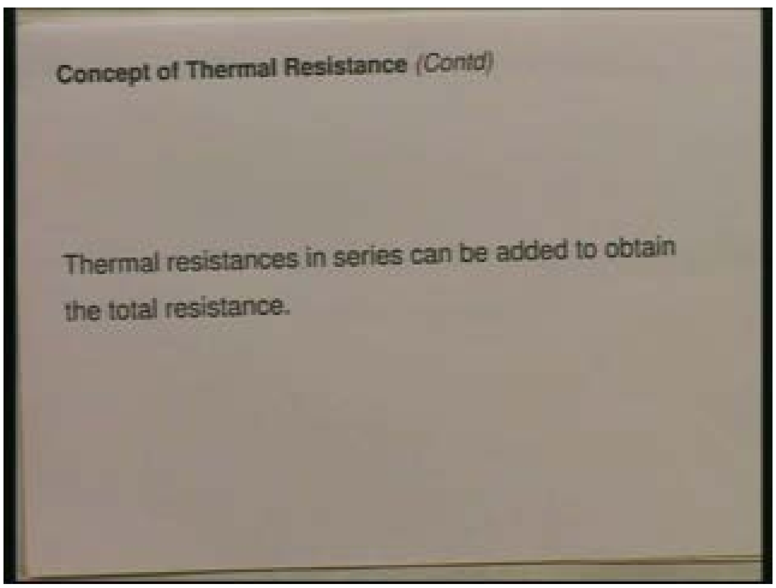

Thermal resistances in series can be added to obtain the total resistance; when we have many resistances in series, the thermal resistance - total thermal resistance - is the sum of the individual thermal resistances. Now with this backdrop, let us look at a composite situation. A composite situation is one in which we have a number of thermal resistances in series.

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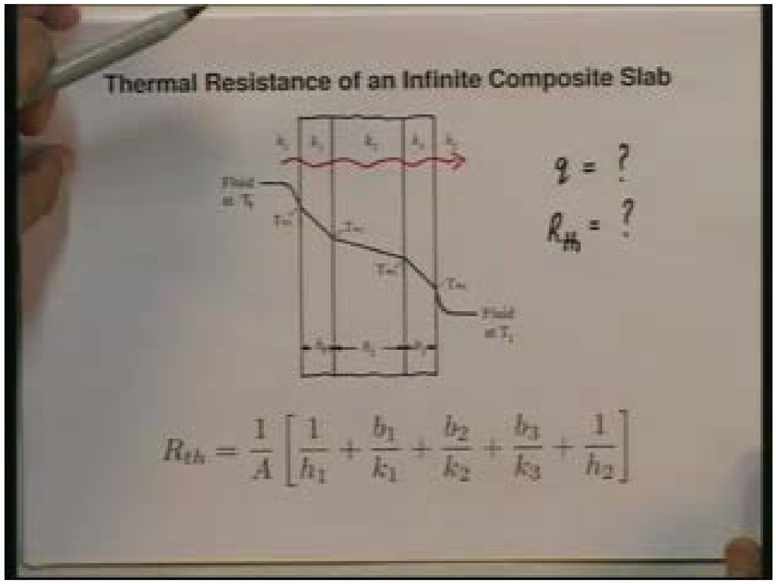

So, we are now going to look at instead of a slab, just one slab that we looked at earlier, we are now going to look at an infinite slab which is composite in nature. Look at this figure; we have now a composite slab consisting of three materials with thermal conductivities k_1, k_2 and k_3 and thicknesses b_1, b_2 and b_3; three slabs attached to each other side by side. The first one is thickness b_1, b_2, second one - b_2, third one - b_3. Their conductivities are k_1, k_2 and k_3 and let us say on the left side here, the slab is in contact with a fluid at a temperature T_1 and on the right side, the slab is in contact with a fluid at a temperature T_2 and arbitrarily I have said let T_1 be greater than T_2. If that is the case, heat will flow from the left side to the right side; if the reverse is the case it will flow in the reverse direction; it doesn’t matter, I have just taken one temperature to be higher than the other.

Now what we want to do; we want to find an expression for the heat flowing across this slab. Heat will flow like this across this slab; I would like to find an expression for the rate at which heat flows across this slab and I would like to find an expression for the thermal resistance of this composite slab; that is what I would like to do. For the moment, let us not at look at this expression; we will come to it in a moment. This is the composite slab; we want an expression for the heat flowing across it and the thermal resistance of this composite slab.

Now the first thing you recognize is that for analyzing this slab, we need to recognize first of all that there are five thermal resistances in the path of the heat flow. What are the five thermal resistances in the path of the heat flow? Starting from the left, we have the heat transfer coefficient on the left side, then the thermal resistance associated with the first with conductivity k_1, then second, then third and then finally the thermal resistance associated with the heat transfer coefficient h_2 on this face. So there are five thermal resistances in the path of the heat flow and for each of them if I first say the intermediate temperatures are T_{w1}, T_{w2}, T_{w3} and T_{w4}. So, for across the first thermal resistance, there is a temperature difference T_1 minus T_{w1} which is governed by Newton's law of cooling; across the next T_{w1} minus T_{w2} I can use the result for an infinite slab which I have just derived a few minutes ago; same for the second and the third slab, I can use the linear distribution and the heat flow rate result which I got a moment ago and finally again for the last thermal resistance where there is a heat transfer coefficient h_2, I can use Newton's law of cooling to put down the expression for heat flow rate for the fifth thermal resistance. So let us do that; what will we get? We will get starting with the left side again

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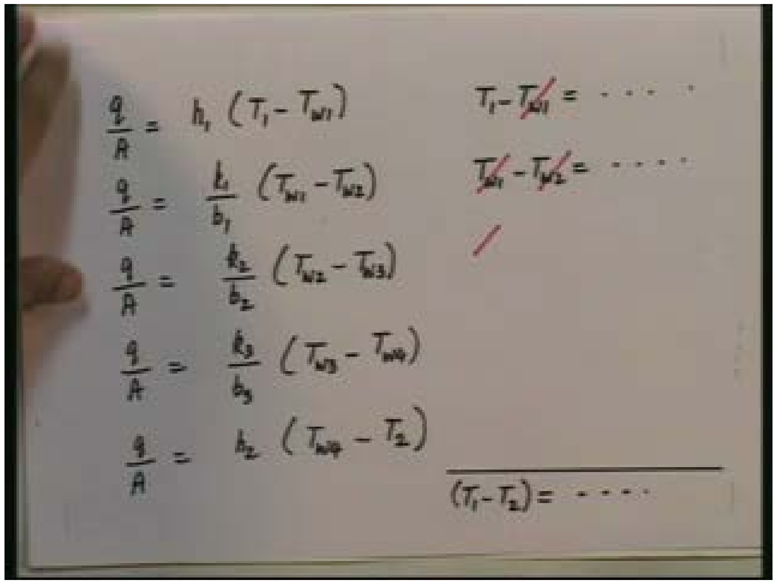

q by A and now going from left to right - is equal to h_1 T_1 minus T_{w1}; T_{w1} is the temperature of the surface on the left side. Then, first slab - q by A is equal to k_1 by b_1 T_{w1} minus T_{w2}; second slab making up the composite situation k_2 by b_2 T_{w2} minus T_{w3}; third one - q by A is equal to k_3 by b_3 T_{w3} minus T_{w4} and finally the last one - this is the heat transfer coefficient q by A is equal to h_2 into T_{w4} minus T_2. These are the five expressions and obviously q by A is the same for all of them. It is not just symbolic; all these q by A have to be equal because it is a steady state situation - heat flowing from left to right.

Now, I can rewrite each of these expressions in terms of a temperature difference alone; so I can write say the first expression as T_1 minus T_{w1} is equal to something; I can write the second one T_{w1} minus T_{w2} equal to something; all the five can be rewritten like this and then I can add them up. If I add up notice what is going to happen. The intermediate temperature T_{w1} will cancel, the intermediate temperature T_{w2} will cancel; all of them will go - T_{w1}, T_{w2}, T_{w3}, T_{w4} - all will go. All that would be left on the left hand side when I had up will be T_1 minus T_2. If I now add up all the five expressions, all the intermediate temperatures will go and I am simply going to get on the left hand side T_1 minus T_2 is equal to something; let us put that down. What we will get is the following.

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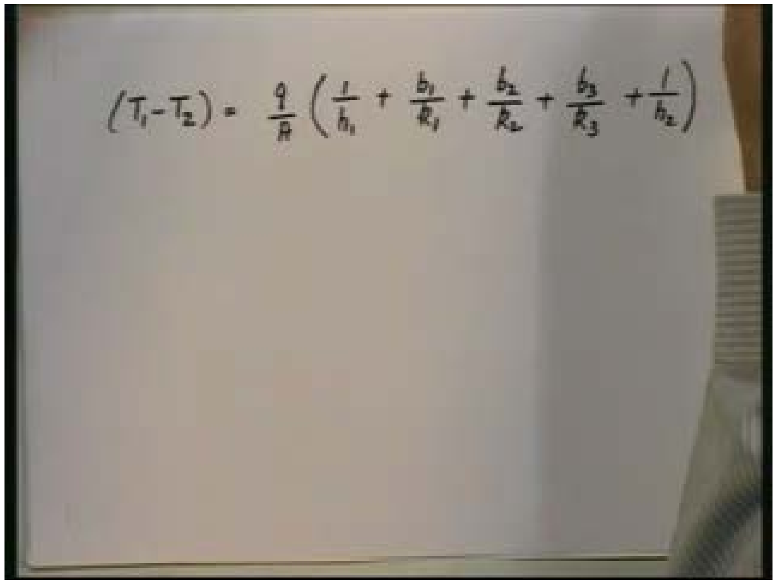

T_1 minus T_2 is equal to q by A multiplied by one by h_1 plus b_1 by k_1 plus b_2 by k_2 plus b_3 by k_3 plus 1 by h_2; that is the expression I am going to get and this is my expression now for the heat flow rate across this composite slab consisting of three infinite slabs joined together and two heat transfer coefficients on the two faces. This would be my expression and now compare this with by definition of thermal resistance for a composite situation. What will it be? The thermal resistance for the composite situation is given by T_1 minus T_2 divided by q; so if I take $T_{1,}$ in this expression that I have derived here, if I take T_1 minus T_2 upon q to the left hand side, what am I left with on the right hand side? Whatever I am left with in the right hand side will be my expression for the thermal resistance; so for the composite case, I get the thermal resistance to be the following, which I had shown you earlier. For the composite case, thermal resistance is

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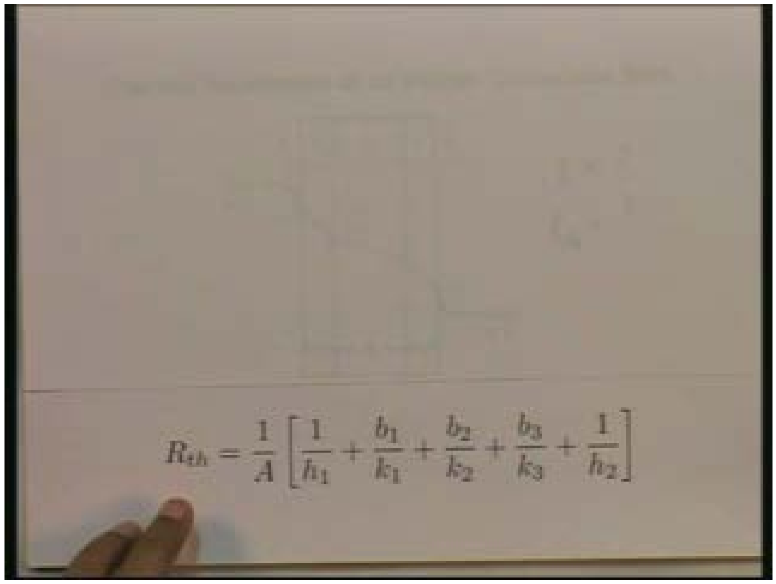

R_{th} is equal to 1 upon A into 1 by h_1 plus b_1 by k_1 plus b_2 by k_2 plus b_3 by k_3 plus one by h_2 and this is nothing but the sum of the five individual thermal resistances which I have got. The first one is 1 by h_{1A}; second one is $b_1$by k_{1A} , etcetera. So, we have not only got an expression for the total thermal resistance of this composite case, we also have shown that when thermal resistances come in series, the total thermal resistance is the sum of the individual components. We have done that as well. Now, at the same time while we are on the composite case, I want to define another term for you and that term is a term called the overall heat transfer coefficient.

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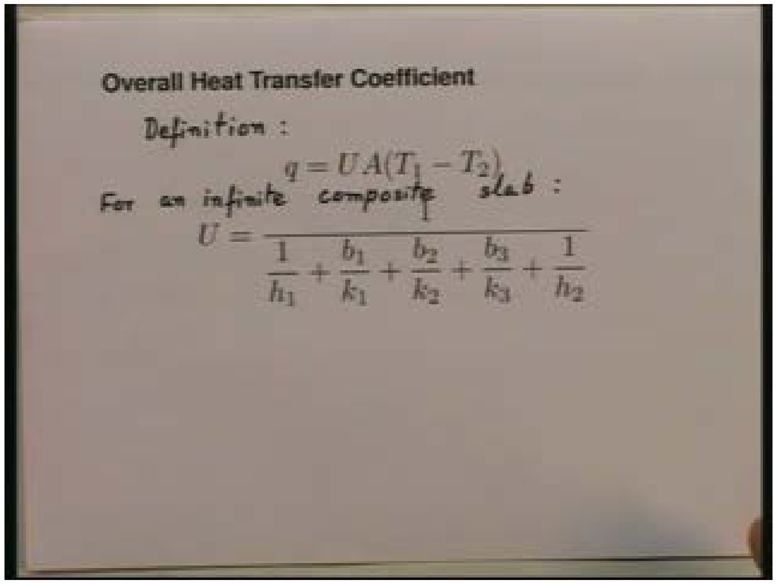

It is an important term which we use quite a lot in heat transfer; so I want to define a second term called the overall heat transfer coefficient. I want to define that term for you. The overall heat transfer coefficient whenever is defined for a situation in which we have more than one thermal resistance for some composite situation. We say the overall heat transfer coefficient for a composite situation may consist of two thermal resistances; it may consist of three or like we did just now, five thermal resistances.

It is defined as follows. We define it as the overall heat transfer coefficient for a composite situation; is defined by the following expression. Let me write down here definition. It is defined by the expression q is equal to UA into T_1 minus $T_{2\,-}$ that is the definition of the term which we have called as overall heat transfer coefficient. U is the overall heat transfer coefficient, q is the rate of heat flow in watts, A is the area across which heat is flowing and T_1 minus T_2 is the temperature difference at the two extremities of the system across which we wish to define this term - overall heat transfer coefficient; that is how we define this term.

Now mind you, if A is a constant as in the case of the infinite slab, it is well defined, there is no ambiguity. But if on the other hand A is varying, then we need to tell on what area A, the quantity U is defined; it is based on some area. So, if A is varying I will have to tell the area A on which U is defined; if on the other hand there is no ambiguity like in an infinites composite slab then of course, I don't need to tell anything on what basis U is defined.

Now, compare this definition with the expression which we had a moment ago for q for an infinite slab. What was our expression for q for an infinite slab? Let me just show that again for a moment; the expression for q for an infinite slab was the following. We got T_1 minus T_2 is equal to q upon A into all this; this was our expression from which I can write q equal to something or the other. So, if I compare this expression for q for a composite, infinite composite slab with my definition of an overall heat transfer coefficient, then for an infinite composite slab I can say for an infinite composite slab, the expression for overall heat transfer coefficient will be U is equal to 1 divided by 1 upon h_1 plus b_1 by k_1 plus b_2 by k_2 plus b_3 by k_3 plus one upon h_2. So what we have done is we have defined a term called overall heat transfer coefficient and for the infinite composite slab case, I have given you what would be the expression for the overall heat transfer coefficient for that case since we have just derived an expression for heat flow rate across an infinite composite slab. Now let us move on to the thermal resistance of a long hollow composite cylinder.

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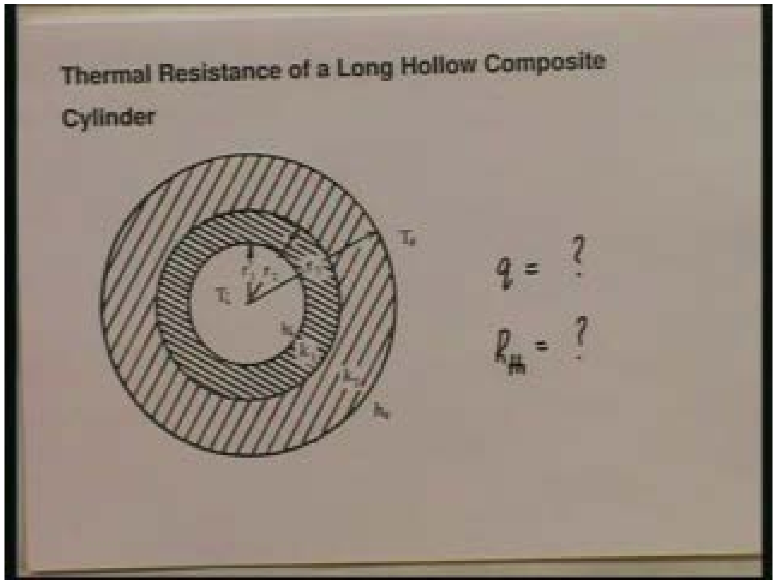

So, instead of a composite slab, now I have a composite cylinder. Let us look at this figure; here I have a composite cylinder - a long composite cylinder - made up of two materials. Thermal conductivity is k_1 and k_2; the inner cylinder has radius r_1 and r_2; the outer cylinder has r_2 and r_3. The inner surface there is - at the inner surface there is a heat transfer coefficient h_i, at the outer surface there is a heat transfer coefficient h_o; the fluid inside the tube is at a temperature T_i and on the outside the fluid which is in contact with this surface is at a temperature T_o. So for this case we have four thermal resistances - a thermal resistance associated with the heat transfer coefficient h_i; a thermal resistance associated with conduction through the first cylinder; then a thermal resistance associated with the heat conduction through the second cylinder and finally a thermal resistance associated with the heat transfer coefficient h_o on the outer surface at radius r_3.

Now, we can go through a derivation for this case again in the same manner as we did for an infinite composite slab and derive expressions for q. What is the rate at which heat flows across this slab, across this composite cylinder and what is the thermal resistance for this composite cylinder? We can do both that in exactly the same way that I did for the slab; that means I will take each one of them one by one and say let the intermediate temperatures be T_{w1}, T_{w2}; then I will say T_i minus T_{w1} is equal to something; T_{w1} minus

T_{w2} is something; T_{w2} minus T_{w3} is equal to something and then finally T_{w3} minus T_o is equal to something. Add them all up, get rid of the intermediate temperatures and I will get an expression for q and for R_{th}. If we do that, which I want you to do on your own because it is exactly a repetition of we did for the infinite slab, you will get the following results.

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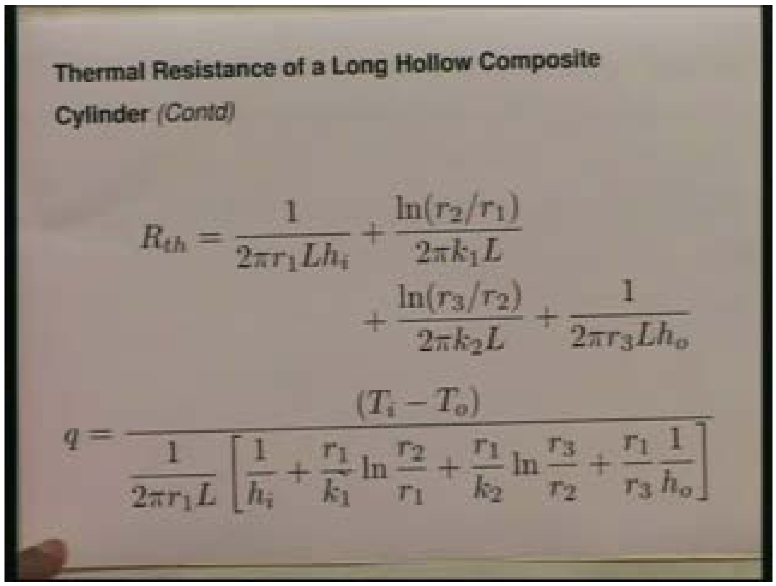

You will get - R_{th} the thermal resistance for the composite case is equal to again the sum of the four thermal resistances which make up that composite situation. Start with the inner one - one upon hA, one upon h_i into the inner area; what is the inner area? 2 pi r_1 L; L is some length that we are taking for the cylinder. Second, thermal resistance of the first inner cylinder log to the base e r_2 by r_1 divided by 2 pi k one L; second cylinder - log to the base e r_3 by r_2; 2 pi k_2 L and finally the thermal resistance on the outermost face 1 upon h_o into 2 pi r_3 L. That is the total thermal resistance of the composite cylinder and obviously if I want expression for the total thermal resistance, it follows that q must be equal to the temperature difference from one end to the other - that is from T_i to T_o - divided by this total thermal resistance. So that is my expression for q straight away. q is given by T_i minus T_o divided by this whole R_{th} out here.

I could also for this case put down an expression for the overall heat transfer coefficient by comparing with my definition of overall heat transfer coefficient. Suppose I want to base my overall heat transfer coefficient on the inner area that is on the radius r_1. What would be the area there? For r_1, the radius would be 2 pi r_1 L.

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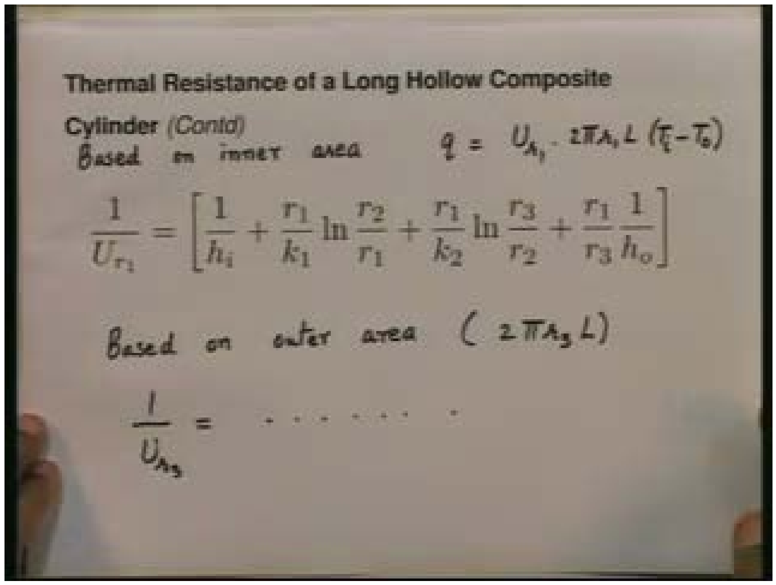

So my U - the overall heat transfer coefficient - then based on inner area, the definition of U would be q is equal to U and since this is U based on the inner area that is on the radius r_1, I will put subscript on it Ur_1 to indicate that it is based on the radius r_1 multiplied by 2 pi r_1 L – that is the area - multiplied by the temperature difference from the inside to the outside and what is the temperature difference? T_i minus T_o; that is my definition of the overall heat transfer coefficient based on the inner radius r_1 where the area would be 2 pi r_1 L. Therefore, if I use this definition and use my expression for q for this case, I straightaway get 1 upon U_{r1} is equal to 1 upon h_i plus r_1 by k_1 log to the base e r_2 by r_1 - this is the inner cylinder, then r_1 by k_2 log to the base e r_3 by r_2 plus r_1 by r_3 1 upon h_o.

My definition of U can be based even on some other radius, say for instance suppose I say based on outer area, what is the outer area? 2 pi r_3 L; in that case, I will get an expression 1 upon U_{r3} is equal to and I want you to write this on your own. Write this expression 1 upon U_{r3} on your own. Remember if I use this value of U_{r3}, I must always multiply by 2 pi r_3 L; if I use this value U_{r1}, I must always multiply by 2 pi r_1 L in order to get q.

So now, we are ready to stop here today. What we have done today; we have done the following. We have first of all started the topic of conduction and I said that while studying conduction in solids, we will be primarily concerned; what we have to do is to satisfy the first law of thermodynamics for a closed system and that too without a work term being involved. We will not have to worry about Newton's second law of motion or the law of conservation of mass because these would be trivially satisfied. Then we considered 2 one dimensional steady state situations. First of all an infinite slab, then an infinitely long hollow cylinder; for both of them we got the temperature distribution; we got the heat flow rate.

Then we introduced the concept of a thermal resistance and I said we introduced this concept because thermal resistances in series are additive. We went on then to a composite situation; we considered an infinite composite slab, got an expression for q and for thermal resistance for an infinite composite slab. We did the same for an infinitely long composite cylinder and for both of them I also defined a term called as overall heat transfer coefficient and said - we define an overall heat transfer coefficient as q is equal to U into A into T_1 minus T_2 where T_1 minus T_2 are the temperature differences at the two extremes and for both these cases - the infinite slab and the infinitely long composite cylinder - we got expressions for the overall heat transfer coefficient U for these two geometries and that is where we have stopped today.