Heat and Mass Transfer
Prof. U. N. Gaitonde
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Lecture No. 35
Introduction to Mass Transfer – 3

Welcome back to this final lecture on mass transfer which will also be the final lecture on this series of lectures on heat and mass transfer. Near the end of the previous lecture, we were looking at a boundary layer for a process of convective mass transfer.

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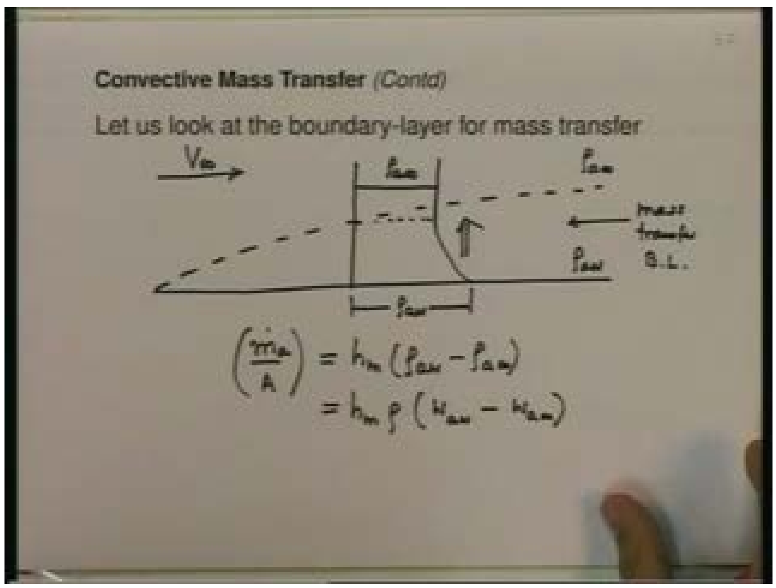

We had just sketched this figure in which - on a plate we considered a mass transfer boundary layer, we had a flow taking place, free-stream velocity V infinity. There was some material coated on the wall and just next to the wall in the fluid, the density of component a was row aw. In the free stream, the density of the same component was different; in this case it is shown lower at row a infinity. Because of this density difference, there will be transfer of mass from near the wall to outside the boundary layer and the mass flux of component a m dot a by A was to be related through the mass transfer coefficient to the density difference of component a near the wall and in the free stream. Our aim is to determine relations for the mass transfer coefficient h_m.

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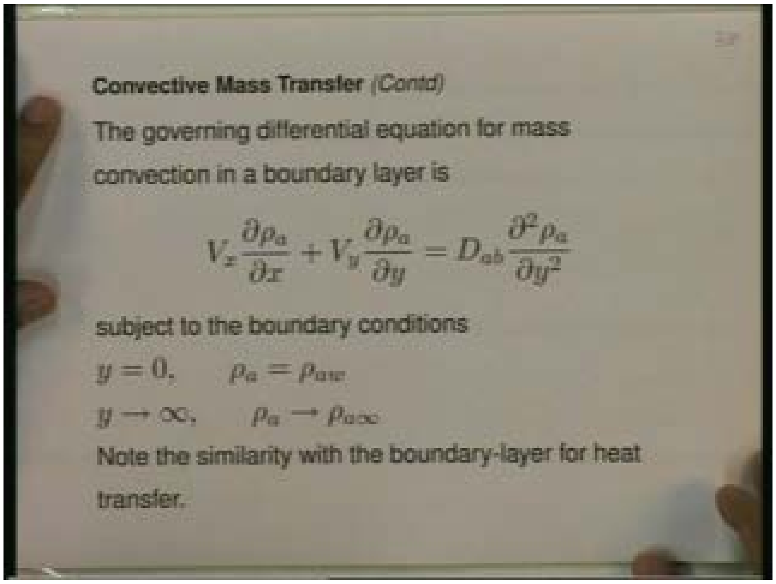

If we derive in detail the governing differential equations for mass convection, this will be a conservation of mass equation but conservation of mass equation for a species a not just the overall gross conservation equation which leads to the continuity equation. This is the equation for conservation of mass of species a. And in the boundary layer form it turns out to have the form V_x into partial of row a with respect to x, V_y into partial of row a with respect to y, the sum of these 2 should equal the diffusion coefficient D_{ab} into second partial of row a with respect to y squared. Because of the boundary layer velocity components V_x and V_y and the variation of row a both in the x as well as the y direction, we have a second order partial differential equation. It is subject to the boundary conditions which we saw in the previous figure - row aw at the wall row a infinity in the free-stream. So we have at the wall where y is 0; row a is row a at the wall and as y becomes large, tends to infinity, row a tends to row a infinity. Why? We should note the similarity between this boundary layer equation for mass transfer to the boundary layer equations for heat transfer and even for fluid flow.

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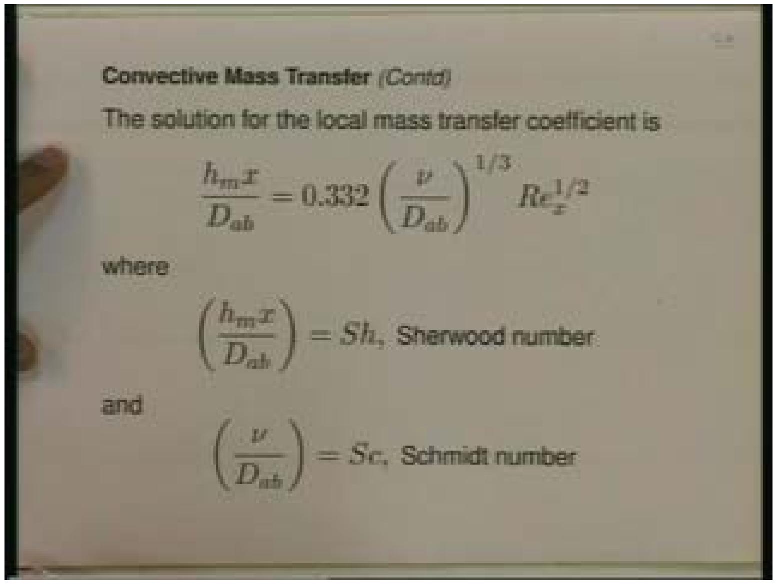

If we solve this equation using some standard methods, we get a solution for the local mass transfer coefficient. In dimensionless form, it can be expressed as mass transfer coefficient into x divided by D_{ab} is .332 into Nu divided by D_{ab} raise to one-third and the local Reynolds number raise to half. We now see 2 new dimensionless numbers – one is Nu by D_{ab}, another is h_m by D_{ab}. This h_m by D_{ab} is known as the Sherwood number, it is a dimensionless number representing the mass transfer coefficient in terms of distance and the diffusion coefficient and the Schmidt number Sc is the ratio of kinematic viscosity to the diffusion coefficient. The Sherwood number bears similarity to the Nusselt number and the Schmidt number bears similarity to the Prandtl number. Let us rewrite these equations in terms of the dimensionless numbers.

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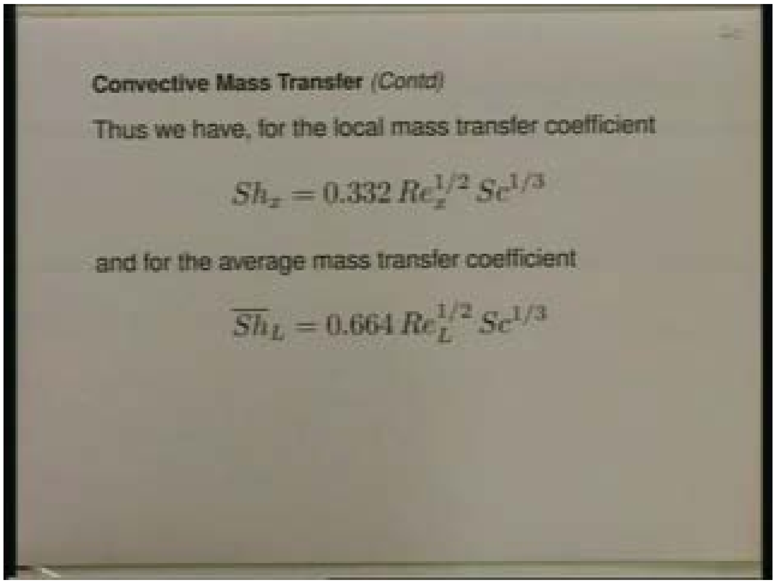

For the local mass transfer coefficient, we have a Sherwood number equal to .332 Re_x raise to half Schmidt number raise to one-third. And if you average out the local mass transfer coefficient and get the average mass transfer coefficient that can be expressed in terms of the mean or average Sherwood number over a length L and that will be .664 into Re_L raise to half into Schmidt number raise to one-third. Notice the similarity of these equations with the equations for Nusselt number in terms of Reynolds number and Prandtl number. This similarity which we have been looking at right from the Fick's law of diffusion leads to an extension of the Colburn analogy which was so far between fluid flow and heat transfer into mass transfer. So, let us now look at the analogy between heat and mass transfer.

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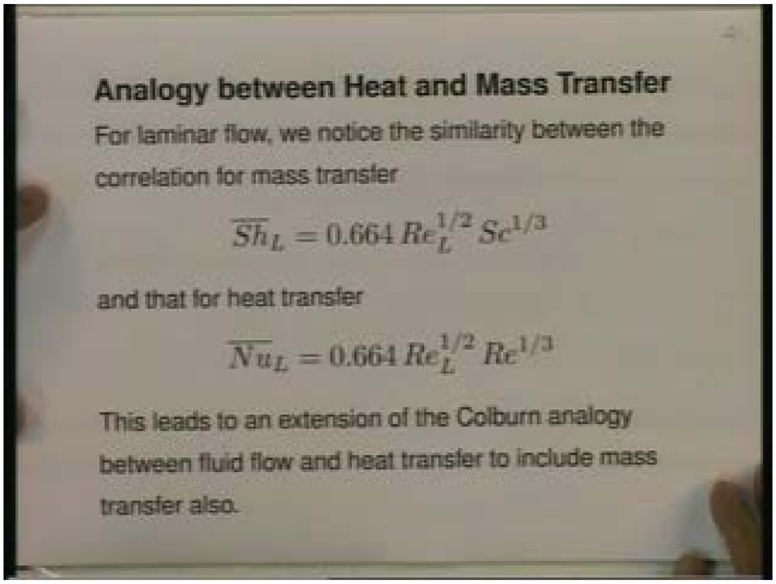

We have just now seen for laminar flow the similarity between the correlation for mass transfer which is reproduced here and that for heat transfer which we had seen earlier during our study of forced convection over a flat plate laminar, forced convection over a flat plate. These 2 relations are so similar that using these we can extend the Colburn analogy between fluid flow and heat transfer to include mass transfer also. So the Colburn analogy extended to mass transfer will be a statement of analogy or a relation of analogy between fluid flow heat transfer as well as mass transfer.

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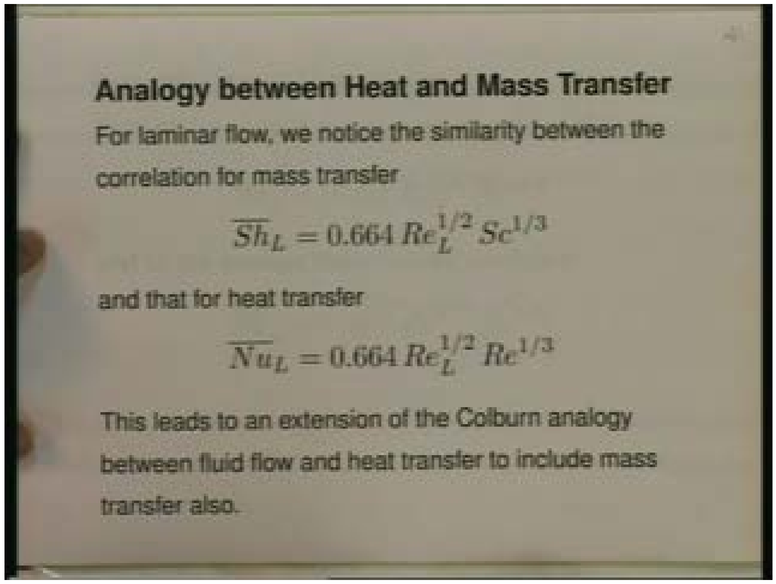

The fluid flow characteristic will be represented by the friction factor the heat transfer by the Nusselt number or the Stanton number and the mass transfer characteristic by the Sherwood number or the Stanton number for mass. Let us look at it properly. The extended Colburn analogy for fluid flow heat transfer and mass transfer can be written down in this relation. The extended Colburn analogy states that the Stanton number for mass transfer multiplied by the Schmidt number raise to 2 by 3 should equal the Stanton number for heat transfer multiplied by Prandtl number raise to 2 by 3 equals the friction factor divided by 2. This is applicable for local mass transfer coefficient, local heat transfer coefficient and local friction factor. It is also applicable to average mass transfer coefficient, average heat transfer coefficient and average friction factors.

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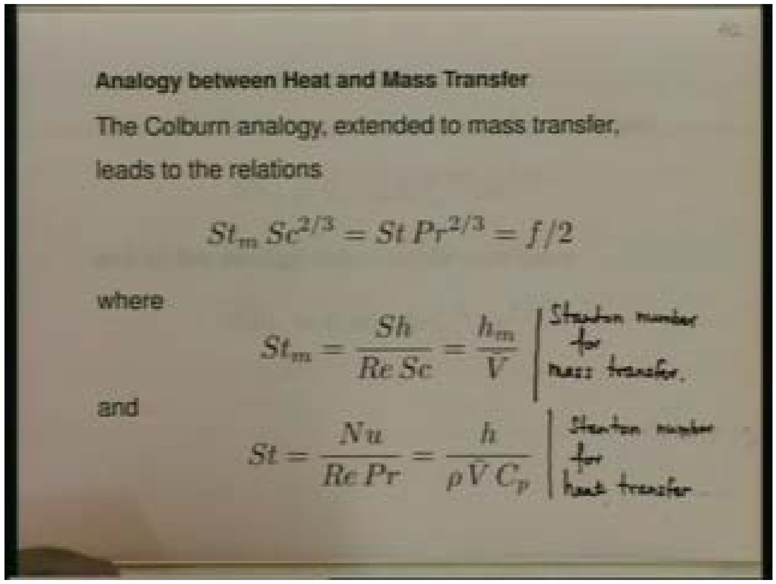

We now have a new dimensionless parameter here and that is the Stanton number for mass; we already know the Stanton number - this was for heat transfer. Now we have a Stanton number for mass transfer and that is St with a subscript m - m for mass transfer; this is Stanton number for mass transfer. The definition of the Stanton number for heat transfer was Nusselt number divided by Reynolds number Prandtl number which made it equal to the heat transfer coefficient divided by row divided by the mean velocity divided by C_p. The Stanton number for mass transfer in an analogous fashion - it is Sherwood number divided by Reynolds divided by Schmidt number and it simply becomes the ratio of the mass transfer coefficient divided by the mean velocity.

We have already seen that the mass transfer coefficient has the dimension of velocity so the mass transfer coefficient divided by the velocity becomes a dimensionless Stanton number for mass transfer. To what extent is the Colburn analogy applicable?

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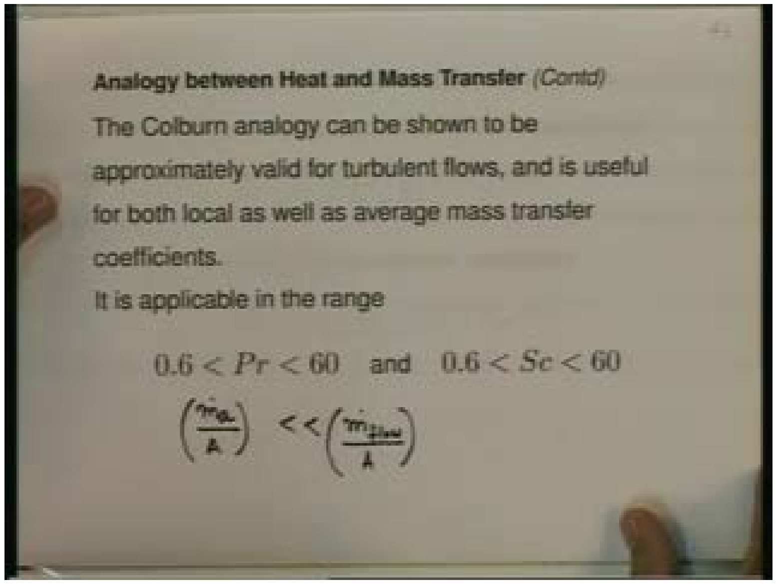

It can be shown using some elementary properties of turbulent flows that the Colburn analogy is approximately valid for turbulent flows and is useful for both local as well as average mass transfer coefficients. It is applicable in the range of Prandtl number between .6 and 60 and Schmidt number between .6 and 60. It should be noted that the approximation is better when the Prandtl number and Schmidt numbers are near 1 and near each other in magnitude. As we move away from one say towards 60 and if there is a significant difference between Prandtl number and Schmidt number, the approximation for the Colburn analogy becomes that much less accurate. The Colburn analog is also applicable when the mass flux say of some species because of mass transfer is much less than the mass flux of the principle flowing medium. We will check this out and illustrate this in a problem later.

Let us look at an application of the analogy between heat and mass transfer and that is in the process of simultaneous heat and mass transfer. The process of drying, the process of evaporation - particularly when there is a temperature difference - requires the study of heat transfer as well as mass transfer simultaneously. One of the situations where simultaneous heat and mass transfer takes place - a common situation is the wet bulb thermometer which is a thermometer with a wick attached and the wick is kept is wet. Let us sketch the situation.

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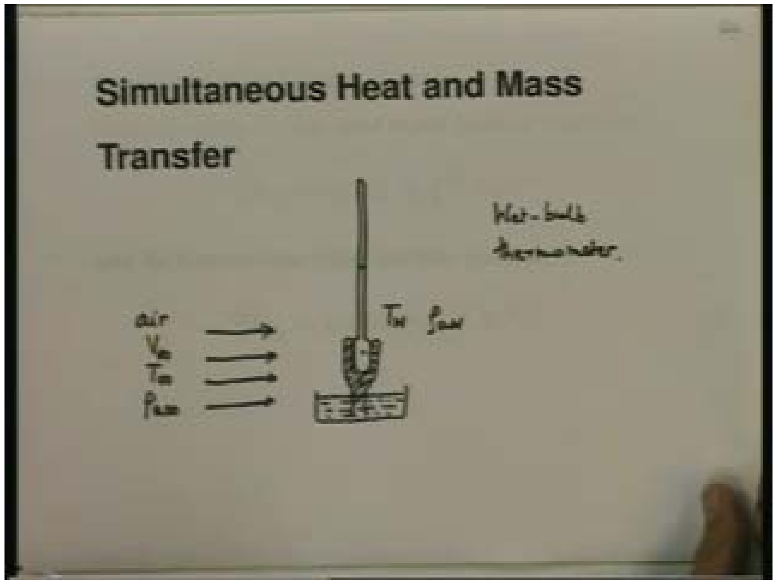

Let us consider a wet bulb thermometer; this is the stem of the thermometer and this is the bulb of the thermometer. The bulb is covered by a wick, a piece of cloth which is dipped in water, so capillary action keeps the wick wet and if there is a flow of air on it, the water vapor from the wick evaporates, takes away latent heat. The wick reduces its temperature and we have the wet bulb temperature measured by the thermometer.

Let us say that air flows over it, let us say V bar or V infinity is the velocity. Let us say T infinity is the pressure and let it contain water vapor, it may not bone dry air. Let us say a is the water vapor as a component and row a infinity is the mass density of water vapor in the air as it approaches the wet bulb thermometer. Let us say that the temperature of the wick and hence what is measured by the thermometer is T_w and let us say that the density of vapor in air just next to the wick as it evaporates is row aw. Let us try to derive a relationship between T_w, row aw, T infinity, row a infinity, V infinity, etcetera.

This is the typical wet bulb thermometer; notice that the bulb is at a temperature lower than that of the surroundings so the bulb will receive heat from the surrounding air by convection. The wick is saturated with water so the density of water vapor next to the wick will be higher than the density of air which is approaching it and hence there will be a mass transfer of water vapor from the wick to the air which is flowing across. Let us write expressions for heat transfer and mass transfer.

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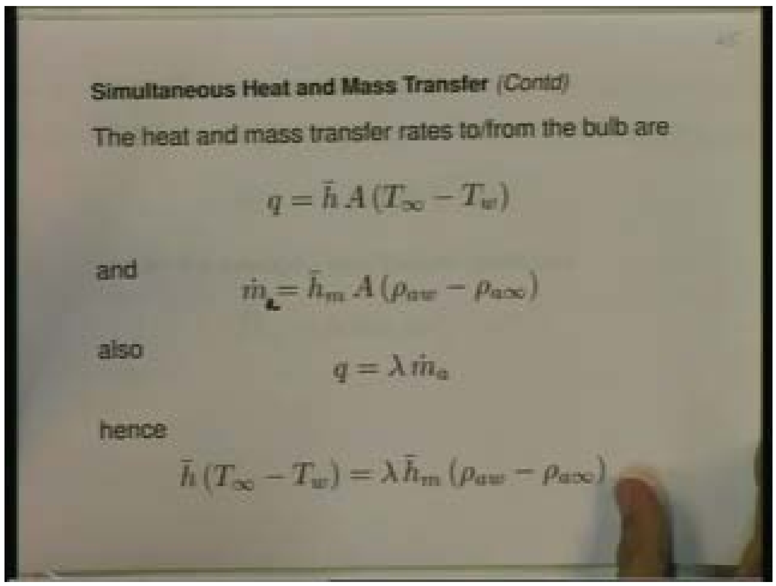

The heat transfer rate from the bulb, from the surroundings to the bulb because the surroundings are at a higher temperature will be q. Let it be represented by the average heat transfer coefficient into the area of the bulb into the temperature difference - in which case the free-stream temperature minus the wet bulb temperature. So this is the heat transfer from air to the bulb. The mass transfer of water vapor from the bulb to the air will be, assuming that the density of air does not significantly change due to this process, will be the mass transfer coefficient, the average mass transfer coefficient, into the area, the same area which is used here multiplied by the density difference for water vapor at the wick w minus row a infinity which is the density of water vapor in the free state. So this is the heat transfer rate from air to the bulb; this is the mass transfer rate from the bulb to the flowing air.

Now this is a simple energy balance. The heat transfer rate must equal the rate of mass transfer, there should be an a here which is missing, multiplied by lambda - the latent heat at the wick temperature. Now if you substitute for q from this equation and m dot a from this equation, we will get a relationship between h bar and h bar m. Now, from the Colburn analogy let me keep this here.

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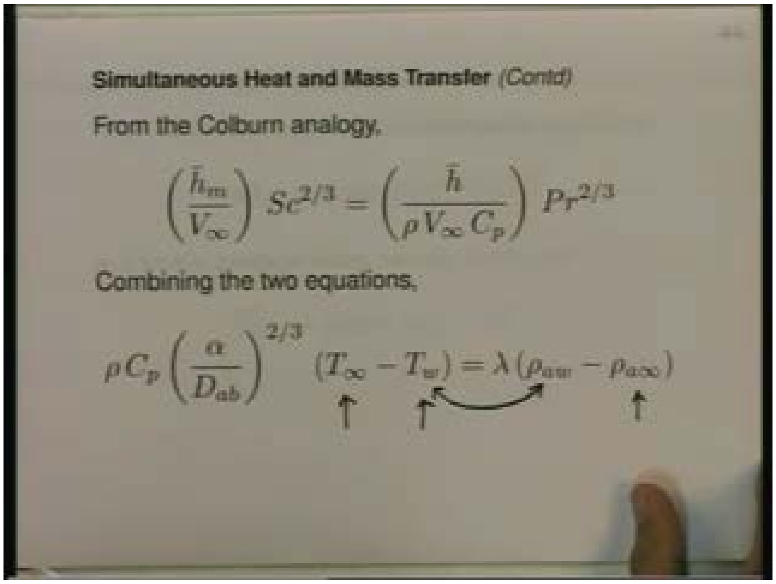

From the Colburn analogy, we have Stanton number for mass transfer into Schmidt number raise to 2 by 3 equals Stanton number for heat transfer into Prandtl number raise to 2 by3. Here, the Stanton numbers have been rewritten in terms of the mass transfer coefficient and the heat transfer coefficient. Now in this equation we have an h bar and an h bar m in this equation also, we have an h bar and then h bar m. So, eliminating this h bar and h bar m from these 2 equations or combining these 2 equations, we will get an equation which relates the remaining parameters as follows. Here we have row C_p, density and specific heat of air, diffusivity of air; diffusion coefficient is the thermal diffusivity of air, diffusion coefficient raise to 2 by 3. And we have the temperature difference between the air and the wick, the density difference for water vapor near the wick and air and the latent heat.

This is the relation which relates these parameters - air temperature, wick temperature and the density of air, density of vapor in the air. The density of vapor at the wick is related to the wick temperature; this is the density of saturated vapor at the wick temperature. So, this equation has 3 unknowns - T infinity, T_w and row a infinity. If we know 2 of these, the third can be calculated and that is what we do when we use the wet bulb thermometer.

Now, it is time for us to solve some problems so that the ideas which were generated are understood properly and also the equations which we have derived are used properly and also we should get a feel for numbers.

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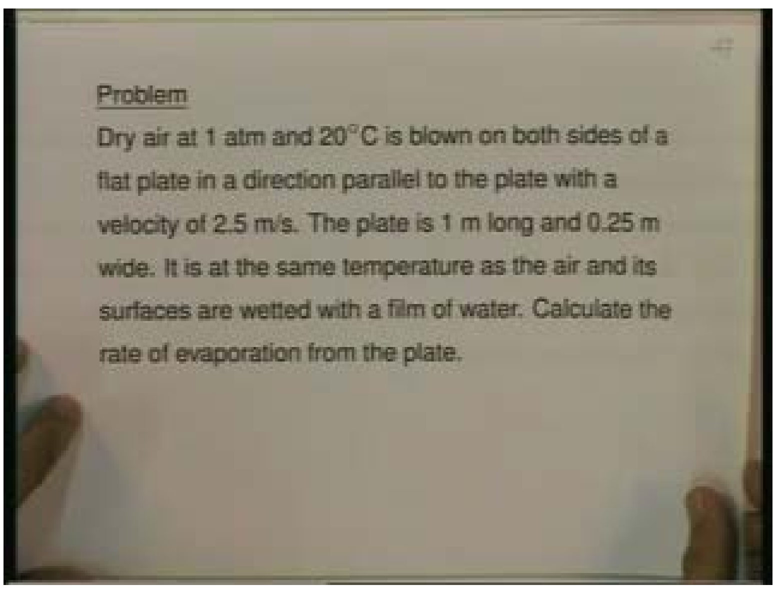

The first problem reads like this - dry air at 1 atmosphere and 20 degrees C is blown on both sides of a flat plate in a direction parallel to the plate. So this is a boundary layer on a flat plate - both sides; so we have either side active. The velocity in the free stream is 2.5 meters per second; the plate is 1 meter long and .25 meter wide. It is at the same temperature as the air so no heat transfer is involved and its surfaces are wetted with a film of water. We have to calculate the rate of evaporation of water vapor from the plate. Let us sketch the situation and then solve the problem.

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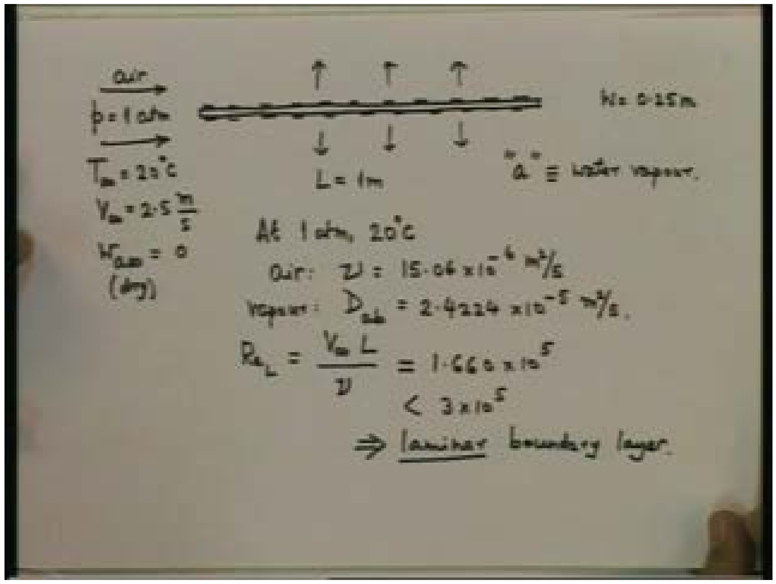

Let us say this is our plate, thin plate - it is wet on both sides, thin layer of water so thin that we don't worry about the thickness and because it is at a different vapor pressure and vapor density water vapor will evaporate from either side into the free stream. The length of the plate is 1 meter, the width of the plate is .25 meter that is a. Air flows across the plate on both sides; the pressure of air is 1 atmosphere, the free-stream temperature is 20 degrees C, the free-stream velocity is 2.5 meters per second and since it is given that it is dry air at 1 atmosphere, the mass fraction of component a - water vapor - in the free stream is 0. This indicates dry air, so here component a is water vapor.

We start by noting or reading off or obtaining properties; so at 1 atmosphere and 20 degrees C for air we will need the kinematic viscosity which is 15.06 into 10 raise to minus 6 meter squares per second and for water vapor we will need the diffusion coefficient, water vapor diffusing in air. We will need to make some interpolation and we will get the value as 2.4224 into 10 raise to minus 5 meter square per second. We first calculate the Reynolds number; we want to determine the total mass transfer rate so we will work with the average mass transfer coefficient and hence let us calculate the Reynolds number based on the length of the plate. This will be V infinity which is 2.5

meters per second into L which is 1 meter divided by Nu which is 15.06 into 10 raise to minus 6 meter square per second.

If you substitute the values, you will notice that this turns out to be 1.660 into 10 raise to 5 which is less than 3 into 10 raise to 5. This implies that we have a laminar boundary layer. The next step - since the boundary layer is laminar, we will use the derived relation for laminar boundary layer in terms of Sherwood number.

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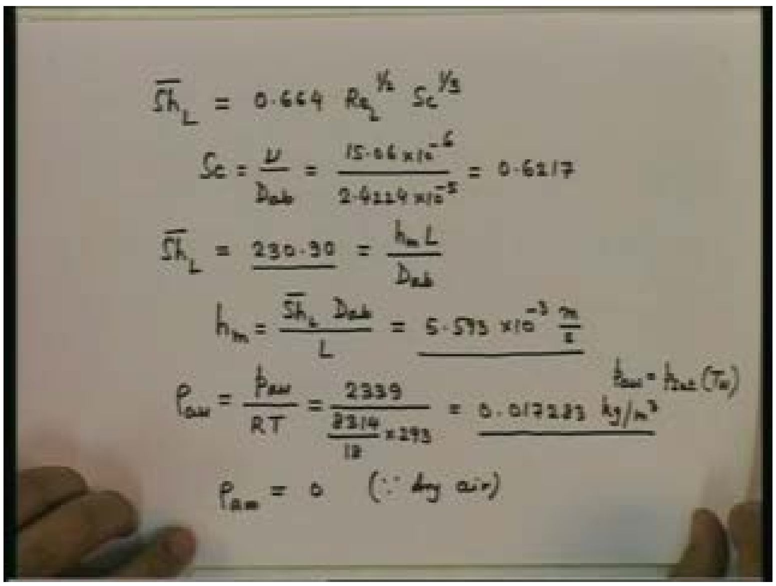

We have the average Sherwood number for a plate over a length L; it is 0.664 into Reynolds number based on the length raised to half Schmidt number raise to one-third. We have calculated the Reynolds number, we have determined the Schmidt number. The Schmidt number is Nu divided by D_{ab} and we have the values 15.06 into 10 raise to minus 6 divided by 2.4224 into 10 raise to minus 5 - this is 0.6217. For gases you will find that the Schmidt number is usually of the order of the Prandtl number. Now substituting the Reynolds number which was 1.660 into 10 raise to 5 and the Schmidt number into this relation, we will get the Sherwood number to be 230.90 and we know that the Sherwood number is nothing but h_m into L divided by D_{ab} from which we get h_m

to be Sherwood number into D_{ab} divided by L. Substituting the values we will get this to be 5.593 into 10 to the minus 3 meters per second.

Now to determine the mass flux, the mass flow rate, we need row a infinity and row a wall. Let us first calculate row a wall. Now row a wall will be partial pressure of component a divided at the wall divided by RT. The partial pressure of water vapor at the wall will be the saturation pressure at the wall temperature because the water vapor is expected to saturate the air near its surface; T_w is 20 degrees C. So this from steam tables turns out to be 2339 Newton per meter squared - this is the gas constant for water vapor. That will be 8314 divided by 18 into 293 and this turns out to be 0.017283 kg per meter cube. What about row a infinity? It is given that it is dry air so row a infinity is 0. Now we have the mass transfer coefficient, we have the density difference

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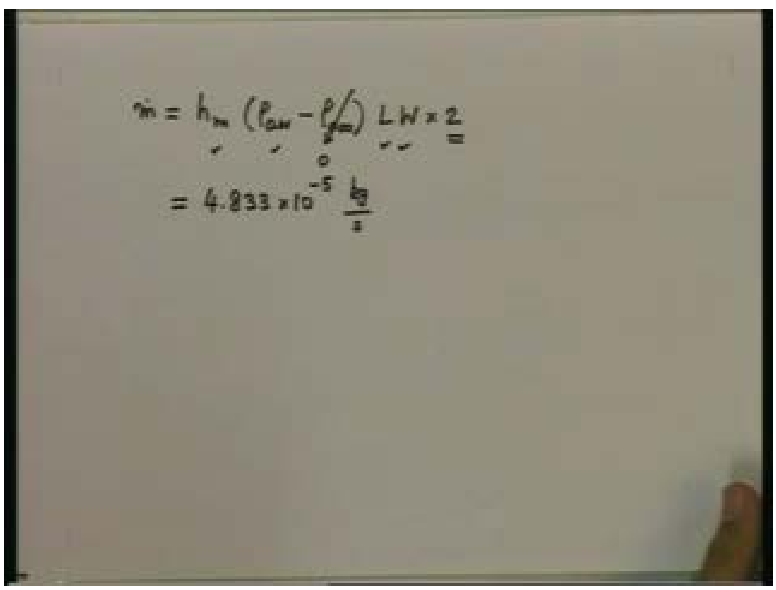

So, the mass flow rate will now be calculated as mass transfer coefficient h_m multiplied by row aw minus row a infinity. That will be the mass flux multiplied by the area which is L into W into 2 because both sides of the plate are wetted and air flows over either side; this is 0 and we have the values for h_m row aw L and W. So if you substitute in this, you will get the mass flow rate or mass transfer rate that is the evaporation of water, rate of evaporation of water from either side of the plate is 4.833 into 10 raise to minus 5 kilogram per second.

Now, let us look at another problem where we are going to make use of the Colburn analogy. This will be a problem of flow of air through a tube the inner surface of which is wetted; covered with a thin layer of water as the air passes over the tube, it will pick up water vapor.

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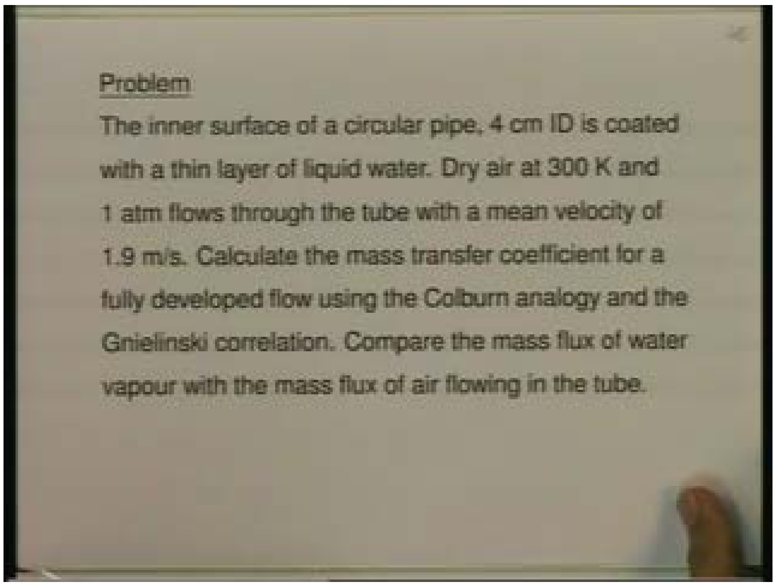

The problem reads as follows - the inner surface of a circular pipe, 4 centimeter ID is coated with a thin layer of liquid water. Dry air, notice dry air so initially containing no moisture at 300 Kelvin and 1 atmosphere flows through the tube with a mean velocity of 1.9 meters per second. Calculate the mass transfer coefficient for a fully developed flow using the Colburn analogy and the Gnielinski correlation which we have used for heat transfer. Compare the mass flux of water vapor with the mass flux of air flowing in the tube. Because we know that the Colburn analogy is applicable when the mass flux of water vapor is significantly smaller than the mass flux of air flowing in the tube, let us first sketch the problem and then solve it.

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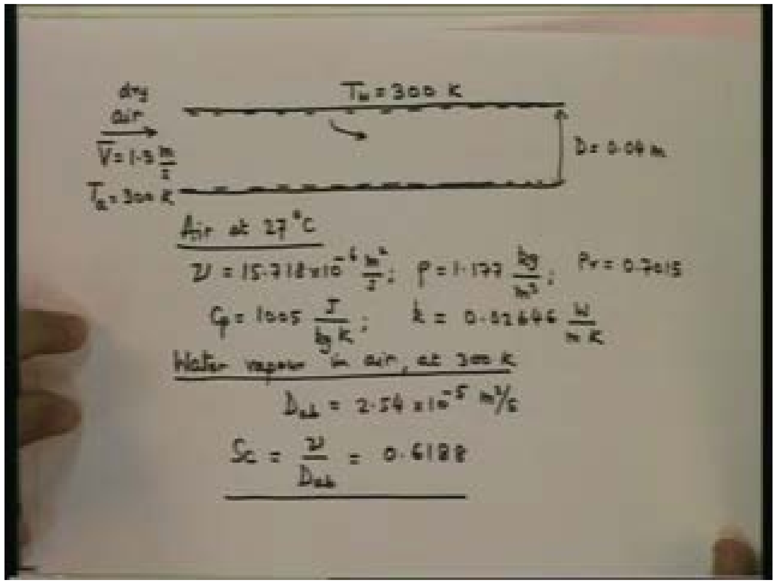

We have the tube, the tube diameter is 4 centimeter; the inner surface of the tube is wetted and let us assume that T wall is also at 300 Kelvin. Air enters the tube it is dry air it enters with a mean velocity of 1.9 meters per second and the air temperature is also 300 Kelvin. We have to determine the mass transfer coefficient and then the mass flux. Let us assume that the mass transfer, it is small, so that the properties of air do not really change because of that small ingestion of water vapor. So we will use properties of air at 300 K which is at 27 degrees C; some interpolation will be required in standard tables. But you get Nu equal to 15.718 into 10 raise to minus 6 meter square per second, row is 1.177 kg per meter cube, Prandtl number is 0.7015, specific heat 10005 joule per kilogram Kelvin, conductivity 0.02646 Watt per meter Kelvin.

For water vapor in air at 300 Kelvin, we have D_{ab} - where a is water vapor and b is air - 2.54 into 10 raise to minus 5 meter square per second. This immediately allows us to calculate the Schmidt number which is Nu divided by D_{ab} that turns out to be 0.6188. The next step - calculate the Reynolds number.

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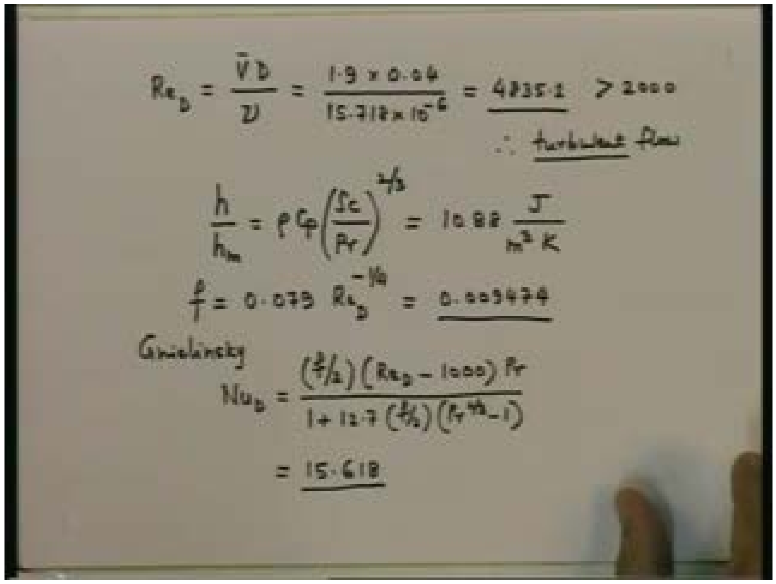

It is a flow in a tube so Reynolds number is based on the diameter that will be V bar D by Nu. V bar is 1.9 meters second, D is .04 meters divided by Nu 15 by 718 into 10 raise to minus 6 meter square per second which is 4835.2. This is greater than 2000 hence we have turbulent flow. Now, using the Colburn analogy, we have h by h_m is row C_p into Schmidt number divided by Prandtl number raise to 2 by3. If you substitute the values here - we know row, C_p, Sc and Pr - we get this to be 1088 joule per meter cube Kelvin. Notice that this is a ratio of a heat transfer coefficient to a mass transfer coefficient so it will have these dimensions associated with it.

Now, at this Reynolds number, let us use the Blasius relation to obtain the friction factor f which is needed to use the Gnielinski correlation. The friction factor f using Blasius correlation .079, Reynolds number based on diameter raise to minus 1 by 4 and this turns out to be 0.009474. Now we use Gnielinski correlation and the friction factor here is determined only to substitute in the Gnielinski correlation gives us the Nusselt number based on the diameter in fully developed flow to be f by 2 into Re_D minus 1000 into Prandtl number divided by 1 plus 12.7 f by 2 into Prandtl number raise to 2 by 3 minus 1. We know the friction factor, we know the Reynolds number, we have already read off the Prandtl number; so if you substitute it here you will get the Nusselt number to be 15.618.

The next step - use the Nusselt number to calculate the heat transfer coefficient and then use the Colburn analogy, this relation to calculate the mass transfer coefficient.

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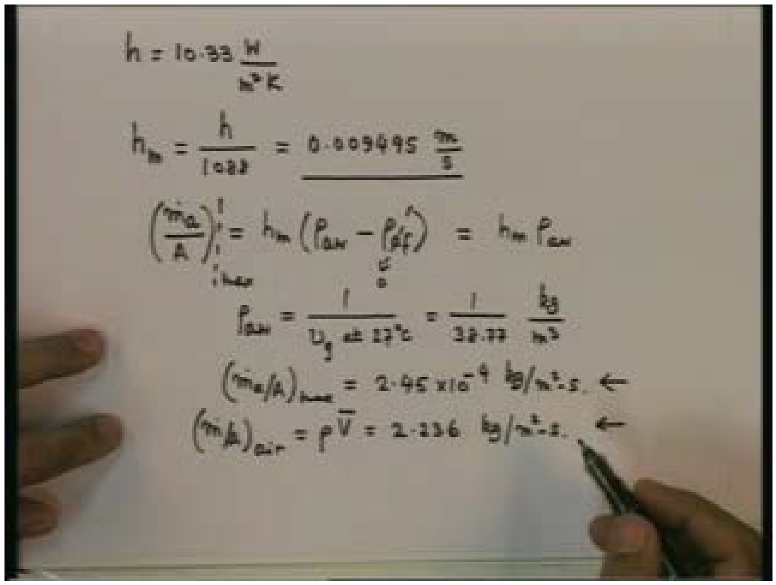

From the Nusselt number the heat transfer coefficient h turns out to be 10.33 Watt per meter squared Kelvin and from the analogy we have derived that h by h_m is 1088 so h_m will be h divided by 1088 in the appropriate units and that turns out to be 0.009495 meters per second. We have been asked to determine the mass flux of water vapor and the mass flux of air. The mass flux of water vapor m dot a by A will be h_m into row a at the wall minus row a in the free stream. Now, row a in the free stream starts with 0. It is dry air which is entering the tube, it will pick but to obtain a maximum value of this, we will use dry air here and that means this will be 0 so the maximum value of this will be given as h_m into row aw.

Now, what is row aw? Density of water vapor at the wall will be 1 over the vapor specific volume of saturated vapor at 27 degrees C which from steam tables is read off as 308.77 meter cube per kg. So row aw will be 1 over 30 8.77 kg per meter cube. This gives us m dot a by a max to be h_m into row aw which turns out to be 2.45 into 10 raise to minus 4 kg per meter square second and now we have m dot by a for air. This will simply be row of air into the mean velocity, the flux of air, and this turns out to be - we know row, we know V bar - this turns out to be 2.236 kg per meter square second. So here we notice that the water vapor mass flux is of the order of 10 raise to minus 4 kg per meter square per second whereas that for air is a few kg per meter square per second. This indicates that the vapor mass flux is roughly 10000 times lower than the air mass flux. this indicates that the Colburn analogy is applicable in this case.

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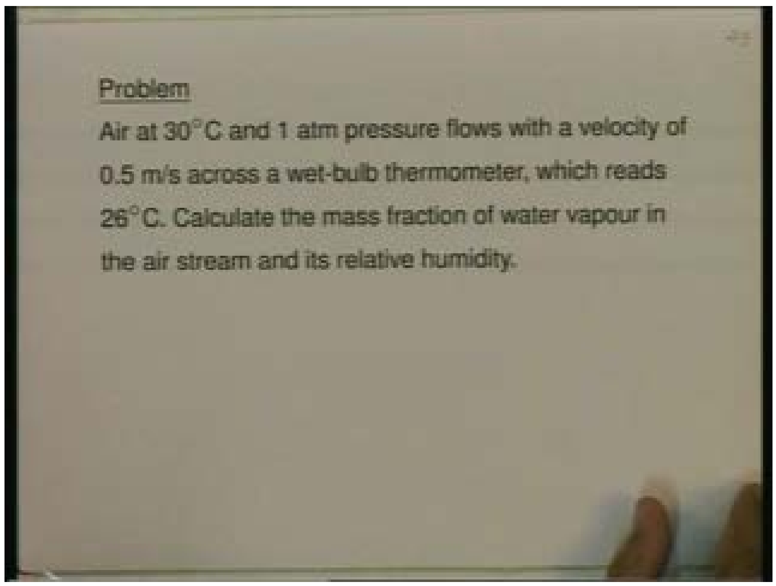

Finally, we look at a problem pertaining to the wet bulb thermometer, we have air at 30 degree C and 1 atmospheric pressure flowing across the wet bulb thermometer. Air velocity is .5 meters per second and the wet bulb thermometer reads 26 degrees C. We have to calculate the mass fraction of water vapor in the air stream and its relative humidity.

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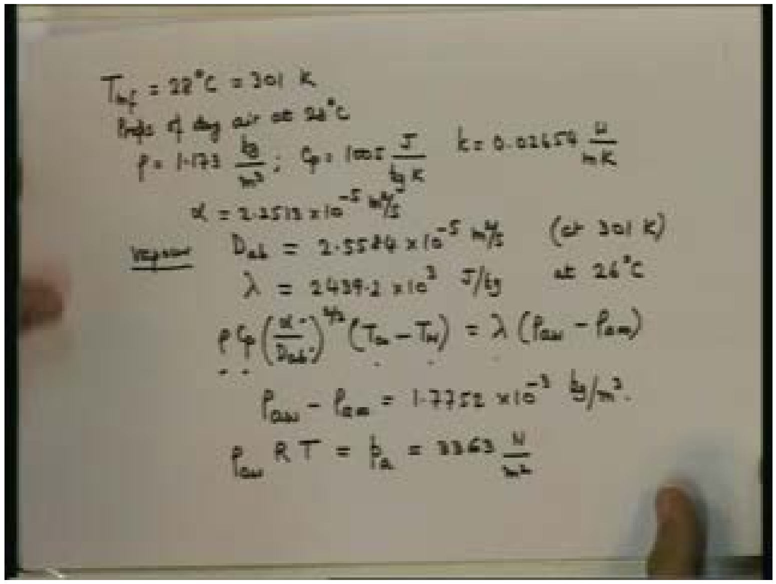

We notice that the mean film temperature is the average of 26 and 30 degree C which is 28 degree C which is 301 Kelvin. The air is moist but the moisture content is small so we will use properties of dry air at 28 degrees C. These are row, C_p, k, alpha and for vapor we obtain D_{ab} at 301 K to be 2.5584 into 10 raise to minus 5 meter square per second; some interpolation is needed. We also need the latent heat which is 2439.2 into 10 raise to 3 joule per kg at the wick temperature which is 26 degrees C.

Now, we have the expression for a wet bulb thermometer, row, C_p, alpha by D_{ab} raise to 2 by 3 T infinity minus T wall equals lambda row aw minus row a infinity. We know row C_p alpha D_{ab} T infinity T_w lambda; everything is known so the only unknown is row aw minus row a infinity. So we get row aw minus row a infinity is 1.7752 into 10 raise to minus 3 kg per meter cube. Now to determine row aw - we have row aw into R into temperature equal to the partial pressure of air, partial pressure of water vapor. Now this is at the wick temperature and at wick temperature this from steam tables is 3363 Newtons per meter square.

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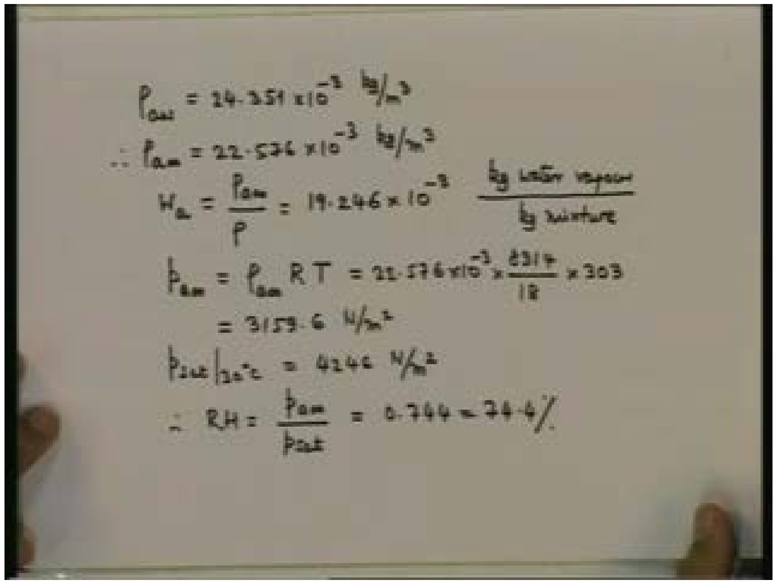

Substitution for temperature which is 299 Kelvin and the gas constant for steam gives you the value of row aw to be 24.351 into 10 raise to minus 3 kg per meter cube and row a infinity is from our difference relation 22.576 into 10 raise to minus 3 kg per meter cube. So, our mass fraction of water vapor in air will be row a infinity divided by row which turns out to be 19.246 into 10 raise to minus 3 units of kg water vapor per kg mixture.

Now, to determine the relative humidity, we have to determine the partial pressure of air in the free stream. The partial pressure of water vapor in the free stream pa infinity is given by row a infinity -which we have already determined - multiplied by R multiplied by T where R is the gas constant for water vapor. This turns out to be 22.576 into 10 raise to minus 3, R is 8314 divided by 18 - the molecular weight of water, temperature in the free stream is 303 Kelvin. So this turns out to be 3159.6 Newtons per meter squared. We have the saturation pressure at 30 degrees C to be 4246 Newton per meter squared and hence the relative humidity is the partial pressure of moisture in the free stream divided by the saturation pressure at the same temperature and that turns out to be .744 or 74.4 percent.

So, that brings us to the end of our study of mass transfer. What did we do during our study?

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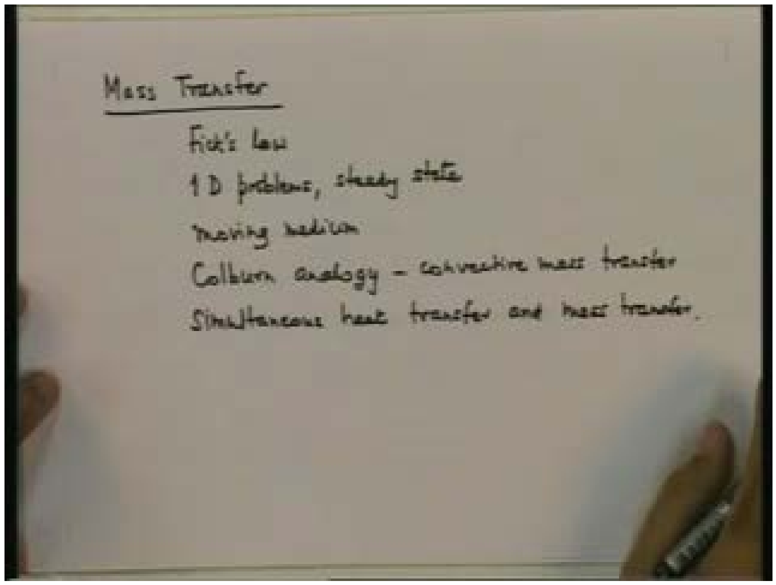

We studied Fick's law, then we studied 1D problems in steady state, then we looked at a moving medium, then we studied the Colburn analogy for convective mass transfer and finally we looked at simultaneous heat transfer and mass transfer and we solved some problems. Now that brings us to the end of our study of mass transfer. Also, this is the concluding lecture of the lecture series on heat and mass transfer.