Heat And Mass Transfer
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Lecture No. 30
Boiling and Condensation-2

We were, when we stopped last time, we were studying heat transfer during film condensation on a vertical plate. I was deriving an expression for the heat transfer coefficient and following the derivation which was first done by Nusselt long ago in 1960. And if you recall where we stopped after making a certain number of simplifying assumptions, we had applied this Newton's second law of motion, that is one of our basic fundamental laws, derived an expression for the velocity profile in the liquid fill. Then, we had derived an expression for the mass flow rate in the film, the liquid mass flow rate in the film and then taken the differential of that to show that if you go from a section z down to a section z plus dz, then the mass flow rate increases by the amount -differential amount - and we had an expression for that and that is where we stopped. So, let me just put down those expressions again for, to recollect.

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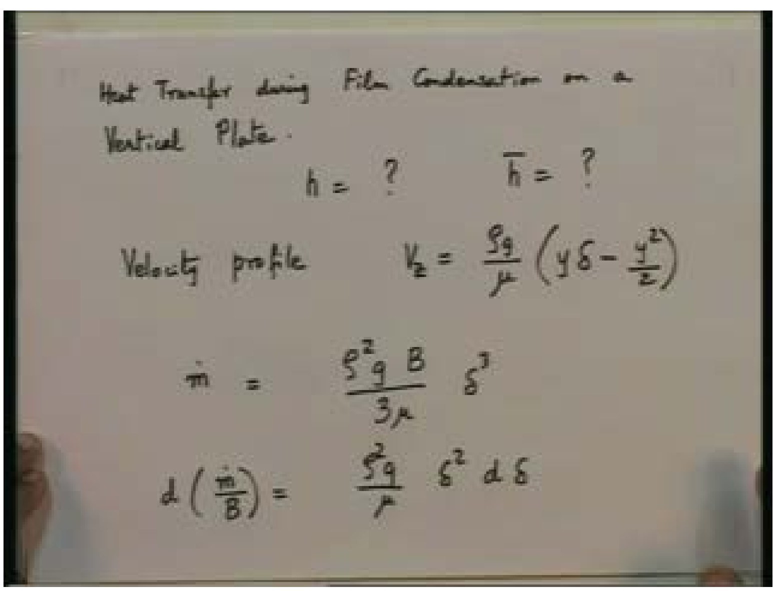

We were studying heat transfer during film condensation on a vertical plate and our objective is to derive an expression for h - the local heat transfer coefficient - and h bar - the average heat transfer coefficient - during the condensation process. When we stopped last time we had come about half way through the derivation. We had derived an expression for the velocity profile based on applying Newton's second law of motion. We had got the velocity profile in the downward direction - the velocity V_z given by rho g by mu y delta minus y squared by 2, that was the first thing we have derived. Then we had calculated the mass flow rate with the help of this velocity profile, the mass flow rate through the liquid fill. And for that we had an expression m dot is equal to rho square g. B is the width of the plate in a direction at right angles to the paper divided by 3 mu delta cubed and taking the differential of that we had got the expression d m dot B.

m dot B is the mass flow rate per unit width of the plate is equal to rho squared by rho square g by mu delta squared d delta - this is where we had stopped last time. And what this really implies is when you get a differential amount, what it implies is that if you are going, if you are moving down the plate. Let us say - let me draw the sketch of the plate again if you are moving down the plate.

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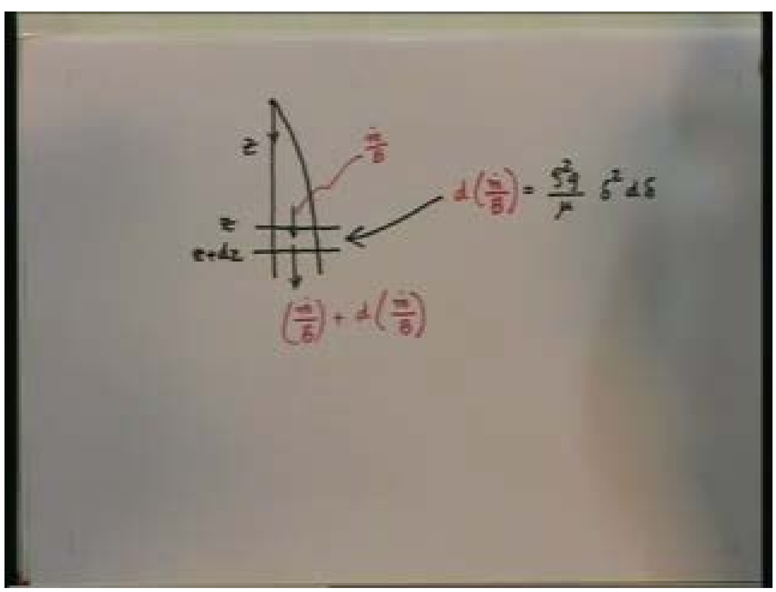

This is the top of the plate, this is the z direction downwards and this is the liquid fill. Then, if you have a cross section and if you have cross section z plus dz, this is at z and this is at z plus dz. Then, from this cross section at z to this cross section at z plus dz the mass flow rate per unit weight increases by an amount rho square g by mu delta square d delta. And where does that increase come from? That increase comes from the fact that we have condensation taking place.

So, out here at the cross section z the mass flow rate is m dot by B; out here at section which is z plus dz you move the distance dz forward, it is m dot by B plus d of m dot by B and obviously the difference between the 2 has been made the condensation which is taking place from z to z plus dz at the liquid vapour interface. That is how the difference is being made up. So, this quantity here which is coming in is d of m dot by B and for this quantity we have the expression d m dot by B is equal to rho squared g by mu delta squared d delta.

So, if conservation of mass is to be satisfied, if the equation of continuity is to be satisfied, it follows that this is the mass balance. Mass flowing, in mass flowing out, the difference is made of by the condensation which is taking place from z to z plus dz. This is how continuity satisfy. Now, what remains is to apply the remaining fundamental law which is the first law of thermodynamics. So, that is what we will be doing next; we will now apply the first law because we have used Newton's second law - the satisfied continuity.

So, now we go on and decide that we will apply the first law of thermodynamics. So, let us do that we say. Let us take a control volume which is a slice and let me show that slice just for the, let us take as our control volume for applying the first law.

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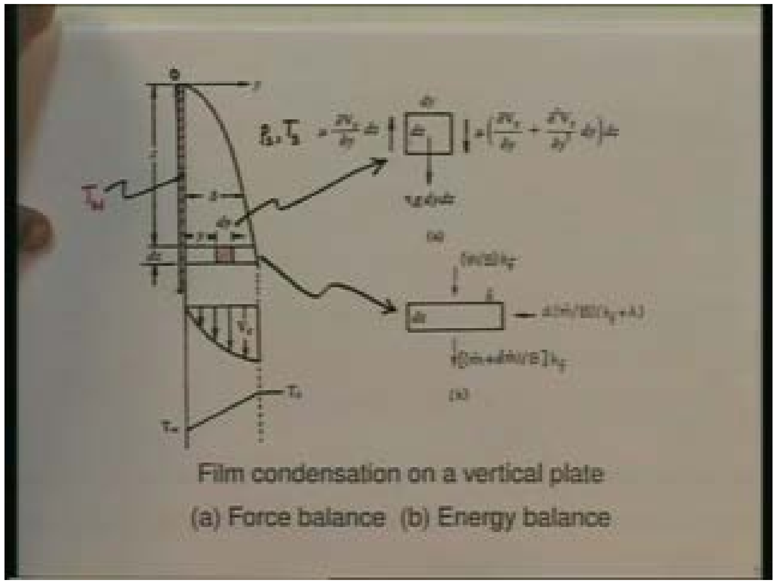

Let us take this slice z and z plus dz, a slice of the film between the section z and z plus dz, and let us apply the first law of thermodynamics to this slice. Now, an enlarged view of this slice is shown here; this is an enlarged view of the slice and out here you have the, first of all you have the mass balance m dot by B which I had shown a moment ago. Then, dm dot by B is the mass flowing in by condensation, that is vapour being converted into liquid, and this is m dot plus dm dot by B which is the mass flowing out that I had just shown a moment ago in another sketch.

Now, if I want to apply the first law to this mass balance, to these masses flowing in and out, I must multiply them by the enthalpy per unit mass. So, let us do that; so out here now, we have m dot by B flowing in, you multiply by the enthalpy per unit mass h F. h_f is the enthalpy of saturated liquid. So, out here I am multiplying by that; remember we have made an assumption that we are going to neglect sub-cooling. So, we will just multiply this by h_f which is the enthalpy per unit mass of saturated liquid. In the flow going out, again it is saturated liquid so again I multiply the mass flow rate by h_f and in the flow coming in, that is, this is the condensation taking place here.

I multiply by h_f plus lambda because out here it is this enthalpy of saturated vapour which I have to take. The saturated vapour condenses at the liquid vapour interface and then enters this control volume. So, it brings with it enthalpy per unit mass h_f plus lambda, lambda being the let in heat of vaporization. So, this is our enthalpy balance, this is the enthalpy flowing out, this is the enthalpy flowing in at the section z. This is the enthalpy flowing in at the liquid vapor interface because vapor is condensing at the interface. Now with this as the background let us supply the first law of thermodynamics to this.

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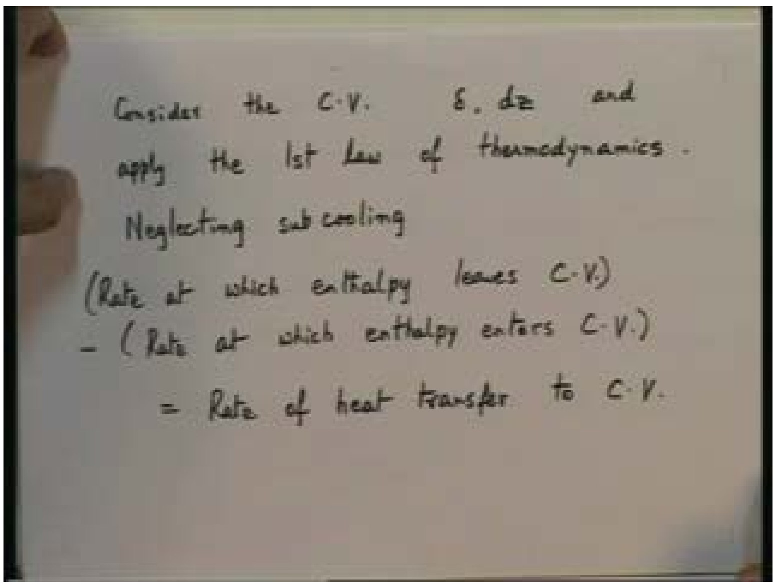

So, we say consider the control volume, consider the control volume delta dz the slice delta dz, neglect sub-cooling. Consider the control volume and apply the first law of thermodynamics; neglect sub-cooling. As I have said that is one of our assumptions - neglecting sub-cooling. Then, rate at which enthalpy rate, now I am applying the first law, rate at which enthalpy leaves control volume minus rate at which enthalpy enters the control volume is equal to rate of heat transfer, rate at which heat flows by heat transfer to the control volume - that is the first law statement.

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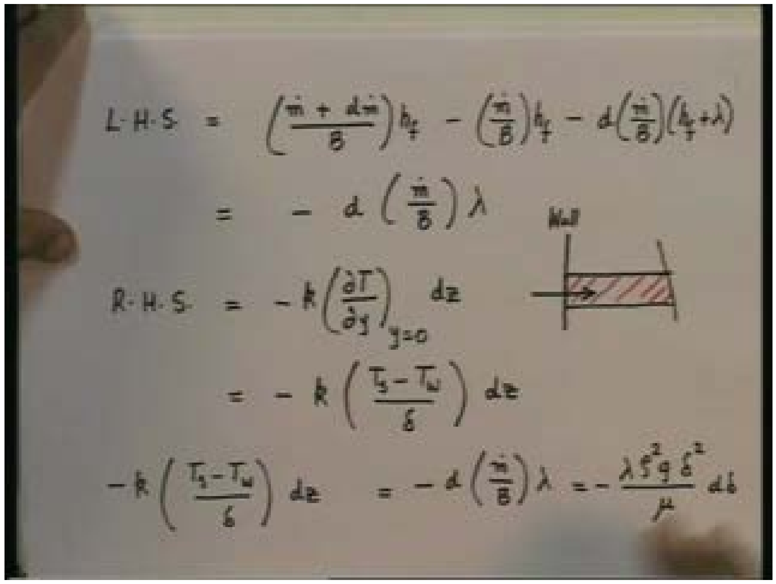

Now, what is the left hand side? The left hand side in this case will be, the left hand side in this case will be m dot plus dm dot by B multiplied by h_f - this is the enthalpy leaving the control volume minus the enthalpy entering at the cross section z that is m dot by B into h_f minus the enthalpy entering where the vapor condenses and becomes liquid multiplied by, this is the rate at which enters and this is the enthalpy per unit mass because this is saturated vapor being converted into saturated liquid so multiply by h_f plus lambda. So, the left hand side is nothing but simplification; it is minus d of m dot by B lambda. Now, the right hand side of the expression into which we want to substitute, the right hand side is the rate of heat transferred to the control volume. This is, let me just draw the control volume here, this is our control volume like this. This is the liquid film here and this is the wall and this is our control volume.

Now, the heat transfer to the control volume takes place through this phase where the liquid comes in contact with the wall. This is the wall; the liquid is flowing down like this, this is the wall here. So, this is where the heat transfer to the control volume takes place and since it is a fluid flowing across a wall we know from our study of convection that the rate of heat transfer will be minus k, Fourier's law will apply at the wall, minus k dT dy at y is equal to 0 multiplied by the area dz and we are taking a unit weight so into y.

So, this is the rate of heat transfer to the control volume and that is we have made an assumption that we are going to take the temperature profile to be linear. So, it is nothing but minus k; now dt dy at y is equal to 0 is nothing but T_s minus T_w. Since we have a linear temperature profile in the liquid divided by delta multiplied by dz, so we have the left hand side an expression here, the right hand side expression here. Now, let us equate the two and we get minus k T_s minus T_w divided by delta the whole thing multiplied by dz is equal to d minus dm dot by B lambda which in turn is equal to - we have an expression for m dot by B dm dot by B. So, we get minus lambda rho squared g delta squared divided by mu d delta, that is our expression which we get by substituting into the first law of thermodynamics.

The left hand side is the rate of heat transfer to the control volume. The right hand side here is the, sorry, the right hand side I have put it the other way. Now, the right hand side I have put on the left side. So, the right hand side is the rate of heat transfer to the control volume; the left hand side is the rate at which enthalpy flows out minus the rate at which enthalpy flows into the control volume. Let us clean that, clean up that expression a little.

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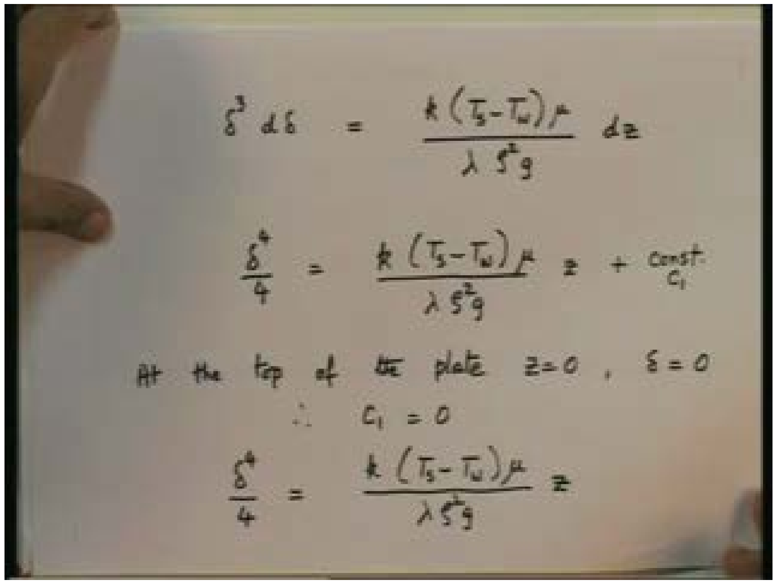

If you clean up the expression a little we get delta cube d delta, delta cube d delta is equal to k T_s minus T_w multiplied by mu divided by lambda rho squared g. The whole thing multiplied by dz - this is what we get on substituting the, on applying the first law of thermodynamics to that slice which is delta multiplied by dz. Now, let us integrate this expression; if I integrate this expression, I will get delta to the power of 4 divided by 4 is equal to k T_s minus T_w mu divided by lambda rho square g multiplied by z plus a constant, a constant of integration.

Now, in this case, we know that right at top, at the top of the plate, at the top of the plate where the condensation begins that is at z is equal to 0, delta must be equal to zero; theoretically at least that is a condition we can apply. So, if you put that in, then this constant which I will call as C1, therefore C1 is equal to zero.

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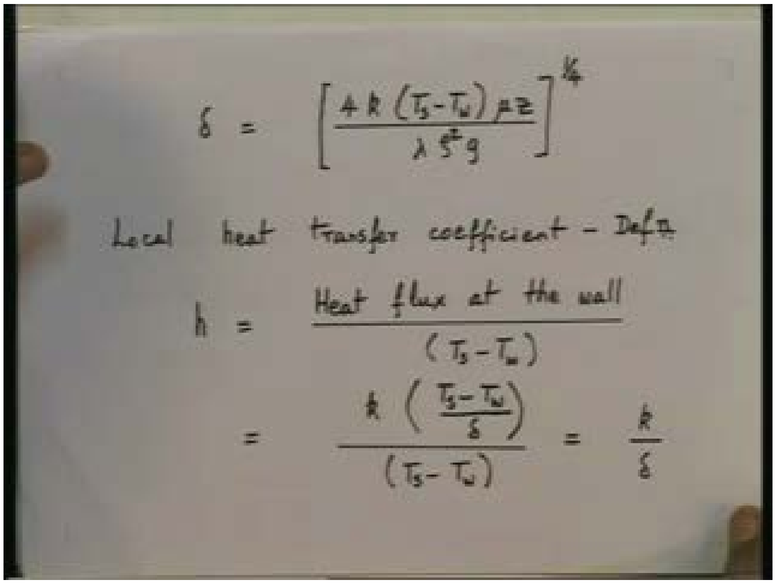

So, on integration we get delta to the power of 4 divided by 4 is equal to the constant being 0 k T_s minus T_w divided by mu divided by lambda rho squared g multiplied by z or if I want an expression for the film thickness I will get - if I want an expression for the film thickness, I will get delta is equal to 4 k T_s minus T_w mu z divided by lambda rho square g divided by lambda rho square g delta is equal to, all this to the power of one fourth. That is the expression we get for the film thickness after applying the first law of thermodynamics.

Now, let us now use this to calculate the local heat transfer coefficient. First of all, let me define, we want an expression now for the local heat transfer coefficient. We want an expression for the local heat transfer coefficient; we will define it as follows. We will say h is equal to, as always they have been consistent, the local heat transfer coefficient is equal to the heat flux at the wall, the heat flux at the wall divided by the temperature difference T_s minus T_w. This is how we will define a local heat transfer coefficient, that is the local heat flux at a cross section z divided by the temperature difference T_s minus T_w.

Now, in this case the heat flux at the wall, we know it can be readily put down in terms of the temperature difference because it is a linear temperature profile. So, we can see this is equal to heat flux at the wall is nothing but k T_s minus T_w divided by delta ,that is the heat flux at the wall because we have a linear temperature profile and that is to be divided by T_s minus T_w. So, this simplifies to nothing but, the expression for h simplifies to nothing but k divided by delta. Now, we have our, we have derived already an expression for the thickness, film thickness delta.

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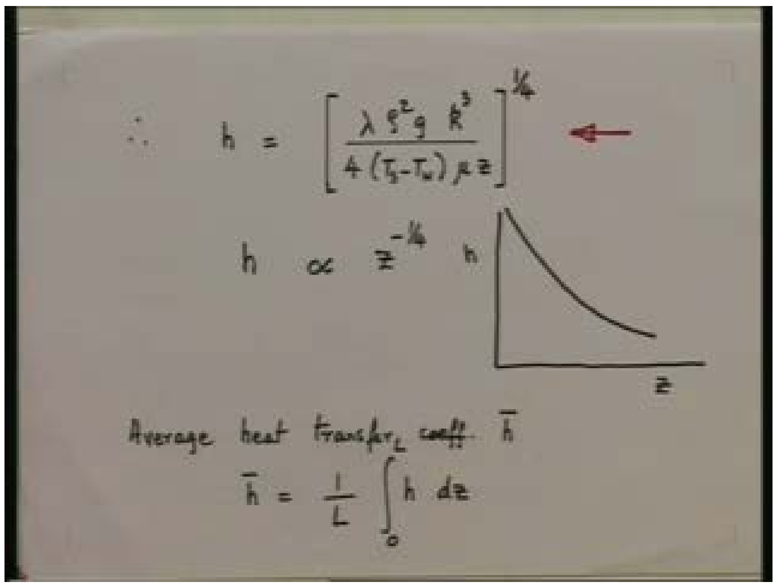

So, we get h is equal to, therefore h is equal to lambda rho square g k cubed divided by 4 T_s minus T_w mu z. This is the the whole thing to the power of 1 by 4 the whole expression to the power of 1 upon 4. So, this is our expression, our expression for the local heat transfer coefficient and this an important expression. We put an arrow on it to indicate that this is what we were looking for, one of the things we were looking for - an expression for the local heat transfer coefficient.

Now, what is its characteristic? Look at it - lambda rho g k T_s minus T_w and mu are all constants. So, effectively what I am saying is that, from this looking at it what you are saying is, h is proportional to z the distance along the plate, counted it from the top downwards to the power of minus 1 upon 4. So that is the type of variation of the local heat transfer coefficient we are getting. Or if I were to plot this; let us say if I had a graph, just sketch it and if I were to plot h against z that is moving downwards along the plate, then the variation of h is going to be something like this. Because it is z to the minus one fourth - that is the kind of variation we are going to get for h.

Now let us get an expression for the average heat transfer coefficient. The average heat transfer coefficient h bar, average heat transfer coefficient h bar would be given by, quite logically would be given by h bar is equal to 1 upon L integral 0 to L h into dz. The h bar is the average value of h over a length L of the plate. So, I want a plate length, I want an average value of h bar over a length L of the plate. All I do is integrate h over that length L and take 1 upon L of that; that gives me the average value of the heat transfer coefficient. So, let us calculate an expression for h bar.

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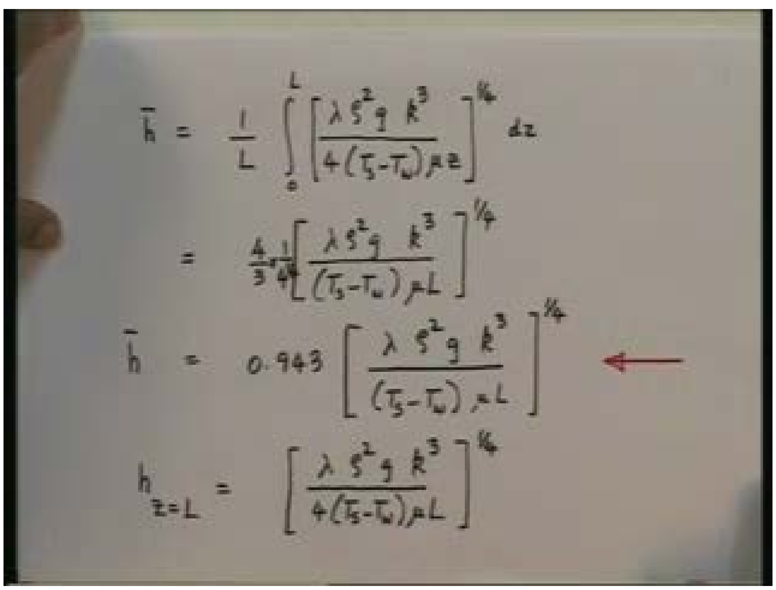

Now, the average for the average heat transfer coefficient, if I put in expression for h, I will get h bar is equal to 1 upon L the integral 0 to L lambda rho squared g k cubed divided by 4 T_s minus T_w mu z the whole thing to the power of 1 upon 4 - this is my expression for h into dz. And on performing the integration which is very simple, I will simply get this is equal to 4 upon 3 lambda rho squared g k cubed divided by T_s minus

T_w mu L 4 upon 3 multiplied by, I missed a term here, multiplied by 1 upon 4 to the one fourth multiplied by 1 upon 4 to the one fourth into lambda rho square g k cube upon T_s minus T_w mu L the whole thing to the power of one fourth. Or it gives, on simplification you get .943 into square bracket lambda rho square g k cubed divided by T_s minus T_w mu L the whole thing to the power of one fourth.

So, this is our expression for h bar and this is the second - the next important expression which we are wanted to derive. We wanted to derive an expression for h and we wanted to derive an expression for h bar. So, h bar is .943 lambda rho squared g k cube upon T_s minus T_w mu L the whole thing to the power of 1 fourth. Now, h the local heat transfer coefficient at the bottom of the plate if I ask you, if it is plate of length L, h at z is equal to L is nothing but lambda rho square g k cubed divided by 4 T_s minus T_w mu L the whole thing to the power of one fourth - that is our value of the local heat transfer coefficient at the bottom of the plate at z equal to L. If I take a plate of length L, so it follows that the average heat transfer coefficient, it follows that the average heat transfer coefficient h bar is related to the local heat transfer coefficient at z equal L. I repeat - the average heat transfer coefficient for a plate of length L is related to the local heat transfer coefficient at z equal to L by the expression.

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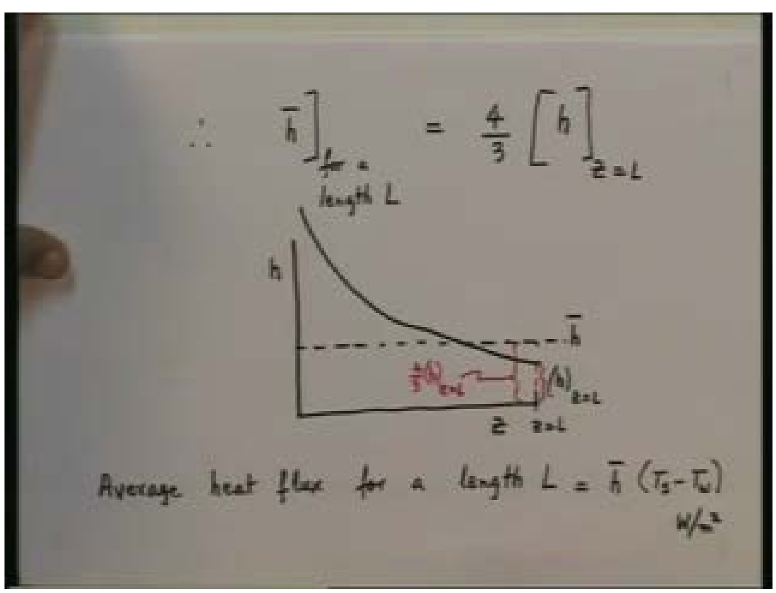

h bar for a length L is equal to 4 upon 3 into h at z is equal to L that is the relationship between the average on the local heat transfer coefficient. Or again, if I were to plot a graph, suppose to show h against z. If I am plotting h again z. And let us say this is z equal to L, this point is z is equal to L and let us say the variation of h, I have already sketched it for you earlier, it is something like this - proportional to z to the minus 1 4. Then what we are in effect saying is that if this is the value of h at z equal to L this is h at z equal to L, then if I take 4 by 3 times this, if I at this point if I go up to 4 by 3 times this so that this quantity is 4 by 3 into h at z equal to L, then if I draw horizontal line here, this will be the value of h bar for a plate of length L. h bar for a plate of length L is 4 by 3 times the value of h at z equal to L, that is at the bottom of the plate, that is what we are saying.

So, these were 2 important results we were looking for, for which we have expressions. Given now a vertical plate with condensation taking place on it, I know how to calculate the local heat transfer coefficient and the average heat coefficient. If I ask you for instance what is the average heat flux for a plate of length L, you can readily tell me now. Average heat flux for a length L will be equal to h bar, the average heat transfer coefficient which is in watts per meter square kelvin multiplied by the temperature difference T_s minus T_w. So, this would be the average heat flux for a plate of length L so many watts per meter square or if I were to ask you what is the average condensation flux for a plate of length L, average condensation flux for a plate of length L, you will say it is nothing but the average heat flux that is h bar into T_s minus T_w so many watts per meter squared divided by the latent heat of vaporization.

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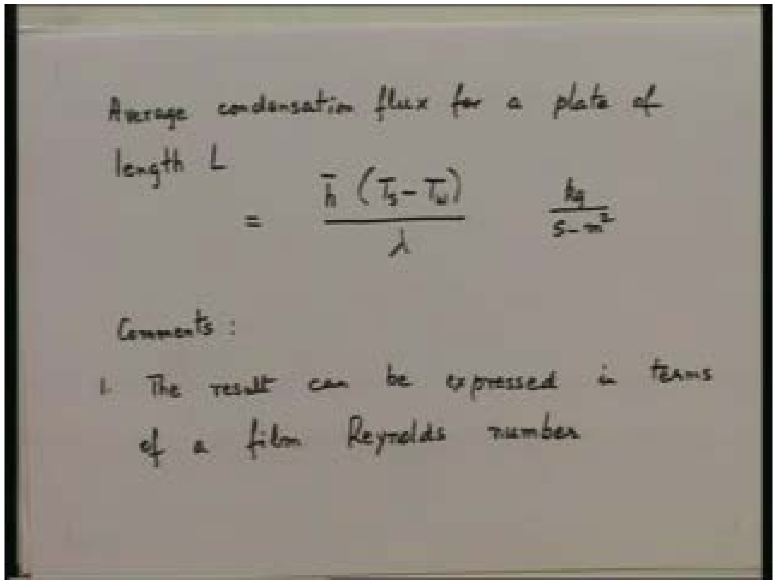

This would the average condensation flux and this would obviously come out in kilograms per second per square meter of the vertical platelet. So, these are results which are consequences of getting expressions for the local and the average heat transfer coefficient.

Now some comments - this is the Nusselt formula which we have derived - the well expression for h and the expression for h bar. This is the famous Nusselt formula as I said derived in 1960 and still used extensively all over for calculations. Now this result, some comments on the Nusselt formula. The first comment we will make is that the result which we have got; the result can be expressed in terms of what is called as a film Reynolds number, in terms of what is called as a film Reynolds number. There is liquid flowing in the film based on that flow; you can define a Reynolds number of the film called a film Reynolds number and the result for h or h bar can be expressed in terms of that film Reynolds number.

So, let me see what is the film Reynolds number first, then give the result in terms of the film Reynolds number.

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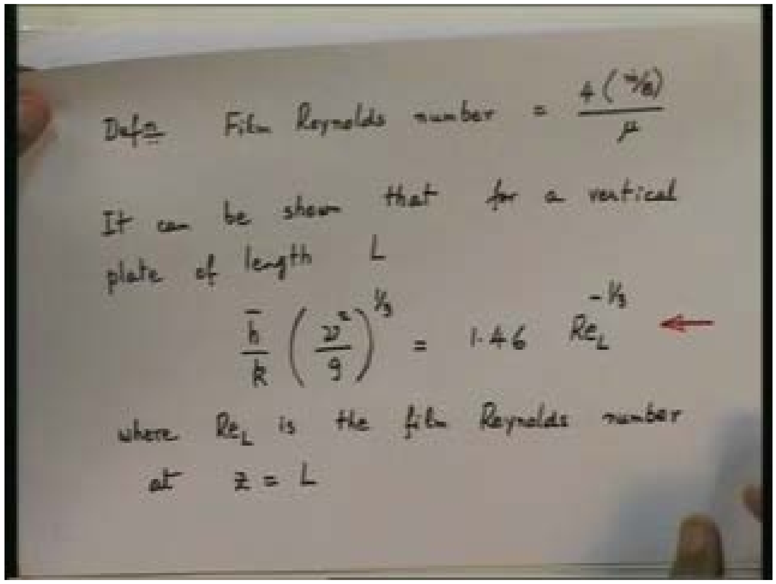

The film Reynolds number is defined as the film Reynolds number - this is the condensate film - is defined by the expression equal to 4 m dot by B divided by mu. m dot by B is the flow rate of liquid in the condensate film at any cross section z and B is of course the width of the plate so m dot by B is the flow rate per unit width. I divide it by the viscosity of the liquid and the 4 is an number which has been by convention used and we get therefore what is called as the film Reynolds number. So now, if you use this definition we know what is m dot by B for in our case.

So, if you use this definition you can show; it can be shown that for a vertical plate of length L, using the results that we have derived, it can be shown that h bar - the average heat transfer coefficient - is related to the film Reynolds number by the expression, by the expression h bar by k into bracket mu squared by g to the one third is equal to 1.46 Re_L the Reynolds number, film Reynolds number to the minus one third where Re_L is the film Reynolds number at the bottom of the plate, where Re_L is the film Reynolds number at the bottom of the plate, at the bottom that is at z is equal to L, that is how Re_L is defined. So, this is an alternative result; this is an alternative expression for h bar which is sometimes useful.

The h expression, the previous expression which we derived for h bar, was in terms of the temperature difference T_s minus T_w. Now, the expression that we have got for h bar is in terms of the film Reynolds number. This is an alternative way of calculating h bar; sometimes this is useful, sometimes the previous expression is useful. You should be able to derive this expression for h bar from the previous expression; I leave that to you as an exercise. It is a very simple algebraic manipulation, shouldn't take much time. So, please derive this expression which we have got from the previous expression. Derive the expression for h bar in terms of film Reynolds number from the previous expression for h bar which we have derived. So this is the first comment we wanted to make on the Nusselt result.

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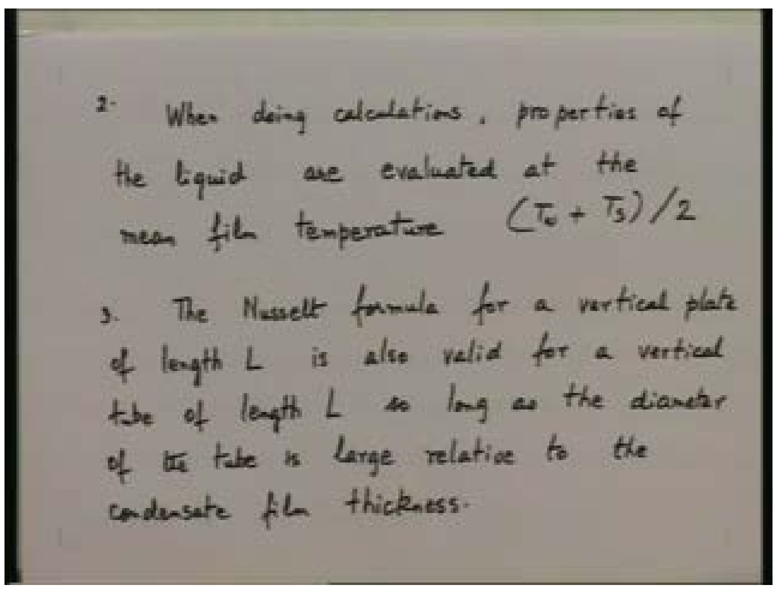

The next comment which we want to make on the Nusselt result is when doing calculations with it, when doing calculations properties are taken of a liquid. You know you need properties like the k, mu, rho, etcetera. When doing calculations, properties of the liquid of the liquid are to be evaluated at, are evaluated at the mean film temperature, at the mean film temperature. And what is the mean film temperature? It will be T_w plus T_w plus T_s divided by 2. So when you are doing actual numerical work with this Nusselt formula, take properties of the liquid at the mean film temperature T_w plus T_s by 2. And this is a very good assumption because the difference between T_w and T_s is usually not all that much. So, taking a mean film temperature works out very well.

A third comment - the Nusselt formula for a vertical plate of length L, that is what we have derived is also valid for a vertical tube of length L so long as the diameter of the tube is large relative to the condensate film thickness. Keep this in mind. The Nusselt formula that we have derived for a vertical plate is also valid for a vertical tube so long as the diameter of the tube is large relative to the condensate film thickness and this is very easily seen. Suppose I take, just look at this now, I have this as a vertical plate here. Here is a vertical plate. Now, look at this vertical plate; this is there now. If I were to fold this into a vertical tube condensation will take place all the way on the outside of this tube. Now, so long as the film thickness, the condensate that is falling on the outside the film thickness, is small related to the diameter, obviously the result for a vertical plate is going to be valid for a vertical tube. The vertical tube is nothing but a vertical plate which has been folded into the shape of a circle. So, keep this in mind that the present result is also valid for a vertical tube so long as a diameter is not small and remember the condensate film thicknesses are very small. We are talking of thicknesses which are usually less than a millimeter in thickness; keep that in mind also.

So, with these comments now, we have derived our formula for condensate condensation on a vertical plate.

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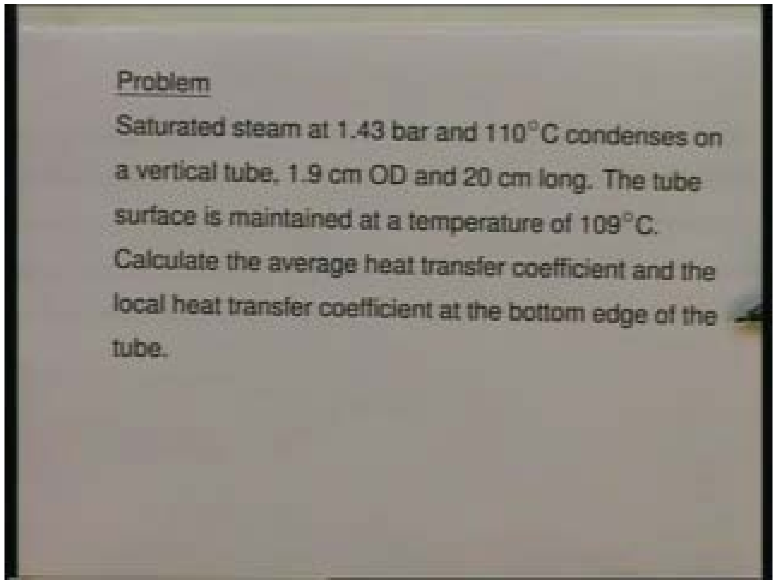

Now, we will do a problem just to illustrate ideas. We are going do the following problem. Please look at the text, take it down if you like. Saturated steam at 1.43 bar and 110 degree centigrade condenses on a vertical tube, 1.9 centimeters OD and 20 centimeters long. The tube surface is maintained at a temperature of 109 degrees centigrade. Calculate the average heat transfer coefficient h bar and the local heat transfer coefficient at the bottom edge of the plate. Calculate the average heat transfer coefficient h bar the local heat transfer coefficient.

h at z is equal to L at the bottom edge of the plate means at z is equal to L, that is the problem. It is a straight forward substitution into the Nusselt formula that we have just derived, no real complications. I just want to illustrate the whole procedure so let us draw the tube, let us draw the tube.

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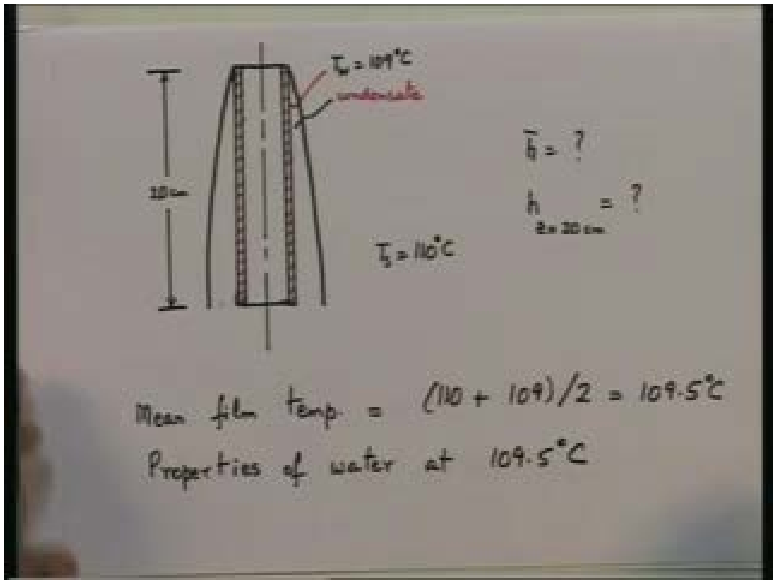

Let us say this, our tube, let us say this is the tube - the vertical tube. Let us say this is the vertical tube and this is the wall of the tube; condensation is taking place on the outside; I will draw the film thickness, exaggerated again. So, I will say this is the condensation taking place on the outside with the film thickness of course exaggerated all round the tube. So, it is like condensation on a vertical plate, so this is condensate film flowing down, this is the condensate but I am exaggerating this film thickness. Keep that in mind. It is much much smaller than this; so this is the condensate film.

Given T_w is 109, T_w is 109 degrees centigrade, then T_s is equal to 110 degrees centigrade and we are told that the dimensions are 20 centimeters long so the tube is 20 centimeters long. The tube is 20 centimeters long; this is twenty centimeters. Now, we would like to calculate h bar and h at z is equal to 20 centimeters. This is what we have to do. So, we have take properties at the mean film temperature; mean film temperature is equal to 110 plus 1099 divided by 2 which is equal to 109.5 degrees centigrade. So, we need the properties of the liquid condensate at 109.5 degrees centigrade. Look them up in the tables for water properties of water at 109.5 degrees centigrade. If you look them up, you will get look up the properties.

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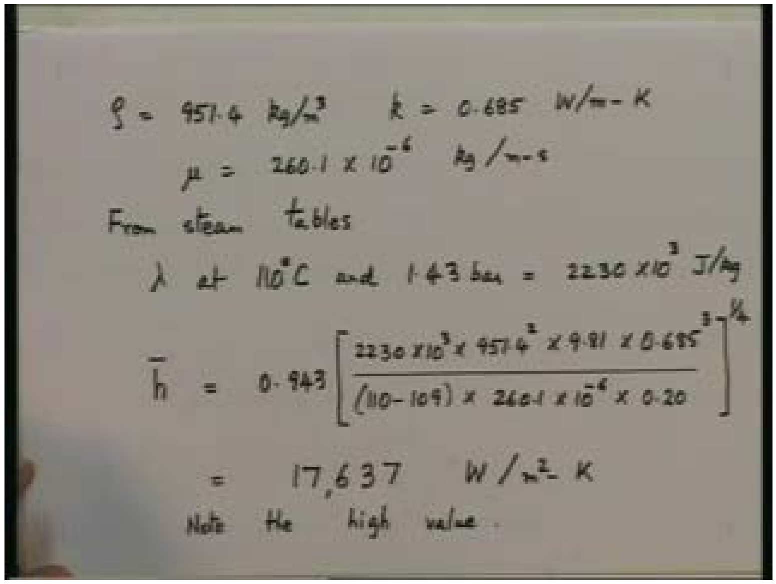

You will get rho is equal to 951.4 kilograms per meter cube, k is equal to .685 watts per meter square watts per meter kelvin and mu is equal to 260.1 into 10 to the power of minus 6 kilograms per meter second. We will also need the value for lambda, so from the steam tables we can look up the value of lambda. The latent heat from steam tables lambda at 110 centigrade and 1.43 bar -this is the saturated pressure corresponding to the temperature of 110 - the value of lambda in the table, this is 2230 into 10 to the power of 3 joules per kilogram, so these are the properties. First, look up the properties.

Now, substitute into the Nusselt formula; so if you substitute into the Nusselt formula you will get h bar is equal lambda rho squared g k cube upon T_s minus T_w of into mu L the whole thing to the one fourth power to be multiplied by .943 - that is our formula, so let us do that. So, we get 943 multiplied by lambda that is 2230 into 10 to the power of 3 rho squared 951.4 whole square multiplied by g - 9.81 - multiplied by k cubed - .9685 cube - divided by, divided by T_s minus T_w that is 110 minus 109 multiplied by mu - 260.1- into 10 to the minus 6260.1 into 10 to the minus 6 and multiplied by the length of the plate length of the tube which is .2 meters the whole thing to the power of one-fourth. And if you work this out, you will get this is equal to 17637 watts per meter squared Kelvin. So you see what a high value you get, note that high value, note the high value. I had started off my talk on condensation by saying that values in condensation and boiling are high so here is an example which shows that the values are always in, generally always in thousands. Now this is our value of h bar. What is our value of h? That is very straight forward.

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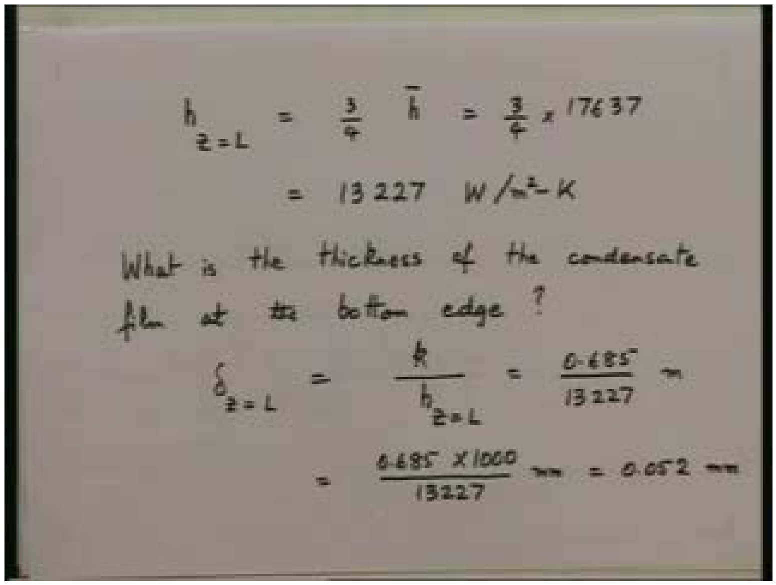

h at z equal to L, h at z equal to L is nothing but 3 by 4 h bar. So, it is going to be 3 by 4 into 17637. So that is nothing but 13227 watts per meter squared Kelvin; so that is the second result we are looking for; first the value of h bar, then the value of h.

Now finally, if I ask you what is the thickness of the condensate film at the bottom edge just to get a feel. What is the thickness of the condensate film at the bottom edge? That is very straight forward - delta at z equal to L is equal to k divided by h at z equal L - because we have an expression for the local heat transfer coefficient. So, all we are going to get in this case is .685 which is the value of k divided by 13227 so many meters or if I want it in millimeters, it will come out to be .685 into 1000 divided by 13227 millimeters which is nothing but .052 millimeter.

So, notice how small the film thicknesses are and that is also worth noting in terms of numbers. We get high values of h - very thin films which are flowing smoothly down the vertical plate or the vertical tube as is the case in this, in this situation. So, we will, next time now, we will talk about condensation on some other geometries. We will talk on condensation on a horizontal tube and a bank of horizontal tubes and put down some expressions per h bar for those geometries also.