Heat and Mass Transfer
Prof. S.P. Sukhatme
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Lecture No. 29
Boiling and Condensation-1

Today we will begin with the topic of condensation and boiling; we will be studying the heat transfer during this change of phase processes. Now, first of all, the obvious question is where are we interested in, what situations are we interested in studying heat transfer during condensation and boiling. Well, if you look, say at a power plant - any thermal power plant - you got change of phase occurring when water is converted into steam in a boiler or sometimes in an evaporator and you got change of phase occurring when the steam which has been expanded through a turbine is condensed in a steam condenser. Or you, if you look, say at a refrigeration system - vapor compression refrigeration system - there also we have change of phase occurring when the refrigerant is condensed in a condenser after having given up its heat in the evaporator. So, there are numerous such situation; I have just mentioned 2 where we are interested in knowing the heat transfer rates during change of phase that is either during condensation or during boiling. So, our purpose in this topic when we are studying this topic is to derive expressions for the heat transfer coefficient in some situations for some geometries and with the help of these expression or with the help of these correlations, one is able to design the equipment in which either condensation or boiling is taking place.

Now, first a general comment; a general comment which is worth noting is the values of h, the values of the heat transfer coefficient, during condensation and boiling are invariably high as compared to the values we are used to in single phase heat transfer. For example, just to illustrate the idea, suppose I have got single phase heat transfer natural convection to a gas. A typical value of h is 5 or 10; h is typically 5 or 10 Watts per meter squared Kelvin. Suppose I have got heat transfer occurring and natural convection to a liquid; typical value of h is00 200 Watts per meter squared Kelvin. On the other hand, suppose I have got forced convection; depending on the velocity the value of h forced convection to a gas maybe 20, 30, 40, may go up to a 100, 150 Watts per meter square

Kelvin, at very high velocities maybe 200 Watts per meter squared Kelvin. And similarly, if I have got single phase to a liquid, typical values of h in forced convection range from a few00 that is 200, 300, 400 at low velocities to maybe a few000 - 3000, 4000 - at reasonably high velocities used in industrial practice.

In contrast to this, in condensation and boiling in most of the types of condensation and boiling that takes place, values of h are invariably in thousands – 5000, 10000, 20000 Watts per meter squared Kelvin are typical values of h during condensation and boiling. There are some exceptions; I don't want to say it is always the case. There are some exceptions which you will point out when we go further but in general keep that in mind, values of h during a change of phase process that is during condensation and boiling are invariably in thousands. 1000, 2000 are low values; typically 5000, 10000, 20000 are values that we encounter all the time so, that is just one general comment.

Now, let us let us first of all study condensation; we are first of all going to study, take up the study of condensation. When does condensation occur? That is the first statement we want to make, that is the first the thing we want to clarify.

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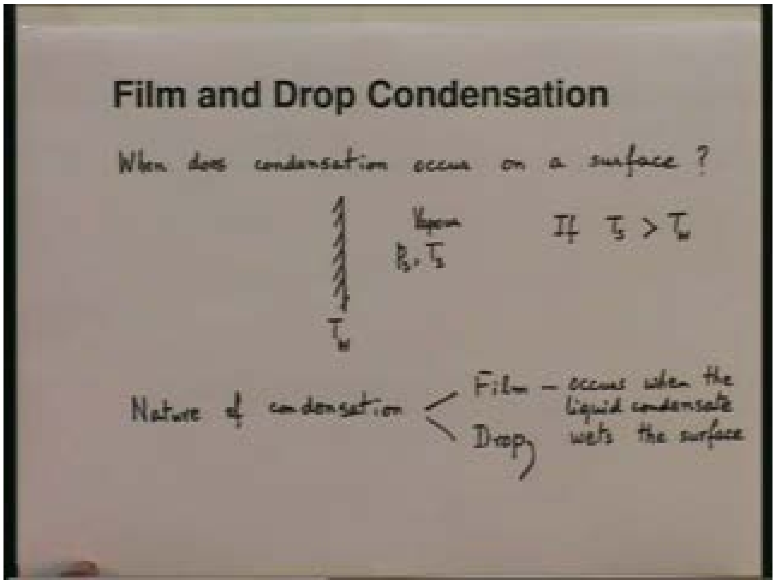

Film and Drop Condensation

When does condensation occur on a surface ?

If $T_s > T_w$

Nature of condensation < Film – occurs when the liquid condensate wets the surface
Dropwise

When does condensation occur on a surface, when? Let us say I have some surface, let us say it is a vertical surface and it is maintained at a temperature T_w with the help of some fluid flowing on this side, the back side. And on the right hand side I have a vapor, some vapor - it maybe steam or it may be some other vapor; it doesn’t matter. And let us say the saturation temperature of that vapor corresponding to the vapor pressure is T_s – it is a vapor which is at some pressure P_s and the saturation temperature corresponding to it is T_s. It is a saturated vapor, the pressure P_s and the temperature T_s.

Now if T_s is greater than T_w, that is the wall is maintained at a temperature lower than the saturation temperature of the vapor which is in contact with it, then this vapor is going to condense on this surface. So, condensation occurs when a vapor with a saturation temperature T_s comes in to contact with a solid surface which is maintained at a temperature T_w; T_w being less than T_s, that is the, that is when condensation occurs. Now, the nature of the condensation which occurs, the nature of the condensation which occurs or the type of condensation which occurs is of 2 types. It maybe of what we call as the film type which we call as film condensation or it may be in the form of droplets which we call as drop-wise or drop condensation.

So, the condensation which occurs on the surface when this condition T_s is greater T_w is satisfied maybe of 2 types - film condensation or drop condensation; film condensation occurs, film condensation occurs when? Occurs when the liquid condensate, when the liquid condensate wets the surface. The surface tension between the liquid and the solid is such that the liquid condensate wets the surface when that happens film condensation occurs. And the word film is descriptive enough; it means that the vapor on condensing forms some kind of film on that solid surface; that is why we call it film condensation. On the other hand, drop condensation or drop-wise condensation occurs when, let me write it on another page.

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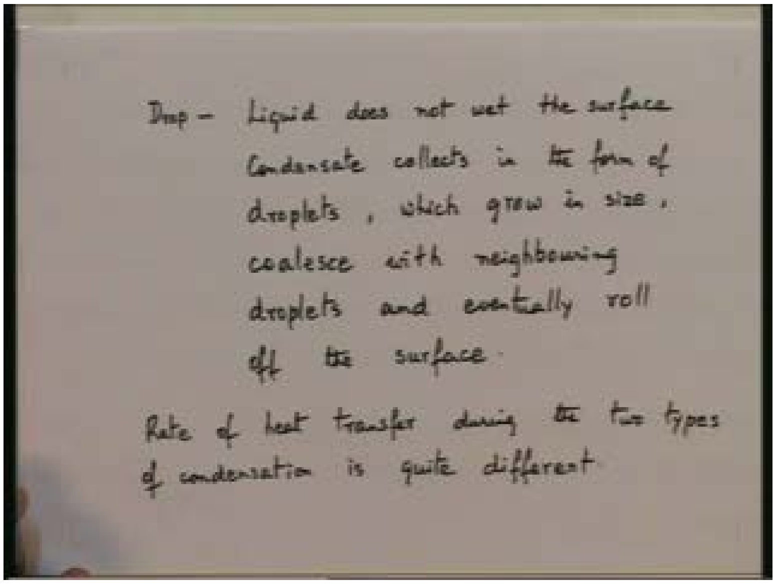

Drop – Liquid does not wet the surface
Condensate collects in the form of
droplets, which grow in size,
coalesce with neighbouring
droplets and eventually roll
off the surface.

Rate of heat transfer during the two types
of condensation is quite different

Drop condensation occurs when the liquid does not wet the surface. When the liquid formed during condensation does not wet the surface then it accumulates in the form of drops on that solid surface and we get drop-wise condensation. So, when the liquid does not wet the surface the condensate collects in the form of droplets. These droplets grow in size as more and more condensate forms on them which grow in size. Sometimes, they join up with each other; if they are near enough, coalesce with neighboring droplets and eventually when they become large enough, these droplets roll off the surface. This is the process of drop-wise condensation.

The requirement is the liquid condensate should not wet the surface; when it doesn't it form as droplets on the surface then the condensate forms, these droplets grow in size. If they are near enough to each other some of them join each other and coalesce and then when they become large enough. They cannot under the force of gravity, they may roll off the surface and drop off completely. So, these are 2 types of condensation which occur, which we observe from our daily needs also we can observe these 2 processes.

Now the interesting thing from a heat transfer standpoint is the rate of heat transfer during the 2 types is quite different, that is what we need to note. The rate of heat transfer during the 2 types of condensation is quite different, that is what we need to note.

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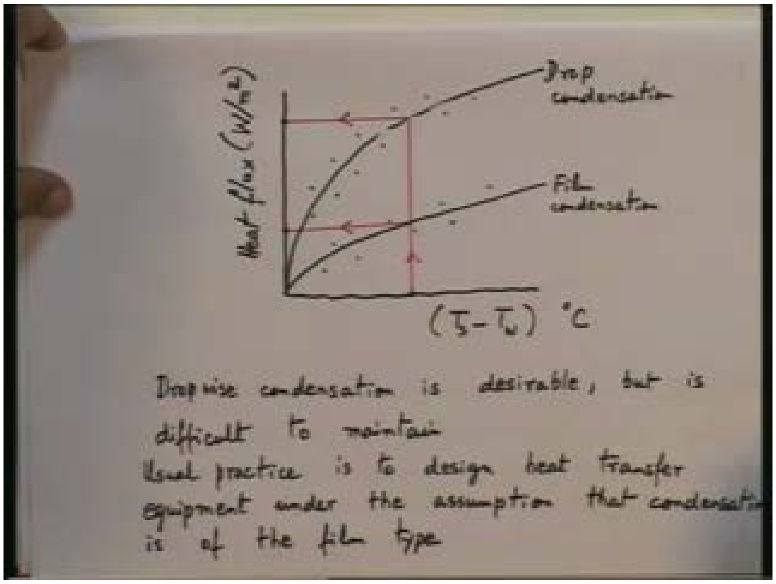

Drop wise condensation is desirable, but is
difficult to maintain
Usual practice is to design heat transfer
equipment under the assumption that condensati
is of the film type

Let us say I plot, let us say I plot a graph showing and on in this graph on the x axis I have T_s minus T_w. T_s is the saturation temperature of the vapor which is condensing on the solid surface and T_w is the wall temperature which is slightly less than T_s. Let us say on the x axis, I plot T_s minus T_w and on the y axis I plot the heat flux transferred during the condensation process, the heat flux in Watts per meter square, typically if you collect experimental data in one case for film condensation occurring on the surface, in the other case for drop condensation occurring on the surface. And through the experimental data you draw some curves typically in drop-wise condensation; you might get a curve something like this in film condensation, you might get a curve something like this. This would be film condensation and this would be drop condensation, these would be typical curves which you would get if you were to draw them through experimental data.

Let us say we will get experimental data like this; we will get experimental data in condensation like this and draw a smooth curve through them. You will get curve something like this. What does this mean? What these graphs, what these graphs are showing is for the same temperature difference; let us say for a particular temperature difference, some value - the heat flux during drop-wise condensation - would be so, much and for the same temperature difference the heat flux during film condensation would be so, much . If I go up for a particular value of temperature difference during film condensation I will get so, much heat flux and during drop-wise condensation I will get so, much heat flux. So, during drop-wise condensation one gets much higher heat fluxes for the same temperature difference. So, obviously from a heat transfer standpoint I would be interested in having drop-wise condensation.

That is the first one can see from this experimental data, that drop-wise condensation is desirable because it gives us higher heat fluxes for the same temperature difference. But what we find during practice is that in reality, suppose we have some condenser - maybe a steam condenser or may be a refrigerant condenser - and we start condensing either the refrigerant or steam or some other vapor on the surface which is being cooled. Typically, what we find is that while one may get drop-wise condensation for a few hours, it is difficult to maintain drop-wise condensation, does not continue to occur if you have the equipment running for hours and hours – hundreds of hundreds of hours of operation.

So, while it is desirable to have drop-wise condensation, it is difficult to maintain drop-wise condensation until one does some special things with the surface on which the condensation is occurring. For typical manufactured surfaces on which condensation takes place drop-wise condensation may occur initially but is difficult to maintain. Therefore, although from a heat transfer standpoint one would prefer to have drop-wise condensation, one designs heat transfer equipment during condensation under the assumption that the condensation is of the film type. So, keep that in mind that we would, drop-wise condensation may look desirable but we design heat transfer equipment under the assumption that the condensation is of the film type so, let me just make a few notes here.

Drop-wise condensation is desirable, that is the point I made. Drop-wise condensation, as we can see from the data shown in this figure but is difficult to maintain, difficult to maintain; I am repeating what I said - difficult to maintain. Therefore, the usual practice is to design heat transfer equipment; the usual practice is to design heat transfer equipment under the assumption that condensation is of the film type - that is what we do. Although it is desirable to have drop-wise condensation, it is difficult to maintain and therefore the usual practice is to design heat transfer equipment under the assumption that the condensation is film condensation. Now - mind you - even in film condensation, although the heat transfer coefficients are less than in drop-wise condensation, they are are still high. It is not as if they are very low; they are high but they are low compared what we get in drop-wise condensation.

So, from now onwards let us focus our attention on film condensation because as I said heat transfer equipment is designed under that assumption. We are going to first of all look at the classical problem of film condensation on a vertical plate. It is the problem which I sketched a moment ago and we will talk about that now in some detail.

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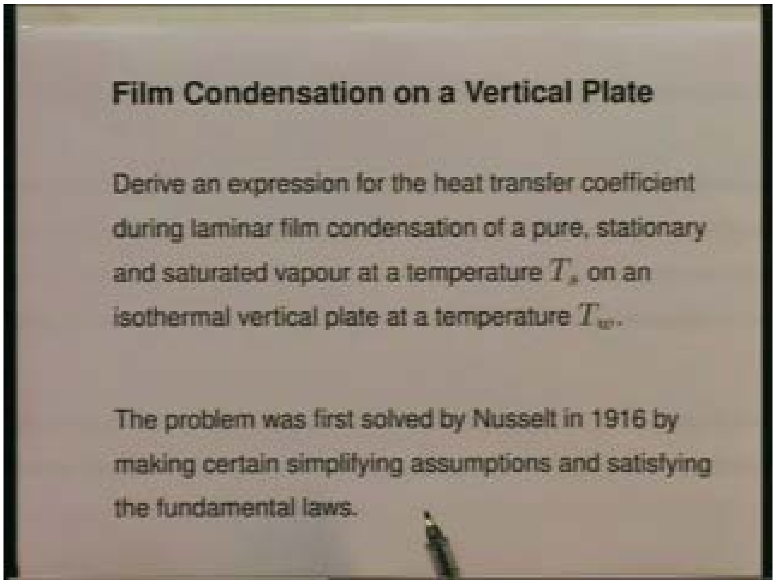

Film Condensation on a Vertical Plate

Derive an expression for the heat transfer coefficient during laminar film condensation of a pure, stationary and saturated vapour at a temperature T_s on an isothermal vertical plate at a temperature T_w.

The problem was first solved by Nusselt in 1916 by making certain simplifying assumptions and satisfying the fundamental laws.

We are going to, let me read out here, we are going to derive an expression for the heat transfer coefficient for h during laminar film condensation of a pure stationary and saturated vapor at a temperature T_s on an isothermal vertical plate at a temperature T_w, that is what we are going to do. This is the classical problem which was, this problem was first solved by Nusselt in Germany in 1916; it is the classical problem of condensation and we are going to derive the expression which was derived then so, many years ago. Let me again repeat that. Derive an expression for h - the heat transfer coefficient - during laminar film condensation of a pure stationary and saturated vapor at a temperature T_s which is condensing on an isothermal vertical surface at a temperature T_w.

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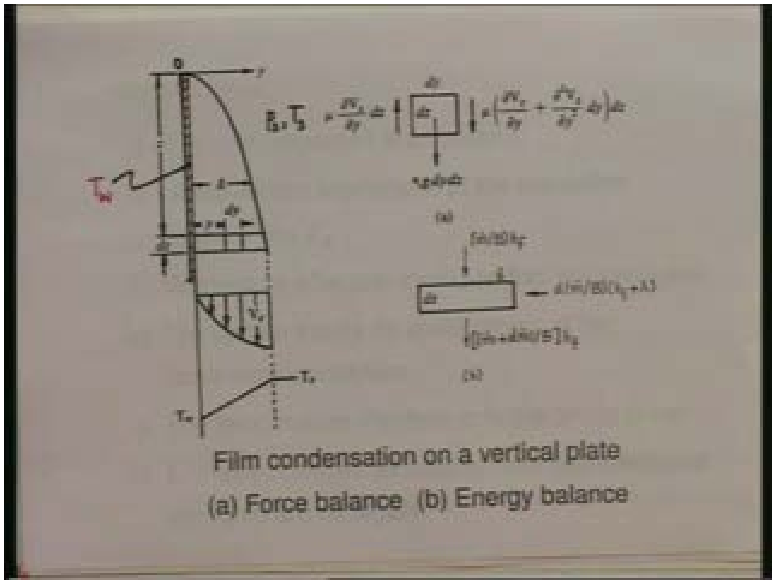

Film condensation on a vertical plate
(a) Force balance (b) Energy balance

Let us just look at the geometry a little for your, for the moment focus your attention on the left hand side not on the whole figure which I am displaying or rather let us cover the rest of the figure so, that your attention is not diverted. This is a plate, a vertical plate here, and just to show the plate let us draw a double line here. This is a plate like this; this is a plate, a vertical plate, and it has, it is being maintained at a temperature T_w, maintained at a temperature T_w, a vertical plate being maintained at a temperature T_w. And on the right side here, on the right side coming in contact with it is a pure stationary saturated vapor at a pressure P_s and a corresponding temperature T_s and of course T_s is greater than T_w .

So the vapor condenses on this plate because T_s is greater than T_w and it condenses in the form of a film. We will assume that film condensation is taking place and the film thickness here is shown, exaggerated actually. It is a very thin film that forms on the surface and flows down but I am showing it here purposely exaggerated because we want to understand the fluid mechanics of that film, its thickness, etcetera. So, in order to study it, we are showing the thick of the film delta as a very exaggerated film thickness here.

The film, obviously condensation begins from the top so, the film will start with zero thickness at the top and will grow in thickness as it as we go downwards. As I say, it is the delta, the thickness of the film is exaggerated in the sketch. The coordinate system we will adopt is y, z; y is direction at right angles to this vertical plate, z is the direction along the plate and the origin is right here at the top. So, z is measured from the top of the plate positive downwards, z is measured from the top of the plate positive in the downward direction and the direction at right angles to the paper, the direction at right angles to the paper, it will be obviously the direction x. And let us assume that the plate is of width B, B at right angles to the paper - some constant value B. So, it is a plate of width B on which condensation is taking place and we want to analyze the process of this condensation.

As I said a moment ago, this problem was first analyzed by Nusselt and he analyzed it by making a number of simplifying assumptions and basically applying the fundamental laws. What are the fundamental laws? The fundamental laws are conservation of mass that is the equation of continuity, the Newton’s second law of motion and the first law of thermodynamics. So, that is all that, I mean, Nusselt applied to this problem with certain simplifying assumptions in order to obtain the value of h for this particular situation. Now, we are going to go through that process of derivation. The assumptions which we are going to make are the following, the assumption which we are going to make are the following. I will first list them one by one and then of course as we go along I will justify why they are being made.

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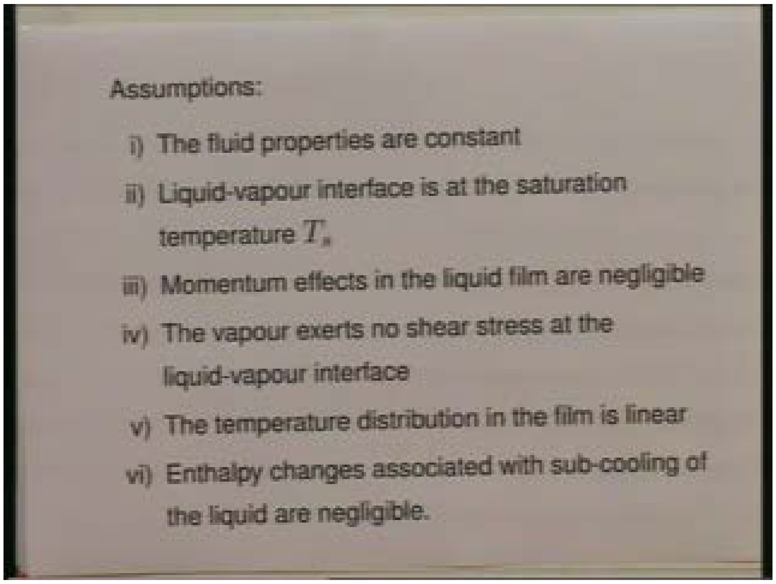

Assumptions:

- i) The fluid properties are constant
- ii) Liquid-vapour interface is at the saturation temperature T_s
- iii) Momentum effects in the liquid film are negligible
- iv) The vapour exerts no shear stress at the liquid-vapour interface
- v) The temperature distribution in the film is linear
- vi) Enthalpy changes associated with sub-cooling of the liquid are negligible.

First of all the assumption we are going to make is - the fluid properties are constant. During the analysis, we will need properties like the conductivity of the fluid, the density of the fluid, the viscosity of the fluid, etcetera. We assume some constant value for these properties. If I go back to the sketch, the temperature of the liquid condensate is going to vary a little inside. Here it will be T_s at this point, T_w at this point. Therefore, if temperature varies, properties are going to vary a little but this variation for properties will be exceedingly small and therefore it is a good assumption to say that we can take some constant value of the properties. It is a very, an extremely good assumption to make for this particular problem.

The second assumption we are going to make is, the liquid vapor interface is at the saturation temperature T_s, liquid vapor interface is at the saturation temperature T_s. What do we mean by this? Let me again go back to the sketch. This is the condensate forming this curve line, this is the condensate film surface so, this is the interface right here. This, at this interface the temperature is the saturation temperature T_s, all along this liquid vapor interface the temperature is T_s which is also a good assumption to me.

The third assumption is that the liquid momentum effects in the liquid film are negligible. Now, what do we mean by this? What we mean is the following - momentum effects in the liquid film are negligible. Here is the liquid film; it grows in thickness and the liquid is flowing downwards like this right in this film, because of the action of the gravity it is flowing downwards along this vertical plane. When it flows downwards, it has a velocity and if something has a velocity it has momentum. What we are saying is when you apply Newton’s second law of motion to a control volume there are terms in that equation which say that the net forces in a particular direction are equal to the momentum change momentum flowing out of the control volume minus the momentum flowing into the control volume. Let me repeat that. Newton’s second law as applied to a control volume says net forces in a particular direction are equal to the momentum flowing out of the control volume minus momentum flowing into the control volume.

What we are saying for this particular problem is that the momentum effects - the change of momentum flowing out minus momentum flowing in - is negligible. In any control volume that I have selected inside this liquid film momentum effects are negligible. Therefore, in effect, what we are saying is the net forces acting on a the surface or the net body forces and surface forces acting on any control volume are equal to zero. Momentum effects need not be taken into account; we will come back to this when we take the particular control volume in a moment. so, we will neglect momentum effects inside of the liquid in the liquid film.

Assumption 4 - the vapor exerts no shear stress, vapor exerts no shear stress at the liquid vapor interface. What we mean again by this is the following. As this liquid flows down, it has a velocity and that velocity is relative to the stationary vapor which is next to it. So, obviously, if something is moving, the liquid is moving and the vapor is stationary, one thing must exert a shear stress on the other. So, there has to be a shear stress at this liquid vapor interface because there is relative motion between the liquid and the vapor in the vertical direction. What we are saying is, this shear stress is negligible again, probably an exceedingly good assumption to make for the physical situation that we are talking about.

Then the next assumption which Nusselt made and which we are making is the temperature distribution in the film is linear. The film is very thin, the liquid film as I said a moment ago, which is condensate film formed is a very thin film and Nusselt said let us assume that the temperature variation in this liquid film is linear, it is a straight line. So, we will make that same assumption that within the liquid film the drop from T_s to T_w over the thickness of the liquid film, which is delta, will be a straight line. Again I have probably an exceedingly good assumption to make.

And final assumption which Nusselt made was, he said enthalpy changes associated with sub-cooling of the liquid are negligible. What he meant was the vapor first condenses at the liquid vapor interface, vapor first condenses at the liquid vapor interface where the temperature is T_s but the liquid inside, its temperature drops from T_s to T_w inside the film and T_w is less than $T_{s\ so,}$ that is the sub-cooling that is taking place in the liquid condensing. Nusselt said these enthalpy changes associated with sub-cooling from T_s to T_w are negligible compared to the latent heat lambda that we will have to take account of during the change for phase and that probably is also an exceedingly good assumption to make.

So, these were the 6 assumptions that Nusselt made - fluid properties are constant, number 2 -liquid vapor interface is the saturation temperature T_s, momentum effects in the liquid film are negligible, the vapor exerts no shear stress at the liquid vapor interface, the temperature distribution in the film liquid film is linear and finally enthalpy changes associated with sub-cooling of the liquid are negligible. These were the assumptions which Nusselt made. Now, straightway you can see that if I look at assumption number 2 which says that the liquid vapor interface is at the temperature T_s and if I look at assumption number 5 which says the temperature distribution in the liquid film is linear and if I combine these 2, then I can see that the temperature profile in the liquid film is going to be like this.

In the vapor space, the temperature is going to be constant T_s and then at the wall it is T_w, $_{so,}$ across the liquid film I am going to get a straight line like this. So, the temperature distribution in the liquid film and in the vapor space based on assumptions 2 and 5 will be as shown. As I am showing you just now, that is where we have used assumption 2 and assumption 5.

Now let us move on; let us apply our fundamental laws. Now let us consider, let us consider this is our liquid film. I go back to my sketch, this is my liquid film. Let us say I consider inside the liquid film a differential control volume dy dz which I am showing just now, shading. That means in the y direction at right angles to the plate inside the liquid film, we take a differential dy and in the z direction that is moving downwards we take a distance dz. In the x direction of course, we have a width B. It is a 2 dimensional problem so, we have a width B in the x direction. so, take a differential control volume dy dz inside the liquid condensate film.

Let us apply Newton's second law to this control volume. What does Newton's second law state? Newton's second law will state - if I want to apply it for the z in direction - the direction of flow - it will, say net z direction forces acting on this control volume on the liquid inside this control volume, net z direction forces acting on the liquid. This control volume are equal to the rate at which z direction momentum leaves this control volume minus the rate at z direction momentum enters this control volume. That is what Newton's second will state if applied to this control volume.

We have already made an assumption; what are that assumptions? The assumption was that, let me repeat it and show it here. The assumption was, number 3 assumption was, momentum effects in the liquid film are negligible. That means we are saying we will neglect the difference between the momentum flowing out and the momentum flowing into this differential control volume dy dz. So, what are we left with? Therefore, if we do make that assumption, we will now get, let me write that down.

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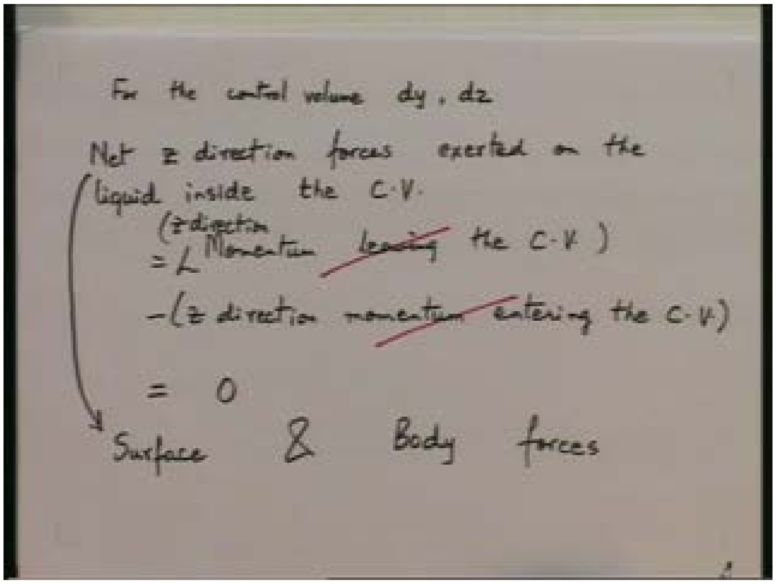

For the control volume dy . dz

Net z direction forces exerted on the liquid inside the C.V.

= (z direction Momentum ~~leaving~~ the C.V.)

−(z direction ~~momentum~~ entering the C.V.)

= 0

Surface & Body forces

Now, what will Newton’s second law as applied to this control volume for the control volume dy dz? Newton’s second law states net z direction forces, net z direction forces exerted on the liquid on the condensate liquid inside the control volume equal to momentum z direction momentum leaving z direction, momentum leaving the control volume minus z direction momentum entering the control volume, that is Newton’s second law as applied to the control volume.

In this particular case the terms on the right hand side of this equation we are saying are, the difference between them is negligible, momentum effects are negligible. Therefore, we are saying net z direction forces exerted on the liquid inside the control volume is equal to zero. Now, what are the net z direction forces exerted on the liquid are of 2 types - surface forces and body force. What are the surface forces in this case? Let me say the z direction forces are surface forces and body forces. Now, let us look at the control volume again. Here is my control volume dy dz; the body forces exerted on this is the force of gravity - the weight of the liquid, what is that? Volume of the liquid - the mass of the liquid multiplied by the acceleration due to gravity; so, what is the mass of the liquid? rho into dy into dz; if I take unit width at right angles rho into dy dz is the mass of the liquid - density into volume - and if I multiply it by the acceleration due to gravity g I get the force of gravity.

So, this is the body force being exerted on this liquid; so, here is an expended view of this, of this control volume. This is an expended view of this control volume and the body force is rho g dy dz. What are the surface forces in the z direction? The surface forces in the z direction are the shear stresses being exerted on the 2 vertical faces which, the 2 vertical dz faces - these are the shear forces, these are the ones in the z direction. So, what are the expressions for shear forces on this face? It will be mu dVz dy - this is our Stokes law relating stress and rate of strain - and on this face, because from this point I have moved dy, a distance dy forward in the y direction, so this mu dVz dy becomes, gets increased by certain amount because I move a distance dy forward in the positive y direction. So, it becomes mu into dVz dy plus d 2 V_z dy squared into dy the whole thing multiplied by the area dz. So, the 2 shear forces acting on in the z direction on the 2 faces - vertical faces of this control volume - are this one which is upwards and this one which is downward. so, these are the forces, the 2 are the surface forces, and this is the body forces acting on in the z direction on the liquid inside this control volume dy dz.

So put all this down and we get our force balance. If I put all these force down, I will get my force balance and I get the equation by applying; I will get, let us put that down. What are the net z direction forces? I will get first the body force.

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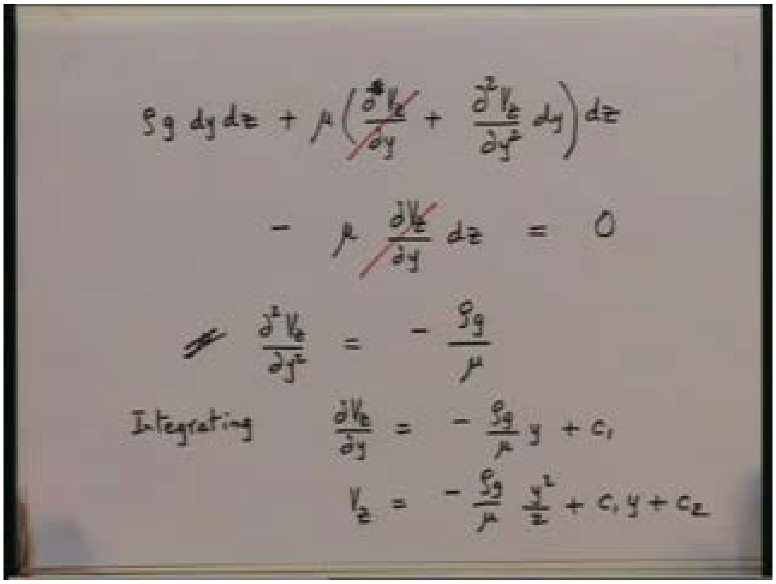

$$\rho g\, dy\, dz + \mu\left(\cancel{\frac{\partial V_z}{\partial y}} + \frac{\partial^2 V_z}{\partial y^2} dy\right) dz$$

$$- \mu \cancel{\frac{\partial V_z}{\partial y}} dz = 0$$

$$\cancel{\mu} \frac{\partial^2 V_z}{\partial y^2} = - \frac{\rho g}{\mu}$$

Integrating $\frac{\partial V_z}{\partial y} = - \frac{\rho g}{\mu} y + c_1$

$$V_z = - \frac{\rho g}{\mu} \frac{y^2}{2} + c_1 y + c_2$$

I will get rho g dy dz, that is the body force in the z direction, plus the shear force in the positive direction mu d 2 V_z dy, sorry, d dVz dy plus d 2 V_z dy squared dy multiplied by dz minus the force which is acting in the negative direction - negative z direction - that is mu dVz dy dz. So, this is the body force, this is the second one, is the shear force in the positive y direction on one face - one vertical face - and this is the other shear force acting in the opposite direction on the other face of the control volume.

Newton’s second law says if you neglect momentum effects the sum of all these must be equal to zero. So, if I simplify this, the obvious, this term will cancel with this term so, if I simplify I am simply going to get mu d 2 V_z dy squared is equal to; sorry, I will take the mu on this side is equal to minus rho g by mu. I am simply going to get d 2 V_z dy squared is equal to minus rho g by mu on simplification. So, the application of Newton’s second law to this control volume dy dz with the simplification that momentum effects in and out of the control volume are negligible then gives gives me this very simple differential equation.

Let us integrate this; integrating it is a second order equation so I will integrate it twice. I integrate it once, I will get dVz dy is equal to V_z – it is of course the velocity in the z direction of the liquid dVz dy; it is equal to minus rho g by mu into y plus c 1. And if I integrate it a second time, I will get V_z the velocity in the z direction of the liquid condensate is equal to minus rho g by mu y squared by 2 plus c 1 y plus c 2, that is what I get on integration. So, I need 2 boundary conditions in order to get the velocity profile of the liquid condensate. What are my 2 boundary conditions? My 2 boundary conditions are at y equal to zero the no slip condition

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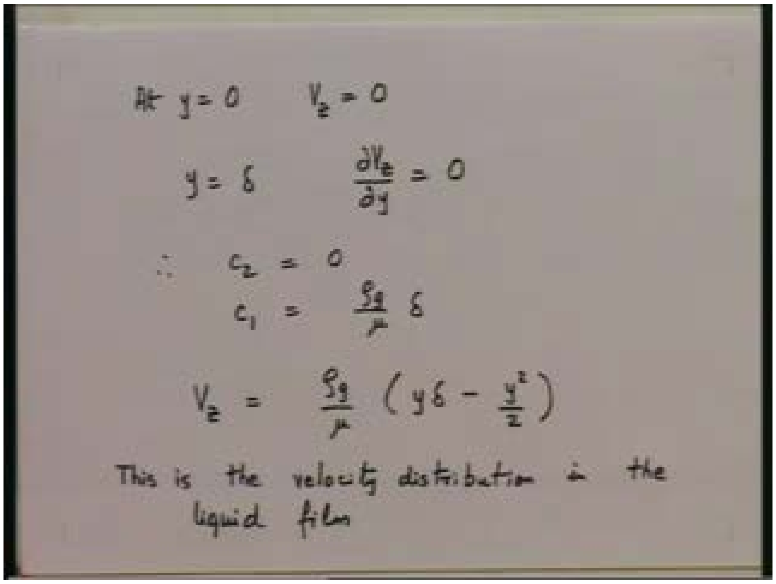

At $y = 0 \quad V_z = 0$

$y = \delta \quad \frac{\partial V_z}{\partial y} = 0$

$\therefore \quad c_2 = 0$

$c_1 = \frac{\rho g}{\mu} \delta$

$$V_z = \frac{\rho g}{\mu}\left(y\delta - \frac{y^2}{2}\right)$$

This is the velocity distribution in the liquid film

So, at y equal to zero, V_z is equal to zero - typical fluid mechanics condition - and at y is equal to delta. Let us now use the fact that the vapor exerts no shear stress at the liquid vapor interface so if there is no shear stress being exerted at the liquid vapor interface, it follows that dVz - by Stokes law - dVz dy, there is no shear stress at the liquid interface, then dVz dy must be equal to zero. So, these are my 2 boundary conditions - one at y is equal to zero and at one is at y is equal to delta which I put in.

If I take both these boundary conditions, then I can show that the constant c_2 - I am now leaving out a few simple steps - you can show that c_2 is equal to zero and c_1 is equal to rho g by mu delta; these are the 2 constants of the integration. Therefore, the velocity profile of the liquid condensate is given by V_z is equal to rho g by mu multiplied by y delta minus y squared by 2 - this is what we get for the velocity profile of the liquid condensate inside the liquid film. This is, let me write that down, this is the velocity distribution of, in the liquid film V_z equal to zero at y equal to zero and V is dVz dy equal to zero at y equal to delta so this is my velocity distribution.

Let me go back to the sketch to show you the liquid, the velocity distribution here it is on this sketch. Now, you can see the nature of the distribution that we have got is V_z is equal to zero at y equal to zero and dVz dy z is equal to zero at y equal to delta so we have a parabolic distribution of this form. That is the equation that we have got for velocity distribution after applying Newton's second law of motion. Now, once I have the velocity profile, I can calculate the mass flow rate at the condensate so what is the mass flow rate of the condensate? Mass flow rate of condensate - if I ask you at a distance z at a cross section z, what is the rate at which mass is flowing in the liquid at a distance z?

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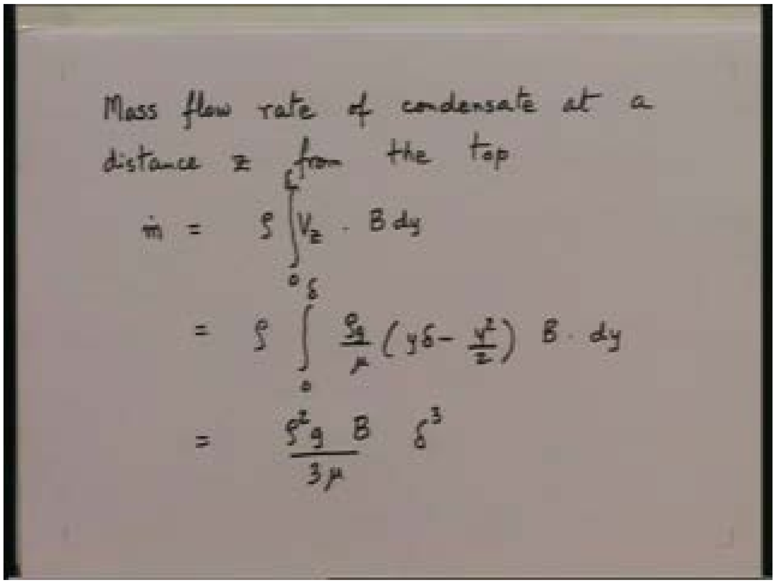

Mass flow rate of condensate at a distance z from the top

$$\dot{m} = \rho \int_0^\delta V_z \cdot B\,dy$$

$$= \rho \int_0^\delta \frac{\rho g}{\mu}\left(y\delta - \frac{y^2}{2}\right) B \cdot dy$$

$$= \frac{\rho^2 g\, B}{3\mu}\,\delta^3$$

Then, you will say mass flow rate using the velocity distribution, mass flow rate of condensate at a distance z from the top - if I call that as m dot - then it will be equal to, let us look at the sketch again now at a distance z I want to know the mass flow rate. Suppose I ask you what is the mass flow rate flowing through this area dy and in this x direction. It is B so B into dy is the area, the velocity is V_z and the density of the liquid is rho so through a distance dy and an area B dy the flow rate will be rho multiplied by V_z multiplied by B dy rho into velocity into area.

I want the value - the mass flow rate in kilograms per second - across the whole thickness delta therefore I will integrate this expression from zero to delta to get the mass flow rate. And if I integrate this expression I will get rho integral zero to delta. For V_z, now I will substitute T velocity distribution that we had got. What was the velocity distribution? rho g by mu y delta minus y squared by 2, that was the velocity distribution for V_z into B which is a constant dy.

Perform the integration. If you perform the integration you will get rho squared g into B - these are all constants - upon mu and I am doing the integration and putting the limits zero to delta. We will get a 3 in the denominator and a delta cubed in the numerator. So, the net result of the integration is rho squared g B delta cubed upon 3 mu, that is the mass flow rate in the condensate film.

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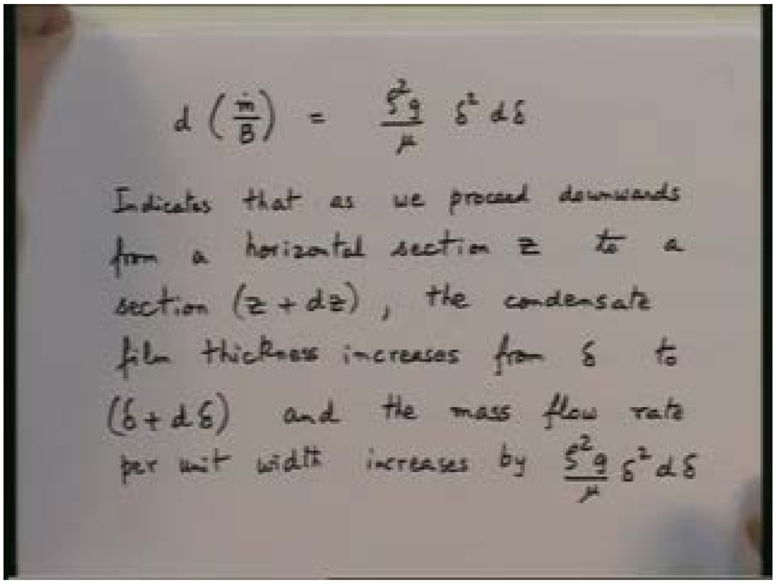

$$d\left(\frac{\dot{m}}{B}\right) = \frac{\rho^2 g}{\mu}\,\delta^2\, d\delta$$

Indicates that as we proceed downwards from a horizontal section z to a section $(z + dz)$, the condensate film thickness increases from δ to $(\delta + d\delta)$ and the mass flow rate per unit width increases by $\frac{\rho^2 g}{\mu}\,\delta^2\, d\delta$

Now let us take a differential of that; if I take a differential of that, take a differential of that, I will get d m dot by B. This is mass flow rate per unit width at right angles to the paper is equal to rho squared g by mu into delta squared d delta if I take a differential. So, what I am saying? Then, I take a differential; what I am saying in effect is the following. I am saying this differential indicates that, as we proceed, indicates this will be the result that we have got, indicates that as we proceed downwards from a horizontal section z to a section z plus dz. If we go down a distance dz in the positive z direction the condensate film thickness, the condensate film thickness increases from delta to delta plus d delta with a differential amount and the mass flow rate per unit width and the mass flow rate per unit width increases by rho squared g by mu delta squared d delta.

That is what we are saying by applying, by taking the differential of the condensate mass flow rate or in other words what we are saying is - let me again go back to the sketch I showed you - if I go from across the cross section z here, I know m dot by B, if I go down a distance here the m dot by B increases by certain amount and why is that increase occurring? Because condensation is taking place, so in other words, the condensation rate per unit width from z to dz is rho squared g delta squared d delta upon mu or in other words what we are saying, let me write that down in other words.

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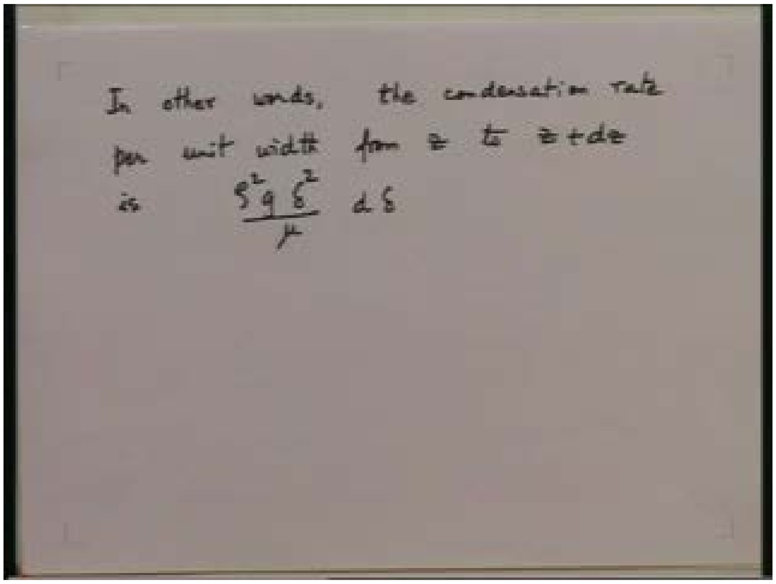

In other words, the condensation rate per unit width from z to $z+dz$ is $\frac{\rho^2 g \delta^2}{\mu} d\delta$

The condensation rate per unit width per unit width from z to z plus dz is rho square g delta squared upon mu d delta rho squared g delta squared d delta upon rho squared g delta squared upon mu into d delta that is the condensation rate per unit width from z to z plus dz.

So, now we know the condensation, we got a velocity profile in the liquid condensate, we have got the mass flow rate so we effectively applied conservation of mass and Newton’s second law. What remains now is to apply the energy equation the first law thermodynamics. We will stop at this stage and next time we will apply the energy equation to get an expression for the heat transfer coefficient and for the thickness of the condensate film.