Heat and Mass Transfer Prof. S. P. Sukhatme Department of Mechanical Engineering Indian Institute of Technology, Bombay Lecture No. 28 Heat Exchanger – 4

So, we have discussed the mean temperature difference method for analyzing heat exchangers. We derived expressions for the mean temperature difference in various flow arrangements, for various flow arrangements like parallel flow, counter flow, all the 3 cases of cross flow. And once we had those expressions or the results in the form of graphs, we were able to show through numerical examples how one does calculations using the basic performance equation q is equal to UA delta T_m . Now today, we will look another approach, discuss a method called the effectiveness NTU method.

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We are going to discuss a method called the effectiveness NTU method. Now, first of all I want to give, discuss the types of problems that we encounter and why the effectiveness NTU method has is at all thought of.

So, let us first of all just state 2 types of problems that we generally encounter in the case of heat exchangers. Generally, we encounter 2 types of problems and these problems are like are as follows. In one type, we are given 2 flow rates. We are given the flow rates and the inlet and outlet temperature on the 2 sides that is m dot h T_{hi} , T_{he} , m dot c T_{ci} and T_{ce} ; given the flow rates on both sides and the inlet and outlet temperatures on the 2 sides that outlet temperatures on the 2 sides and we are given; for U all these quantities are given to us and we asked find the value of A. What is A? That is a typical problem and that is the type of problem which we have done in the previous lecture.

Let me just outline this a little so, that it is seen clearly. We are given the data which I am going out, just put red boundary on, red lines around it, this is the data given and we have to find the area A of the heat exchange. This is one type of problem; I call this, as let us say call it type 1 and as I said we have already solved problems of this type using the mean temperature difference approach with no real difficulty. The second type of problem, let us call that as type 2. We are given in the second type of problem, we are given 2 fluids. When I say 2 fluids it means we know the properties of the fluids. You know we may need the specific heat or whatever other properties might be needed density, etcetera.

So, we are given 2 fluids, we are given a heat exchanger A, we are given some heat exchanger whose area is specified, a heat exchanger whose area A is specified. We are given the flow rates on the 2 sides - m dot h and m dot c - and we are given the inlet temperatures on the 2 sides - T_{hi} , T_{ci} . And we are asked to find the outlet temperatures on the 2 sides - T_{he} , T_{ce} ; that is a second type of problem. Let me again just outline, put a red boundary around it to show that this is the second type of problem we are talking about. Given 2 fluids, given a certain heat exchanger with an area A and given 2 fluids entering with flow rates m dot h and m dot c and 2 inlet temperatures T_{hi} and T_{ci} , what are the exit temperatures of these 2 fluids? So, if you want to describe in words what these 2 problems are - type 1 is a problem in which we are sizing the heat exchanger for a given job; we are wanting a certain amount of heat to be transferred and we are saying what is the area of A required for this job. Type 2 is a problem in which we are given a heat

exchanger of a certain size and we are asked to calculate its performance for certain given inlet conditions and flow rates, inlet conditions of temperature and flow rates. Now, let us try to apply the mean temperature difference approach to both these problems.

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For the first type of problem, apply the mean temperature difference approach for both these problems, for the first type our basic equation is q is equal to U A delta T_m and if I want to calculate the area A, I will say A is equal to q upon U delta T_m . In the previous lecture, we have such done such problems by the mean temperature difference method. We have calculated the mean temperature difference depending on the flow configuration. Once we have got it, we have used this basic equation to get the area A of the heat exchanger.

Now, let us look at the second type problem; for the second type of problem if use, put down again my basic equation. A basic performance equation - q is equal to U A delta T_m . Well, you can see U is known, from my knowledge of convection A, is known because the size of the heat exchanger is given but q. In order to calculate q, I will need an exit temperature on one side or the other that is not known and delta T_m cannot be calculated because both the exit temperatures are not known. So, both q and delta T_m are

unknowns in this equation and I will need, we will need a trial and error approach to solve this type of problem. What do I mean by trial and error approach? What I mean is the following.

Assume let us say a value of one of the exit temperatures - T _{he}. The moment I assume T_{he} , I can calculate T_c by heat balance; the transferred from the hot side is equal to heat gained by the cold side. So, if I assume one exit temperature, the other can be calculated. Once I get both the exit temperatures and I know both the inlet temperature, I calculate delta T_m . Then, I see from my basic equation whether q is equal to UA delta T_m , whether q on the left hand side is equal to the right hand side. If it is not, go back and assume some other value of T_c ; so, I will need a trial and error approach. Guess value of T_c , go through the calculation, check if the left hand side is equal to the right hand side.

Now, the trial and error approach can be avoided if we adopt the effectiveness, the alternative method which is the effectiveness NTU method. So, one reason for introduction of this new method is that we need, don't need a trial and error approach for this second type of problem. So, let me write that down; the trial and error can be avoided if we adopt the alternative method called the effectiveness NTU method.

So, now let us look at what this method is all about. So, this is one reason why we are looking for an alternative method that for certain types of problems, the q equal to UA delta T_m method would require a trial and error approach. Effectiveness NTU approach would not require a trial and error approach. But, that is not the only reason. By the way, the effectiveness NTU method has some advantages. It is a dimensionless way also of looking at the performance of a heat exchanger and dimensionless approaches always have certain benefits compared to dimensional approaches. So, that is also an added reason why we developed this new approach called the effectiveness NTU method; now let us start with the method.

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The Effectiveness - NTU Method (Contd) Maximum possible heat transfer rate $\frac{\dot{m}_{h}C_{ph}(T_{hi} - T_{he})}{(\dot{m}C_{p})_{s}(T_{hi} - T_{ci})}$ $\frac{\dot{m}_{c}C_{pc}(T_{cc}-T_{ci})}{(\dot{m}C_{p})_{s}(T_{hi}-T_{ci})}$

The first quantity, we want to define a quantity first called the effectiveness. We say effectiveness of a heat exchanger is equal to the rate of heat transfer in the heat exchanger divided by the maximum possible heat transfer rate, that is what, how we define the effectiveness of heat exchanger - the actual rate of heat transfer in the heat exchanger divided by the maximum possible heat transfer rate. We will use the symbol epsilon for effectiveness so, we will say epsilon is equal to q, the heat transfer rate in the heat exchanger, divided by maximum possible which we will call as q max.

Now, what is q? q is nothing but m dot h C_{ph} into T_{hi} minus T_{he} or it is m dot c C_{pc} into T_{ce} minus T_{ci} - that is the loss in enthalpy on the hot side, the gain in enthalpy on the cold side. So, the numerator can be specified either in terms of the hot side flow rate and it is change in temperature or the cold side flow rate and it is change in temperature - both are equivalent expressions. Now, in the denominator notice what we have put down is m dot C_{ps} multiplied by T_{hi} minus T_{ci} . T_{hi} is the entering temperature on the hot side, T_{ci} is the entering temperature on the hot side - these represent the maximum and the minimum temperatures in the heat exchanger.

Let us say if I draw a scale here - this is T_{hi} and this is T_{ci} - this is where the hot, this is the temperature level at which the fluid enters; this is the temperature level at which the cold fluid enters. All other temperatures in the heat exchanger must be within this range so, T_{ce} or T_{he} have to be within this range; obviously, the maximum possible heat transfer rate therefore is obtained. If I get the change of temperature of one of the fluids to be T_{hi} minus T_{ci} so, I say, what is the maximum possible change of temperature in the heat exchanger T_{hi} minus T_{ci} ? And what flow rate and heat capacity do I multiply it by? I must multiply it by the lower of the 2 m dot C_{ps} that is I have m dot h C_{ph} , m dot c C_{pc} the smaller of the 2 is what I need to multiply it by not the larger of the 2, to get the maximum note. Why is that? If I take the larger of the 2, this stands for the smaller of the 2 m dot C_{ps} . If I take this larger of the two, you will quickly find out that either T_{he} or T_{ce} will lie outside this band of T_{hi} to T_{ci} .

Take any situation, specify 2 values of T_{hi} and T_{ci} , take some values of m dot h, m dot c, take the larger of the 2 here and you will quickly see if I take the larger of the 2 in the denominator you will; either T_{he} or T_{ce} will lie outside this band; they will not lie within this band which is an upper and lower limit for temperatures in the heat exchanger. So, q max is equal to the smaller of the 2 - m dot C_p ; smaller of the two means calculate m dot h C_{ph} . Calculate m dot c C_{pc} . The smaller of the two is m dot C_p is multiplied by the maximum possible change in temperature that is T_{hi} minus T_{ci} . So, that is how we define the term called as effectiveness. Now, let us say that m dot h C_{ph} is less than m dot c C_{pc} . (Refer Slide Time: 16:31)

Hence if
$$\overline{m}_{h} C_{ph} < \overline{m}_{e} C_{pc}$$
, then $\overline{m}_{h} C_{ph} = (\underline{c} C_{p})_{s}$
 $\varepsilon = \frac{T_{hi} - T_{hi}}{T_{hi} - T_{ci}}$
if $\overline{m}_{e} C_{pc} < \overline{m}_{h} C_{ph}$, then $\overline{m}_{e} C_{pc} = (\underline{c} C_{p})_{s}$
 $\varepsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$
Note:
i Two definitions are equivalent when $\overline{m}_{h} C_{ph} = \overline{m}_{e} C_{pc}$
2. By definition $0 \le \varepsilon \le 1$

Let us say we have a situation in which m dot h C_{ph} is less than m dot c C_{pc} . Therefore, m dot h C_{ph} is equal to m dot C_{ps} - it is a smaller of the two so, it is m dot C_{ps} . In that case, looking back to the definition you will see quickly epsilon becomes equal to T_{hi} minus T_{he} divided by T_{hi} minus T_{ci} - that is how definition of effectiveness. On the other hand, if m dot c C_{pc} is less than m do h C_{ph} then m dot c C_{pc} is equal to m dot C_{ps} and the definition of epsilon becomes epsilon becomes equal to T_{ce} minus T_{ci} divided by T_{hi} minus T_{ci} - that is how the definition of effectiveness. So, depending upon which is the smaller of the heat capacity rates, m dot h C_{ph} or m dot c C_{pc} , we will get this definition of effectiveness or this definition of effectiveness.

Note the two definitions are equivalent when m dot h C_{ph} is equal to m dot c C_{pc} why because than T_{hi} minus T_{he} will be equal to T_{ce} minus T_{ci} so, it doesn't matter which definition you use they are equivalent. Also note, by definition, this is one thing; second thing to note is, by definition epsilon is dimensionless and epsilon is a number which is by definition bounded between zero and 1, zero less than or equal to epsilon less than or equal to 1. So, epsilon is a dimensionless quantity telling us something about the performance of the heat exchanger and it must lie between zero and 1 because in the denominator we have the maximum possible heat transfer rate in the heat exchanger. So, now we have defined a term called the effectiveness and what we are going to do next is the following.

We are going to say, suppose I have a parallel flow heat exchanger.

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Let us say I have a parallel flow heat exchanger. I want to derive an expression for the effectiveness of a parallel flow heat exchanger derive and expression for the effectiveness in parallel flow. So, let us start, let us say assume m dot h C_{ph} is equal to m dot C_{ps} , then it follows that epsilon is equal to T_{hi} minus T_{he} divided by T_{hi} minus T_{ci} , that is the definition if m dot h C_{ph} is the smaller heat capacity rate. Multiply the numerator and denominator by - in this case by 1 plus m dot h C_{ph} divided by m dot c C_{pc} .

So, multiply this by 1 plus m dot h C_{ph} upon m dot c C_{pc} and divide by, I can always do that, 1 plus m dot h C_{ph} upon m dot c C_{pc} multiplying and dividing by the same expression. Now, this is equal to - now multiply this end T_{hi} minus T_{he} into 1 gives me T_{hi} minus T_{he} upon T_{hi} minus T_{ci} . I am multiplying by 1 than multiplying T_{hi} minus T_{he} upon T_{hi} minus T_{ci} into m dot h C_{ph} upon m dot c C_{pc} . The 2 numerators multiply T_{hi} minus T_{he} into m dot h C_{ph} is q, q divided by m dot closed C_{pc} is T_{ce} minus T_{ci} . So, I can say this is plus T_{ce} minus T_{ci} divided by T_{hi} minus T_{ci} the whole thing divided by 1 plus m dot h C_{ph} upon m dot c C_{pc} which I can further write as equal to 1 minus T_{he} minus T_{ce} later divided by T_{hi} minus T_{ci} the whole thing divided again by 1 plus m dot C_{ph} upon m dot c C_{pc} .

Now, if we go back to our derivation of the mean temperature difference in parallel flow, if I go back to notes, there you will see we derived an expression at that point. Based on the integration which we performed there we had an expression for delta T_e by T_i . And the expression which we had derived earlier was T_{he} minus T_{ce} that is delta T_e upon T_{hi} minus T_{ci} that is delta T_i is equal to the exponential minus 1 upon m dot h C_{ph} plus 1 upon m dot c C_{pc} ; we have derived this earlier. If you go back to your notes, you will see this derived earlier when we were getting the expression for delta T_m in parallel flow. so, now, substitute this expression into the one above for epsilon.

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Substituting

$$\begin{aligned} & \in \left\{1 - \exp\left[-\left(1 + \frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{m_{h}}\right]\right\} / \left(1 + \frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \text{If we had assumed initially } \frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}} = \left(\frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}}\right), \\ & \text{Hen} \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{m_{h}}\right]\right\} / \left(1 + \frac{m_{h}}{m_{h}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \text{We combine the two expressions } for \in \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}}\right) \\ & \in \left\{1 - \exp\left[-\left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{c_{\mu}}{c_{\mu}}\right) \frac{UA}{c_{\mu}}\right]\right\} / \left(1 + \frac{c_{\mu}}{c_{\mu}}\frac{$$

We will get, substituting we will get epsilon is equal to - I will do a little, I will put it in its final form 1 minus exponential minus 1 plus m dot h C_{ph} m dot c C_{pc} the whole thing multiplied by UA upon m dot h C_{ph} 1 plus m dot h C_{ph} upon m dot c C_{pc} - this is the expression you will get. I have a skipped a line or 2 of algebraic simplification so, this is the end result that we are getting.

Now, suppose we have made the reverse assumption if we had assumed initially m dot c C_{pc} equal to m dot C_{ps} ; suppose we had taken it to be the smaller. Then, we would have got epsilon is equal to the expression and that you can verify yourself by going through the derivation. You would have got 1 minus e to the power of minus 1 plus m dot c C_{pc} upon m dot h C_{ph} UA upon m dot c C_{pc} divided by the whole thing divided by 1 plus m dot c C_{pc} upon m dot h C_{ph} . So, we will combine both these now; rather than writing 2 expressions, we will say combine the 2 expressions. We combine the 2 expressions for epsilon and we say epsilon is equal to 1 minus exponential of minus 1 plus m dot C_{ps} upon m dot C_{pb} that is the larger of the 2 multiplied by UA upon m dot C_{ps} the whole thing divided by 1 plus m dot C_{ps} upon m dot C_{pb} . So, m dot C_{ps} is the smaller of the 2 heat capacity rates than m dot C_{pb} ; it is the larger of the two. In this way, we have written one expression for both instead of 2 expressions for epsilon for the 2 different cases. Now, let us now define two new, two further, two new dimensionless parameters.

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We are going to define one, first of all a parameter called as the capacity rate ratio for which we will use the symbol capital C and second we are going to a dimensionless parameter called the number of transfer units for which you will we will use the symbol NTU. C is defined as the smaller of the heat capacity rates divided by the larger of the heat capacity rates m dot C_{ps} upon m dot C_{pb} and NTU is defined as UA upon m dot C_{ps} .

Note : dimension less NTU > 0

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So, note you will note by definition that first of all for these two dimensionless parameters, you will note both are dimensionless. Let me write that down to emphasize the point; like epsilon these two parameters are dimensionless also, also that the quantity the parameter C is dimensionless and has to be between zero and 1, zero less than or equal to C less than or equal to 1. And NTU is a positive dimensionless number which can be any number greater than zero. So, we have defined two further parameters and our whole performance now of the heat exchanger is going to get defined by these dimensionless parameters.

Now, put in these definitions; put these definitions into the expression for parallel flow the definition of C and the definition of NTU - put these in and what do we get? We get a nice neat simple expression; we get the expression. Let me show that. (Refer Slide Time: 30:26)



Say for parallel flow, epsilon is equal to 1 minus; for parallel flow epsilon is equal to 1 minus e to the power of minus 1 plus C into NTU divided, the whole thing divided by 1 plus C - we get a nice neat expression. The performance of a parallel flow heat exchanger is expressed in terms for 3 dimensionless parameters – epsilon, C and NTU; a very neat expression for the effectiveness. Suppose I want to plot this; what would I get? I will get something like this.

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Suppose I want to plot effectiveness against NTU with C as a parameter; C can range between zero and 1. Notice for C equal to zero, I would get a line like this. It is effectiveness increasing with the NTU and then asymptotically going into 1. Then for C equal to .25, again effectiveness increases with NTU and asymptotically goes into a value of .8 so on. At C equal to 1 that is when both the m dots C_ps are equal, the effectiveness increases with NTU and asymptotically goes into a value of .5, this is .5 here.

So, in each case, the effectiveness increases with the NTU and then asymptotically goes into some particular value depending on the value of C - whether C is zero, it goes into 1, asymptotically into 1, C is 1, it asymptotically goes into value of .5. And this graph, I mean this expression that we have derived for parallel flow is known shown graphically in this form. We could do the same thing for counter flow as well and I am not going to do go through that whole derivation; I will leave that as an exercise for you. Exactly along identical lines, start with a definition of effectiveness

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Take one of the m dot C_p smaller, go through the whole process, you will get the expression which I have shown here at the bottom for the effectiveness. In counter flow, you will get in counter flow, you will get effectiveness is equal to 1 minus e to the power of minus 1 minus C into NTU divided by the whole thing by 1 minus C into e to the power of minus 1 minus C NTU - this is the expression and I would like you to derive this on your own. So, now we have got effectiveness as a function of NTU with C as a parameter that is what we have got graphically. Let me show what it looks like in the case of counter flow; this is what it look like in counter flow.

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The effectiveness increases with NTU for any given value of C; C equal to zero asymptotically goes into 1, for C equal to 1 it also and goes on increasing and the NTU is larger enough than this value of epsilon will eventually go into 1, it will take a long time but will eventually go into 1. So, in the case of counter flow the effectiveness asymptotically goes into 1 but with C equal to 1 or C equal to .75 you need very larger values of NTU to asymptotically go into 1. Such values are normally not used in practice where we generally have heat exchangers with NTU ranging from zero to about 5; that is why this graph has stopped at a 5 here in this case. So, these are the same expression the same equations plotted graphically. Now, the same calculations have also been done in cross flow. In cross flow, you cannot derive a closed form expression like we derived in parallel flow or counter flow.

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In cross flow you have to do numerical integrations. This has been done for cross flow for all the 3 cases that we have described - both fluids unmixed, one fluid mixed the other unmixed and both fluids mixed. And the expressions have been derived; I mean the calculations have been done numerically and the effectiveness against NTU graphs are as you are seeing them here. This is the case of both fluids unmixed in cross flow, effectiveness plotted against NTU. Effectiveness goes on increasing depending on the value of C and asymptotically goes on approaching some high value as the value of NTU increases; this is the case of both fluids unmixed.



The next graph I am showing you is effectiveness against NTU for the case of 1 fluid mixed and the other unmixed.





This is the second case of cross flow and the third case of cross flow is when both fluids are mixed, that is this 1 and in this case. You can see that the values of effectiveness do not asymptotically go into 1 as the NTU goes on increasing. I will not spend time explaining that it is not necessary, it is important that you should know that these graphs are available - plotting effectiveness again NTU - and we will use them just now for solving one particularly problem. Now, a special case again which we have discussed earlier in the case of mean temperature difference but I would like to discuss again.



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Suppose the capacity rate on one of the side - hot side or cold side - is infinite; capacity rate m dot C_p is infinite either on the hot side or cold side. It follows immediately C is equal to zero because the denominator is going to infinity in the definition of C. For this situation, you can show that for all flow arrangements epsilon is equal to 1 minus e to the power of minus NTU. You get this nice simple relation between effectiveness and NTU when C is equal to zero. So, keep this in mind that for this special case, it doesn't matter what is the flow arrangement; you get the same value of effectiveness.

You will recall from our delta T_m discussion that I said that the delta T_m - when C is equal to zero - the delta T_m is the same whether you have parallel flow, whether you have counter flow or whether you have cross flow. Now, let us do a problem a numerical problem so, that we get these ideas written down. The problem we are going to do to illustrate the use of the effectiveness NTU method is the following.

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Problem 35 15 5000 Mg mate. 5075 Ay/h JAN

We say in a gas turbine power plant heat is transferred in an exchanger, in a plate film heat exchanger from the hot gases leaving the turbine to the air leaving the compressor. We utilize the energy available in the exhaust gases of the turbine to heat up the air which has just been compressed before combustion takes place. This helps to increase the efficiency of the gas turbine cycle. Given the air flow rate, the air flow rate is 5000 kilograms per hour, and the flow rate of hot gasses; flow rate of hot gases is 5, it is a little more than that because of the fuel added on at the time of combustion so, 5075 kilograms per hour. Further, given T_{hi} is equal to 450, the hot gas is leaving the turbine, the air which is to be heated up, inlet temperature is 170. The value of U is 52 Watts per meter squared Kelvin, the value of A is equal to 50 square meters, C_p on both side is equal to 1050 Joules per kilogram Kelvin. This is given.

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Now further given that the arrangement - flow arrangement - is single pass cross flow with both fluids unmixed. A plate film heat exchanger with films on both sides would ensure this both fluids unmixed. Now, this is the arrangement. It is a plate film heat exchanger single pass cross flow, both fluids unmixed, inlet temperatures and flow rates given on both sides, find the 2 exit temperatures. Find T_{he} , find T_{ce} and find the quantity of heat transferred in the heat exchanger. This is a problem which it would be ideal to solve by the effectiveness NTU method, you will agree on that.

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And just to refresh your idea, here is the core of the platelet film heat exchanger which I am showing you just now. So, one fluid will be entering through these passages, say let us say the air or the hot gases, the other fluid will be entering through these passages. So, you have got a situation of cross flow; both fluids unmixed because of the films which are in the parallel plate channels. That is what I want you to see so, the films ensure that both fluids stay unmixed on both sides. Now, let us apply the effectiveness NTU method to this. So, let me just draw a sketch of what we have before us; it is useful always to put down the given numbers in a sketch.



So, we say here is the heat exchanger; let us say it is, this is the area A of the heat exchanger - 50 square meters. Then, we have the fluid to be heated at 170 degree centigrade and its flow rate m dot C is 5000 kilograms per hour; we would like to find out its exit temperature. And on the other side, we have the hot side inlet temperature 450, these are the hot gases leaving the turbine after expansion, the flow rate on the hot side 5075 kilograms per hour and we would like to find out the exit temperature on the hot side T_{he} , T_{ce} and T_{he} .

So, let us get the quantities now - C the capacity rate is equal to m dot C_{pc} upon m dot C_{pb} , m dot C_{pc} is obviously on the cold side so, it is 5000 into 1050 divided by 5075 into the specific heat happens to be the same on both sides which is, so, we get .985 to be the value of C. NTU is equal to UA divided by m dot C_{pc} that is 5000 upon 3600 so, many kilograms per second into C_p 1050 which gives me 1.783.

Now, go to the figure for effectiveness against NTU with both fluids unmixed. You recall we had shown that figure and let me just show it to you again. This was the figure that we had - effectiveness against NTU, both fluids unmixed. We go to this figure and in this figure which I have shown, in this figure we will have to do an interpolation. (Refer Slide Time: 44:53)



This is our figure, cross flow, both fluids, unmixed for this is NTU and this is epsilon. We will have to go to the graph for C equal to .985; there is a graph for C equal to 1 so we will have to go a little below it. And then NTU is equal to 1.783 so we say 1.783 NTU and it is intersection with C equal to, the graph for C equal to .985 would give epsilon is equal to .60. So, from the figure, from the results in the figure, the graphs in the figure for cross flow, both fluids unmixed, we get epsilon is equal to .6 and epsilon in this case is nothing but T_{ce} minus T_{ci} upon T_{hi} minus T_{ci} . So, if you substitute the numbers, I will get T_{ce} minus 170 upon 450 minus 170 and I get, on calculation, T_{ce} is equal to 168 plus 170 which is equal to 330vdegree centigrade. I get q which is nothing but the heat transferred - 5000 upon 3600, (that is the flow rate in kilograms per second) into 1050 multiplied by the change in temperature - 450 minus, no sorry, 338 minus 170 (change in temperature on the cold side) - that comes out to be 245000 Watts. And finally I have to calculate T_{he} which I can just get from a simple heat balance from the two sides. Heat given, heat gained by the cold side is equal to heat given up by the hot side.

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5075 × 1050 × (450-The) = 5000 × 1050 × (338-170) The = 284.5°C

So, I get 5075 multiplied by the specific heat 1050 multiplied by 450 minus T_{he} - this is q is equal to 5000 multiplied by 1050 multiplied by 338 minus 170. So, I get T_{he} minus is equal to 284 .5 degree centigrade. So, these are the 4 answers we have got, T_{he} the one exit temperature and from the previous page, if I go back we have got the exit temperature and this is the heat transfer rate q so many 245000 Watts to be changed, to be the heat transferred in the heat exchanger. So, these are 3 answers that we were looking for.

Now, if we have adopted the q equal to UA delta T_m approach we would have needed to do a trial and error. In this case, that is a good reason why we have adopted this particular method. Now, before we close this topic of heat exchangers, I have discussed both the methods which is what we wanted to do. Let me say that heat exchanger design is not just a matter of applying the q equal UA delta T_m equation or effectiveness NTU; there is more to it. There are many more other considerations designing heat exchanger and I just want to list them for you, not really take them up in any detail.

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So, here are some other considerations, other considerations in designing heat exchangers. Look at these. Pressure drop on either side - it is a very important consideration; sometimes there are restrictions on how much should be the pressure drop. Sometimes, size restrictions are there; heat exchanger should be of a certain size. Very often, stress considerations are very important; there are high temperature changes in the heat exchanger. It is a pressure vessel; you have to design it according to pressure vessel code. So, stress considerations are extremely important to ensure that the heat exchanger stands up to the required stresses - either mechanical stresses because of pressure or thermal stresses because of temperature difference.

There are servicing requirements - we have talked of fouling; fouling takes place, you must be able to clean the heat exchanger. So, servicing requirements means you must be able to open out the heat exchanger and be able to clean it effectively not keep it out of service for too long. Materials of construction are important; if you don't use the right materials the heat exchanger may corrode rapidly, erode rapidly; whatever it is materials of construction are important. Then, you have to keep in mind system operation. What do you mean by that? Heat exchanger is part of a whole system; it is never working in isolation. Sometimes transients come into a system; we would like to ensure that - the system operation means that the heat exchanger works as part of a whole system.

Finally, of course, every heat engineer has to worry about the cost of a heat exchanger as part of an overall system. So, keep in mind that there are many other considerations apart from thermal considerations which go in designing heat exchangers - the pressure drop on either side, size restrictions, stress considerations, servicing requirements, materials of construction system operation and cost.

Now, let me just quickly review the main topics that we have done in this, the main subtopics that were have done in this overall topic of heat exchangers. So, let us just do a 2 minute review what have we covered in this topic on heat exchangers - these 4 lectures that we have had.

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Heat Exchangers Types of heat exchangers Direct transfer Classificati

First of all, if you recall correctly, we discussed various types of heat exchangers then we said we will focus on the direct transfer type and in the direct transfer type of heat exchanger; we described tubular heat exchangers, plate heat exchangers, extended surface heat exchangers. Then, we talked about classification by flow arrangement and you will recall here that we talked about parallel flow, counter flow and cross flow in which again we had 3 cases for consideration. Case 1 - both fluids unmixed, case 2 - one mixed and

the other unmixed, case 3 - both fluids mixed. Then, we talked about the overall heat transfer coefficient and the fouling factor, how fouling is taken account of by introducing a term called a fouling factor and then we spent a lot of time talking about the 2 methods for analyzing the thermal performance of heat exchangers. What were the 2 methods?

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temperature difference UT N - UA

One method was the mean temperature difference method in which we said there is a basic performance equation q is equal to UA delta T_m and we derived expressions for the mean temperature difference in parallel flow, counter flow and cross flow; cross flow again 3 cases. For parallel flow and counter flow, we had expressions for cross flow. Based on numerical integration, we had graphs. We also discussed some special cases - the case of the m dot C_p going to infinity on either side. Then we came to the effectiveness NTU method as another approach for solving heat exchanger problems. In this method, we defined the 3 dimensionless parameters. What where they? Effectiveness, capacity rate ration C and number of transfer units. And then towards the end having defined these terms, we got expressions for the effectiveness and NTU.

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We derived expressions, effectiveness NTU relationship in parallel flow, counter flow and presented results for the cross flow - the 3 cases of cross flow. For both the mean temperature difference and the effectiveness NTU method, we solved some numerical problems. Finally, just for a minute or two, I mentioned that there are the other design considerations apart from the thermal considerations which went into our discussion and these can be often be just as important. So, with these we have to come to the end of what we would like to do with the topic of heat exchangers.