Heat and Mass Transfer
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Lecture No. 26
Heat Exchangers-2

Now in today's lecture we begin with the topic of mean temperature difference in a heat exchanger. We are going to first define the term - what we mean by the mean temperature difference in a heat exchanger - and then we are going to derive expressions for the mean temperature difference for various flow classifications, that is for parallel flow, counter flow, the 3 cases of cross flow and so on. Let us consider some heat transfer surface.

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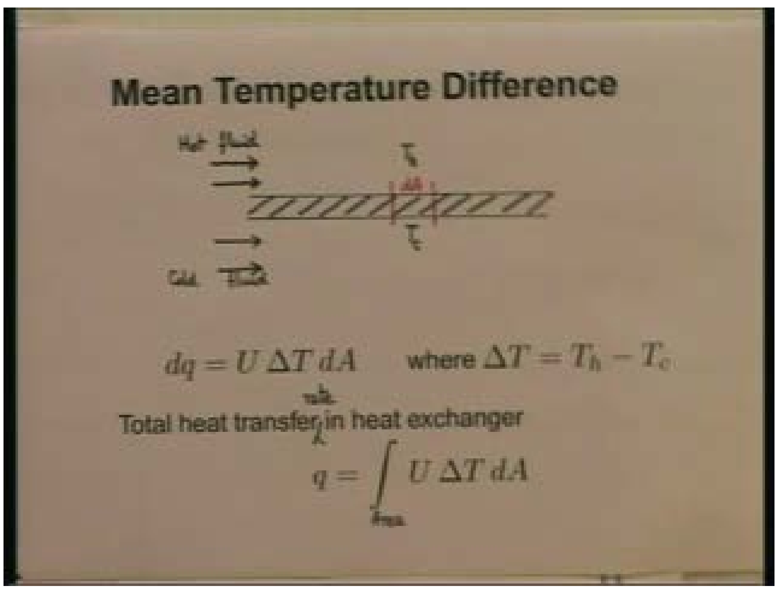

Let me draw a heat transfer surface like this across which heat is flowing, some heat transfer surface. A fluid - a hot fluid - let us say is flowing on the top side. Let us say this is the hot fluid and let us say the cold fluid flowing on this side showing on the other side. This is the heat transfer surface and let us take some elementary area dA on this heat transfer surface. Let us say somewhere on this heat transfer surface, we take an elementary area dA anywhere on this heat transfer surface. Let us assume that the temperature of the hot fluid at dA is T_h on this side of dA and the temperature of the cold fluid on the other side of dA is T_c.

Now, we know from our knowledge of heat transfer that the heat transfer rate dq across this elementary area dA, the heat transfer rate dq is given by U - the overall heat transfer coefficient. May consider fouling multiplied by T_h minus T_c which we are calling as delta T multiplied by dA - this is our expression for the heat transfer rate. And if I ask you what is the total heat transfer rate across this heat exchanger, you will say, well all I need to do is to integrate this expression over the whole area A of the heat transfer surface. So, the total heat transfer rate in the heat exchanger, the total heat transfer rate - let me add the word rate here - in the heat exchanger q will be given by the integral U delta T dA where the integration is carried out over the whole area A of the heat exchanger. So, the integration is carried out over the whole area of the heat exchanger that is the total heat transfer rate. Now, if I assume that the overall heat transfer coefficient is a constant not varying over the surface of the heat exchanger if U is assumed to be a constant, some constant value, then we can say the expression for q simplifies to, then q is equal to U.

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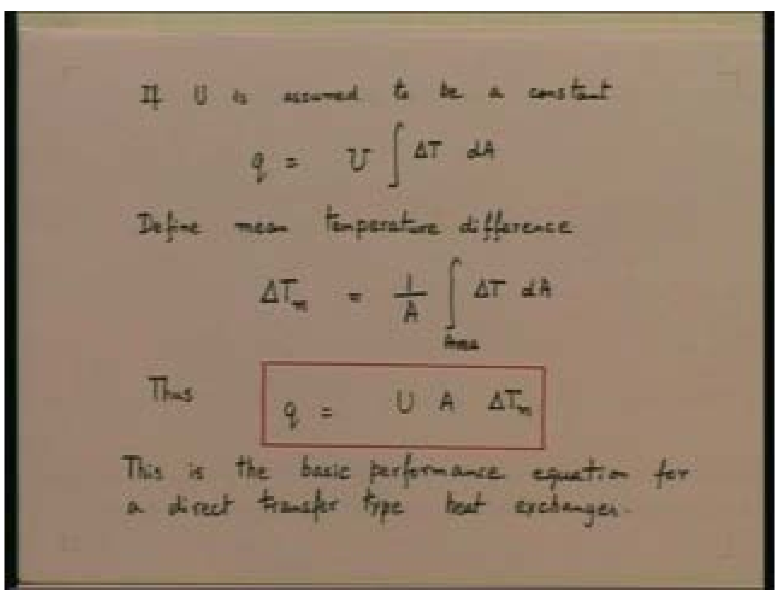

U can be taken outside the integral delta T integrated over the area A. Now I define the quantity mean temperature difference as follows. I say define mean temperature difference by the expression delta T_m. delta T_m stands for the mean temperature difference in the heat exchanger and I define it as - delta T_m is equal to 1 upon A the integral of delta T dA, the integral being over the whole area A of the heat exchange. Thus, if I introduce this definition into my expression for q, I get - thus q is equal to U A delta T_m - that is the expression that I get. And what we have here is the basic performance equation of a direct transfer type heat exchanger.

So, let me just sort of circle it to give it its, give importance. I will put a circle, put a rectangle round it to say this is an important equation which we are going to use for calculating the performance of a direct transfer type heat exchanger; this is our basic performance equation. Put a rectangle angle to give it, give importance and say this is the basic performance equation. This is the basic performance equation for a direct transfer type heat exchanger. So, suppose I have a direct transfer type heat exchanger whatever it be, shell and tube plate, fin plate, whatever it is, if I want to calculate the q - the amount of heat transferred per unit time in that heat exchanger - I will need to know the value of U, the overall heat transfer coefficient. I will need to know of course the heat transfer area in that heat exchanger and I will need to know the mean temperature difference for the heat exchange. I will need to have some expression for calculating the mean temperature difference, the mean temperature difference which depend upon the temperatures of the fluids, 2 fluids, the hot fluid and the cold fluid at the inlet and the outlet. So, I need to find ways of calculating delta T_m based on my knowledge of T_{hi}, T_{he}, T_{ci}, T_{ce} - that is what I am going to do next.

Now, our next job is find expressions or ways of calculating delta T_m for parallel flow, for counter flow, for cross flow - all the 3 cases of cross flow, that is all going to be our next job. We have defined a delta T_m. How do we get? It is a value for different flow situations so let us look first at parallel flow. Let us say now, our next job is to derive an expression - mean temperature difference in parallel flow.

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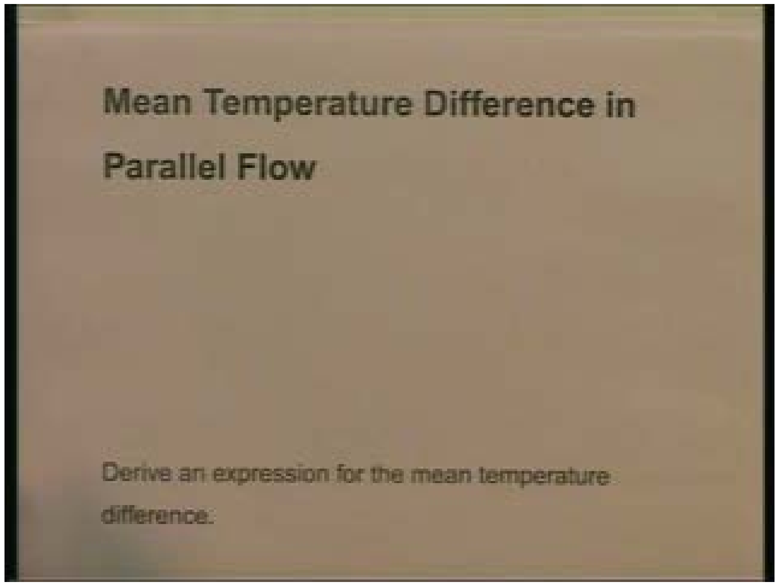

Derive an expression for the mean temperature difference in parallel flow - that is our next job. Now, let us first look at a parallel flow heat exchanger, a typical parallel flow heat exchanger. Let us say, let me for the moment not to confuse you, let us put a cover on the lower side so that we focus attention on the upper part of the figure. Here, we have a parallel flow heat exchanger

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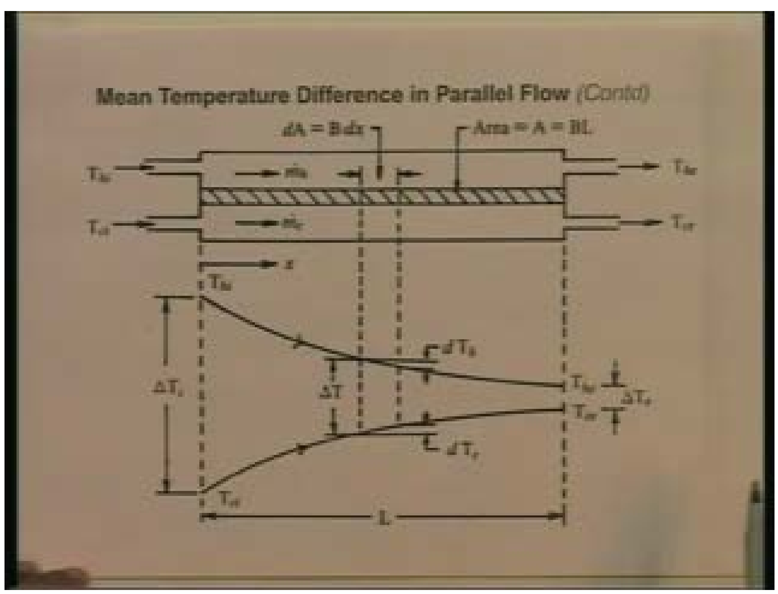

The flow rate - this is the shaded, the hatch lines indicate the heat transfer surface area is A and this heat exchanger has a length L in the direction x. x is measured from this side, the left hand side - it has a length L, a breadT_h B at right angles to the paper. So, the area A of this parallel flow heat exchanger is A is equal to BL. The flow rate on the hot side is m dot h. You can see it is shown here - the flow rate on the cold side is m dot c shown here. The hot fluid enters with the temperature T_{hi} and leaves with a temperature T_{he}, the cold fluid enters with the temperature T_{ci} and leaves with a higher temperature T_{ce}. We would like to find, derive not find, derive an expression for the delta T_m, the mean temperature difference for this parallel flow heat exchanger, that is our job.

Now, let us now before we start deriving the expression, let us just for a moment qualitatively see the picture before us. This is the, these are temperature profiles on the hot side and the cold side. On the hot side, the fluid is entering with a temperature T_{hi} and as it goes along the length of the heat exchanger it is giving up heat and reducing in temperature to value T_{he}. On the cold side, the fluid is entering with the temperature T_{ci} and as it goes along the length of the heat exchanger it is increasing in temperature to a value T_{ce} at the exit. So, these, let me put arrows to show the directions of the temperature profiles, the flow directions.

So, the temperature difference between the hot and the cold fluid at the inlet is T_{hi} minus T_{ci} and that is delta T_i we will call that as delta T_i. Similarly, the temperature difference at the outlet of the heat exchanger would be T_{he} minus T_{ce} and we will call that as delta T_e so that is the position. Now, the delta T_m - the mean temperature difference for this heat exchanger - is going to lie between these 2 extremes. One extreme is delta T_i and the other extreme is delta T_e. delta T_m is going to lie between these 2 extremes, some value in between these. We want an expression for delta T_m in terms of T_{hi} T_{ci} T_{hi} and T_{ce}. Consider, let us now derive the expression so let us consider some elementary area dA of this heat exchanger.

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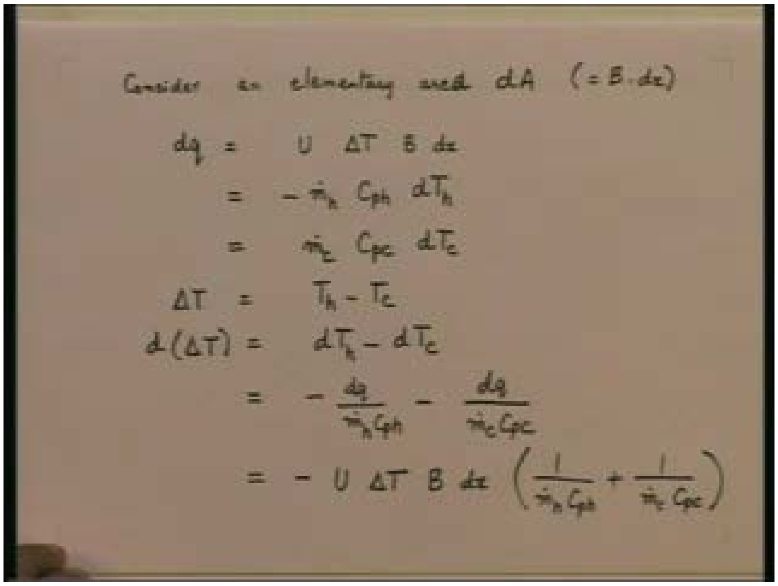

Consider an elementary area dA, elementary area dA, which will be equal to B the breadT_h of the heat exchanger which is a constant in to dx. Let us consider some elementary area and the heat transfer occurs across it. Before we do that, let us make 2 assumptions which are reasonable assumptions. Let me put those down and then start continue with the derivation. We are going to make 2 assumptions; let me show those.

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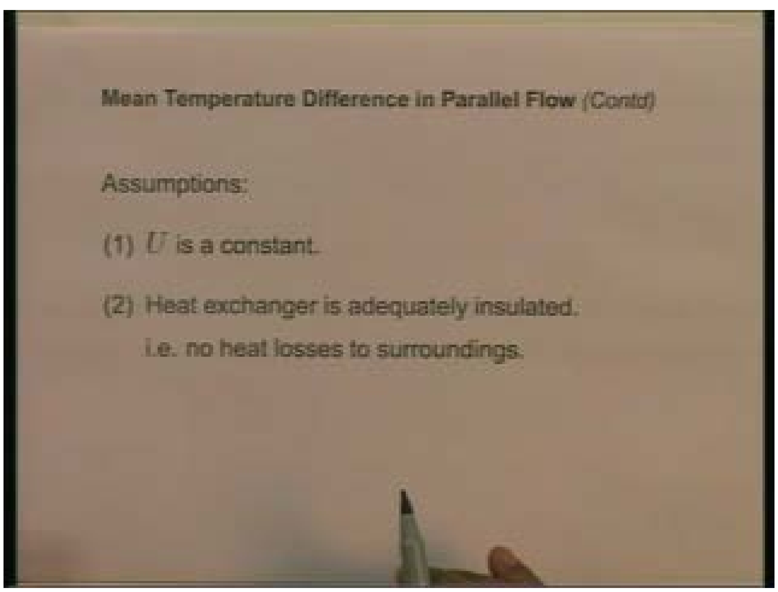

First, we will make the assumption that the overall heat transfer coefficient U is a constant; the value of U for this heat exchanger all over the area of the heat exchanger is a constant so U is a constant - that is assumption 1. The second assumption we are going to make is the heat exchanger is adequately insulated so that there are no heat losses to the surrounding. What we mean by this is the hot fluid, whatever heat it gives up, whatever loss of enthalpy it has, all that energy flows to the cold fluid; none of that energy is lost to the surroundings. And usually this is a good assumption because that is the purpose of a heat exchanger - to transfer heat from a hot fluid to a cold fluid not to lose the energy to the surroundings. So, heat exchangers are adequately insulated so that the heat losses to the surroundings are in fact negligible. So, let us make the 2 assumptions: U is a constant and heat losses are negligible to the surroundings therefore enthalpy change of the hot fluid will be equal to enthalpy change of the cold fluid.

Now let us continue with our derivation. I said to you, let us take an elementary area dA of the heat exchanger which is equal to Bdx. What is the heat transfer rate across this areas dA? We know that already from earlier. dq is nothing but U delta T dA; dA is nothing but B dx that is so many Watts is the heat transfer rate across the elementary area dA. Now, when this heat transfer takes place - because this heat transfer takes places from the hot side to the cold side - the enthalpy of the hot side flow decreases by this, by this amount, and the enthalpy of the cold fluid increases by this amount. The enthalpy decrease of the hot fluid is shown, is seen by the decrease in temperature. So, over the length dx the temperature of the hot fluid decreases by dT_h. So, we will say this must be equal to the change in enthalpy on the hot side m dot h C_{ph} dT_h and that must be equal to the increase in enthalpy on the cold side which is equal to m dot C C_{pc} dT_c.

Notice I have put a negative sign on the dT on this first one, why? Because dT_h will be negative in the positive direction which I am measuring from the inlet x, is being measured from the inlet, so dT_h - the temperature of the hot side - will go on decreasing in the positive direction. dT_h will be a negative number so I need to put a negative sign in order to get an enthalpy change and equate this to the heat transfer rate across the area dA. So, these are all equal expressions; this is the heat transfer rate {acr} (17:00) across the area dA this is the enthalpy change on the hot side this is the enthalpy change on the cold side and they have to be all equal.

Now, delta T is nothing but T_h minus T_c delta; T is T_h minus T_c therefore d delta T - if I take a differential - is nothing but dT_h minus dT_c. I. can write this, as now I have an expression for dT_h out here and an expression for dT_c out here. So, instead of dT_h I can write minus dq upon m dot h C_{ph} and instead of dT_c I will get minus dq upon m dot c C_{pc} using these 2 expressions. And further for dq, I can write the expression.The first expression so I can say this is equal to minus U delta T B dx; I am substituting for dq from this expression here the first one and into bracket 1 upon m dot h C_{ph} plus 1 upon m dot c C_{pc} - that is what I will get. So, d delta T is equal to minus U delta T Bdx in to 1 upon m dot h C_{ph} plus 1 upon m dot c C_{pc}.

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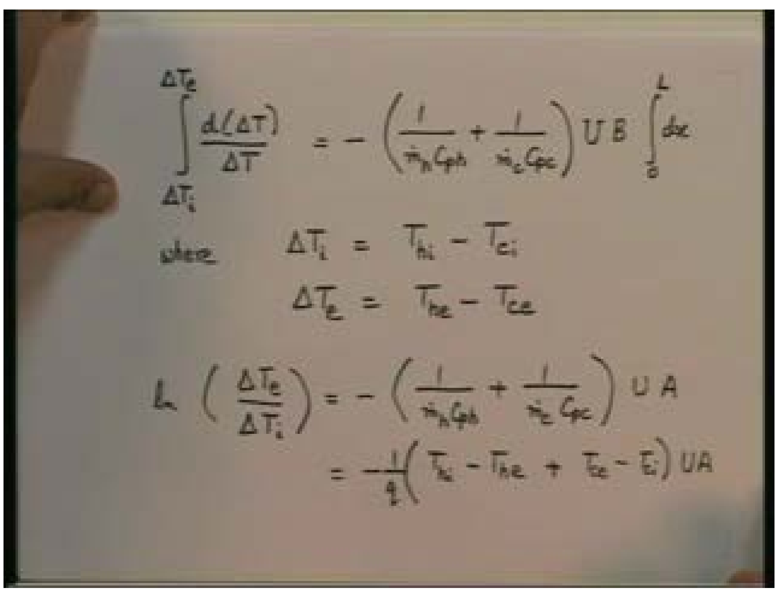

Let us integrate this expression from inlet to outlet; so let us say, let us rewrite this expression as d delta T upon delta T, bring the delta T to the left hand side and integrate this expression from inlet to outlet, that is integrate inlet; at the inlet the value of delta T is delta T_i at the outlet it is delta e. So expression integrated from inlet to outlet - this must be equal to minus 1 upon m dot h C_{ph} plus 1 upon m dot c C_{pc}. It is a constant; it comes outside the integral, U is a constant so it comes outside the integral; B is a constant, comes outside the integral. Only the dx remains inside so integral of dx 0 to L - the whole length of the heat exchange L - and if I do this and integration. Now, by the wa,y before I go further let me say we should be careful here. delta T_i is T_{hi} minus T_{ci} - I mentioned that earlier but I am writing it down now delta T_e is equal to The minus T_{ce}. delta T_i is the temperature difference between the hot and the cold side at the inlet end; delta T_e is the temperature difference between the hot and the cold side at the exit.

Now, therefore we get with the integration, I will get log to the base e delta T_e by delta T_i - it is the left hand side - is equal to minus 1 upon m dot h C_{ph} plus 1 upon m dot c C_{pc} multiplied by U multiplied BL which is nothing but A the area A of the heat transfer surface and that is equal to, I can write that further as equal to minus, now for m dot hC_{ph} I can write T_{hi} minus T_h e and divided by q which I will take outside.

So, I will take 1 upon q plus for m 1 upon m dot c C_{pc}, I can write T_{ce} minus T_{ci} and again a q which I take outside; that is the total heat transfer rate in the exchanger is nothing but m dot h C_{ph} T_{hi} minus T_h e and multiplied, the whole thing multiplied by UA. So, I can further simplify and finally get my expression which I am looking for; I get

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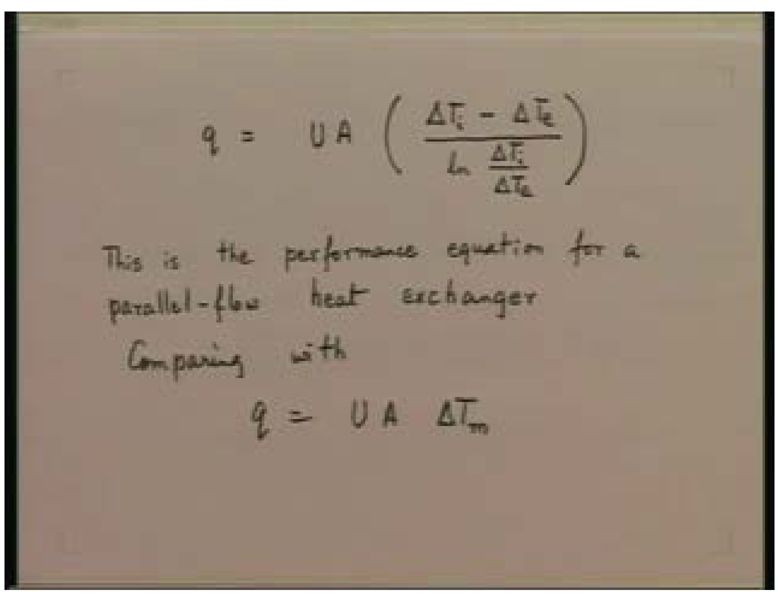

q is equal , now the expression in the final form is - q is equal to UA delta T_i minus delta T_e divided by the logarithm to the base e delta T_i by delta T_e - that is the final expression that I get. This is the performance equation for a parallel flow heat exchanger and if you compare this; compare without basic performance equation for a direct transfer type heat exchanger, what was that? Comparing with the general equation q is equal to UA delta T_m; if I compare this, this is our general equation for any direct transfer type heat exchanger.

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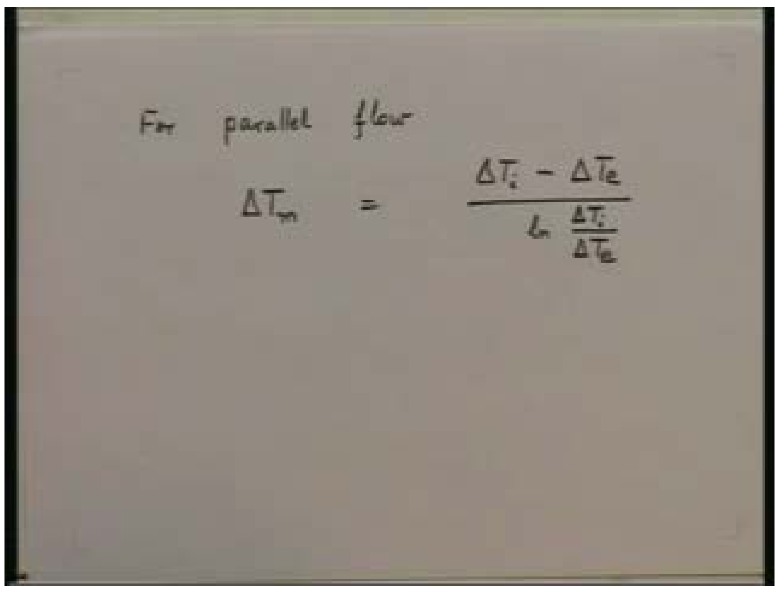

Then you can see for parallel flow, for parallel flow, comparing with this we say for parallel flow delta T_m is equal to delta T_i minus delta T_e divided by logarithm to the base e delta T_i by delta T_e and delta T_i is T_{hi} minus T_{ci} delta T_e is T_h e minus T_{ce}. So, you achieved our objective now for parallel flow heat exchanger if I know the temperatures of the 2 fluids at the inlet and outlet. I know delta T_i, I know delta T_e, I can calculate delta T_m for parallel flow. Once I have this, go to my basic equation q is equal UA delta T_m and I will get the heat transfer rate in the heat exchanger. this is what we have achieved.

Now, we are going to do the same thing for counter flow one by one; as I said, first we will do for parallel flow then for counter flow. So, our next job is to proceed for counter flow in a very similar manner so I will not spend that much time in this derivation.

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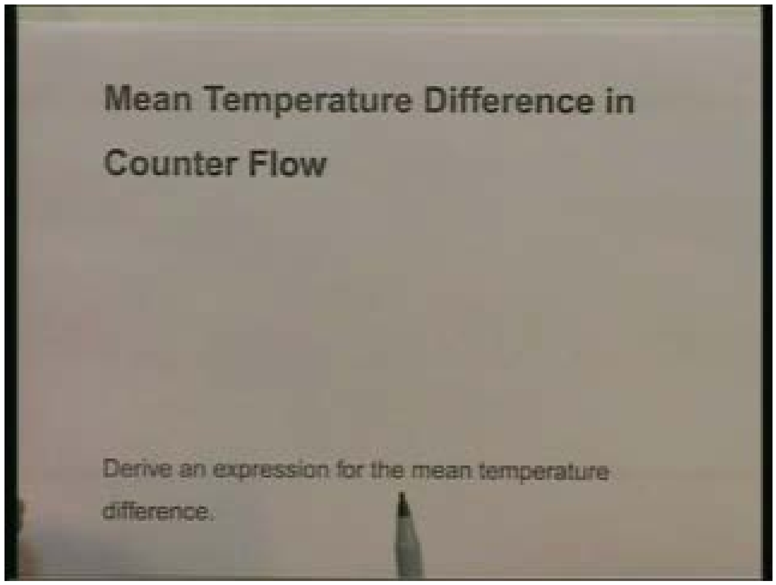

Our next job is to derive an expression for the mean temperature difference in counter flow; derive an expression for the mean temperature difference. We will proceed more or less in the same fashion with some small difference which I will point out as we go along. First, let us look at a counter flow heat exchanger - a sketch of it - as we did for parallel flow and then put down the equations again. But we will move a little faster because as I said the derivation is somewhat similar. Let us look again at the top half of this figure rather than the whole figure.

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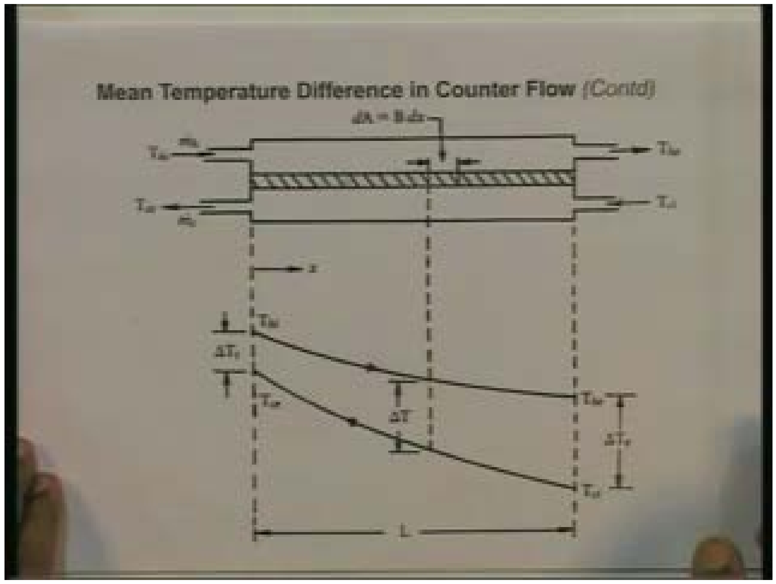

Here, there are counter flow heat exchanger length L breadth B at right angles to the paper flow rate on the hot side m dot h, flow rate on the cold side m dot c same as earlier. But, notice however fluid entering on the hot side enters with a temperature T_{hi} leaves with the temperature T_h e. The fluid on the cold side enters from the other side has a temperature T_{ci} and leaves with the temperature T_{ce}. So, now the flow, the directions of flow are reversed. We will again consider an elementary area dA which is equal to Bdx. So, we have got the same sketch except in that the 2 fluids are flowing in opposed directions so T_{hi} and T_{ci} are on the 2 sides of the heat exchanger.

Let us look at the sketch of the temperature profiles; here are the 2 temperature profiles. On the hot side, this is the fluid entering at T_{hi}, decreasing in temperature as it goes along the length of the heat exchanger and leaving at a temperature T_{he}. On the cold side, it is entering on the other side that is where T_{he} is entering on this side and as it goes along the length of the heat exchange, then the cold side fluid picks up and leaves at a temperature T_{ce}. So, the 2 temperature profiles now look something like this and we will define delta T_i as T_{hi} minus T_{ce}, which is different from the earlier definition of delta T_i. delta T_i will be in counter flow, will be T_{hi} minus T_{ce} and delta T_e in counter flow will be T_{he} minus T_{ci}, so keep that in mind as we go along.

Now, let us proceed with the derivation. As I said the derivation will be more or less on the same lines; there will be hardly any difference. We can move a whole lot faster.

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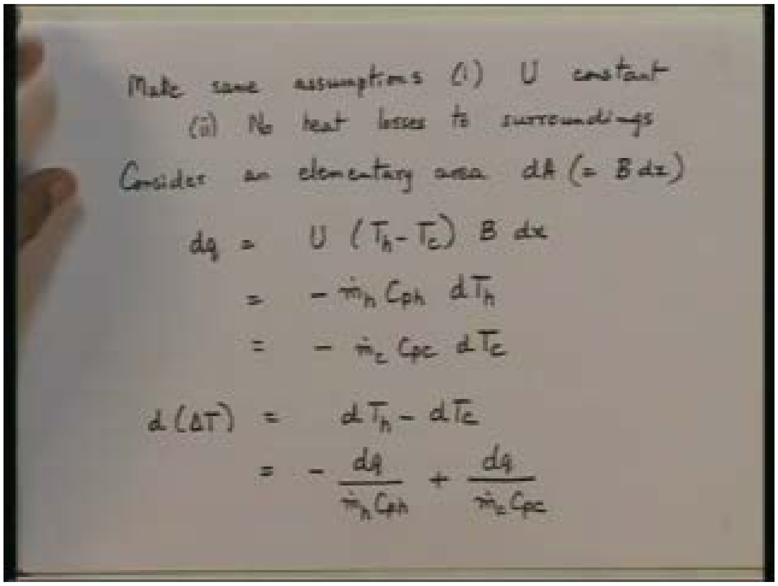

We will make the some assumptions, make some assumptions. What were they? Number 1, U is a constant, and number 2, no heat loss heat losses to the surroundings. Change of enthalpy on the hot side is equal to change of enthalpy on the cold side; the decrease in enthalpy on the hot side is equal to the increase in enthalpy on the cold side. Consider again an elementary area dA, consider an elementary area dA equal to Bdx, dq - the heat transfer rate. Proceed in more or less in the same fashion as earlier. Dq - the heat transfer across dA - will be U T_h minus T_c; temperature difference from hot side to the cold side multiplied by Bdx, that is equal to minus m dot h C_{ph} dT_h. Notice the negative sign because dT_h will be negative in the positive x direction and that is equal to minus m dot c C_{pc} in to dT_c.

So, the difference now is I have a negative sign out here; also because in the positive x direction, the temperature on the cold side is also decreasing. Let me show that sketch again just to illustrate what I mean. This is the x direction, the positive x direction in this direction, hT_h is decreasing, T_c is also decreasing. So, dT_h and dT_c are negative quantities therefore I need to put a negative sign on them to get a positive value of dq. So, that is the difference between parallel flow and counter flow, this negative sign. Now, we proceed with the same algebra. We can write d delta T is equal to dT_h minus dT_c which is equal to minus dq upon m dot h C_{ph} plus dq upon m dot c C_{pc} and using again the first and second and the third equations which we had earlier.

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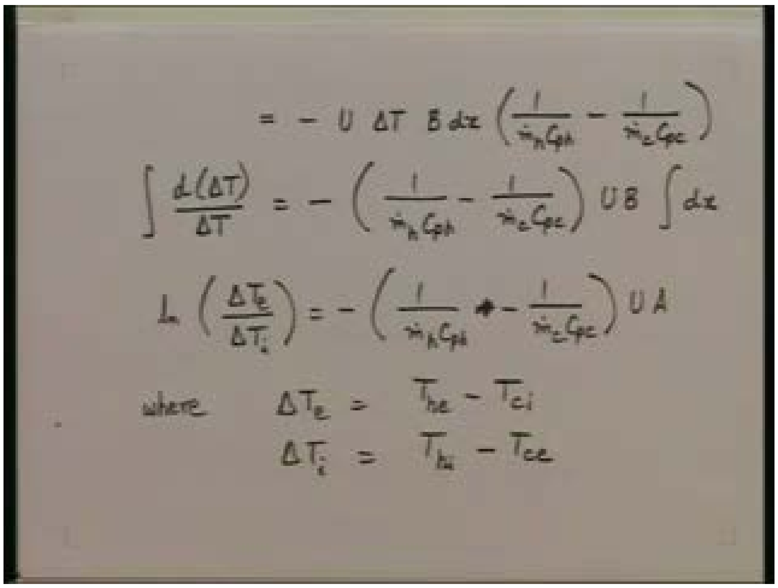

If we do a little algebra we will get this is the equal to, further this is equal to minus U delta T minus U delta T Bdx in bracket 1 upon m dot h C_{ph} minus 1 m dot c C_{pc} and then I integrate from inlet to outlet so I will get d delta T upon delta T is equal to minus 1 upon m dot h C_{ph} minus 1 upon m dot c C_{pc} UB the integral over dx. And performing the integration I will get a logarithm to the base e delta T_i by delta T_e is equal to minus 1 upon m dot h C_{ph} plus, sorry, minus 1 upon m dot c C_{pc} multiplied by UBL which is nothing but UA, where keep in mind however that in counter flow delta T e is T_h e minus T_{ci}. Note this difference from parallel flow and delta T_i is equal to T_{hi} minus T_{ce} - these definitions are different from parallel flow, that you should note.

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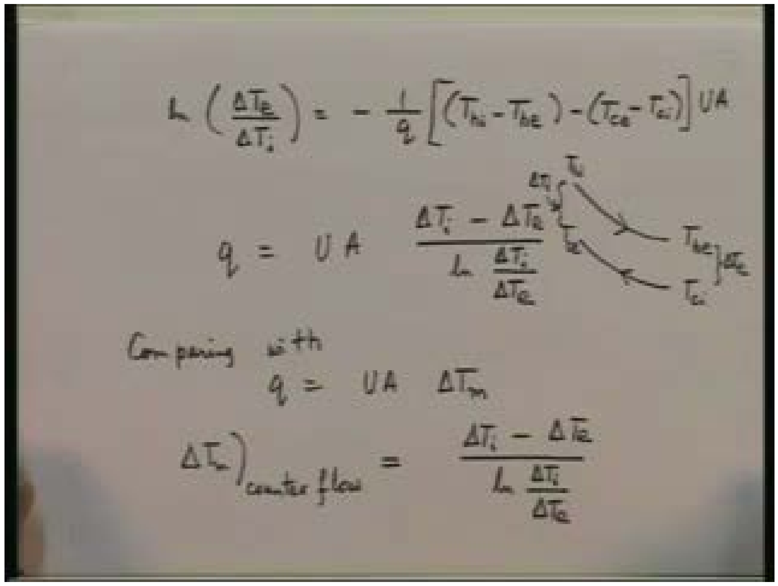

Therefore, we can write further - this log to the base e delta T_e by delta T_i is equal to minus 1 upon q in to bracket T_{hi} minus T_h e minus T_{ce} minus T_{ci} - these are the changes in enthalpy on the hot side and cold side - the whole thing multiplied by UA. Or finally our basic performance equation q is equal to UA delta T_i minus delta T_e divided by logarithm to the base e delta T_i by delta T_e. Notice we have got the same equation as we got for parallel flow; it is the same equation, there is no, in terms of symbols we have the same equation. But delta T_i and delta T_e mean different things in parallel flow and in counter flow there are defined differently; that has to be always kept in mind.

In fact, let me just draw a sketch here so that again there is no confusion. These are the temperature profiles on the hot side and on the cold side; this is T_{hi} decreasing to T_{he} and on this is, on the cold side T_{ci} increasing to T_{ce}. This difference T_{he} minus T_{hi} T_{ci} is equal to delta T_e; this is delta T_e and this difference out here is delta T_i; that is what has to be kept in mind - T_{hi} minus T_{ce} is delta T_i. Now if I again compare with our basic performance equation, comparing with our general performance equation, q is equal to UA delta T_m. For any direct transfer type heat exchanger I will get delta T_m in counter flow is equal to delta T_i minus delta T_e divided by the logarithm to the base e delta T_i by delta T_e where delta T_i and delta T_e are as defined just a moment ago. So, this is my expression for the mean temperature difference in counter flow.

So now, if I have counter flow heat exchanger, I know the inlet temperatures in the hot side and cold side, the outlet temperature on the hot side and cold side. I know how to calculate delta T_m for that counter flow heat exchanger. The moment I have got the delta T_m, I can say the heat transfer rate for that counter flow heat exchanger is U - the overall heat transfer coefficient into the area A into the delta T_m in counter flow as calculated from this expression.

Now we go to the next case. No. Before we go to the next case of cross flow, let me look at one special case of counter flow which is worth talking about.

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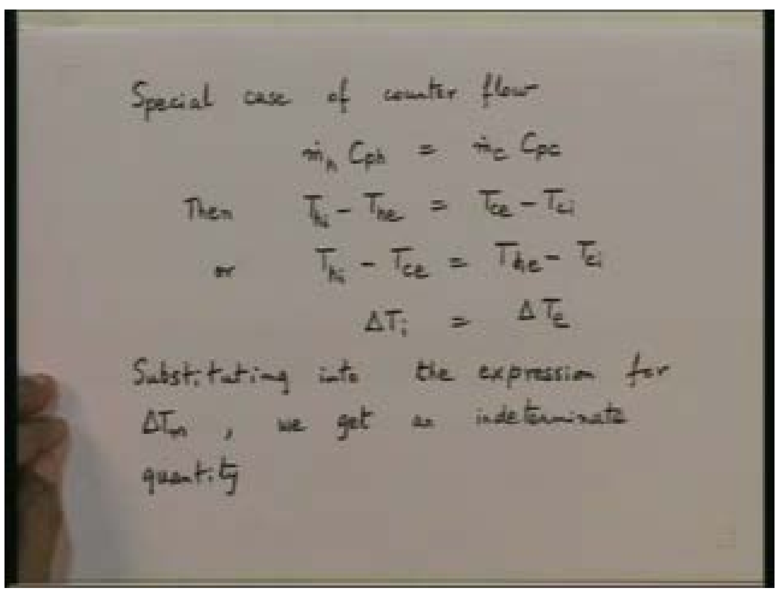

So, for a moment let us talk about one special case of counter flow before we move on to the cross flow situation, a special case of counter flow. What is that? The special case of counter flow which we want to look at occurs when m dot h C_{ph} is equal to m dot c C_{pc} - suppose we have this situation. The product of the flow rate and the heat and the specific heat on the hot side equals the value on the cold side. Then, it follows since q the change of enthalpy on the hot side and the change of enthalpy on the cold are numerically equal. It follows that T_{hi} minus T_h e must be equal to T_{ce} minus T_{ci}; the changes of temperature on the 2 fluids must be equal or I can rewrite that as T_{hi} minus T_{ce} is equal to T_c T_h e minus T_{ci}. T_{hi} minus T_{ce} is nothing but delta T_i in counter flow and T_h e minus T_{ci} is nothing but delta T_e so we have delta T_i equal to delta T_e for this special case.

Now, if I substitute into my expression for the mean temperature difference that delta T_i is equal to delta T_e, substituting into the expression for delta T_m we get, if you do it you can see straight away without even, I am substituting it on paper, we get an indeterminate quantity. We will get 0 upon 0; you get, we get an indeterminate quantity – 0 upon 0. So what we do if we have a situation like this? Calculus or knowledge of calculus tells us that if you want to calculate the value we need to differentiate the numerator and denominator and apply what is called as L'Hopital's rule.

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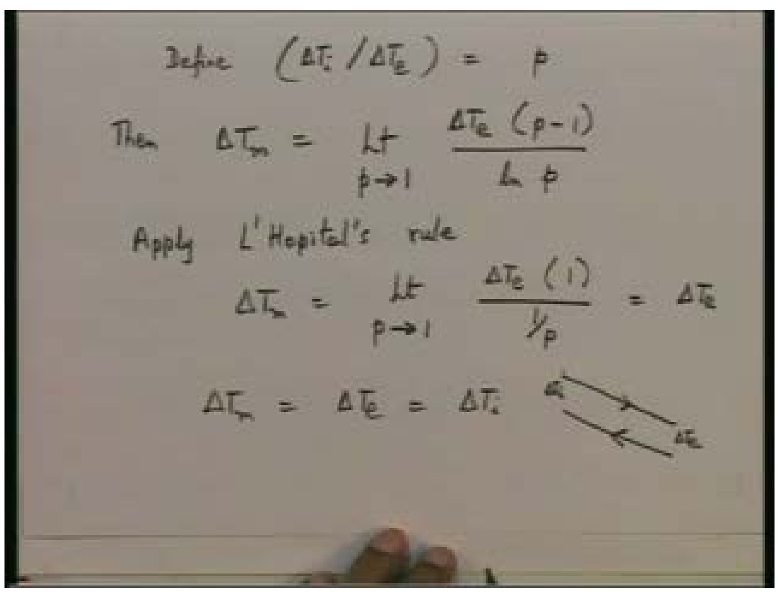

So, let us define - delta T_i divided by delta T_e, let us say this is equal to some quantity p in which case our expression for delta T_m becomes delta T_e p minus 1 divided by the logarithm to the base e of p. And we want to find this in the limit as p tends to 1 as delta T_i tends to delta T_e. L'Hopital's rule, say apply L'Hopital's rule - this is from our knowledge of calculus. L'Hopital's rule says differentiate the numerator and denominator with respect to p so you will get delta T_m is equal to the limit p tends to 1. I am differentiating now the numerator, the denominator with respect to p. delta T remains same; p minus 1 when differentiated with respect to p gives me 1. The logarithm of p when differentiated with respect to p gives me 1 upon p and if I now put the limit p equal to 1 I will get this is equal to delta T_e.

So, the application of L'Hopital's rule gives me the value of this indeterminate quantity delta T_m and what we find is that delta T_m is equal to delta T_e which is nothing but delta T_i. And in fact, in this particular case, what really happens is the following. What we really get is the temperature profiles - on the two sides are two parallel lines like this; this is on the hot side, this is on the cold side. This is delta T_i, that is T_{hi} minus T_c, this is delta T_e which is T_{he} minus T_{ci} and if you have 2 parallel lines it follows for the temperature profiles. If you have two parallel lines, it follows that delta T_m must be equal to delta T_e must be equal to delta T_i - that is really what we are getting. So for the special case, the temperature profiles tend to be straight lines - 2 parallel lines - and delta T_m becomes equal to delta T_i or delta T_e. So, this is one special case of counter flow worth talking about.

Now let us go on to cross flow. We want to now get expressions or get value for the mean temperature difference in cross flow. We have 3 cases there. You recall we have both fluids unmixed, that means we have fins on both sides which prevent the fluid from moving in a direction and mixing with itself. Then, we have one fluid mixed the other unmixed or we have both fluids mixed. So, let us take first case one in cross flow - both fluids unmixed.

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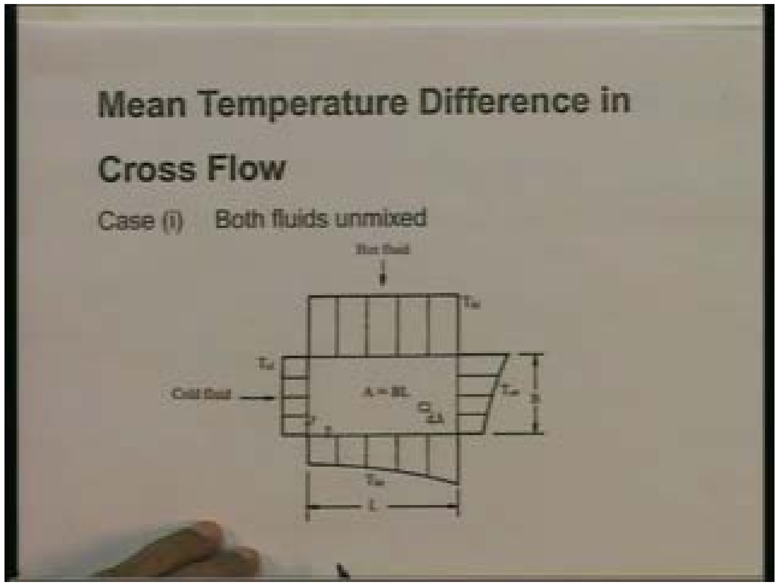

Now, this is how the temperature profile is going to look. In this case, this is the hot side fluid and this is the cold side fluid, this is the hot side fluid, this is the cold side fluid. The temperature on the hot side fluid is T_{hi} - uniform everywhere. While entering this is T_{hi} in the cold side; while entering it is uniform but now since it is a fluid in which no mixing takes place - it is both fluids unmixed - any fluid particle at T_{ci} which enters here is going to come into contact with hot fluid at T_{hi} and is going to heat up the most.

On the other hand, a fluid particle entering at this side at T_{ci} will come into contact with hot side fluid. On the other side which is at a lower temperature, it has been cooled in moving through the heat exchanger; so this fluid particle is going to heat up less. So why, leaving the heat exchanger the cold side profile is going to be something like this. In the same way, you can see the hot side fluid is going to be something like this. So, the temperature of the cold and the hot fluids will now be functions of x and y. Keep that in mind. Now, let us write down the expression that we get and see how far we can go.

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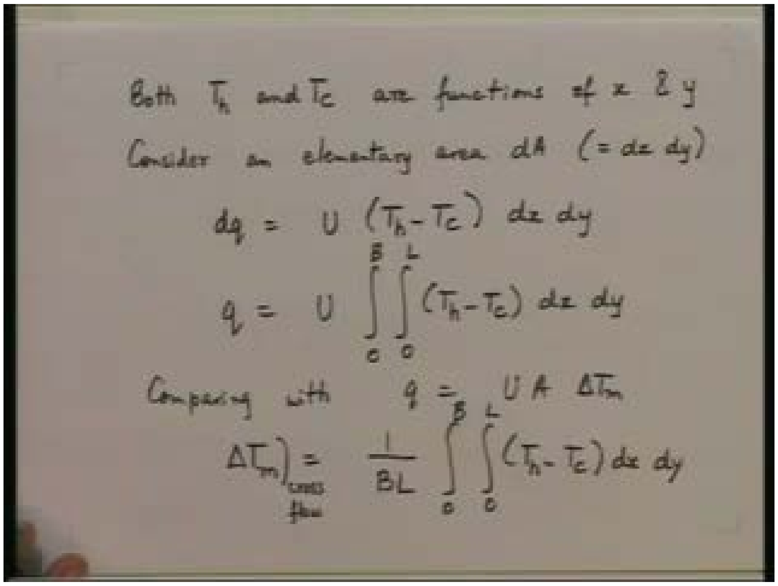

So, let me first make a statement both for this case - cross flow, both fluids unmixed, both T_h and T_c are functions of x and y; x and y are what we had earlier. Let me go back to the sketch - x is this direction that means the cold fluid is moving; y is the direction in which the hot fluid is moving. We need two directions now; it is a two dimensional problem unlike parallel flow and counter flow which have one dimensional problems. So, we had only the x direction there, that is important that you keep that in mind.

Now we have variations in x and in the y direction. So, T_h and T_c are functions of x and y. Consider again an elementary area dA; consider an elementary area dA which will be equal to dx dy. It is a 2 dimensional problem so we can say dq - the heat transfer rate of a - ur d across dA is equal to U T_h minus T_c into dx dy. If we assume U to be a constant and integrate over the whole area A of the heat exchanger we will get q is equal to U integral; we will now have to do a double integral T_h minus T_c dx dy where the integration over x is from 0 to L and the integration over y is from 0 to B.

Let me go back to the sketch that we had and show again this is the area A of the heat transfer surface; hot fluid flowing on one side, cold on the other side. In the x direction, the length is L in the y direction, the length is B, flow length is B so BL is the heat transfer area. x has to be integrated from 0 to L; y has to be integrated from 0 to B. That is what we are doing in order to find out the total heat transfer rate. Therefore, we can see straightaway comparing with our basic equation, comparing with q is equal to UA delta T_m - that is our basic equation.

We have delta T_m in cross flow is equal to: this is for cross flow delta T_m, for cross flow is equal to 1 upon BL the double integral in 0 to B, 0 to L, T_h minus T_c multiplied by dx dy - that is our expression for delta T_m in cross flow. So, you can see it is more complicated than we had earlier; we have now got a double integral which has to be handled. So, the determination of delta T_m is going to be much more complicated and it has been done numerically so we don't have to worry about it; we will get the results. But it is not as straight forward as it is for parallel flow or for counter flow which were one dimensional problems. Now let us look at the other case - case two. What is case two?

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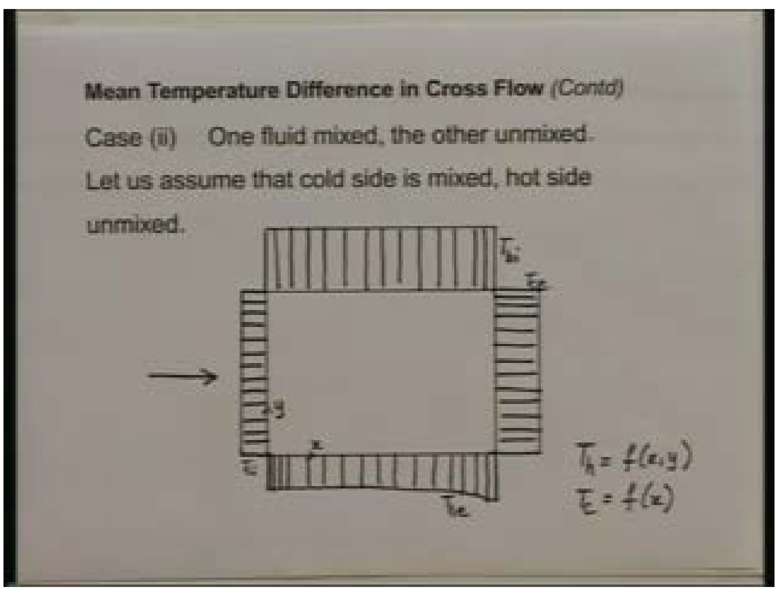

One fluid mixed, the other fluid unmixed. Let us assume the cold side fluid is mixed and the hot side fluid is unmixed. Now what we are going to get is something like this. This is our cold side fluid. Let me draw its temperature profile. This is our cold side fluid uniform while entering which is T_{ci} and this is where the fluid is entering, cold fluid entering. This is my direction x; this is my direction y. Let us say the hot side fluid, let me draw again a sketch there of the hot side fluid, this is the temperature profile at the inlet uniform while entering so this is T_{hi} that I am showing, fluid temperature profile while entering on the hot side this is T_{hi}.

Now, the hot side fluid is unmixed therefore its temperature profile will be non-uniform at the exit, something let us say like this. The fluid on this side which is unmixed will get cold more, the fluid on this side will get cold less, and we will have some non-uniform temperature profile with T_h being a function of x and y; T_h will be some function of x and y. On the other hand, on the cold side since we have got mixing, any fluid particle that enters here and gets heated; any cold particle that enters here and gets heated. The movement that happens, they can also mix transversely with each other and get two uniform temperature, then proceed further and again receive heat from the hot side.

So, on the cold side we are going to have a uniform temperature profile; because of the mixing taking place on the cold side we are going to have a uniform temperature profile like this on the cold side. This is the temperature profile on the cold side like this - this is T_{ce}, this was T_{he}, here which is a function of x and y, T_h which was function of x and y and T_{he} which was varying.

So, we should note that whereas T_h is a function of x and y, T_c - because of mixing - is a function of only of x. So, this is going to be the situation when we have one fluid mixed and the other unmixed and the third case which we have is when both fluids are mixed in which case we are going to get something like this. Let us draw that situation.

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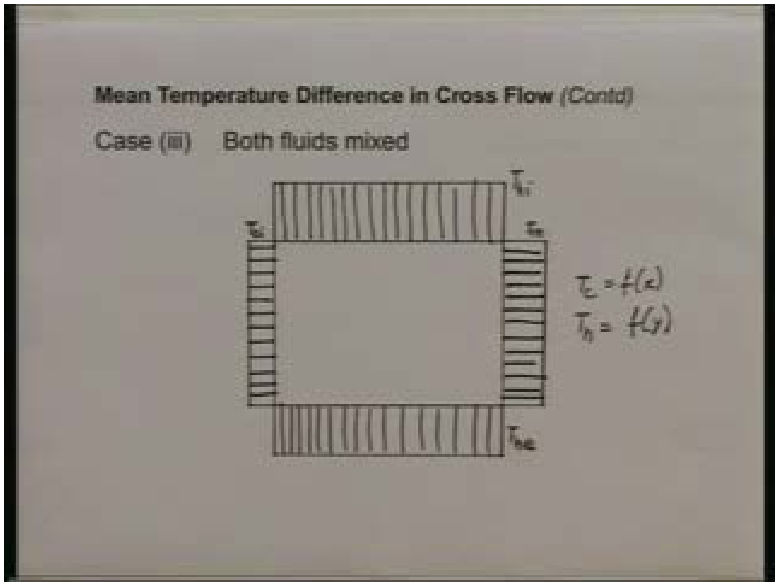

We are going to have mixing on both sides so we will have the fluid on that side, uniform temperature while entering and uniform temperature while leaving mixing on both sides. So, I am going to have now a temperature profile because of the mixing on both sides which is something like this. So, this is T_{ci}, this is T_{ci}, this is T_{ce} also uniform because mixing is taking place on the cold side. Similarly, T_{hi} like this on the hot side and T_{he} on the cold side, so this will be T_{hi}. The temperature profile on the hot side, this is T_{hi} and this will be $T_{he,}$ uniform while leaving. Because of mixing this will be The, so in this case now T_c will be a function of x and T_h will be a function of y.

So, it is still a two dimensional problem but the T_h is a function of y only and T_c is a function of x. So we have got more complicated situation. These integrations which I - for finding out the mean temperature difference for these 3 cases - the integrations to find out delta T_m have been done numerically.

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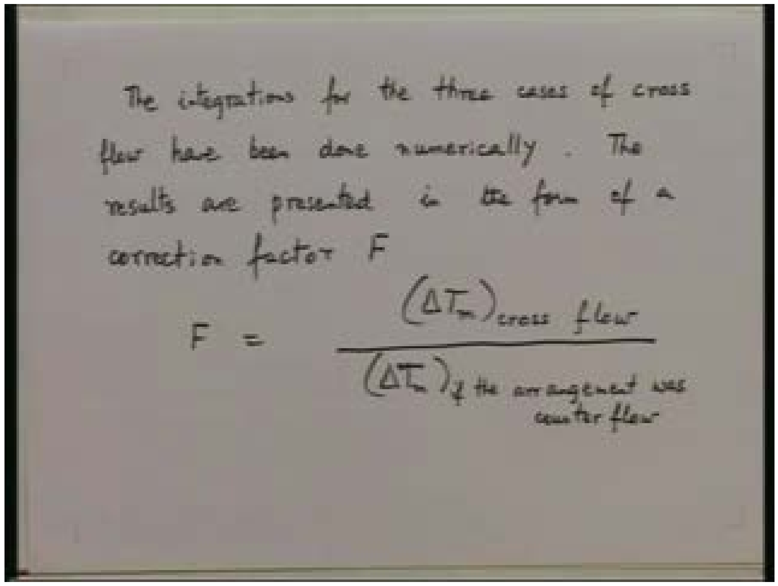

The integrations for the 3 cases of cross flow have been done numerically and the results are presented in the form of a correction factor F; the results are presented in the form of a correction factor F. F is defined as - F is equal to the delta T_m in cross flow divided by the delta T_m if the arrangement had been counter flow. That is how the results are presented in the form of correcting factor F which is delta T_m cross flow upon the delta T_m if the arrangement had mean counter flow.

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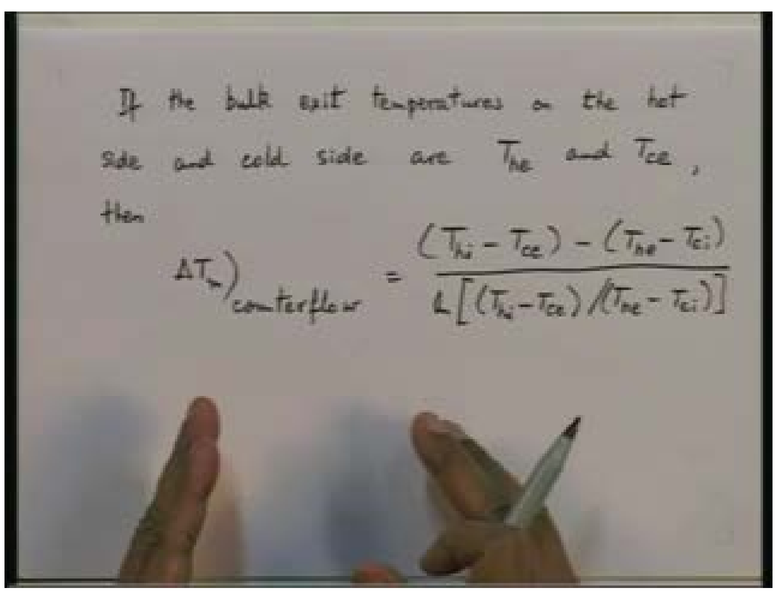

If the bulk exit temperatures on the hot side and the cold side, if the bulk exit temperatures - they may be non-uniform but if I get a mean value and call it the bulk exit temperature, if the bulk exit temperatures on the hot side and cold side are T_{he} and T_{ce}, then delta T_m - if the arrangement had been counter flow - we know that delta T_m in counter flow will be T_{hi} minus T_{ce} minus T_he minus $T_{ci.}$ The logarithm to the base e T_{hi} minus T_{ce} divided by T_h e minus T_{ci} delta T_i upon delta T_e. So the final results based on this numerical integration are given in the form of a correction factor F. F is equal to the delta T_m in cross flow upon the delta T_m; if the arrangement had been counter flow, we know how to calculate the delta T_m in counter flow.

Now next time, I am going to tell you how F has been determined and how it is presented graphically so that once we get a value of F we can get the delta T_m in cross flow.