Heat and Mass Transfer
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Lecture No. 24
Natural Convection – 3

At the end of the last lecture we were looking at the situation of horizontal plates.

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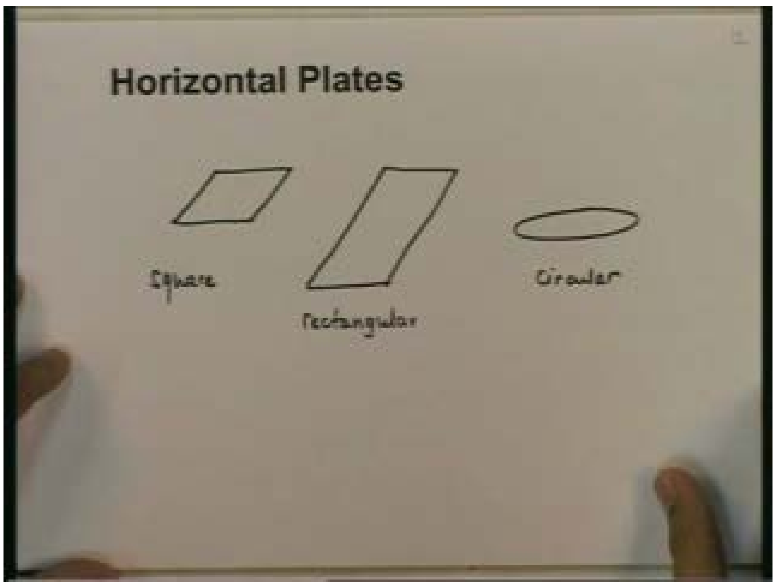

We considered a square platelet or a rectangular plate or a circular plate held horizontal and losing heat either from its top surface or its bottom surface or from either side.

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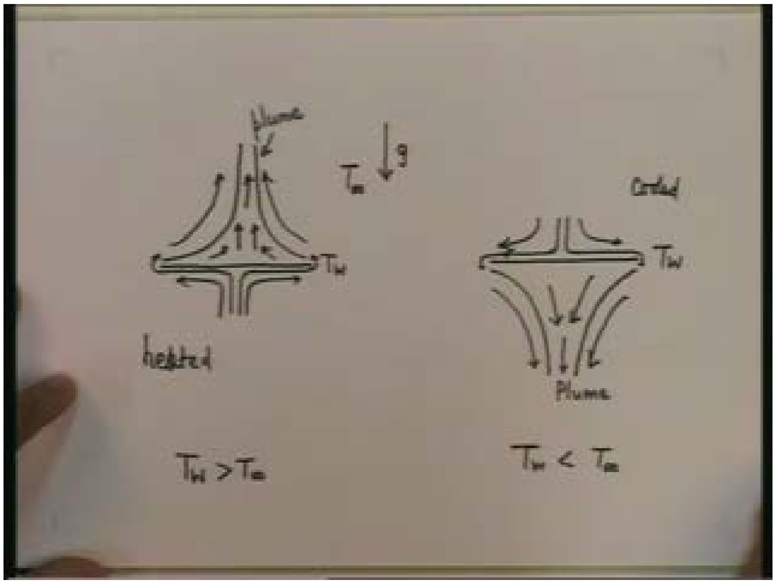

We also considered a typical horizontal surface and we said that if the surface is hot, then the fluid over it will try to rise in the form of a plume whereas the fluid below it will tend to stick to the surface but will try to escape from the sides. The flow pattern will be different on the top surface and on the bottom surface. However, if the surface is cooler compared to the surroundings, then the fluid below the surface will become denser and we will try to descend. So this is the plume which moves down in this case whereas the fluid near the top of such a surface will try to stick to the surface, will drift over the edges and will try to escape downwards. In case of heated surface, the general tendency of fluid surrounding the surface will be for moving up in case of a cooled surface, the general tendency of the fluid surrounding it will be to move downwards and because of this we should appreciate that the situation of a heated surface facing upwards will be that of a heated surface facing off a cooled surface facing downwards. And similarly a cooled surface facing upward will have a similar flow pattern compared to a hot surface facing downwards. Consequently, we expect to see 2 set of correlations, one for a hot surface facing upwards and one for a hot surface facing downwards. These 2 situations would be applicable respectively to a cold surface facing downwards and a cooled surface facing upwards.

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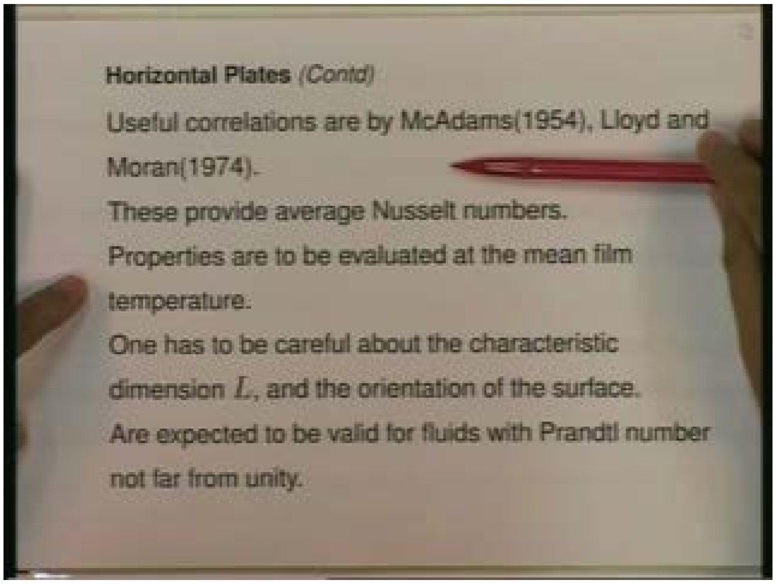

Based on experimental data for such horizontal plate situation, useful correlations are provided by McAdams and by Lloyd and Moran. We should note that these proved average Nusselt numbers not local ones so we have the average heat transfer coefficient. Properties are to be evaluated at the main film temperature. One has to be careful about the characteristic dimension used in the dimensionless Grashof number or Rayleigh number because it differs; definition of L differs for each shape of the plate and it also depends on whether the plate is facing upwards or facing downwards. These are not valid for a very wide range of Prandtl number; the experiments have been done essentially with air, sometimes with water and hence the correlations are expected to be valid for fluids with Prandtl number not far from unity. Air and water are okay but they may not be valid for say liquid metals or for highly viscous oils.

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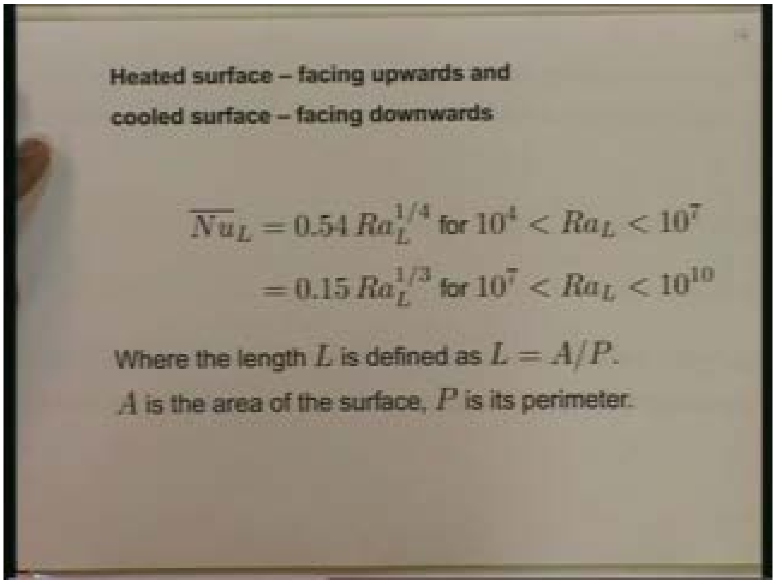

Let us first look at a correlation which is for a heated surface facing upwards or for a cooled surface facing downwards. These are similar situations and you will notice that the Nusselt number is provided as a function of only the Rayleigh number; there is no separate existence of the Prandtl number. And for 2 ranges of Rayleigh number from 10 raise to 4 to 10 raise to 7 and from 10 raise to 7 to 10 to raise to 10, we have Nusselt number as some constant into Rayleigh number raised to a small exponent. Notice that for these correlations - heated surface facing upwards or cooled surface facing downwards - the characteristic length L is defined as area of the surface of the plate of one side of the plate divided by its perimeter. And this is applicable for square plates, rectangular plates as well for circular plates; you will notice that for a circular plate, the characteristic length is not the diameter. For the other situation - a hot surface facing downwards or a cooled surface facing upwards.

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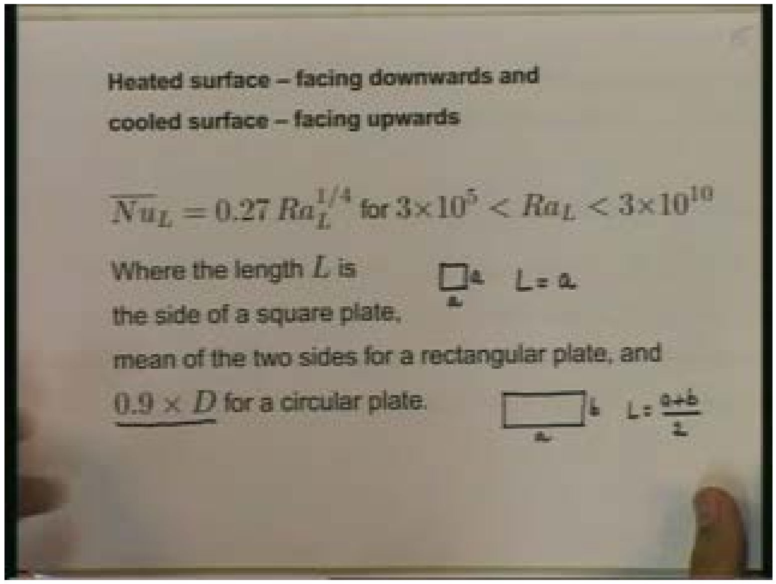

We have a single range of Rayleigh number and a simple correlation similar to the earlier ones with a different value of the constant. Now see it is the L which is defined in a different way, for a square plate it is side of the square, so if you have a square say side a by a, then the characteristic dimension is a; mean of 2 sides for a rectangular plate - if you have a rectangular plate a by b, then L equals a plus b by 2 and if you have a circular plate it is 0.9 time the diameter of the circular plate. Let us take an example so that we are exposed to solving problems with these correlations.

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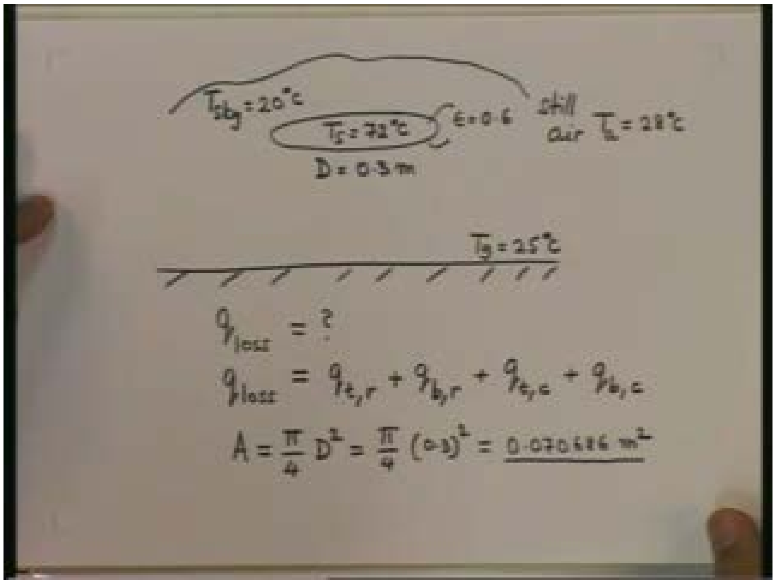

The situation we are going to look at is this - we have a circular plate kept horizontal in still air, it is maintained at a surface temperature of 72 degrees C, the diameter of the plate is .3 meters. Either surface of the plate - top as well as bottom - has an emissivity of .6, the ambient air is at a temperature of 28 degrees C. Well, the plate is at some distance from the ground and the ground temperature is 25 degrees C and the top surface of the plate is exposed to the so called sky where you can say the temperature of the sky is 20 degrees C. Now the tip is warmer than the surrounding temperatures and hence it will lose heat by natural convection and by radiation; our task is to determine what is the heat loss from the plate.

We will have to split this into 4 components since it losses heat both by radiation as well as by convection and both from the upper surface as well as from the lower surface. We can write 4 components - the total loss is from the top surface by radiation plus the bottom surface by radiation plus that from the top surface by convection plus that from the bottom surface by convection. Let us first calculate the area of the plate which we will be needing again and again; this is pi by 4 D squared which is pi by 4 into .3 squared - this turns out to be 0.070686 meter square. We will use this again and again so we will calculate it once and finish it off. Let us compute the components one after another.

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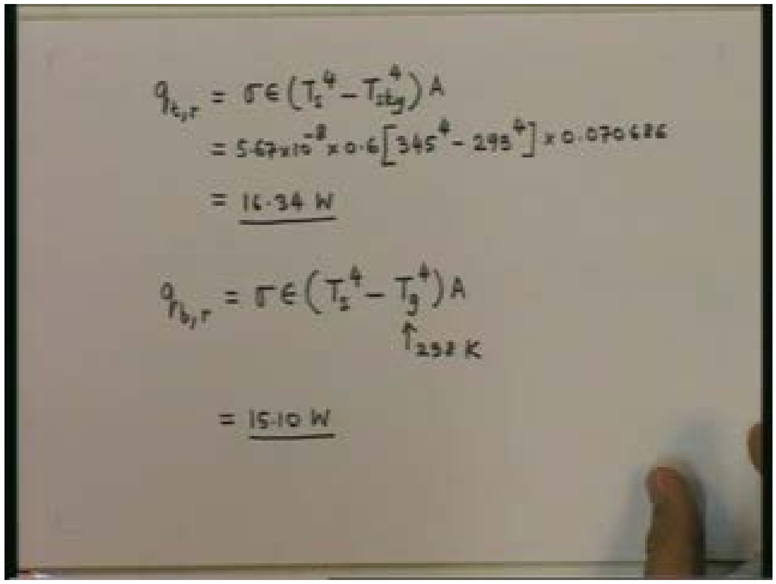

Heat loss from the top by radiation - this will be sigma epsilon surface temperature raise to 4 minus sky temperature raise to 4 multiplied by area. This is 5.67 into 10 raise to minus 8, epsilon is .6, now the surface temperature is 72 degrees C which is 345 Kelvin raise to 4, sky temperature is 20 degrees C - that will be 293 Kelvin raise to 4, multiplied by area 0.070686 and this turns out to be sixteen.3 4 watt 1 component.

Similarly, let us calculate the heat loss from the bottom by radiation; this will again be sigma epsilon but now T surface raise to 4 minus T ground raise to 4 multiplied by area. Now, between this expression and this expression, the only difference is T ground is 25 degrees C which is 298 Kelvin and if you substitute that you will get this to be 15 by 10 watt after having completed the task of obtaining the radiative components from the top surface. And from the top surface, let us move on to compute q top by convection and q bottom by convection.

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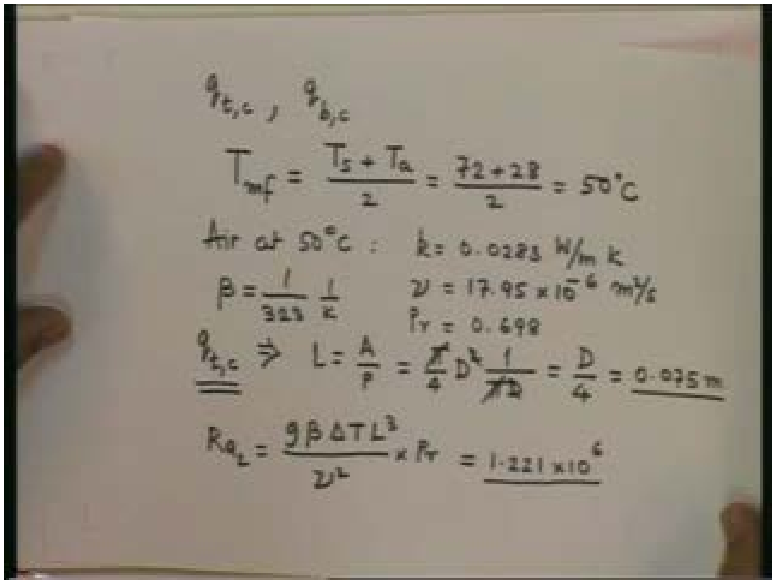

For this we will have to calculate Rayleigh number and for calculating that we need properties at the mean film temperature which is T surface plus T air divided by 2. Surface temperature is 72 plus 28 divided by 2 which is 50 degrees Celsius. If you read off properties of air at 50 degrees Celsius, you will get conductivity of 0.0283 watt per meter Kelvin. nu is 17.95 into 10 raise to minus 6 meter square per second and Prandtl number is 0.698.

Now for determining the top, heat transfer from the top by natural convection, we require – it is a circular plate, the definition of L is area by perimeter. Area is pi by 4 D squared perimeter is pi D so we get diameter by 4; the characteristic length here is 0.075 meter. If you substitute these values in the definition of the Rayleigh number g beta delta T L cube by Nu squared into Prandtl number, you will get this to be 1.221 into 10 raise to minus 6. Remember we need beta as a property but then beta will be 1 divided by T_{mf} in Kelvin, that will be 1 divided by 323, 1 over Kelvin. So, we have everything here to compute the value of the Rayleigh number. After having computed the Rayleigh number, we use the correlation for the heat transfer from the top in that range and that turns out to be - Nu bar L is 0.54 into Rayleigh based on L raise to 1 by 4.

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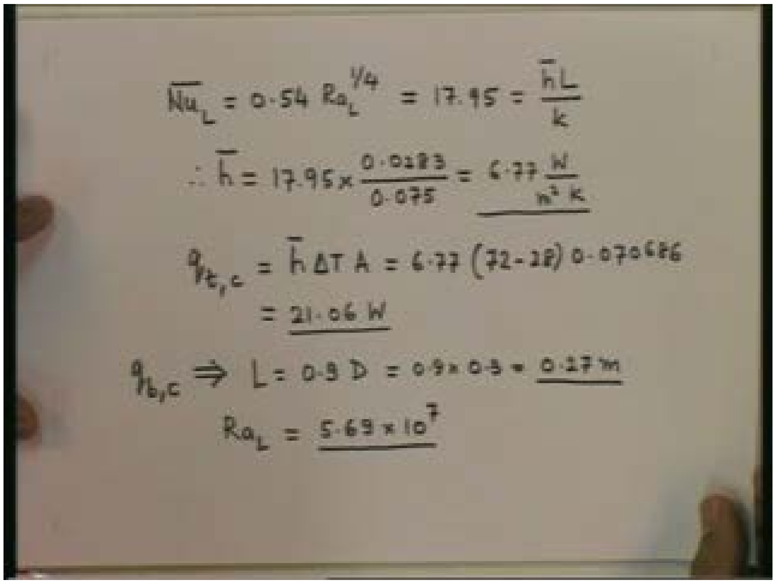

Substituting our value of Rayleigh number which is 1.221 into 10 to the 6, we will get this to be 17 by 95 which is h bar into L by k. Hence h bar is 17 by 95 into k - 0.0283 - divided by characteristic length 0.075; this turns out to be 6.77 watt per meter squared Kelvin and hence our heat loss from the top surface by convection turns out to be h bar into delta T into A which is 6.77, delta T is 72 minus 28 and area is 0.070686 and this turns out to be 201.06 watt .

Now, we need to determine q from the bottom by convection; for this, since it is a circular plate, L is defined to be .9 times diameter. So, this turns out to be .9 into .3 which is .27 meter; notice that this is significantly different from that for the upper surface. This gives us a Rayleigh number to be 5.69 into 10 raise to 7; it is different because the characteristic length is different, all other components for the Rayleigh number remain the same.

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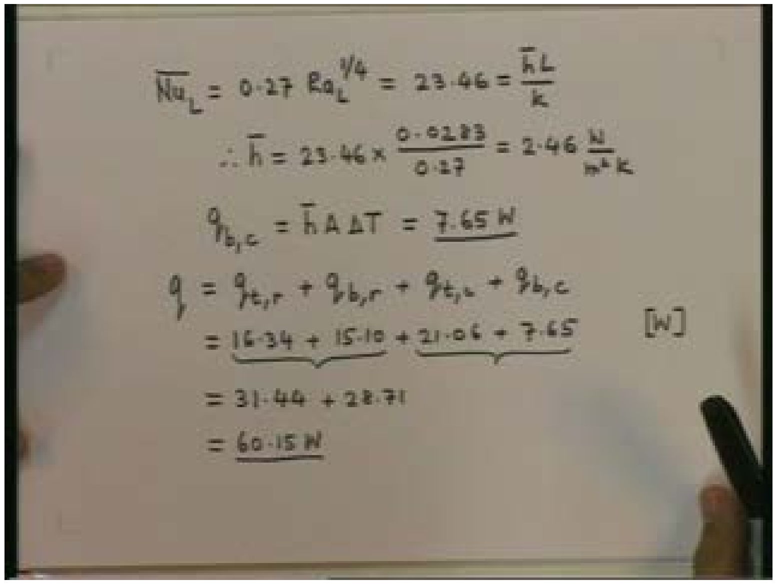

Now, using this Rayleigh number, we get by the correlation for the average Nusselt number which is .27 Rayleigh number based on raise 2 Ray based 1 L raise to 1 by 4 is 23.46 which is h bar L by k. Hence h bar turns out to be 23.46 multiplied by k - 0.0283 - divided by L which is .27 and this turns out to be 2.46 watt per meter squared Kelvin. And from which heat transfer from the bottom by natural convection is h bar A delta T which turns out to be 7 by 65 watt. So our total heat loss q which is q from top by radiation plus q from bottom by radiation plus q from top by convection plus q from bottom by convection, let us write down these terms 16.34 plus 15.10 plus 21.06 plus 7.65 - everything in watts.

Let us compute the s of the radiative component and that of the convective component separately. So, radiative component total is 31.44 watt s, convective component total is 28.71 watt , so the total is 60.15 watt and this indicates that in this situation, the radiation is slightly more that half of the total. Heat loss radiation does not predominate but it is as significant as natural convection, slightly more so.

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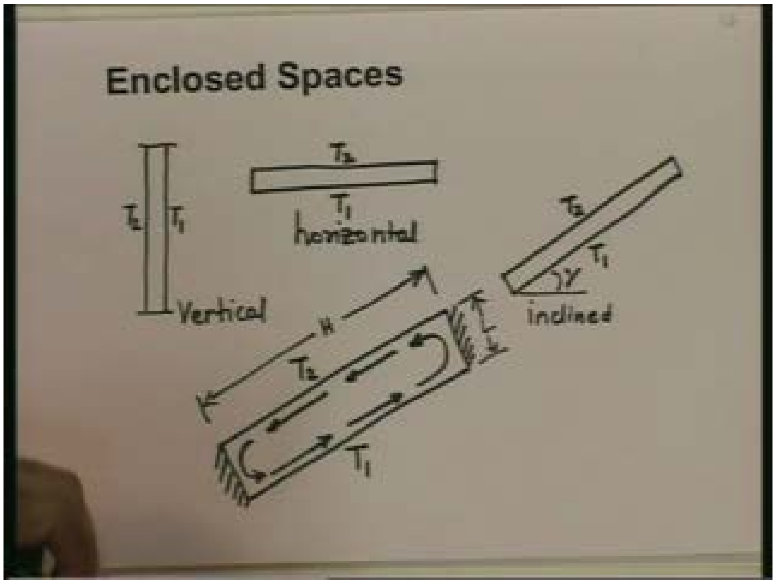

Now let us look at the last situation and that is enclosed spaces - natural convection in enclosed spaces and these enclosed spaces occur in many applications, say vertical enclosed space occurs between double glazed glass windows common on air-conditioned rooms - one side is at T_1, another side is at T_2. Sometimes, they occur in horizontal situations T_1 T_2. In solar collectors, we have a hot plate and a, which absorbs radiation and there is a glass cover on top which suppresses convection so you have a gap - one surface is at T_1, another surface is at T_2. Depending on the orientation - whether it is vertical, horizontal or inclined at some angle gamma from the horizontal - we will have different flow patterns. For example, in a typical inclined cavity where we have T_1 and T_2 as the 2 surfaces and the remaining 2 surfaces - the smaller ones - are assumed to be insulated, the flow along the hot surface will move up; it will take some sort of a u turn near the edges, will move down along the cooled surface and complete a circuit; some sort of circulation pattern will be set up.

In case of inclined cavities, the circulation pattern in often a single circulation cell whereas in case of horizontal cavities there will be a large number of small cavities formed known as benard. In case of a vertical thing, you may sometimes have a single cavity, sometime single cell, sometimes multiple cell, depending on the ratio of the width

L to the height H of the cavity. This situation is a very common situation in solar collectors, double-glazed windows and a number of applications of interest in industry and hence it is a situation which is reasonably well studied both experimentally and analytically. Correlations are available for long cavities where the ratio of H by L is large; a cavity which does not look like a square but looks like a long rectangle.

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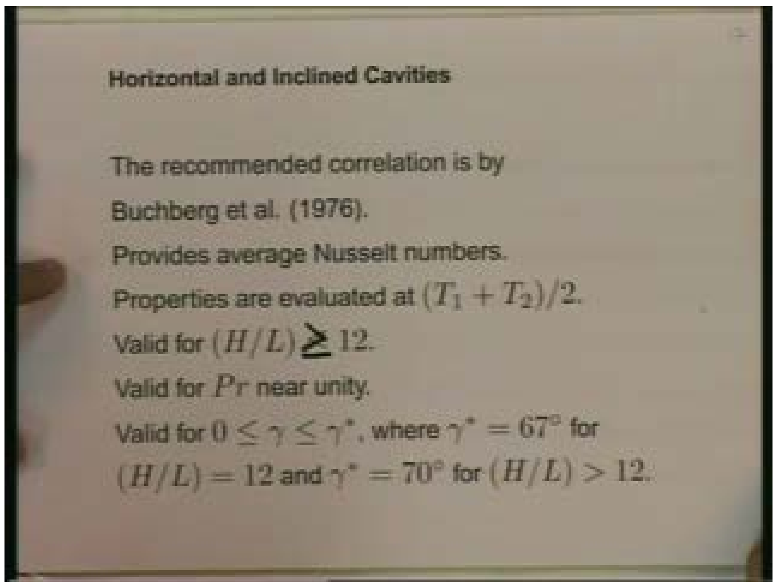

The recommend correlation is by Buchberg and associates, it provides average Nusselt numbers. The properties are evaluated at T_1 plus T_2 by 2 where T_1 is the - a hot surface usually expected to be below the cooler surface - and T_2 is the temperature of the cooler surface. So the temperature difference delta T will be T_1 minus T_2; properties are to be evaluated at T_1 plus T_2. It is valid for long cavity that is H by L greater than 12, greater than or equal to 12. Since the data is essentially with air, it is expected that these are valid for Prandtl number near unity for air or for water. Once, correlation is valid for horizontal cavity and cavities inclined at approximately 67 degrees to 30-70 degrees - that is the range of the inclination of the Buchberg correlation. This upper limit of inclination is 67 degrees if H by L is 12 and 70 degrees if H by L is greater than 12; the correlation is in different ranges.

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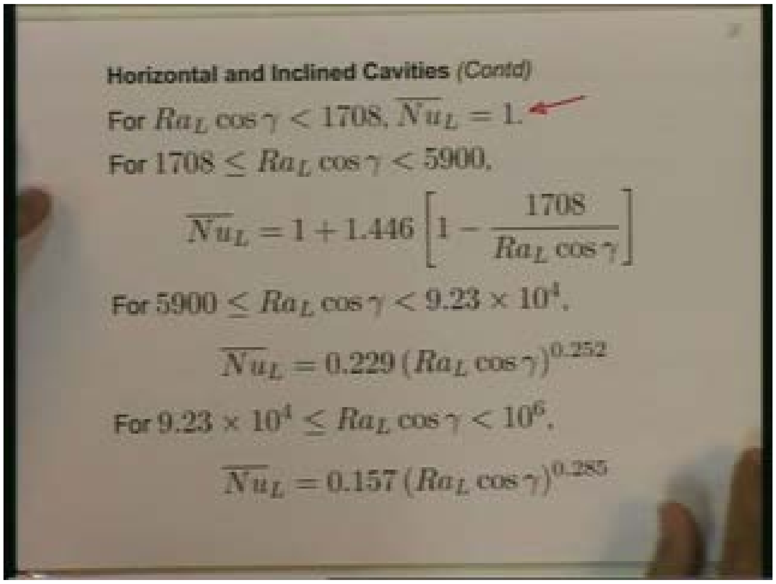

If Rayleigh number into cosine gamma is less than 1708, the Nusselt number is 1 and this indicates, 1 indicates that heat is transferred essentially by conduction. Then for different values of the parameter Rayleigh number into cosine of gamma, we have different correlations up to 5900 one, then up to 9.23 into 10 raise to 4 another and up to 10 raise 6 a third one. We will solve a problem based on this soon. There is a correlation by El Sherbiny and associates.

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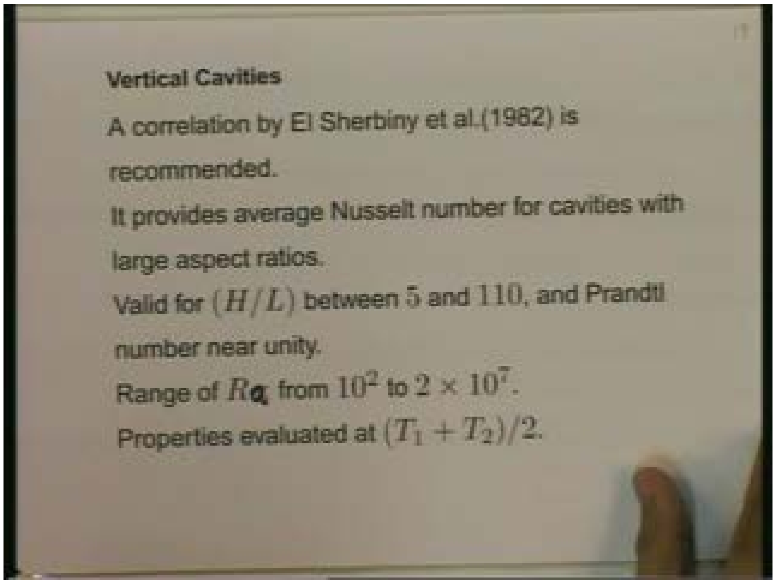

Again it provides average Nusselt number for cavities with large aspect ratios - typically the aspect ratios expected to be between 5 and 110; again Prandtl number is expected to be near unity. The range of Rayleigh number is from 10 raise to 2 to 2 into 10 raise to 7 and properties are evaluated again at the mean temperature T_1 plus T_2 by two. Let us look at the correlation.

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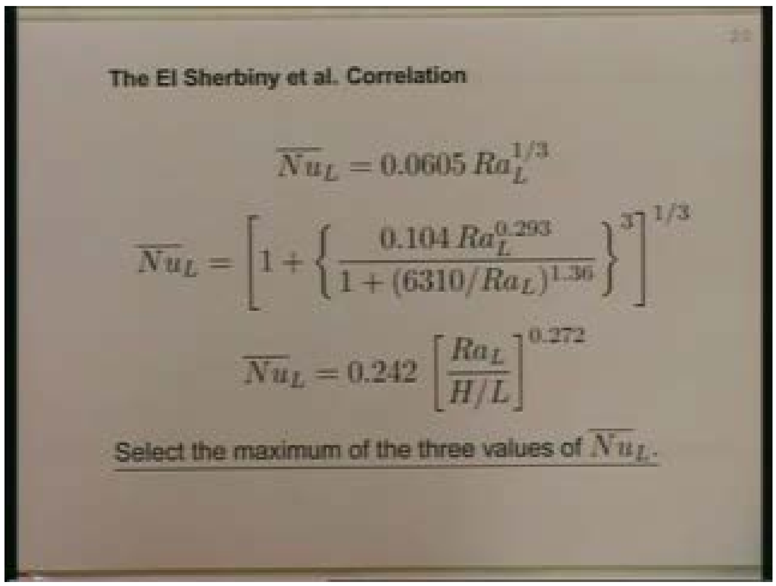

Here, there are 3 expressions provided for Nusselt number - expression 1 which provides Nusselt number as a function of Rayleigh number, a second expression again providing Nusselt number as a function of Rayleigh number - a more complicated one, and a third one though simple but it provides Nusselt number as a function of Rayleigh number and the aspect ratio H by L. And we have to select the maximum of the 3 values of Nusselt number obtained by using 3 correlations. So we must calculate say a Nu bar L_1, a Nu bar L_2 and a Nu bar L_3 and select the maximum of these as the average Nusselt number.

Now, the earlier correlation by Buchberg gave us correlation or Nusselt number up to an inclination of about 70 degrees. So from horizontal to approximately 70 degrees, we have the Buchberg correlation, for vertical we have the El Sherbiny correlation.

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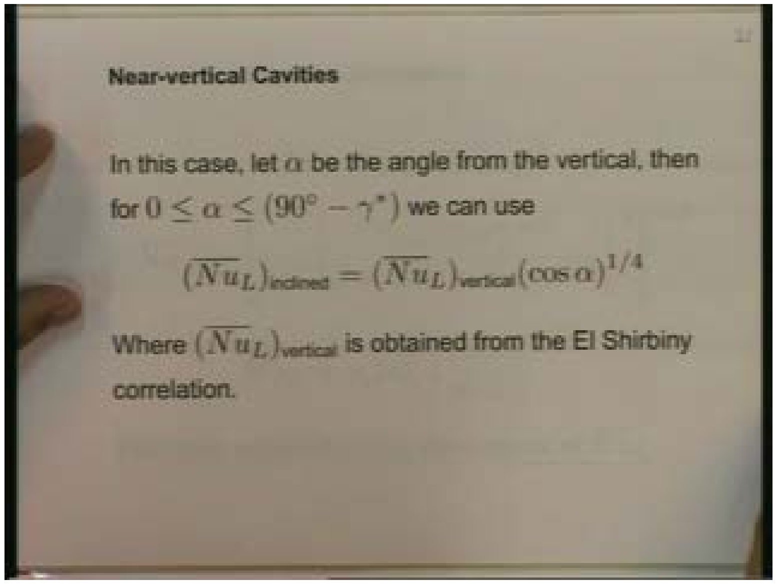

So for angles between 70 degrees and the vertical it turns out that we can use the Nusselt number determined for the vertical correlation and multiplying by the fourth root of cos alpha where alpha is the inclination with respect to the vertical. So alpha will actually be 90 degrees minus gamma; so with this, the El Sherbiny correlation and the Buchbereg correlation provide us almost all cases for horizontal inclined and vertical cavities. Let us solve an example.

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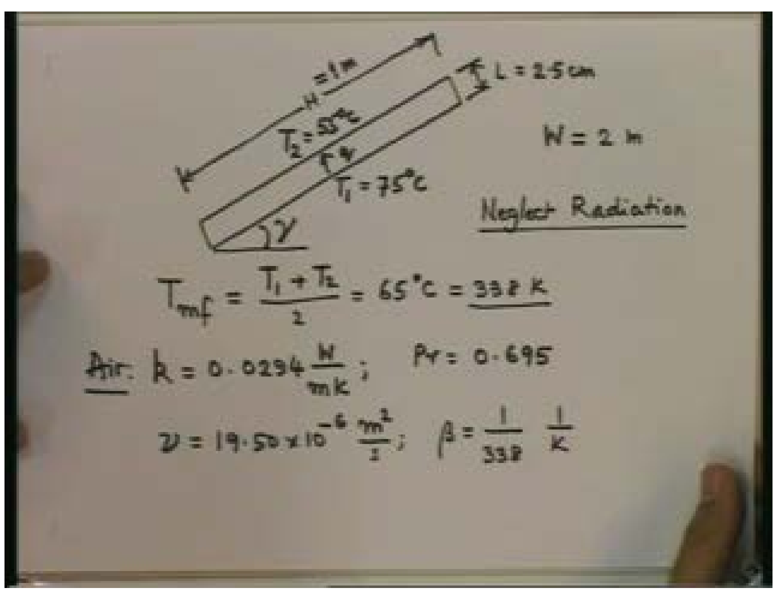

Let us say that we have a cavity which is a part of a solar collector; let us say it is inclined at some angle gamma. We will obtain solutions for some different values of gamma. This is the collector plate which is at 75 degree Celsius and the glass surface is at 55 degree Celsius. The inclined height of the collector H is 1 meter and the gap between the observe plate of the collector and the glass is 2.5 centimeter, the width of the collector say perpendicular to the plane of the picture is 2 meters. So, the collector area will be 1 meter by 2 meters 2 square meters and we would like to determine what is the heat loss from the hot surface to the cold surface q for different values of gamma.

Let us try to cover as much of the range from 0 degrees to 90 degrees as possible. We will have to determine the main film temperature that is T_1 plus T_2 by 2 which is 65 degree C which is 338 k. To begin with, we will neglect radiation in the comparison; we will assume radiation doesn't take a part. However, we should that radiation will not be depending on the inclination of the surface so finally we will determine a value for the radiative component. At this temperature, let us determine properties of air: conductivity - 0.0294 watt per meter Kelvin, Prandtl number is .695, kinematic viscosity 19.50 into 10 raise to minus 6 meter square per second, beta is 1 over 338 per Kelvin. Let us calculate the Rayleigh number Ra_L.

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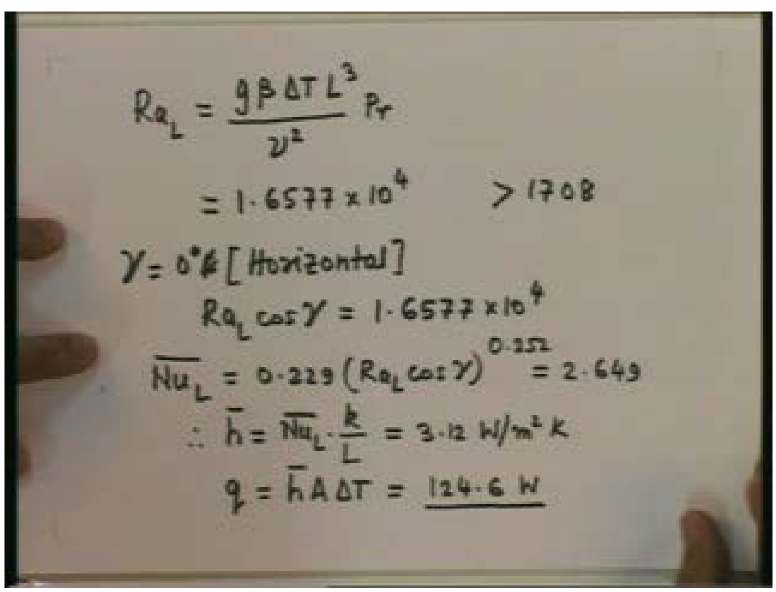

This is g beta delta T L cube by Nu squared into Prandtl number. We know everything here; value of g is assumed to be 9.81 meters per second per second. This turns out to be 1.657 seven into 10 raise to 4, this is greater than 1708 so there will be natural convection in the gap. Let us compute at gamma equal to 0 degree - so horizontal in this case; Rayleigh number into cosine gamma cosine gamma is 1 so this is 1.6577 into 10 raise to 4. We have a correlation for the average Nusselt number which is 0.229 Ra_L cosine gamma raise to .252 and this turns out to be 2.649 and hence the average heat transfer coefficient which is Nu bar L into k by L, this turns out to be 3.12 watt per meter squared Kelvin and the heat flow rate will be h bar into A into delta T. h bar is just computed, area is 2 square meters, delta T is 75 minus 55 which is 20 degrees Celsius. This turns out to be 1204.6 watt. Proceeding in the same way, one can calculate at different values of gamma.

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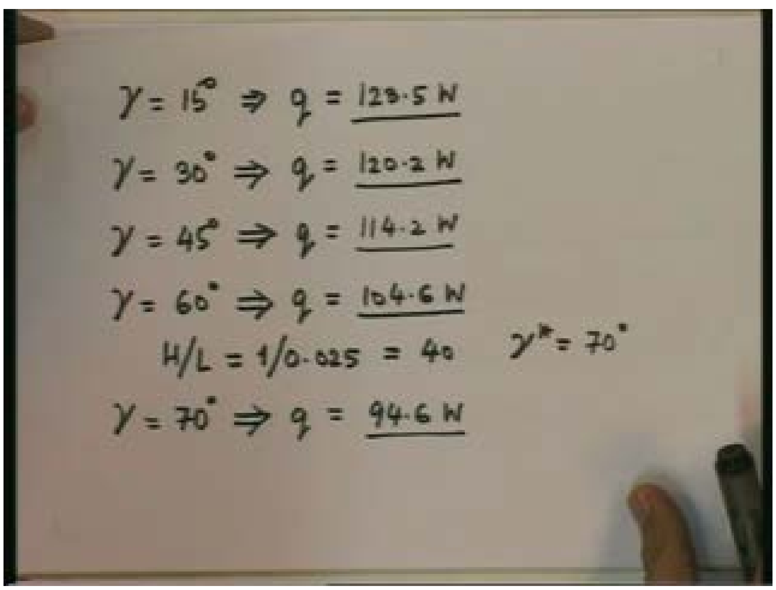

For example, at gamma is 15 degrees, you will get the heat loss to be 123.5 watt , only slightly smaller than the 124.6 watt which we obtained just now. Gamma equal to 30 degree - we will obtain 120.2 watt, gamma equal to 45 degree - we will obtain 114.2 watt. Notice that upto 45 degrees there isn’t any significant decrease in the amount of heat loss. At 60 degrees, q is 104.6 watt; since H by L is 1 meter divided by .025 meters which is 40, our gamma star - the limit for inclination for use of the Bushberg correlation is 70 degrees. And at that, let us calculate q and that turns out to be 94.6 watt. So from horizontal to the inclination of 70 degrees, the heat loss reduces from about 125 watt to about 95 watt - a decrease of around 25 percent.

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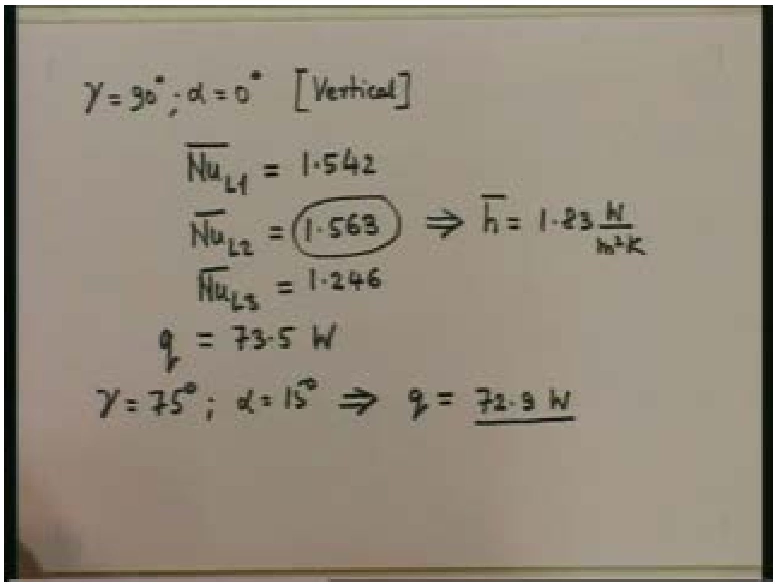

Now let us look at the other end- gamma equals 90 degrees or alpha equals 0 degrees and that is vertical orientation; in this we have 3 correlations and we have to determine the maximum of the 3 - a Nu bar L_1, a Nu bar L_2 and Nu bar L_3. And if you do that, you will get from the first correlation the value of the Nusselt number to be 1.5422. From the second correlation you will get 1.563 and from the third correlation you will get 1.246. We select the highest of the 3 so this is the Nusselt number which gives us an average heat transfer coefficient of 1.83 watt per meter squared Kelvin and a heat loss of 73.5 watt and using the inclination relation at 75 degree which means alpha is 15 degrees, we obtain q equal to 72.9 watt. So, if you tabulate the results you will notice the following.

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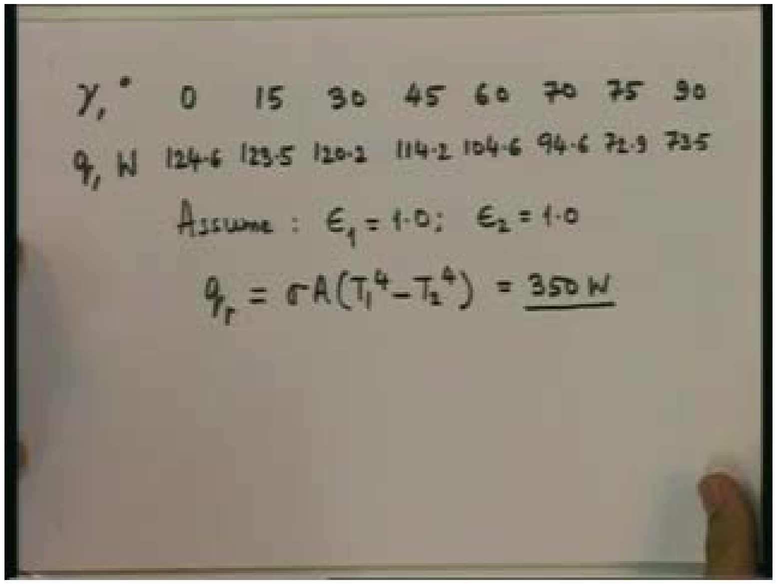

This is a gamma in degrees and q is watt. We have result for 0 degrees, 15, 30, 45, 50, 60, 70, 75 and 90 – 124.6, 123.5, 120.21, 114.21, 104.6, 94.6, 72.9 and 73.5. So, we get the highest heat loss when the cavity is horizontal, reduces to about 72, 73 watts, when it is 75 degrees - increases slightly to 73.5 watts.

How does this compare with radiation? Remember in any natural convection situation, radiation plays a dominant role. So let us assume that for the plate surface, the emissivity is 1 for the glass surface, also the emissivity and absorptivity are 1. For the glass surface this is a good assumption, for the plate surface this is not that good an assumption - this will typically be .8. However assuming 1 will give us the highest value of radiative heat transfer. So assuming this our heat loss from the observer plate to the glass surface by radiation turns out to be sigma A T_1 raise to 4 minus T_2 raise to 4 and this turns out to be 350 watt.

Even if the emissivity is, were slightly lower, for example this would be 1; if this is .5 even then, this is 175 watt and that means in this situation whatever be the inclination, the radiation is going to dominate the natural convection. Now that brings us back to the end of the study of natural convection, what we did in this set of lectures for natural convection.

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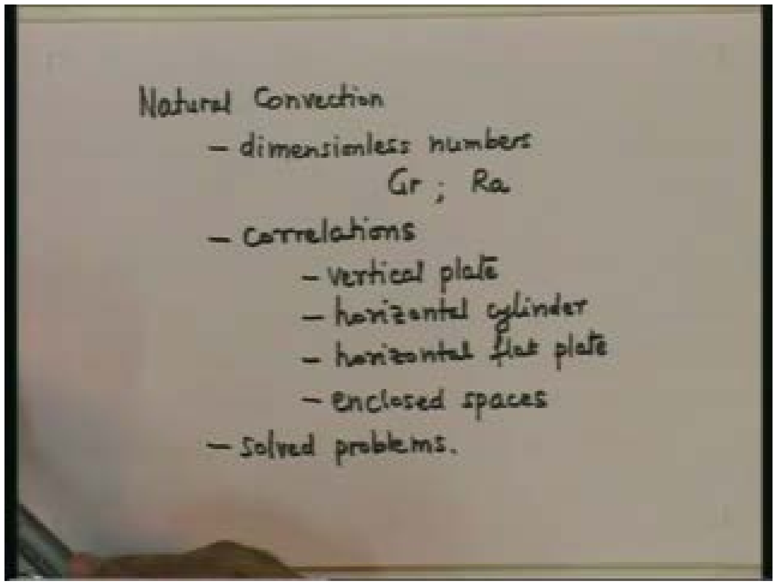

So, we looked at natural convection as a phenomenon, we looked at the dimensionless numbers, particularly the numbers of importance for us are Grashof number and related to it the Rayleigh number. Then, we looked at correlations and the situations we looked at were vertical plate, horizontal cylinder then horizontal flat plates - both facing upwards as well as facing downwards - and finally enclosed spaces. We looked at correlations and we solved problems using these correlations.

Notice that in a large number of situations of natural convection, radiation dominates so whenever you solve a problem in natural convection in any situation, use the proper natural convection. Obtain the natural convection heat transfer but simultaneously also compute or estimate the amount of heat transfer by radiation because in natural convection that will play an important role.