Heat and Mass Transfer
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Lecture No. 23
Natural Convection-2

Welcome back to the second lecture on natural convection. In the previous lecture which was the first1on natural convection we saw what was natural convection all about. We looked at the dimensionless numbers which arise in natural convection, particularly the Grashof number and the Rayleigh number. Then we looked at1situation - that of a vertical flat plate - and looked at the recommended correlation of Churchill and Chu for that situation. In the last part of the lecture, we solved a problem on natural convection heat transfer from a vertical flat plate and noticed that radiation will often play or will usually play a very significant role in processes where heat transfer takes place by natural convection.

In the typical problem that we came across, the contribution to the total heat transfer by the radiative component turned out to be very significant; it was predominantly by radiation. In other cases, it need not be so but radiation will play a reasonably important role in almost all natural convection situations. Although we have a correlation for a vertical flat plate, suppose we have a situation where we have a vertical cylinder of diameter D and height L and it is vertical.

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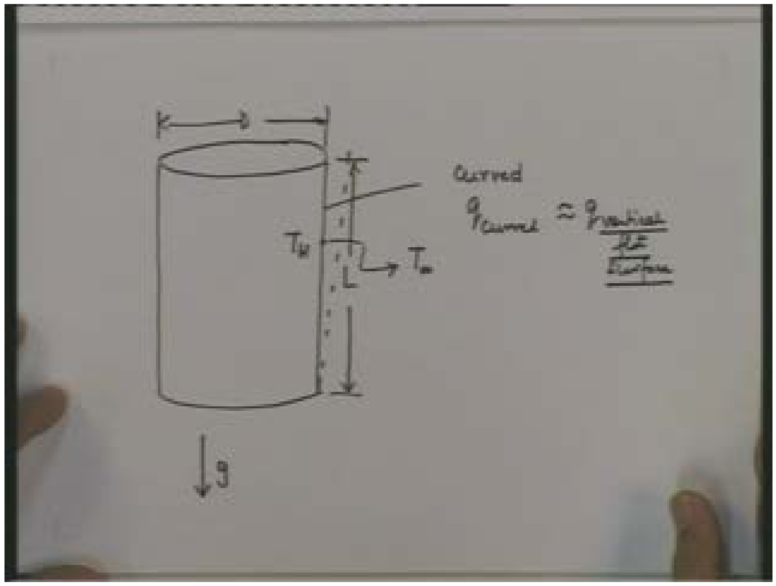

Then the heat transfer from this surface - suppose the curved surface is at a temperature T_w, the bulk is at T infinity - then this heat transfer by natural convection can be calculated by using a vertical flat plate correlation. Although this surface is curved, we can say that q curved surface is approximated by q vertical flat surface provided the diameter D is large compared to the thickness of the boundary layer. The thickness of the boundary layer in natural convection is usually of the order of a few millimeters, not more than that, and hence so long as the diameter happens to be a few centimeters - 8 centimeters, 10 centimeters or larger than that - you can assume that the curvature of the vertical surface is negligible and continue using the correlation for the vertical flat surface.

I think that is about enough about vertical flat surfaces. Now let us move on to some other interesting situations; the next situation that we are going to look at is that of the horizontal cylinder - radiator tube, a thick rod across which there is no flow. It is in in closed room or a closed enclosure; it will lose heat by natural convection.

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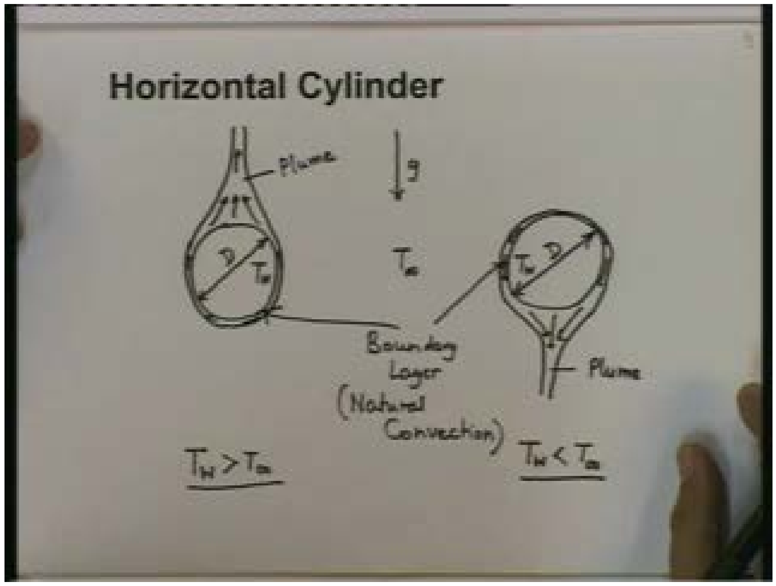

Let us look at what happens; let me draw1cylinder and let me draw another cylinder. Let us say gravity in either case acts downwards. Each cylinder had some surface temperature T_w and a free stream temperature T infinity for either. In one case, let us assume that the wall temperature is higher than the free stream temperature. Here this case, assume that the wall temperature is lower than the bulk temperature of the fluid to which it is exposed. In this case, there will be a boundary layer which will develop and the fluid near the tube will tend to move upwards. So we will have what is known as a plume on the top side and there will be a reasonably thin boundary layer surrounding the tube.

In the boundary layer fluid will move upwards along the circumference of the tube; at some stage the boundary layer will become thick and will form a plume. The plume will move upwards; the thickness of the boundary layer where it has not converted itself into a plume will be much smaller than the diameter of the tube - similar to a forced convection situation except that instead of a wake we have a plume. And the wake of a forced convection situation will have complex structures like vortices may be attached or may be shed periodically from either side. Here it is a much simpler situation; we have a much more stable plume in this case.

This is a hot cylinder exposed to a cold fluid and a mirror image of this - more or less a mirror image of this - will occur when we have a cold cylinder exposed to a warmer fluid. Here the plume will be at the bottom and the boundary layer will be developing from the top ending up into a plume - something like this. Fluid will move around the cylindrical surface in the boundary layer, will detach and form a plume. Here, the plume moves downwards because the wall temperature is lower than the free stream temperature. The fluids near the wall will become cooler, will become denser and will move down.

In this case, when the wall is warmer, the fluid near the wall becomes hotter so less dense and hence tends to move up. Here as well as here, we have the boundary layer caused by natural convection. Again when we look at the correlations, we will notice that Rayleigh number plays a part but here the Rayleigh number will be based on the diameter of the tube instead of the length of the vertical place as we saw earlier. The recommended correlation in this case is again that by Churchill and Chu - another correction published in 1975 for a horizontal cylinder.

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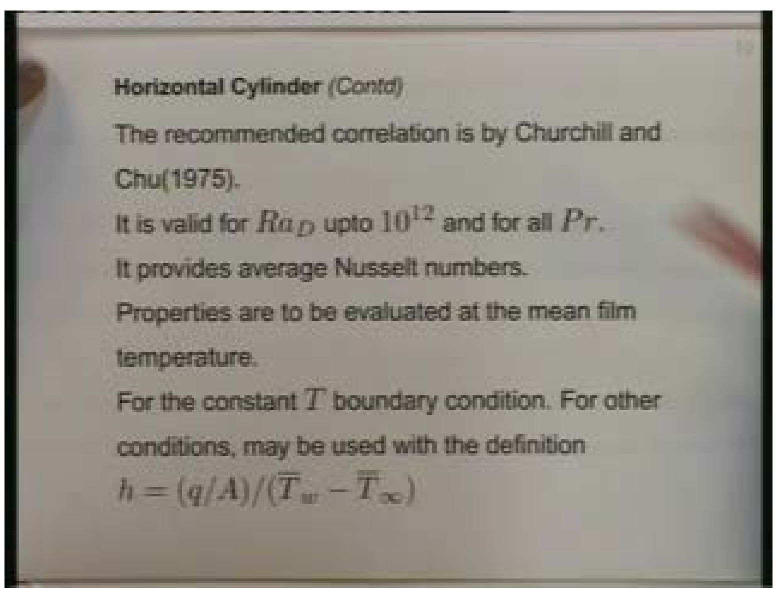

It is valid up to Rayleigh number of 10 raise to 12 and for a wide range of Prandtl number. It provides average Nusselt numbers and hence we can compute average heat transfer coefficient. Again, properties are to be evaluated at the mean film temperature; the correlation is derived using uniform and constant temperature as a boundary condition. However, just as we can use the vertical plate correlation for other boundary conditions, this correlation can also be used for boundary conditions other than constant temperature boundary condition. It can be used for the constant heat flux boundary condition or for a situation - a real life situation - where neither the wall temperature nor the heat flux will be uniform.

In such a general case, the average heat transfer coefficient will have to be defined as heat flow rate divided by area, that will be the average heat flux divided by the average temperature difference between the wall and the fluid. These conditions are very similar to that of the correlation for a vertical flat plate and if you look at the correlation you will find that the correlation in form is also very similar to the Churchill and Chu correlation for a vertical flat plate.

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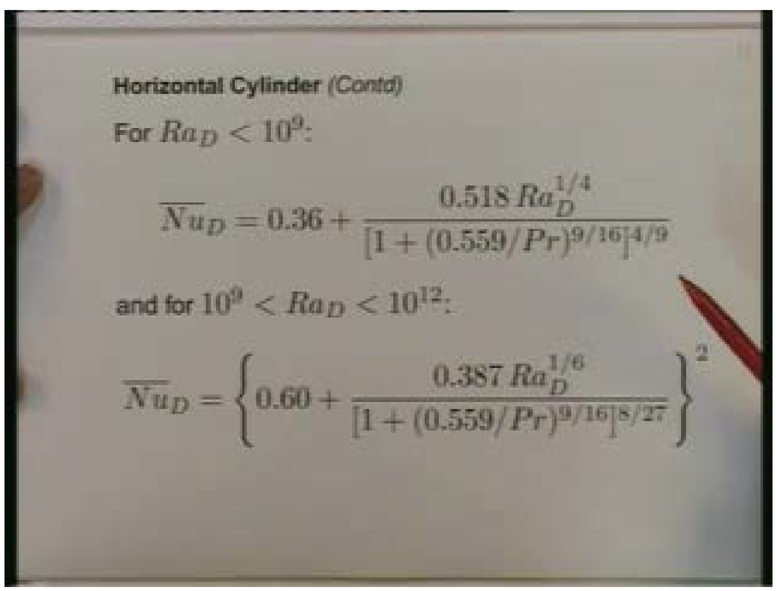

There are 2 ranges of Rayleigh number 1 up to 10 raise to 9 and 1 from 10 raise to 9 to10 raise to 12 and you have 2 different expressions for Nusselt number. When the Rayleigh number is 10 raise to 9 you can assume that the boundary layer and the plume or the part of the plume near the tube near the horizontal cylinder are essentially laminar whereas in this case, when the Rayleigh number exceeds 10 raise to 9, part of the boundary layer and the plume will be in turbulent flow zone. Again, you will notice that Nusselt number is a function of Rayleigh number and Prandtl number. It is the average Nusselt number which is computed from these correlations; the Rayleigh number is based on the diameter and so is the Nusselt number. This is something to be noted otherwise the form or structure of the correlation remains essentially the same.

Now, let us look at an illustrative problem in which we will be using this correlation. In this problem we will notice that radiation is playing a role but we will also notice something about measurement of temperature in a situation where convection and radiation both play a reasonably significant role. The situation we consider is like this - there is a gas say air in an enclosure; the air is at 1 atmosphere but we want to measure the temperature of air. For this we insert a thermocouple

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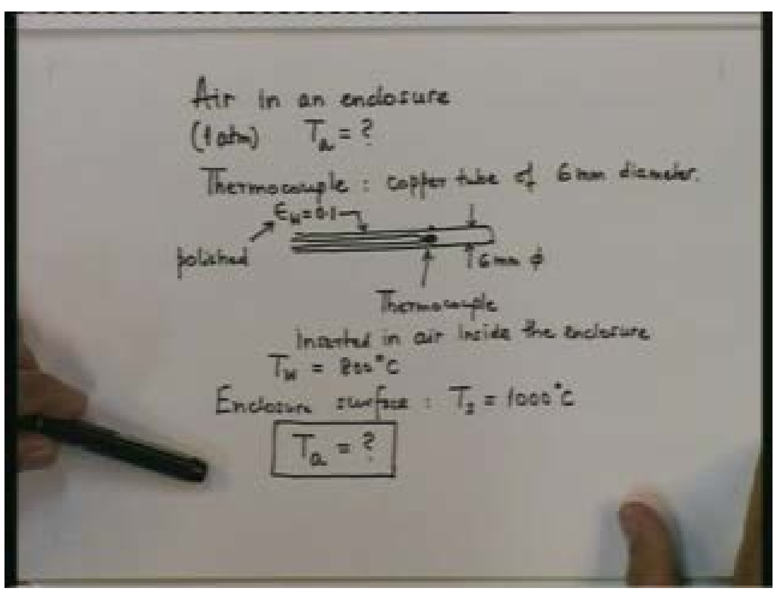

The thermocouple is inside a copper tube of 6 millimeter diameter so we have a copper tube; the thermocouple bead is somewhere here, the wires will be taken out and the diameter of this is 6 mm. This is the thermocouple, this is inserted in the enclosure. Now the thermocouple, let us say T_w - measures the wall temperature here - shows a temperature of 800 degrees C. However, the enclosure surface is at a temperature of 1000 degrees C with a temperature difference of 200 degree C between 800 and 1000 degree C; there will be a significant amount of contribution by radiation to heat transfer and because of this, to reduce the radiation heat transfer, the outer surface of the copper is polished. And let us say that the emissivity here is .1 - polished copper tube.

What we want to determine is what is the actual temperature of air? Is it 800, is it 1000 or is it something in between? Let us sketch the situation properly. Let us say this is the enclosure - large enclosure - with surface; all inner surfaces are at a temperature of 750 degrees C which is 1023 K.

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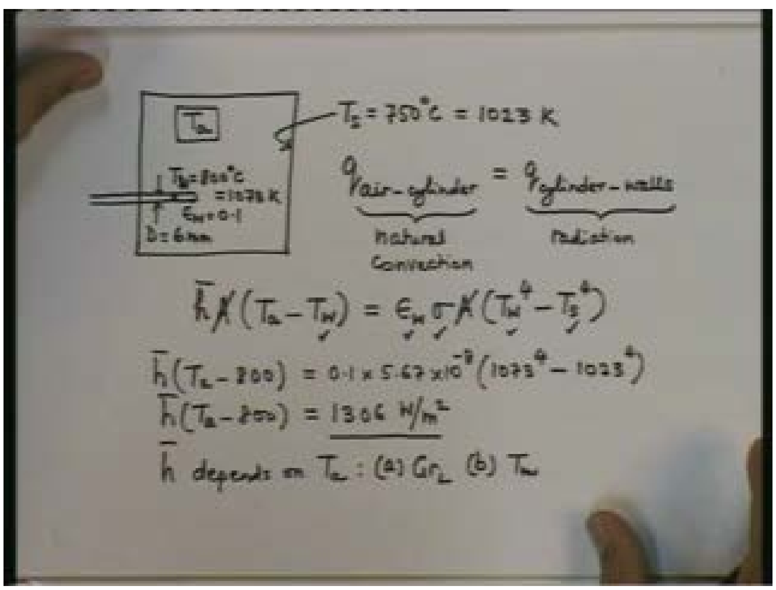

Let us say that this is the thermocouple which is inserted in it. This is the thermocouple bead; this is the copper tube with a diameter D of 6 mm. The surface temperature measured here is - wall temperature T_w is 800 degrees C which is 1072 Kelvin. The wall emissivity is .1. The temperature of the air is T_a which we have to determine.

Now let us look at it this way. The air is at some temperature; there will be a heat transfer by natural convection between air and the wall, then the wall of this thermocouple tube - copper tube - will exchange heat by radiation with the inner surface of the enclosure. And in steady state, we will have heat transfer from the air to the cylinder will equal the heat transfer from the cylinder to the walls. The heat transfer from the air to the cylinder - because it is in enclosure there is no bulk flow; this will be by natural convection and the heat transfer from the cylinder to the walls will be by radiation.

We will assume as is usual that air is a transparent gas for radiation and does not participate in radiation. Let us say that h bar is the heat transfer coefficient for natural convection so the air to cylinder heat transfer can be modeled as average heat transfer coefficient multiplied by area of the cylinder multiplied by air temperature minus the wall temperature of the cylinder and this should equal the heat transfer by radiation, that will be the surface emissivity of the wall - Stefan Boltzmann constant area of the cylinder multiplied by the wall temperature raise to 4 minus the enclosing surface temperature raise to 4; the area is common on either side.

In this expression we know the wall temperature that is the temperature measured by the thermocouple, the emissivity of the wall is given, we know the Stefan Boltzmann constant, we know the wall temperature, we know the surface temperature, so if you substitute we will get h bar into T_a minus 800 where T_a is in degree Celsius. This would equal .1 into 5.67 into10 raise to minus 8 into - since these are fourth powers we have to use Kelvin temperatures, wall temperature would be 80000 degrees C that is - 1073 K raise to 4 minus surface temperature is 1023raise to 4. Calculating the right hand side turns out to be 1306 watt per meter squared.

Now, notice that on the left hand side, we don't know h bar and we don't know T_a but h bar depends on T_a through 2 links. The first link is it is a process of natural convection because it is a process of natural convection; the Nusselt number will be a function of Grashof number so the heat transfer coefficient will depend on the Grashof number. The

Grashof number depends on the temperature difference between the fluid and the wall hence it will depend on the temperature of the fluid T_a.

So, this is one dependence is through the Grashof number and the second dependence is for calculating properties; we will have to use the mean film temperature and that mean film temperature also will depend on the air temperature. So, this equation cannot be directly solved; again i will write down the equation. The equation to be solved is h bar T_a minus 800 equal to 1306 watt per meter squared

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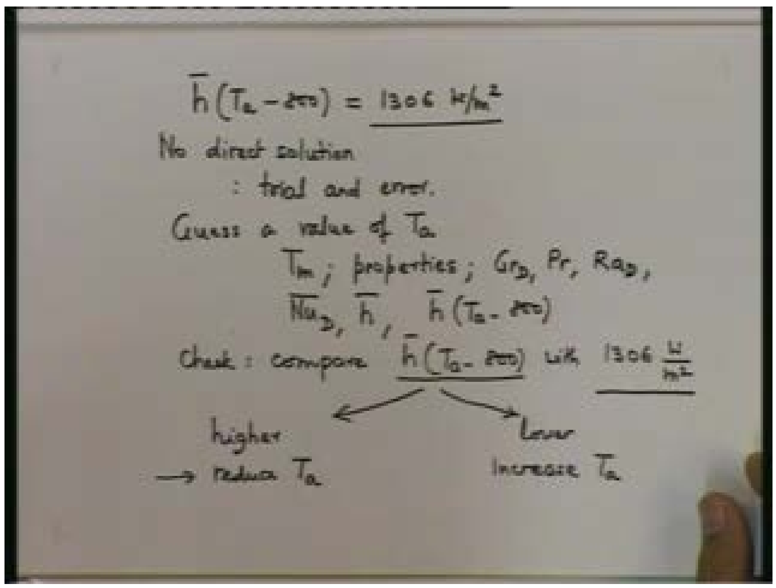

So, since there is no direct solution we will use a process of trial and error so what we will do is we will guess a value of T_a. Using that, calculate Tm, calculate properties, calculate Grashof number, Prandtl number, Rayleigh number based on diameter, it is a cylinder. Using the correlation, calculate the average Nusselt number based on the diameter; from that get the average heat transfer coefficient and from there calculate the value of h bar into T_a minus 800.

Now, we will notice that as T_a increases h bar will change, it will generally increase, and hence we will find that if T_a increases the value of the left hand side will increase. If T_a is lower, the value of the left hand side will lower, so what we will do is check. We will compare h bar T_a minus 800 with1306 watt per meter squared; it is possible that this is higher in which case we will reduce our guessed value of T_a. If it is lower we will increase our guessed value of T_a and when we do this by a reasonable number of trials we should be able to get sufficiently close to the required value of 1306 watt per meter squared.

The temperature of the thermocouple is 800 degrees C so it makes sense to assume that the air temperature will be higher than 800 degrees C, reasonably higher. Let us take as our first guess - a value of air temperature to be 1000 degree Celsius.

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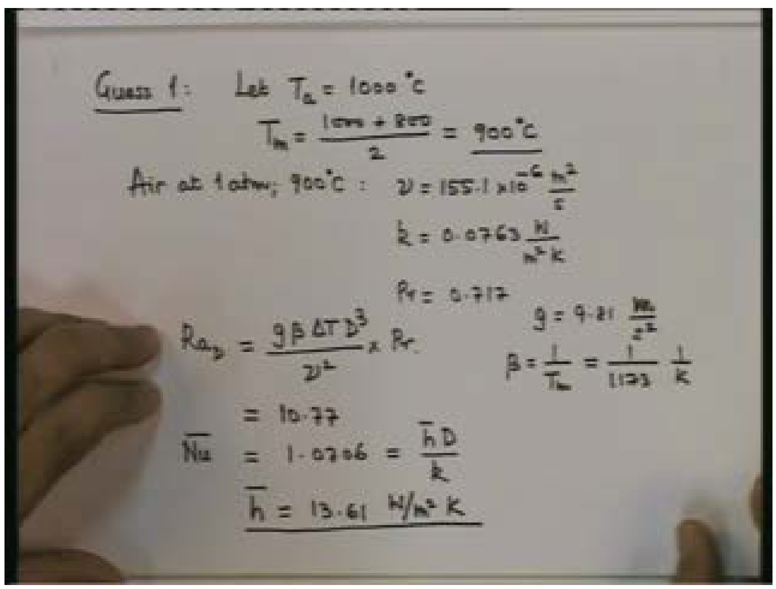

So, the mean temperature mean film temperature will be 1000 plus the surface temperature 800 divided by 2 which is 900 degrees C. Now for air at 1 atmosphere, 900 degrees C we will list out the properties: nu - 155.1 into 10 raise to minus 6 meter square per second, k - 0.0763 watt per meter squared Kelvin, Prandtl number is 0.717. We need the Rayleigh number; this is g beta delta T D cube by nu squared multiplied by Prandtl number. We have rate of the Prandtl number, we know the diameter - 6 m, we have rate of the Kinematic viscosity, we will use g the gravitational acceleration 9.81 watt, sorry, meter per second per second beta, we will use at 1 over T_m assuming air to be an ideal gas and this turns out to be - T_m in Kelvin that is 900 plus 273, 1 over 1173 Kelvin. If you substitute this here you will get a value of Rayleigh number based on the diameter to be 10.77. This gives you a Nusselt number from the Churchill and Chu correlation, average Nusselt number to be 1.0706 which is h bar D by k which gives you the value of h bar to be 13.61 watt per meter squared Kelvin. So we have h bar is 13.61 watt per meter squared Kelvin.

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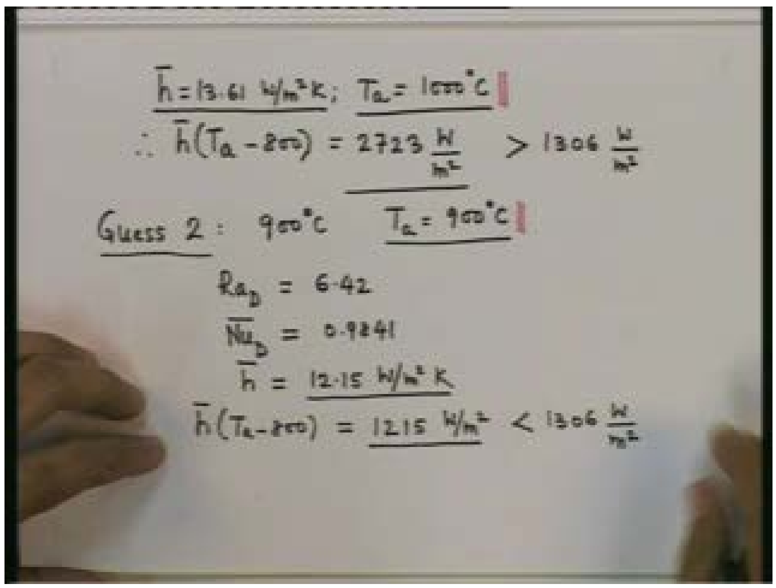

This was based on an assumption of T_a of 1000 degree C. Hence, now we calculate h bar into T_a minus 800 - this turns out to be 2723 watt per meter squared which is significantly higher than the 1306 watt per meter squared which we needed for a solution. This only means that our guess of T_a of 1000 degrees C is not a very suitable guess. We are bit too high so now we reduce our guess. Since it is too high, we go significantly below 1000 but remain above 800. So let us take the second guess as 900 degrees C, so we now assume T_a to be 900 degrees C. Again our new value of T_m will have to be calculated which will be 900 plus 800 by850 degrees C.

You will have to obtain or interpolate properties at that; the same procedure is to be followed. So, I will not write the detail but you will get your Rayleigh number based on the diameter to be 6.42 which will give you - using the Churchill Chu correlation - a Nusselt number based on the diameter of .9841 and hence a value of the average heat transfer coefficient to be 12.15 watt per meter squared Kelvin.

Notice that we reduce T_a from 1000 degrees C to 900 degrees C so the heat transfer coefficient value reduced from 13.61 watt per meter squared Kelvin to 12.15 watt per meter squared Kelvin which gives us h bar into T_a minus 800 to be equal to 1215 watt per meter squared and notice that this is now smaller or lower than the 1300 watt per meter squared - the value which we really need. That means with one guess we had a higher value, with another guess we have a lower value and that means that the temperature of the gas air will have to be between 1000 degrees Celsius and 900 degree Celsius. We also notice that with 1000 degrees Celsius it is significantly higher than 1306. At 900 degrees Celsius it is not that significantly lower but is lower than 1306. Hence, the third guess which we should look at should be much nearer to 900 than to 1000 degrees Celsius. So keeping this in mind that 900 does not seem to very far off 1215 is just 100 watt per meter squared lower than 1306 watt per meter squared. Let us take the third guess.

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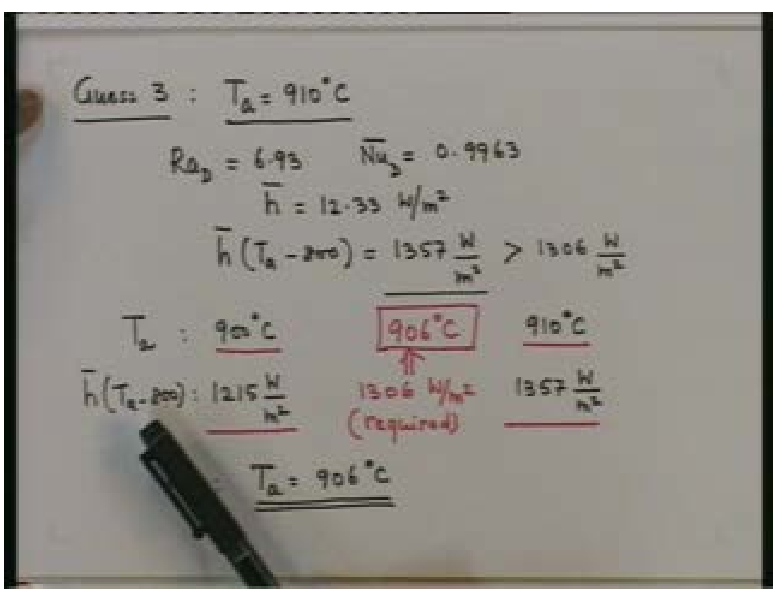

The third guess we take T_a to be 910 degree Celsius; increase T_a by just 10 degree Celsius. Again we will have to determine properties; they would have undergone slight change. We will now get the Rayleigh number to be 6.93which gives us an average Nusselt number based on diameter of 0.9963 which gives us an average heat transfer coefficient to be 12.33 watt per meter squared which gives us h bar into T_a minus 800 to be 1357 watt per meter squared which is, well, still higher than1306 watt per meter squared.

Now, see we assumed a T_a value of 900 degree C and we obtained the value of h bar into T_a minus 800 to be1215 watt per meter squared, then we assumed T_a to be 910 degrees C and we obtained this value h bar into T_a minus 800 to be 1357 watt per meter squared. Our required value of h bar into T_a minus 800 is 1306 watt per meter squared - this is the requirement. So if you interpolate between 1215 and 1357 and 900 and 910 to get this to be 1306 watt per meter squared, you will get this value to be almost exactly 906 degree Celsius.

We can check out again by doing calculation whether this 906 degree Celsius is good enough for our purpose. It may not give you 1306 exactly but it will give you within a few watt per meter squared of 1306 and remember that our estimate of h bar - computation of h bar - is based on a correlation, it gives you data matching with experimental data but not exactly matching and hence it is not necessary for us to do an exact match of h bar into T_a minus 800 to 1306 watt per meter squared. So, this means that our solution to this or the answer is - the actual temperature of air would be 906 degrees C. Indicated temperature by thermocouple is 800 degrees C but because the walls are at a lower temperature and the heat transfer between air and the thermocouple tube, the copper tube which has the thermocouple, is by natural convection which has a low heat transfer coefficient. It turns out that there is a significant error of more than 100 degrees C introduced in the measurement of temperature.

So, what have we learnt from this particular illustrative example is that when you have a gas in an enclosure and whose temperature is measured using some measuring device which is exposed to the gas and which is also exposed by means of radiative exchange to the enclosure inner surface which is exposed to the gas. If the gas temperature and the enclosure inner surface temperatures are different, then it is likely that significant errors will be introduced in the measurement of the temperature of the gas. If this were a situation of forced convection, then our h bar would be much higher and hence the temperature difference between the actual temperature and the actual temperature of air and the indicated temperature of air would be significantly lower.

Before we proceed to some other situation at this stage, it would be of interest to notice how does the heat transfer coefficient in natural convection depend on the temperature difference? In forced convection, if the mean film temperature is maintained, the heat transfer coefficient does not depend on the temperature difference but in natural convection you will notice that the Nusselt number or the average Nusselt number is proportional to Rayleigh number raised to some n where n is usually of the order of one-fourth or one-third or something like that, it is hardly ever outside this range.

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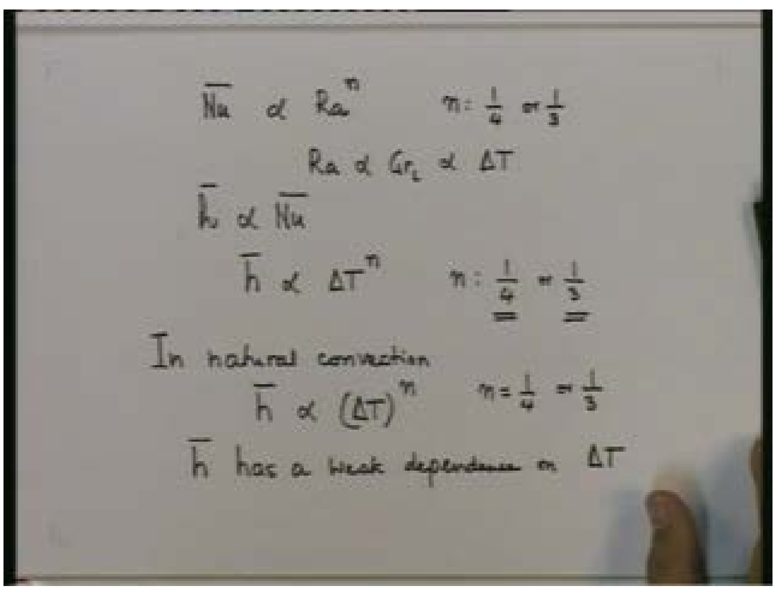

Now remember that Rayleigh number in turn is proportional to Grashof number which in turn is proportional to the temperature difference whereas Nusselt number or the heat transfer coefficient is proportional to the average Nusselt number. And hence in natural convection, the heat transfer coefficient which is proportional to Nusselt number which is proportional to Rayleigh number raise to some n will in general be proportional to delta T raise to n where n is one-fourth or one-third depending on the correlation and the range of Rayleigh number you are involved with. At lower, Rayleigh number, it will be somewhat like 1 by 4; at higher Rayleigh number it will be somewhat like 1 by 3 and that means in natural convection the mean heat transfer coefficient is proportional to delta T raise to n where n is one-fourth or one-third depending on the Rayleigh number. And hence, h bar has a weak dependence on delta T and since h bar is proportional to delta T raise to say one-fourth or one-third, the heat flow will also not be directly proportional to delta T; it will be proportional to delta T raise to 1.25 or 1.33.

After having looked at this illustrative example, let us look at the next situation of interest and that situation is that of horizontal plates. The top surface of a hot plate - you have a box or an ingot which is hot, the top surface of that or a cold circular tin taken out of a refrigerator or a cold space and kept outside the top surface of that which will be at a lower temperature than the surroundings. These are situations where a horizontal plate or a horizontal surface exchanges heat with the surrounding fluid by natural convection.

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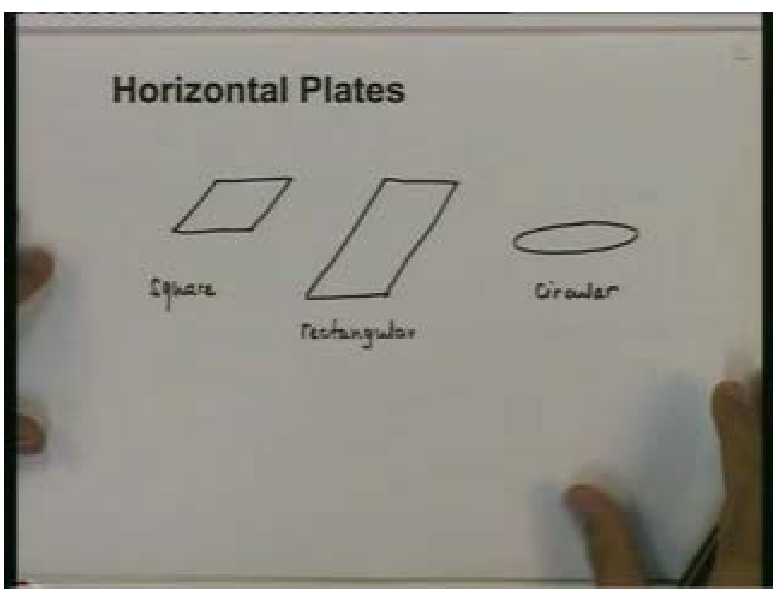

The situation may that be of a square plate or that of a rectangular plate or could even be that of a circular plate - these are top surfaces of something - or it could be even a bottom surface of something hanging in the air. The shape need not be as regular as a square or a rectangle or a circle but these are the 3 typical shapes that we come across. And if the fluid surrounding is at a different temperature than the surface, then naturally we will have heat transfer by natural convection.

We will now look at situation where we will first study the flow pattern around these and then look at a correlation or a set of correlations for predicting the heat transfer by natural convection. We will look at corrections which are applicable for a square plate or a rectangular plate or a circular plate; we will also look at situation where the surface is hotter than the fluid surrounding it, the surface is cooler than the fluid surrounding it. Whatever be the shape of the plate, let us assume that it is one of these shapes and let us look at it in its plane. So let us say that this is a surface.

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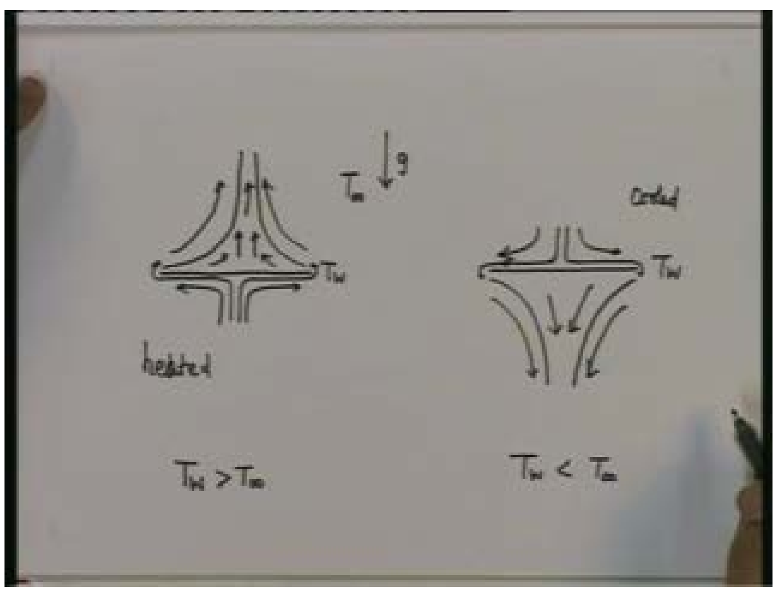

At some temperature T_w, gravity is acting downwards and the surrounding fluid is at some temperature T infinity. And let us look at the first situation where the plate is warmer than the surroundings; let us first look at the upper surface. The layers near the surface will warm up, the density will reduce, buoyancy forces will tend to move them upwards, there will some upward tendency to move. This fluid will then be replaced by fluid which rushes in from the sides and you will have a flow pattern something like this - a plume. What happens on the lower side? The surface will be hot, the layers will be at a lower density but they can't cross the surface so they will accumulate and will tend to move to the side and move up and as they move they will be replaced by cooler fluid from the surroundings so that pattern on the top surface and the bottom surface will be different.

If you look at a surface with a temperature lower than that of the free stream, the situation will reverse. You will have a big plume going down on the lower side whereas on the upper side the cooler fluid will stick to the surface, will try to move out from the edges, will be replaced by warmer fluid from the top. So this is a heated surface and this is a cooled surface. Again, we will see that situation on the top surface here is similar to that on the bottom surface here and vice versa.

In the next lecture, we will look at correlations for predicting the heat transfer by natural convection from the top surface as well as from the bottom surface. See you again.