Heat and Mass Transfer
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Lecture no. 22
Natural Convection 1

Welcome to this sixth topic on heat and mass transfer. In the next few lectures, we will be studying natural convection in these lectures; we will restrict ourselves to single phase natural convection.

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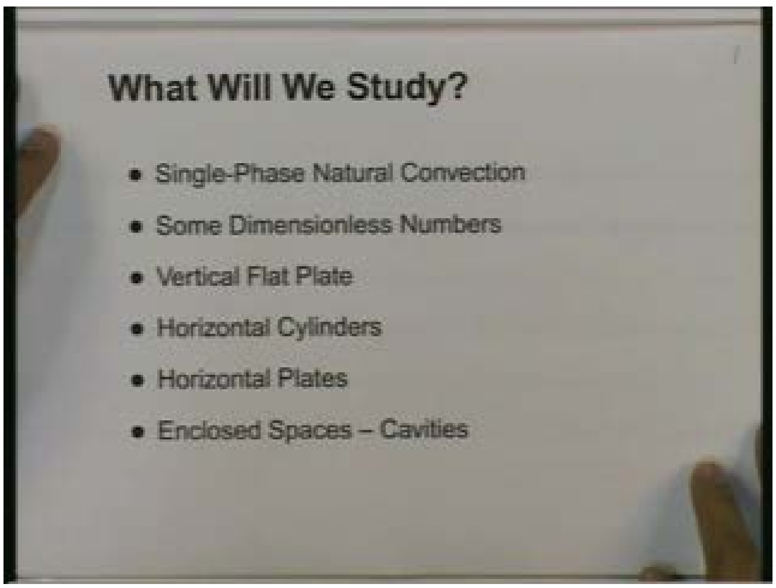

Two phase flows, condensation and boiling will be taken up in a later chapter. The equations of natural convection, those governing the phenomenon of heat transfer and fluid flow during natural convection tend to be complex because of the presence of buoyancy forces and hence we are not going to look at those equations nor shall we study the methods for solving those equations. We will essentially look at correlations based on either analysis or experiment or combination and use them to calculate heat flow rates during natural convection. We will look at dimensionless numbers using which those correlations are built up and then we will look at some situations which are very common in practice and for which very nice correlations are available.

The situations that we will study are a vertical flat plate, a horizontal cylinder, a horizontal plate of some typical shape like a square plate, a rectangular plate or a circular plate and finally natural convection in enclosed spaces or cavities. If we look at a situation of natural convection, we will see the following.

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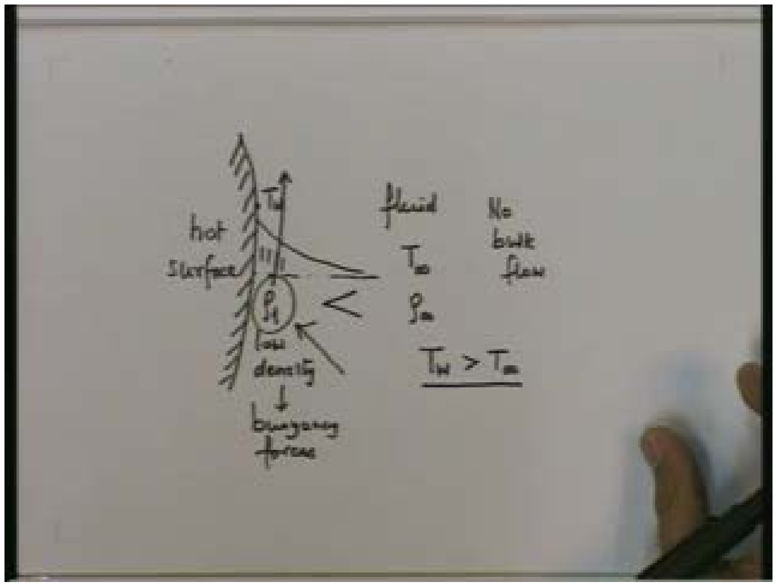

There is no bulk flow during natural convection so if I have a surface - just showing some surface which is say hot - so we have a hot surface say at some temperature T_w and it is immersed or exposed to a fluid at say T infinity away from the plate uninfluenced by its presence, let the temperature be T infinity. There is no bulk flow that means if the hot surface were not to be at a different temperature from the fluid, the fluid would remain stagnant. But now let us assume that the hot surface has a wall temperature which is higher than the fluid temperature. The fluid is not flowing so because of conduction from the hot surface layers near the hot surface, we will have their temperature approaching T_w so some sort of a temperature gradient will be established.

Now the lamina near the wall are at a higher temperature so the density of these layers near the wall - let me call them rho1. Compared to the density in the free stream we will have rho1 less than rho infinity because we have assumed the wall temperature to be higher than the free stream temperature. So, that means, in this bulk fluid there is a zone of low density; this zone of low density leads to buoyancy forces acting on this and hence this zone of low density tends to move up thus setting up the process of natural convection. It moves up is replaced by fluid from the bulk and the process establishes itself as a process of natural convection.

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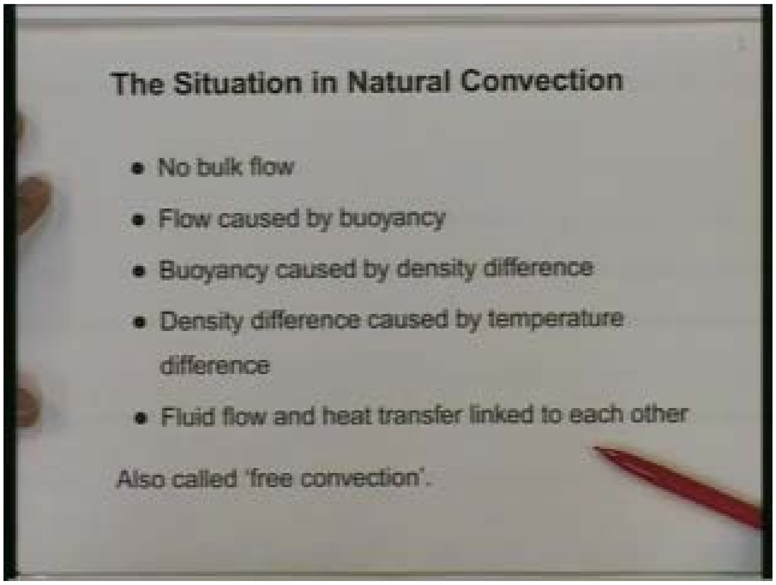

So, in short, the situation in natural convection is that there is no bulk flow; whatever is the flow caused is due to buoyancy. The buoyancy in turn is caused by a density difference and the density difference in turn is caused by the temperature difference between a surface and the fluid surrounding that surface. And because of this temperature difference causing density difference causing buoyancy forces causing flow - because of this link - the fluid flow and heat transfer is linked to each other; this is the main characteristic of natural convection. The natural convection is also called free convection and we will soon see why.

Let us look at the advantages and disadvantages of natural convection as a heat transfer process.

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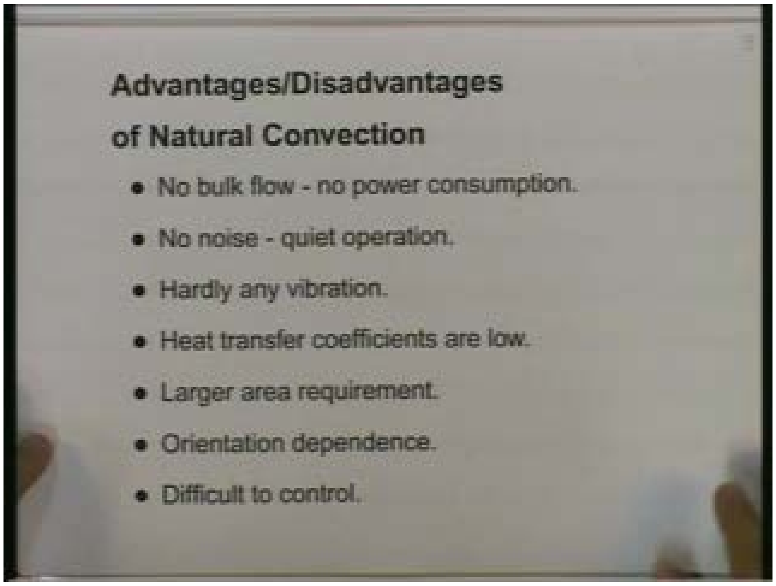

First, there is no bulk flow in natural convection so we don't have to consume any power to drive a pump or rotate a compressor or blower. Perhaps, because there is no power consumption required the natural convection may perhaps be called as free convection. Because there is no fluid flow equipment involved, associated with natural convection, there is no noise process of natural convection - almost invariably proceeds in a quiet fashion. Because there is no bulk flow, there is hardly any vibration associated with flow phenomenon or with an equipment nearby.

The disadvantages compared to forced convection are - heat transfer coefficients in natural convection are low typically by an order of magnitude. If you look up the problems we have solved with the forced convectio0,n forced convection with air gave us heat transfer coefficient of the order of maybe 7500 Watt per meter squared Kelvin. With natural convection, the order of heat transfer coefficients with air would typically be10-12 Watt per meter squared Kelvin.

With water, forced convection would easily provide heat transfer coefficients of the order of a 1000 Watt per meter squared Kelvin or even higher. Water during the process of natural convection will lead to heat transfer coefficient of the order of a few tens of Watts per meter squared Kelvin - maybe 20,30, 40 of that order perhaps 10000 and 50 that is about it.

Because the heat transfer coefficients are low, the area required for a given amount of transfer of heat is large. Because natural convection depends on density differences and gravity, the direction of gravity and the orientation of the surface plays a role and because there is no fluid flow, no blower to switch on or off, no velocity to adjust, natural convection is difficult to control.

Now let us look at some dimensionless numbers associated with the phenomenon of natural convection.

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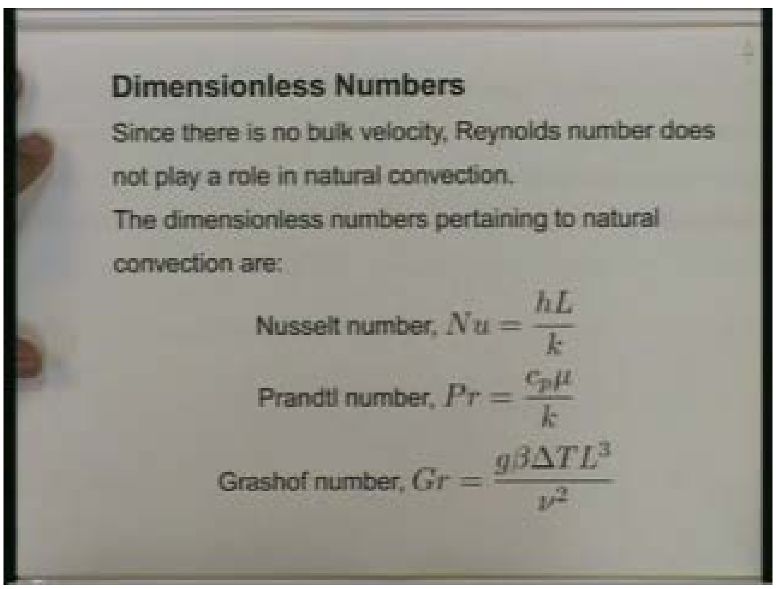

First, we should note and we will always or quite often compare the process of natural convection to that of forced convection. Since there is no bulk velocity in natural convection, there cannot be a physical reference velocity and hence the Reynolds number will not play a role in a situation of natural convection. However, instead of Reynolds number we have a different number; we will soon come to that. We are interested in a heat transfer coefficient and the dimensionless number Nusselt number would represent the heat transfer coefficient in the dimensionless form. We have come across the Prandtl number during our studies of forced convection; we have the Prandtl number present in almost all correlations of natural convection.

Since Reynolds number doesn’t exist but the whole process is a process of natural convection, is caused by temperature difference leading to density difference leading to buoyancy force and flow, the temperature difference would be represented in a dimensionless number and that dimensionless number , a very common number in natural convection correlation, is the Grashof number. It has the gravitational acceleration coefficient of expansion, temperature difference, length and viscosity. Let us look at the Grashof number; it is significance and how it comes about.

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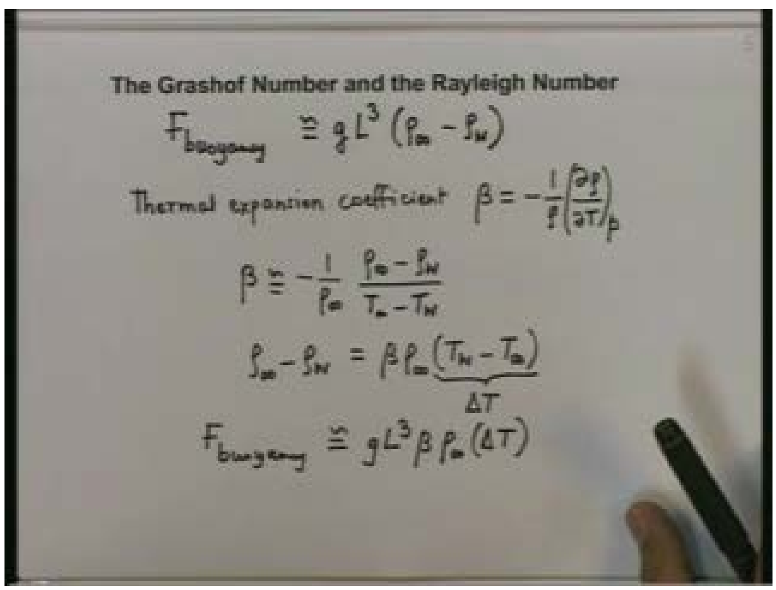

Notice that the force of buoyancy would equal the density difference between the fluid and the surrounding multiplied by the volume of the fluid multiplied by the gravitational acceleration. The gravitational force will be acting downwards, buoyancy force would be acting upwards so the net force of the buoyancy will be the gravitational acceleration volume. Let us represent it by cube of some reference length multiplied by the density difference, say the bulk density in the free stream is rho infinity and say for the layers near the wall, the density is lower. Assuming the wall is hotter than the fluid, rho infinity minus rho w, so this would be representing the buoyancy force.

Now, the density difference is related to the temperature difference. We define the volumetric thermal expansion coefficient; this is usually given the symbol beta and this is defined as1over; rho partial of rho with respect to T at a constant pressure. The negative sign is included to obtain a positive value of beta because as temperature increases density decreases for almost all fluids, that is a common occurrence. So partial of rho with respect to T is a negative number so this negative sign here gives us positive values of beta for tabulation and calculation.

Now, this can be approximated as minus1over - say the bulk density rho infinity multiplied by ratio of rho infinity minus rho w to T infinity minus T_w. And which gives us rho infinity minus rho w to be equal to beta rho infinity T_w minus T infinity; this temperature difference between the wall and the fluid is usually represented by delta T. And now if you look at the buoyancy force force of buoyancy in terms of g, beta, etcetera would now be g L cube beta rho infinity into delta T.

Now, when you include this force of buoyancy and use the Buckingham’s pi theorem, you will get a dimensionless number which we will, we call the Grashof number, symbol Gr.

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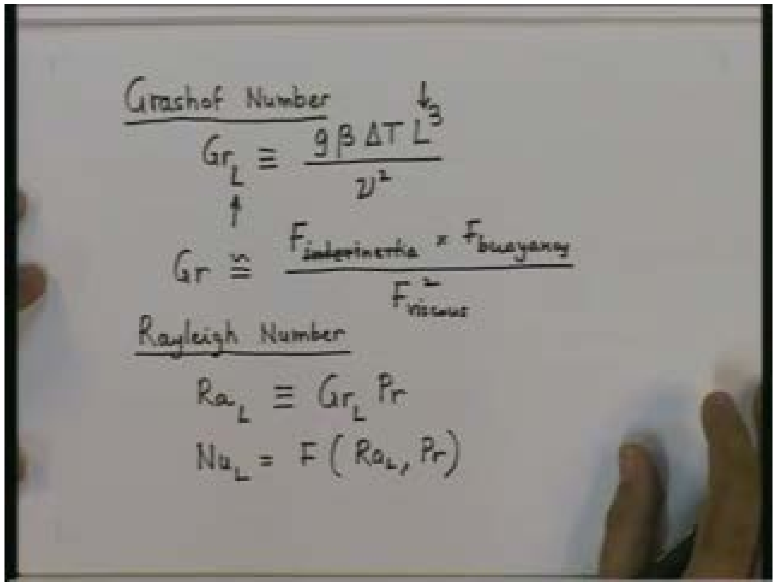

This is defined as g beta delta T L cube, terms which we come across in the force for buoyancy divided by nu squared. Notice that beta and nu would be properties, g is acceleration due to gravity, L is the characteristic length and delta T is the temperature difference between the surface and the fluid. Delta T is always defined as a positive number; if the surface is hotter than the fluid it will be T wall minus T fluid, if the fluid is hotter than the surface it will be T fluid minus T surface; so the Grashof number is always expressed as a positive number.

L is the characteristic length and quite often to represent that it is based on a specific characteristic length; we will use L as a subscript on the Grashof number. Just the way we have appreciated the significance of Reynolds number as a ratio of inertia forces to viscous forces, if you expand the Grashof number it can be shown that the Grashof number represents the ratio of forces which are like this. You put inertia force, multiply it by the buoyancy force and divide it by the square of the viscous force - this is the representation of the Grashof number, the significance of the Grashof number.

If the viscous force is dominant Grashof numbers would be small, if the buoyancy force is dominant Grashof number will be large. Apart from Grashof number, we have a

Rayleigh number related to the Grashof number; it is related to Grashof number because we find that quite often in correlations for natural convection, we come across the product Grashof number multiplied by Prandtl number. This product is so common that we define it as the Rayleigh number symbol Ra. And again if the Grashof number is based on a certain characteristic length L, we base the Rayleigh number also on the same characteristic length L and we will find that the Rayleigh number has a subscript L quite often associated with it indicating that it is based on a particular characteristic length. And because of this the correlations for natural convection would usually be written down as Nusselt number as some function of either Grashof number or Rayleigh number and Prandtl number

Almost all natural convection correlations will have this general form which is similar to the general form for forced convection where we will have Reynolds number and Prandtl number. Now, let us move on to situations of interest to us and let us look at correlations for natural convection in those situation.

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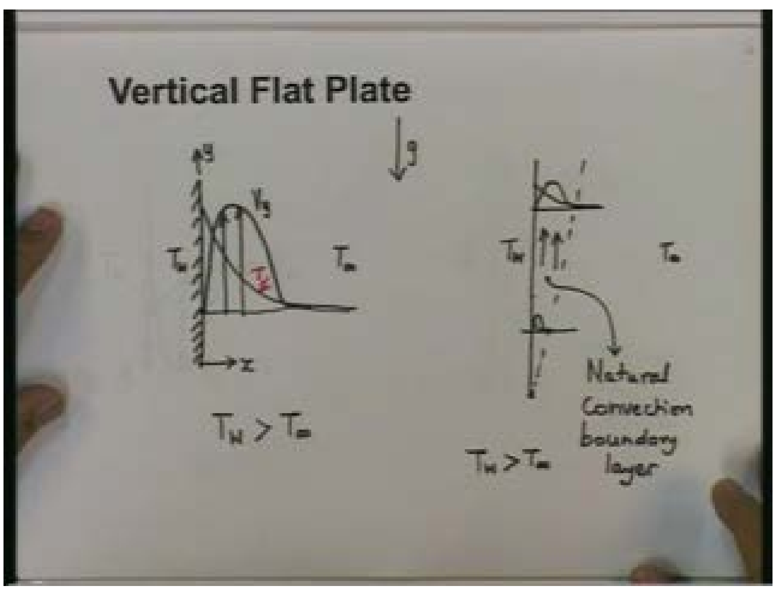

Let us first look at a vertical flat plate; a common enough situation - external walls of buildings, walls of heated packages in cuts which are vertical or neat vertical. This would be the situation representing that. It need not be a flat surface; if the surface is curved but not very sharply curved even then these correlations will be applicable. Let us look at a situation where we have vertical flat surface; let us say it is at some temperature T_w and is exposed to some fluid at T infinity. Let us - for the sake of illustration - assume that the wall temperature is higher than the fluid temperature; the wall is warmer or hotter than the fluid. Let us also assume that the gravity is acting downwards - that is our default assumption in any case.

As we go away from the wall, we will notice that the temperature reduces from the wall temperature to the free stream temperature. Because of the higher temperature near the wall density will reduce, fluid would tend to move up. At the wall itself, the velocity will be 0 but away from the wall, slightly away from the wall, velocity will increase but as it goes away the temperature difference reduces. So the density difference also reduces, so buoyancy forces decay off and you will end up with a velocity profile something like this. If this is the y direction, this would be the velocity profile V_y - it would be a function of x and this is the temperature profile T. Now if you look at this situation, at different values of y, let us say this is y equal to 0 and we are looking at various situations. Near the wall bottom edge, there will be a very thin effect of this boundary layer but that boundary layer will start growing and we will end up with a zone near the wall like this which we would call the natural convection boundary layer. At any.in the boundary layer we will have a temperature profile, we will have a local velocity profile.

Out here, we will have a slightly different temperature profile and a slightly different velocity profile. Fluid in this zone will be moving up and this zone will be known as the natural convection boundary layer. It is similar to the forced convection boundary layer but with the difference that this is the zone in which the temperature is significantly different from the free stream temperature; the effect of the wall is seen because the temperature is different. The density is different because of density difference; there are buoyancy forces which cause local flow of fluid. In the free stream, the V_y will tend to be 0 because the bulk is not moving at all; it is a natural convection situation.

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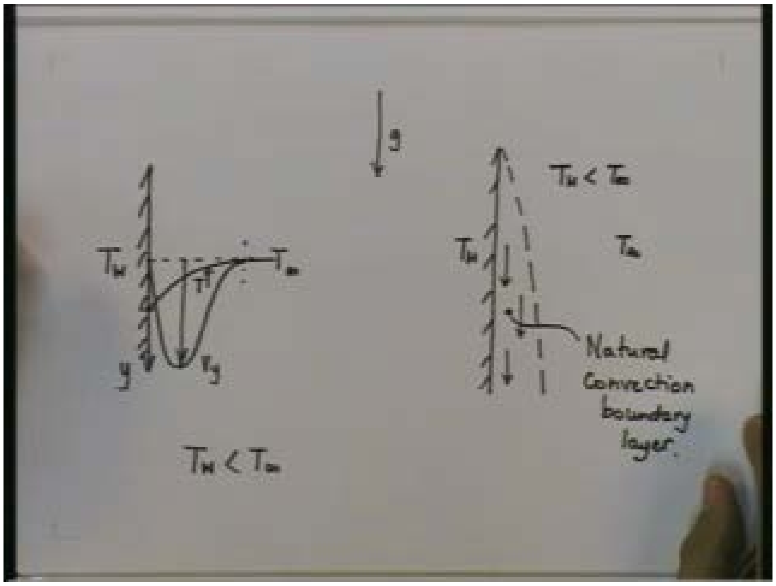

The situation is slightly different but very similar if we have the plate at a temperature lower than the free stream temperature. Let us assume now that the wall temperature is lower than the free stream temperature, now what happens? The wall temperature is lower, some layers near the wall cool down; that will be a thermal boundary layer and because of this lower temperature the density here will be higher, the fluid will tend to sink because of higher density. The net effect of buoyancy would be to move denser fluid layers downwards. So, we will have a velocity profile like this - if you call y axis downwards, then V_y will be positive downwards and this is the temperature profile which will start from the wall temperature near the wall and increase to T infinity in the free stream.

The overall view - the boundary layer will now start developing from near the tip, the top edge, grows as you move downwards and fluid in the boundary layer would start going down. Here also the same situation - wall temperature and free stream temperature, the wall temperature is assumed to be lower than the free stream temperature and this zone is the natural convection boundary layer. Compared to the first situation where we assumed the wall temperature to be higher than the free stream temperature, the boundary layer here began with the bottom edge and became thicker as we move upwards in the boundary layer, the fluid was drifting upwards. When the wall is cooler than the surroundings the boundary layer starts from the top edge, grows as you go down, the fluid tends to move down in that boundary layer. These situations are more or less symmetric to each other in the vertical direction.

If you assume that gravity is acting downwards, the situation of a hot wall facing a fluid is similar to the situation of a cold wall facing a fluid with the vertical direction reversed. Here fluid tends to move up, here the fluid tends to move down. After having looked at the basic process for a vertical flat plate, we look at a correlation - the recommended correlation - for a vertical flat plate.

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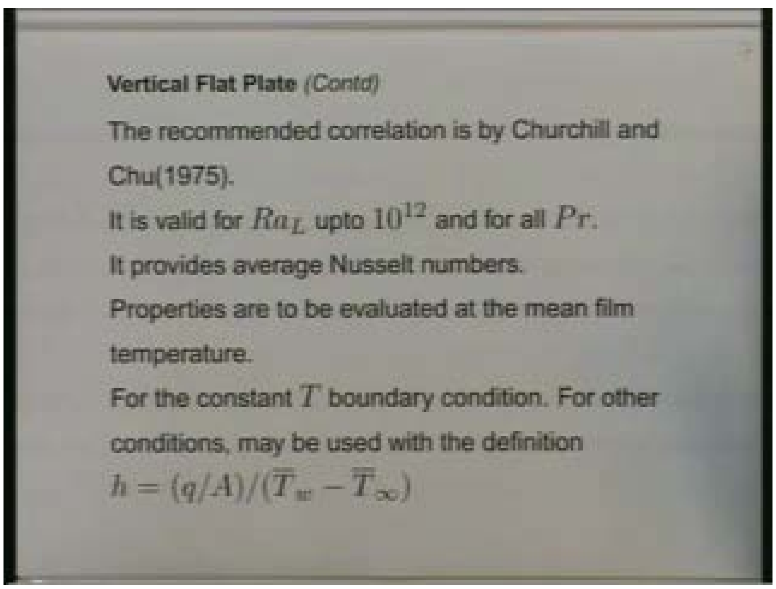

This being a basic situation, number of correlations are available since long; however the currently recommended correlation which is validated against experimental data over a wide range of Rayleigh numbers and Prandtl number is that by Churchill and Chu published in 1975. It is valid for Rayleigh number up to10 raise to 12 from near 0 and for all Prandtl number; for all Prandtl number, we mean Prandtl number of practical importance. So this typically means Prandtl number from something like .0001 to a Prandtl number of something like 1000 or 1200. This is the extreme range of Prandtl numbers that will we will come across from the liquid metal range of low Prandtl numbers to heavy oils range of high Prandtl number.

This correlation provides average Nusselt numbers; we will note that in our study of natural convection, we are not going to look at local Nusselt numbers or local heat transfer coefficient here. We will look at only correlations pertaining to average heat transfer coefficients and average Nusselt numbers. Properties are to be evaluated at the mean film temperature that is the arithmetic average of the wall temperature and the bulk fluid temperature and this correlation is essentially applicable or derived using data on constant temperature boundary condition. So, it is assumed that the surface is at some uniform temperature T_w, the heat flux may vary, heat transfer coefficient local may vary but the local surface temperature is at a unique uniform value. However it turns out that for other situations, say for a constant heat flux situation, we may still use the correlation. It will give you reasonably good results provided you define the heat transfer coefficient - the average heat transfer coefficient - as the average heat flux divided by the average temperature difference. Here is the correlation itself.

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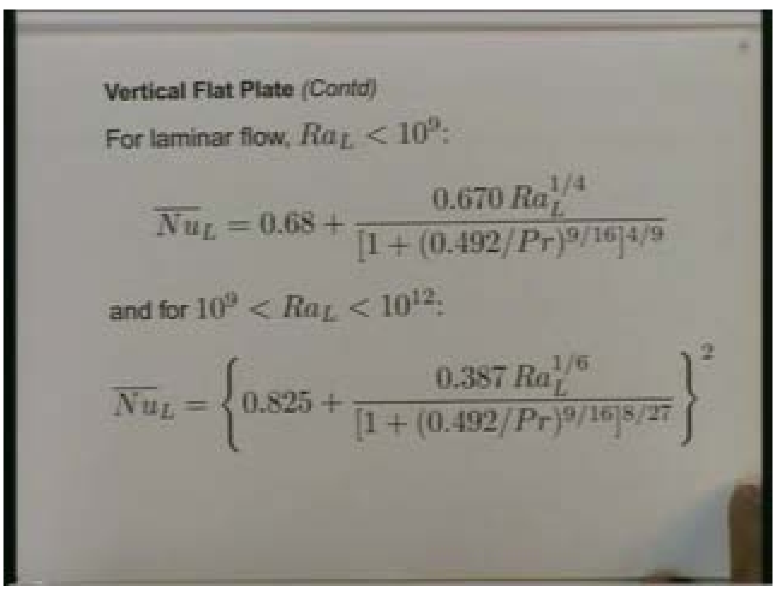

The correlation is provided in two ranges – one, Rayleigh number less than 10 raise to 9 and the whole of the boundary layer tends to be a laminar boundary layer and another for Rayleigh number higher than 10 raise to 9 up to its range of applicability, limit of applicability which is a Rayleigh number of 10 raise to 12. In this zone, the boundary layer tends to be turbulent at least over a major part of the length of the plate. In either case, you will find that the average Nusselt number is related to the Rayleigh number as well as the Prandtl number and as you go to lower and lower Rayleigh numbers the heat transfer coefficient decreases significantly.

Here you will notice that for low Rayleigh number the Nusselt number is somewhat proportional except for this constant to the fourth root of the Rayleigh number whereas for higher Rayleigh number you will notice that the Nusselt number is somewhat dependent on the cube root of the Rayleigh number. Although we have Rayleigh raise to one-sixth, here the whole bracket is squared up so there will be the major term, will be the Rayleigh number raise to 1 by 3 if you expand this bracket. Of course, the Prandtl number is also sitting there as well as in the Rayleigh number because we have seen that the Rayleigh number is nothing but Grashof number multiplied by Prandtl number. That combination comes up so often that it is convenient for us to define the Rayleigh number. After having looked at the correlation, now let us look at an illustrative example.

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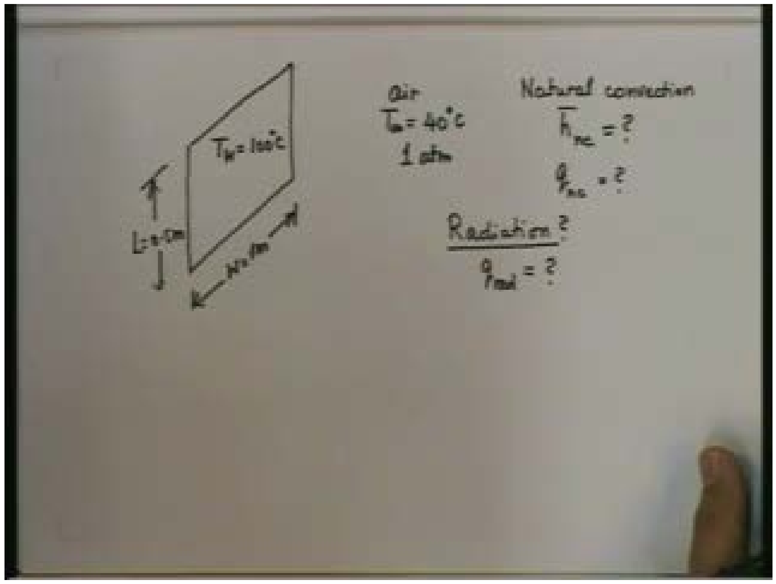

Let us solve a problem where we have a vertical plane. I will first show an isometric view of say height L .5 meters and width w of 1 meter; it is a vertical flat plate. Let us say that the plate temperature is maintained at a uniform 100 degrees C; maybe on the other side steam is condensing at a$T_m$ospheric pressure. It is exposed to air and the air temperature is 40 degrees C, pressure of air is1 a$T_m$osphere. This air is stagnant so we have a situation of natural convection.

We have to determine the average heat transfer coefficient by natural convection and we have to determine the heat transfer by natural convection. We will see while working out this problem that radiation may play a role because when a surface is exposed to a fluid and maybe there are other surfaces surrounding it. The second part of the problem - we will study the effect of radiation; also, we would like to know what is the heat transfer by radiation or what is the likely heat transfer by radiation and is it significant or insignificant compared to the heat transfer by natural convection. After having obtained the specification of the problem, it is time for us to draw a neat sketch.

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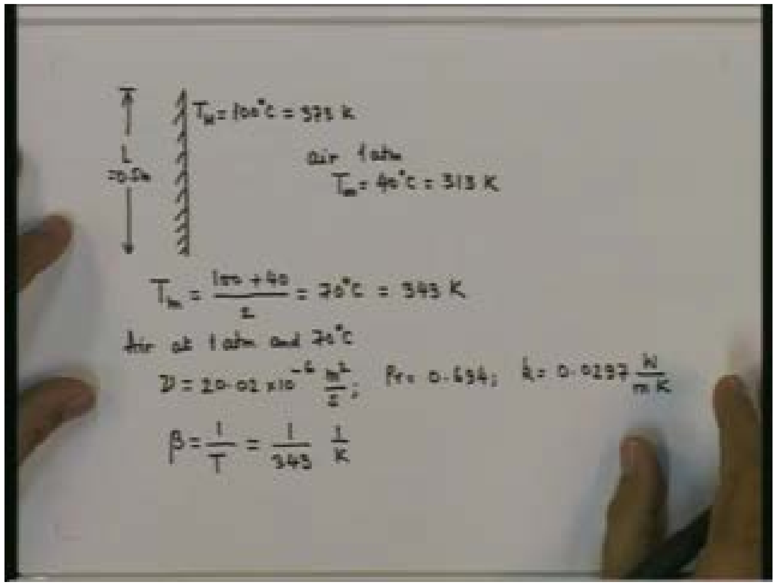

We know that the width of the plate will not play a major role; height is .5 meter, width is 1 meter, wider than the height and let us neglect any edge effects. So we will essentially consider a plate with a vertical extent L of .5meters. Let us say T wall is 100 degrees C; since we are going to consider radiation, we will also write the corresponding Kelvin temperature - this will be 373 K. It is exposed to air which is at 1 a$T_m$osphere, T infinity is 40 degrees C which is 313 K.

The first thing we have to do is to read off properties at the main film temperature; the main film temperature here T_m is wall temperature plus fluid temperature divided by 2 which is 70 degrees C which turns out to be 343 K. So for air at 1 atmosphere and 70 degrees C we will now read off from a table of properties. We need the kinematic viscosity which is 20.2 into10 raise to minus 6 meter square per second, Prandtl number is 0.694, thermal conductivity is 0.0297 Watt per meter Kelvin. We also need to determine beta required for computing the Grashof number or the Rayleigh number.

We assume air to behave like an ideal gas; so a very good assumption at 1 atmosphere and 70 degrees C. Beta turns out to be equal to1over T temperature at which beta is to be determined; this would be now the mean film temperature in Kelvin and this turns out to be 1over 343, unit is of1over Kelvin. With all the properties computed, we will calculate the Grashof number.

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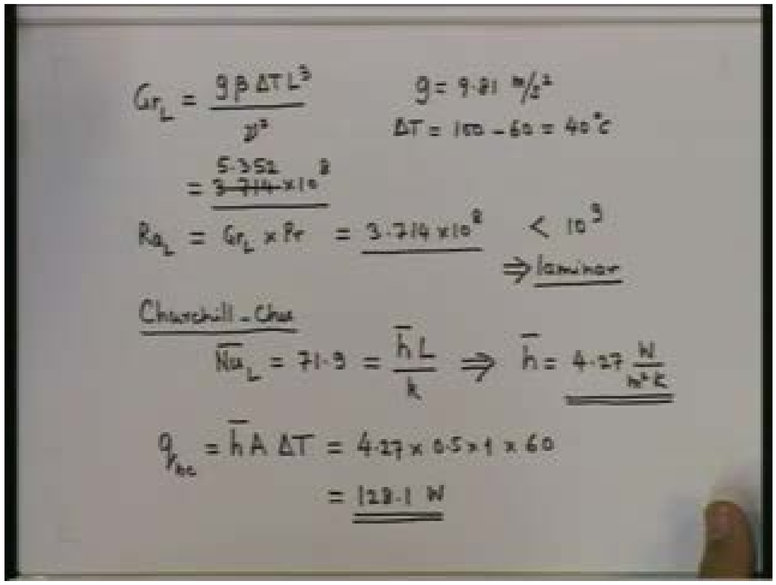

Based on the vertical extent, L g beta delta T L cube by nu squared. We know L, we have determined beta and nu, we will use the value of gas 9.81 meter per second per second - the standard acceleration due to gravity, delta T is 100 minus 60 which is 40 degrees C. If you substitute these numbers, you will get the Grashof number to be equal to 3.714 into10 raise to 8 and the Rayleigh number to be equal to Gr L into the Prandtl number. We have already read off the Prandtl number so the Rayleigh number turns out to be 3.714 into 10 raise to 8. A small mistake - the Grashof number was 5.352.

Now this Rayleigh number is less than 10 raise to 9; that means according to the Churchill and Chu correlation, this implies laminar flow. And we have to use the first of the two Churchill Chu relations; RaL is less than10 raise to 9 so we will be using this particular correlation. If we substitute into that using the Churchill Chu correlation, we will get the average Nusselt number based on L to be 701.9 which is defined as the average heat transfer coefficient into L divided by conductivity of air. We know length; we have read off the conductivity so only unknown is the average heat transfer coefficient and this gives us the average heat transfer coefficient to be 4.27 Watt per meter squared Kelvin. Notice the low value - not even 5 Watt per meter squared Kelvin.

So the heat transfer by natural convection will be average heat transfer coefficient into the area of the plate multiplied by the temperature difference between the plate and the surroundings fluid. This is 4.27, area of the plate .5 meters by 1 meter height into width multiplied by temperature difference which is 60 degrees C and this gives us the natural convection heat flow rate to be 128.1 Watt so about 125-130 Watt.

Now let us look at the radiation effect.

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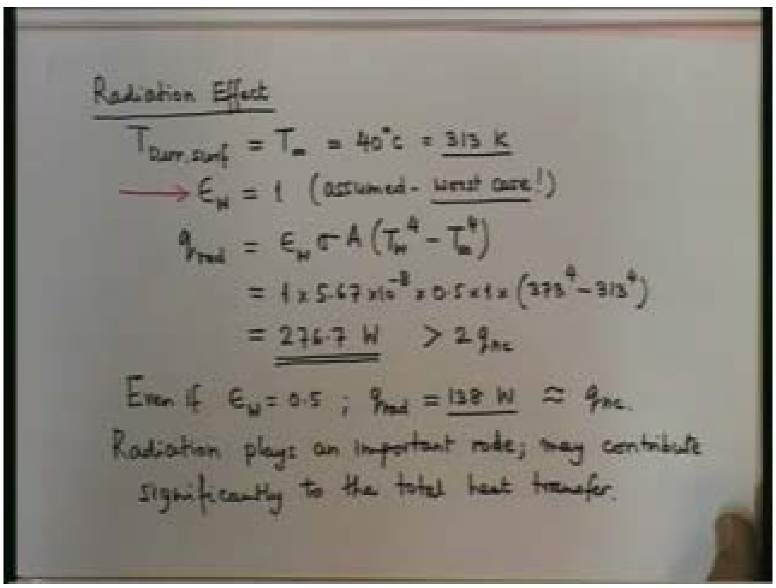

We will assume that the temperature of surrounding surfaces is the temperature of the fluid surrounding the plate, no different from that although in principle it could be different but let us assume that it is so. So this turns out to be 40 degrees C which is 313 K, this is an assumption. Also we need the emissivity of the wall; let us assume the worst case. In the worst case let us assume that the wall is black so the emissivity is one. This also is assumed as a worst case; it can’t be higher than this.

Now with these assumptions, our heat transfer by radiation will turn out to be the wall emissivity sigma which is the Stephen Boltzmann constant multiplied by area of the plate multiplied by the wall temperature raise to 4 minus the surrounding surface temperature raise to 4. We have assumed that to be T infinity so this gets T infinity raise to 4. We have assumed this to be one, this is 5.67 into 10 raise to minus 8 Watt per meter squared Kelvin raise to 4. Area is .5 into 1, temperature of the wall 100 degrees C - 373 Kelvin - raise to 4 minus 313 Kelvin raise to 4 and this gives us the heat transfer by radiation to be 276.7 Watts.

Notice that this is more than twice the heat transfer by natural convection; that means in natural convection processes radiation may play a very significant role. Now, here we have assumed that the emissivity is 1 as a worst case; we will notice that even if emissivity were less than 1; 1 means it is a properly blackened surface and its emissivity is that of a black body but a typical surface, metallic surface, non-polished, oxidized, a general surface which we come across which has not been treated would typically have an emissivity of around .5 - half of this. So even if the wall emissivity is .5 you will notice that the heat transfer by radiation will be around 138 Watt which would still be of the order of the heat transfer by the natural convection. And this means that in almost all natural convection situation, radiation plays an important role and may contribute significantly to the total heat transfer.

Look at this particular case and in other illustrative examples also, we will return to that case.

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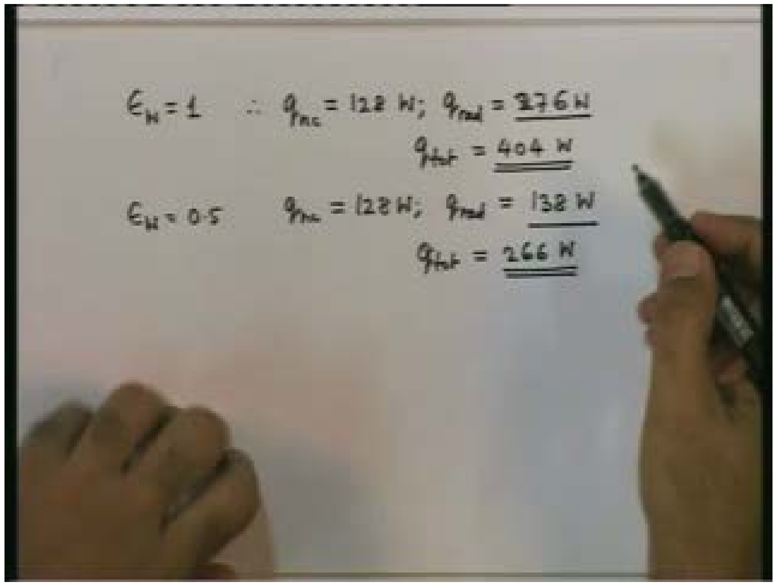

We consider epsilon wall equal to 1. In that case, we had q natural convection to be 128 Watt, q radiation to be 3276 Watt; the total turned out to be 404 Watts. Notice that in this case the radiative contribution was roughly two-thirds. Even with the wall epsilon equal to .5, the natural convection heat transfer would remain at 128 Watt. The radiative component would reduce to 130 Watt and the total heat transfer would then be 266 Watt. Even then, you will notice that the radiation plays at least a 50 percent role - this is one particular case. Depending on the situation, the natural convection heat transfer, this contribution may not be 50 percent or more than 50 percent but it will be a significant contribution. Whenever we solve problems in natural convection, we should keep in mind that radiation may play a role and take account of it.