**Heat and Mass Transfer**
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**Lecture No. 21**
**Forced Convection – 4**

Welcome back. We, today, have the fourth lecture on heat transfer by forced convection. In the previous lecture we had completed a study of flow through pipes; now it is time for us to move on to study the heat transfer from a flat plate. The situation is similar to that of fluid flow impact, fluid flow would exist.

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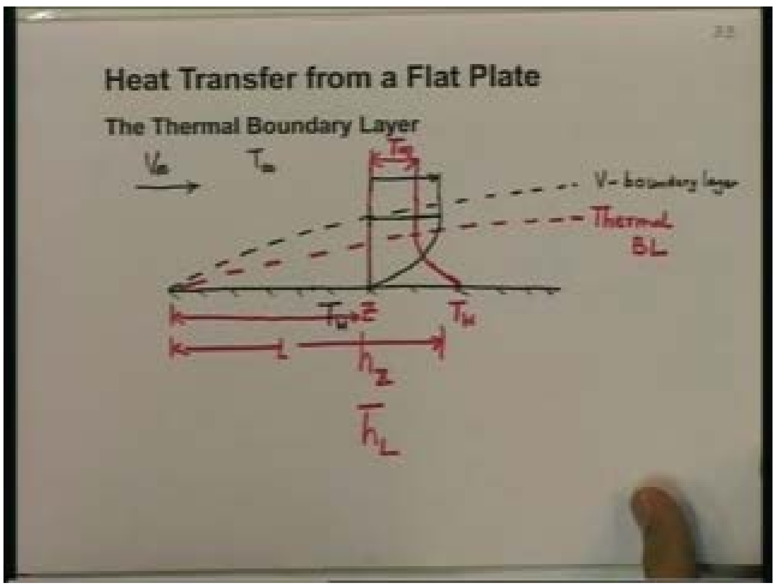


If this is the flat plate let us just look at 1 side flat plate. There is a flow; the freestream velocity is V infinity but now the temperature in the freestream is T infinity. Of course, the velocity of fluid on the wall will be 0 but let us assume that the temperature of the wall is $T_w$. For illustration, let us consider a wall which is at uniform temperature $T_w$ and again for the sake of sketching, I will assume that $T_w$ is greater than T infinity. We will have a velocity boundary layer; I am showing it thick just to be clear about it, in practice it will be pretty thin. So this is the velocity boundary layer.

At some point - let us say you take a section here at the edge of the boundary layer - the velocity would be V infinity; till the edge it will remain V infinity, it will reduce to 0 at the valve and it is a different color to show you the temperature boundary layer. Let us say that is, this is the reference axis. Let me say this is T infinity, the wall temperature will be represented by $T_w$ and the temperature would increase as you approach the wall from T infinity to $T_w$. It is not necessary that the thickness of the velocity boundary layer and the thickness of the thermal boundary layer where the temperature or thermal effects are fed will be the same. It is possible that the thermal boundary layer is thinner than the velocity boundary layer or it could be thicker than the velocity boundary layer.

The relative thickness is, it is something which we will study in detail later because the temperature of the wall is higher, we have a temperature gradient from the wall to the fluid so heat will get transferred from the wall to the fluid. Our aim in the analysis of the thermal boundary layer to determine the local heat transfer coefficient hz at a particular location z from the leading edge and also the average heat transfer coefficient over a certain length L, some length L. Let us look at the energy equation for the boundary layer.

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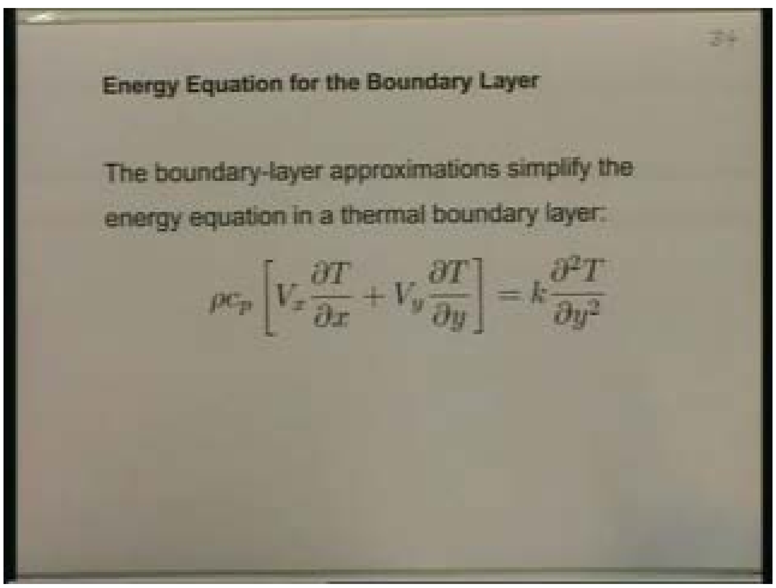

The energy equation for the boundary layer looks similar to the conservation of momentum equation for the boundary layer; in fact, the similarity is striking. It is a simplified equation, it uses the boundary layer approximation and hence you do not have a d square T by dx square term here. This equation needs because it is a second derivative in y, it needs 2 boundary conditions – one, a boundary condition at the wall and another, a boundary condition in the free stream.

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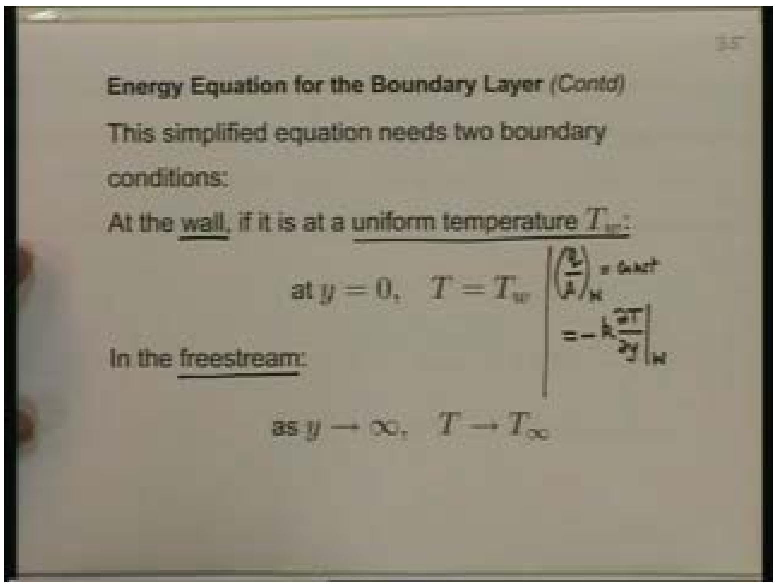


For the purpose of illustration, I have assumed a uniform temperature at the wall. So at y equal to 0 which represents the wall, the temperature is, the wall temperature that is one boundary condition. The second boundary condition would be in the free stream - as y becomes large outside the boundary layer, the temperature approaches the free stream temperature. If instead of a uniform temperature boundary condition on the wall, if we were to have a q by A wall equals constant and specified. Then the boundary condition would be replaced by the wall; heat flux from the wall to the fluid would equal minus k partial of T with respect to y at the wall, that is at y equal to zero. This boundary condition would be replaced by this boundary condition; if the plate is at a uniform temperature, the free stream is also at a uniform temperature and uniform velocity and if the flow is laminar the energy equation in the boundary layer can be solved. We need the velocities and the velocities come from the solution of the momentum equation.

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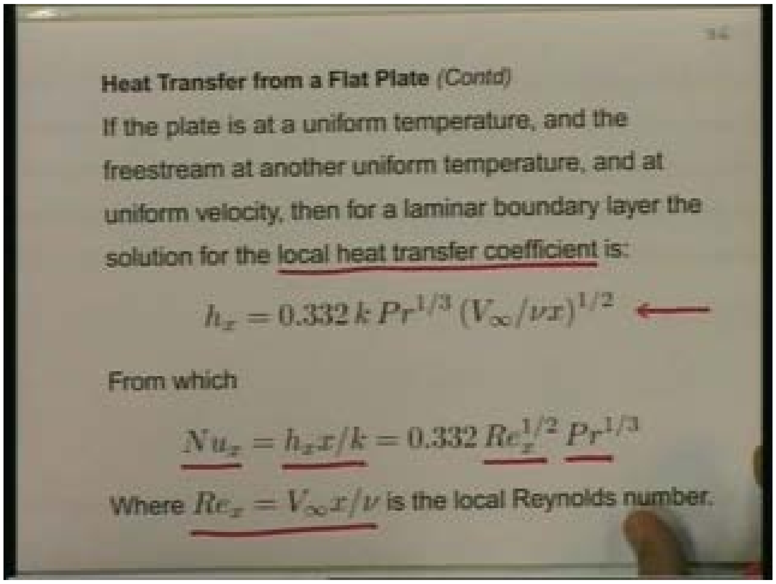


We get a relation for the heat transfer coefficient in terms of the Prandtl number, thermal conductivity k, free stream velocity, kinematic viscosity and location along the plate. This is the local heat transfer coefficient. Of course we would like to represent it in terms of the Nusselt number which the local heat transfer coefficient into its position divided by k. And you will find that the correlation is in terms of the Nusselt number - local Nusselt number, in terms of the local Reynolds number and the Prandtl number. The local Reynolds number is defined as in case of the fluid flow V infinity x divided by the kinematic viscosity.

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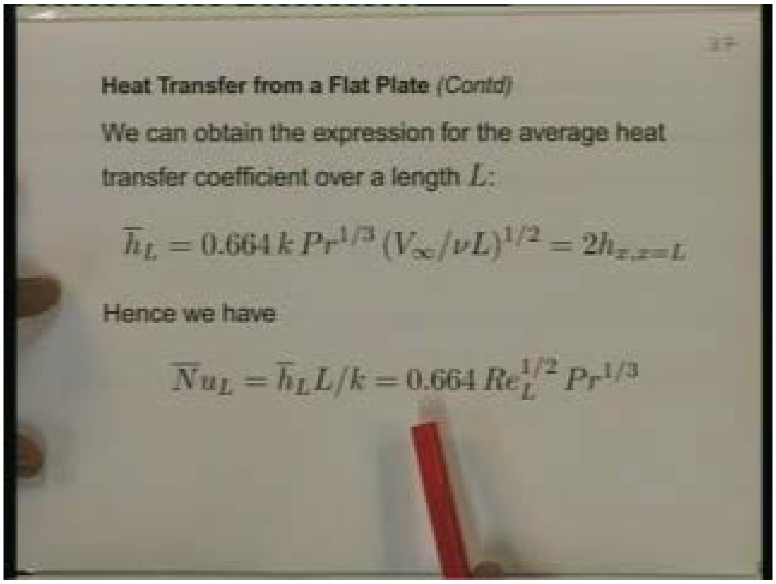


By integrating this equation from say x equal to 0 to x equal to L, we obtain an expression for the average heat transfer coefficient over a length L from the leading edge of the plate and that expression is similar to the expression for the local heat transfer coefficient except that the coefficient is double and instead of x we have an L. So the average heat transfer coefficient over the first length from the leading edge L is equal to twice the heat transfer coefficient local at x equal to L, that is something to be remembered.

In terms of Nusselt number we have a very similar correlation. Here this is the Reynolds number based on the length L and constant in the correlation instead of .332 turns out to be double that - . 664. Before going to the turbulent flow over a flat plate, it is proper for us to look at the analogy between heat transfer and mass transfer because a number of correlations particularly those pertaining to turbulent flow are derived using this analogy between heat transfer and mass transfer.

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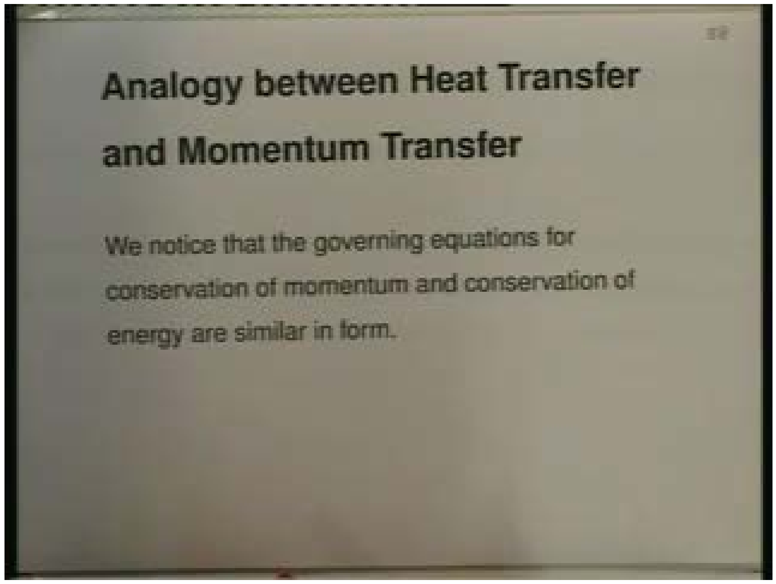


The analogy begins by us noticing that the governing equations for conservation of momentum and conservation of energy are similar in form. Let me go back to an earlier sheet where I have written the governing equation for the thermal boundary layer; this is the energy equation.

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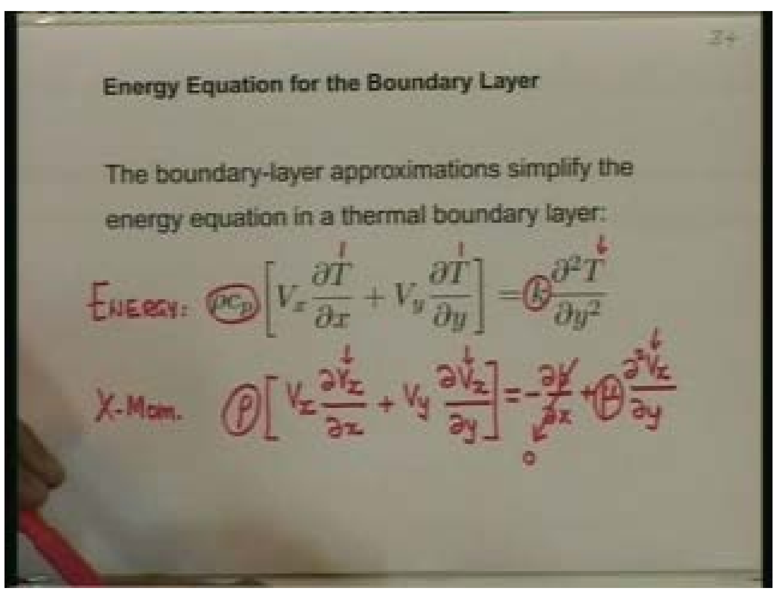


I will write below this the equation which we have seen in the previous lecture during fluid flow which is the X momentum equation. In that equation is rho $V_x$ partial of $V_x$

with respect to x plus $V_y$ partial of $V_x$ with respect to y equal to minus partial of p with respect to x plus mu second derivative of $V_x$ with respect to y. With a uniform free-stream this is 0 and now you notice that there is a term by term correspondence between the 2; all that happens is wherever you see T that is replaced by $V_x$ wherever $V_x$ x $V_y$ y occur they remain as they are. rho $C_p$ is in place of rho and mu gets replaced by k. Even the boundary conditions are similar, one value of velocity at the wall, another value of velocity in the free-stream, one value of temperature at the wall, another value of temperature in the free-stream.

Consequently we expect that the solution for this equation would be very similar to the solution for this equation and hence the results which we obtain would also be similar except that because of the dynamics of k, mu and $C_p$ which occur at different places in different equations, Prandtl number will play a role to some extent. Let us look at the similarity in the final correlations; we have for drag on a flat plate and for the heat transfer from it, of course in laminar flow.

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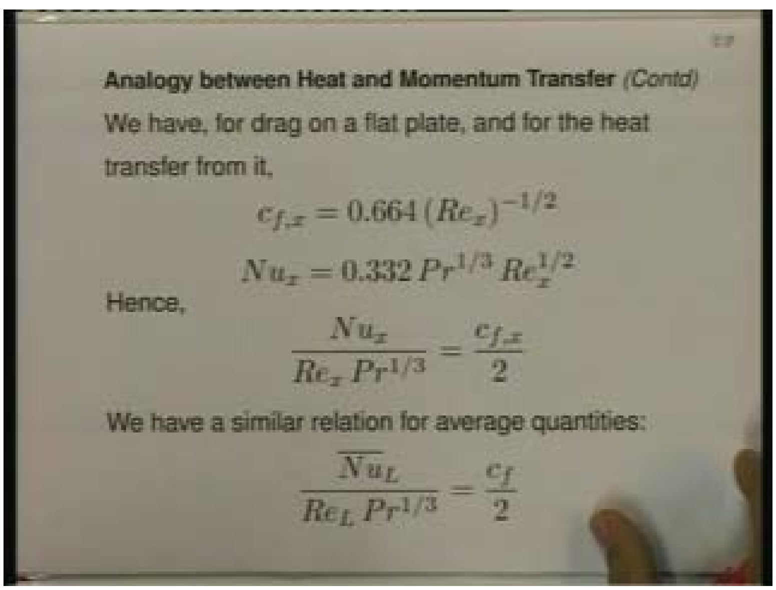


This is the equation for the local drag coefficient; this is the expression for the local Nusselt number and if you divide one by the other, you can show that the Nusselt number divided by Reynolds number divided by Prandtl number raised to one-third is nothing but the skin friction coefficient or drag coefficient divided by two. You compare similarly the expression for the average drag coefficient and the average Nusselt number; a very similar relation is obtained even there. This similarity is generalized in what is known as the Colburn analogy.

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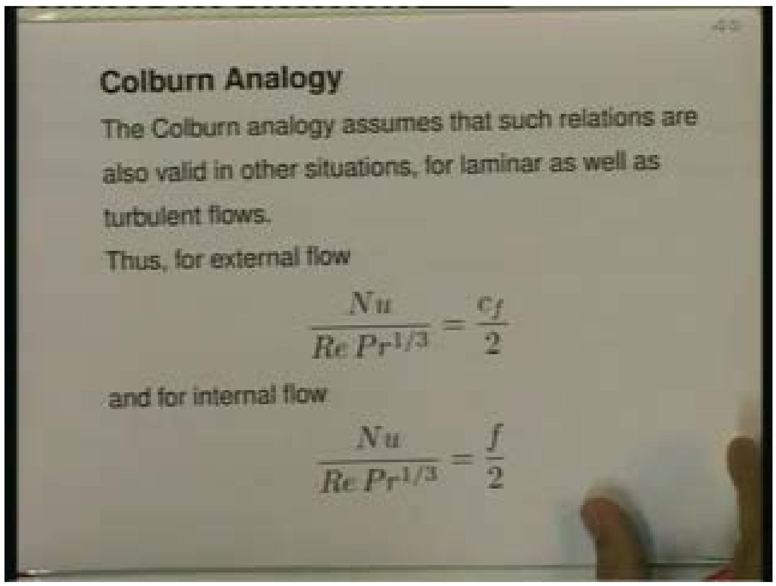


The Colburn analogy assumes that such relations are applicable in other situations also independent of geometry provided the geometry in both cases is similar. Whether the flow is laminar or turbulent, the Colburn analogy assumes that a relation like this would exist. According to the Colburn analogy for external flow where you talk of a drag coefficient, Nusselt number divided Reynolds number into Prandtl number raised to 1one- third would be the drag coefficient divided by 2 and for internal flow the same ratio Nusselt number divided by Reynolds number into Prandtl number raised to one-third will be friction factor divided by 2. And you would notice if you look at it in detail that these two comes out because in the friction factor, in the denominator we have rho V squared by 2; that 2 in the denominator continues to appear in this relation. Now using this, we will now derive the equation for heat transfer from a flat plate in turbulent flow.

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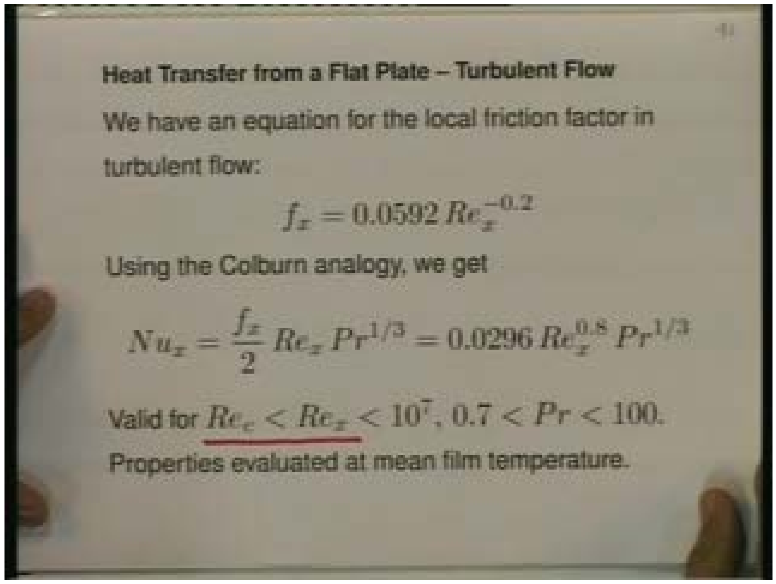


We already have an equation for local friction factor in turbulent flow; we have seen this earlier. Friction factor local is .0592 into Reynolds number at x raised to minus .2. Using the Colburn analogy, we will say Nusselt number, local Nusselt number would be local friction factor divided by 2 multiplied by local Reynolds number multiplied by Prandtl raised to one-third and we will find that this turns out to be .0296 $Re_x$ raised to .8 Prandtl raised to one-third. It turns out that this equation agrees well with experimental data in this range. For the critical Reynolds number where the flow becomes turbulent up to 10 raised to7 and Prandtl number from .7 to 100, for this the properties need to be evaluated at the mean film temperature. Now notice that this is applicable only when the flow is turbulent, when the flow not turbulent, one of two things may happen when the flow is not fully turbulent.

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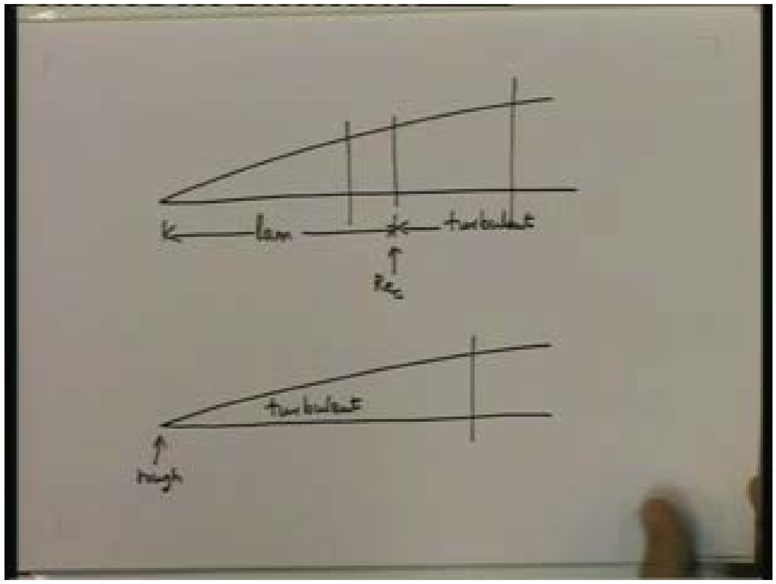


It is possible that the boundary layer may be laminar over some leading zone and turbulent thereafter. If the leading edge is smooth and sharp, this location is given by the critical Reynolds number which is typical 3 into 10 to the 5 but if the leading edge is rough it is possible that you will have a turbulent layer right from the leading edge and depending on whether r length is here or here or a situation like this, we will get slightly different form of the average heat transfer coefficient.

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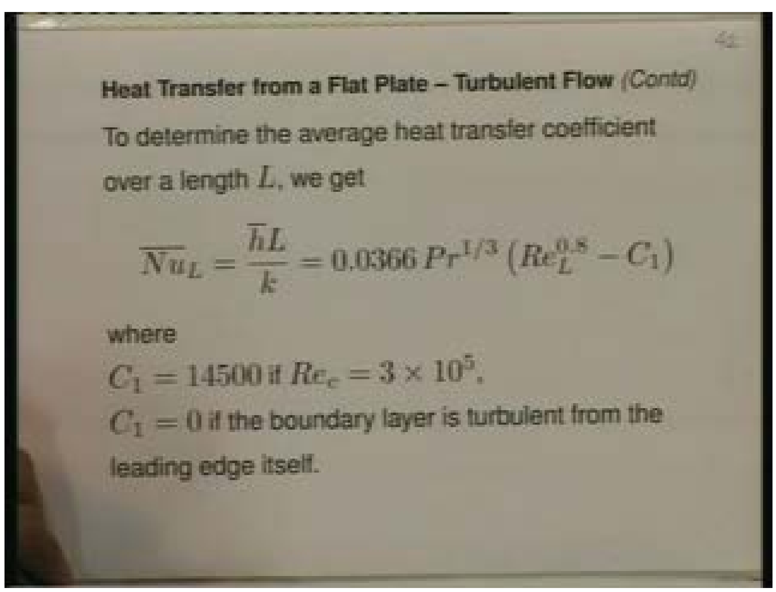

So, the average Nusselt number by proper integration can be shown to be .0366 Prandtl raised to one-third into Reynolds number raised to.8 minus some constant $C_1$ where C depends on whether the flow becomes turbulent when the critical Reynolds number is reached at 3 into 10 raised to 5 and $C_1$ is 0 meaning everything is turbulent. If the boundary layer is turbulent from the leading edge itself, these ideas would be clear when we take an illustrative example which is our next task.

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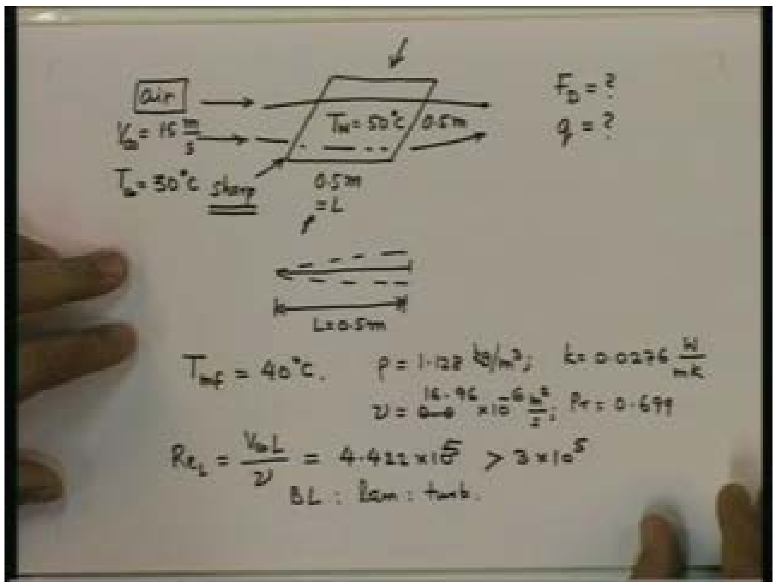


We take an example where we have a square plate size of.5 meters L by .5 meters. It is exposed to air flowing over it both on the upper part as well as on the lower part either side. Air approaches it with a freestream velocity of 15 meters per second. Assume that the leading edge is sharp and smooth, meaning the boundary layer will definitely start off as a laminar boundary layer. The free stream temperature is 30 degree C and the wall temperature is 50 degree C. We have to determine the drag forced and we have to determine the heat transfer rate. Remember that the action is on either side of the plate, this is the first part of the problem. If you now see edge on from this side or from this side we will have a boundary layer on either side and the length of the plate is .5 meters.

To determine properties, we have to determine the mean film temperature which 30 degree C plus 50 degree C divided by 2 which is 40 degree C. At this temperature the fluid is air so we can read off the properties: density - 1.128 kg per meter cube, kinematic viscosity is 16.96 into 10 raised to minus 6 meter square per second, conductivity .0276 watt per meter Kelvin, Prandtl number of .699. We first calculate the Reynolds number $Re_L$ V infinity L by mu which - we know the freestream velocity, we know the length L, the kinematic viscosity is here - this turns out to be 4.42 into 10 raised to 5 which is definitely greater than 3 into 10 raised to 5. This means that because there is a sharp leading edge, the boundary layer is laminar followed by turbulent and we will have to take care of that.

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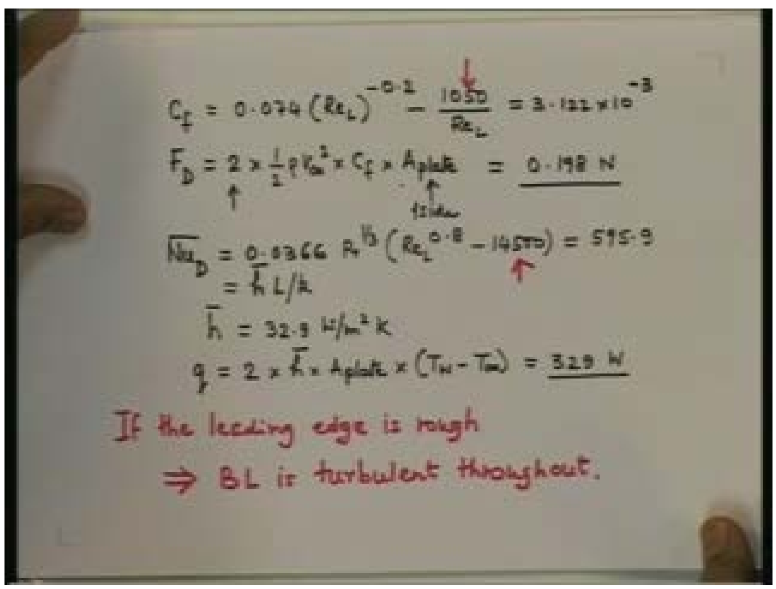


Using the skin friction noted earlier with a transition at the critical Reynolds number of 3 into 10 raised to 5, the skin friction coefficient is .074 $Re_L$ raised to minus .2 minus 1050 divided by $Re_L$ and this turns out to be 3.122 into 10 raised to minus 3 and hence the drag forced on the plate, I will multiply everything by 2 because the action is on either side of the plate, multiplied by half rho V infinity squared, multiplied by skin friction coefficient multiplied by area of the plate, area of 1 side of the plate; remember that is because I already have a 2 here and substituting in this you will get this to be 0.198 Newton.

To determine the heat flow rate, we use the average Nusselt number correlation 0.0366 Prandtl raised to one-third $Re_L$ raised to .8 minus 14500 and this turns to be 595.9. This is nothing but h bar L by k which gives you h bar equal to 32.9 watt per meter square Kelvin and which gives you the heat flow rate as 2 for either side of the plate. Multiplied by h bar into A plate multiplied by T wall minus T infinity and substituting into this we get 329 watt.

Now let us modify the problem; the modification is if the leading edge is rough. This would imply that the boundary layer is turbulent throughout and if the boundary layer is turbulent throughout, these 2 expressions are still valid but this expression without this term and this expression without this term. So naturally since both these terms subtract something and we will not be subtracting that, we will get a higher value of $C_f$ and a higher value of Nu bar D. Let us see how much higher.

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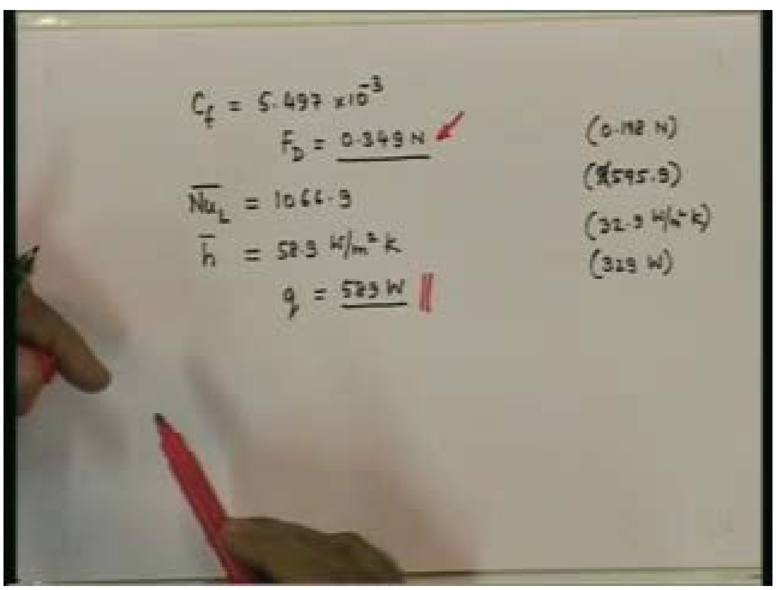


I will not rewrite those expression but it turns out that in this case $C_f$ is 5.497 into 10 raised to minus 3 giving you a drag forced of .349 Newton instead of the earlier value of .198 Newton for comparison. And your Nu bar L turns out to be 1066.9 compared to 595.9 earlier. h bar turns out to be 58.9 watt per meter square Kelvin, the earlier value was 32.9 watt per meter square Kelvin and hence your q turns out to be 589 watt, the earlier value was 329 watt. So notice that by making the leading edge rough or putting bar or it bur or a tripping wire or something like that - to see to it that the boundary layer becomes turbulent right from the leading edge - we are able to increase the value of the heat transfer coefficient and the heat flow rate. So we are able to provide better cooling but even the drag forced is higher.

We now go and look at a study of the heat transfer from a cylinder in cross flow. A cylinder in cross flow has been looked at in fluid mechanics; now look at the heat transfer aspects. A cylinder in cross flow - a situation of basic interest, applied interest, and hence a large amount of studies both analytical and experimental have been conducted on this situation. One of the earlier correlations historically well known is that by Hilbert for heat transfer but we don't use it anymore basically because not only is it old data but is based only on data for air.

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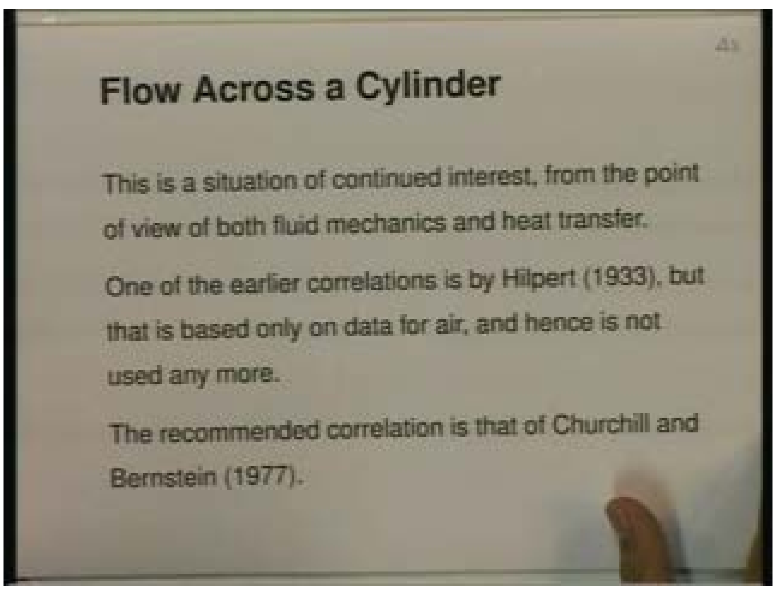


So, may be you can use it for gases which have similar properties to that of air but you can't use it for liquids. The correlation which has been recommended recently and which has been found to agree with a wide range of experimental data is that of Churchill and Bernstein. Let us look at that correlation.

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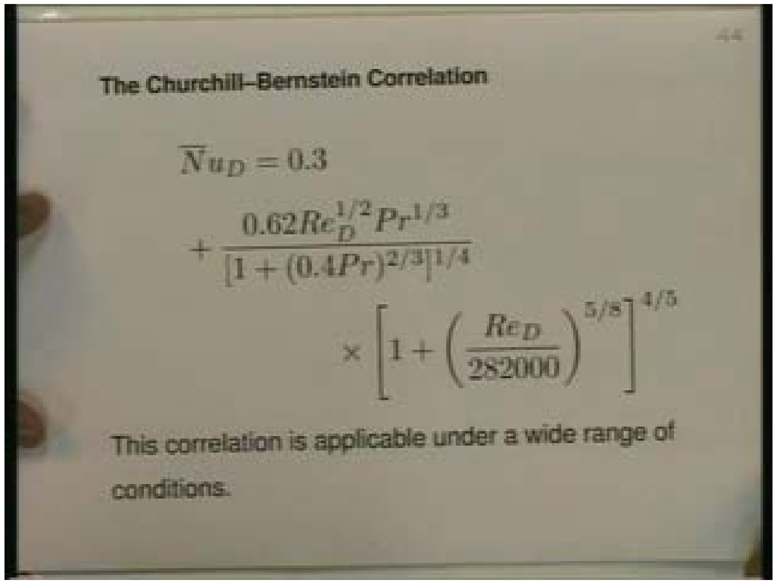


Notice that the correlation occupies almost one whole page of this display. I am not going to read it out because that would take quite a few minutes but you should notice that it is a correlation for the average Nusselt number. It depends on the Reynolds number based on the diameter and the Prandtl number and you have all sorts of factors - Reynolds number raised, half Prandtl number raised to one-third, Prandtl number raised to two-third, Reynolds number divided by something raised to five-eighth.

I think if you want to calculate you write a small program and let the calculations be done by that; it looks complicated but it is applicable under really a wide range of conditions. Let us look at those conditions; in fact, the conditions are really useful because the equation can be applied under a wide range of parameters.

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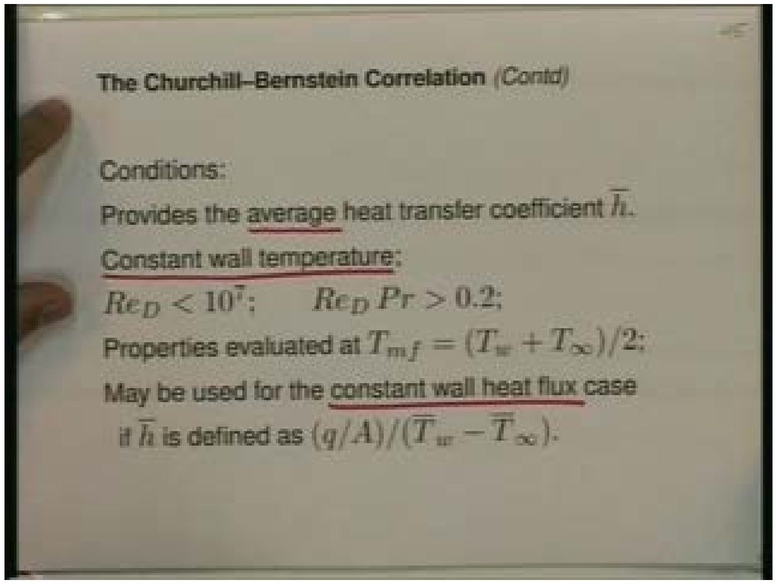


First thing we should know that it provides the average heat transfer coefficient, no local value is provided. It is essentially for the constant wall temperature case; there is no upper limit, there is an upper limit for Reynolds number which is 10 million. There is no specific lower limit mentioned, there is no separate limit for Prandtl number but your Reynolds number into Prandtl number should be greater than 2. So for a given Reynolds number, there is a range of Prandtl numbers which one can use.

The properties for this correlation need to be evaluated at the mean film temperature which is the arithmetic average of the wall temperature and the freestream temperature and then it has been found that although it is for constant wall temperature it can be used even for the constant wall heat flux case provided the average heat transfer coefficient is defined as average heat flux divided by the temperature difference between the average wall temperature and the freestream temperature.

You should not expect to take the average of the local heat transfer coefficient; you take the total heat flow rate divided the area, take the average of the wall temperature, subtract from it the mean, the freestream temperature and then you get the average heat transfer coefficient and if you want this, then the Churchill and Bernstein correlation is a good correlation to use even in the constant wall heat flux case.

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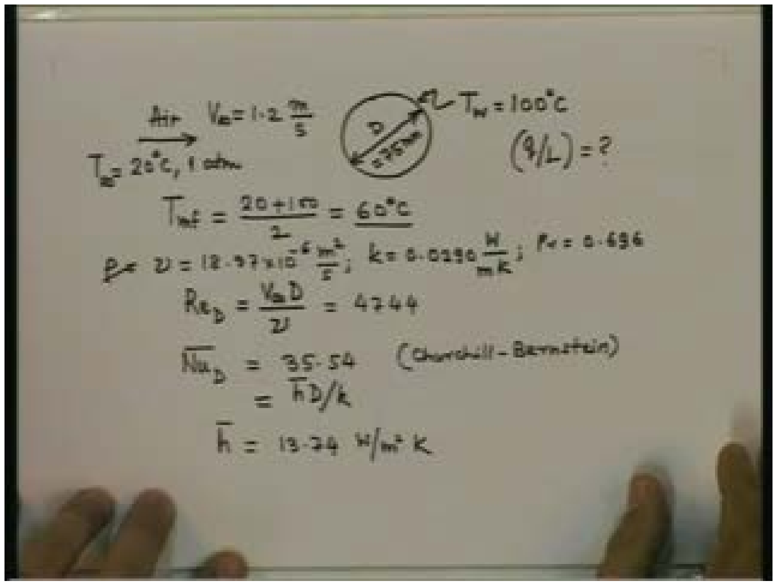


Let us take an illustrative example; here we have a problem where we have a cylinder of diameter D which is 75 millimeter. Air flows across it, the approach velocity is 1. 2 meters per second, air conditions of air are - 20 degree C, 1 atmosphere. It is given that - let me write this as T infinity - the wall temperature is maintained at a uniform value of 100 degree C. May be it is thin pipe and steam at 1 atmospheres is condensing inside. We have to determine what is the heat transfer from the cylinder to air per unit length of the cylinder; unit length perpendicular to the plane of the picture.

First, properties are to be determined at the mean film temperature which is 20 plus 100 by 2 which is 60 degrees C. We have air as the working fluid so we read off the properties from a table; density - we don’t really need density, kinematic viscosity – 18.97 into 10 raised to minus 6 meter square per second, thermal conductivity .0290 watt per meter Kelvin, Prandtl number 0.696. First calculate the Reynolds based on the diameter; this will be V infinity D by nu and this turns out to be 4744. Notice that the

Reynolds number into Prandtl number - it is about 4500 into .7, definitely more than 3000. So we have nowhere near the limit of that Re pr.

Reynolds number is less than 10 to the 7 so we are within the range of the Churchill Bernstein correlation. So I am going to write the correlation again but we will get the value of the heat transfer coefficient Nusselt number to be 355.54 from the Churchill Bernstein correlation. This equals the average heat transfer coefficient into diameter divided by conductivity and this gives us h bar to be 13.74 watt per meter square Kelvin.

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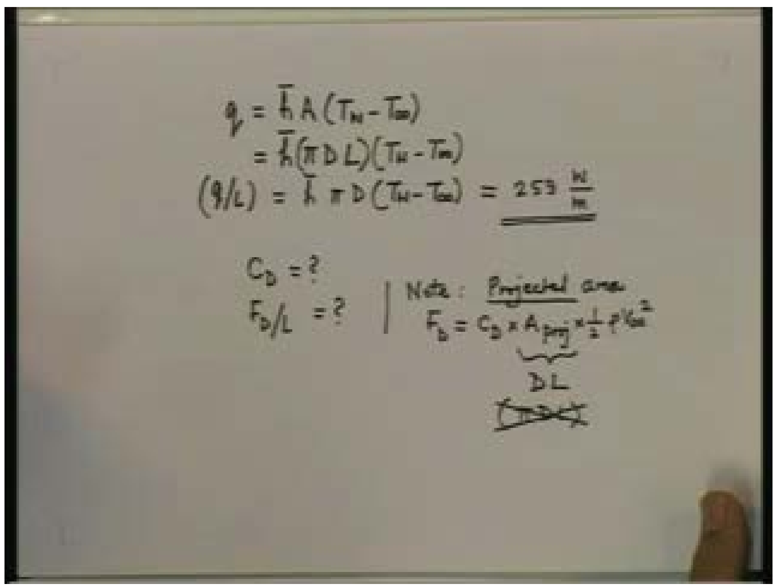


So, the heat flow rate will be, notice that for heat transfer the heat flow rate is always average; heat transfer coefficient into area - the area across which heat transfer takes place - multiplied by wall temperature minus the freestream temperature. In this case, the area happens to be pi into D into whatever is the length L of the cylinder multiplied by T wall minus T infinity and which gives us the heat transfer per unit length will be h bar into pi D into T W minus T infinity which turns out to be 259 watt per meter. I leave it to you to calculate the drag coefficient on this and hence the forced of drag per unit length.

Notice that when you determine the forced of drag, note that you have to use the projected area and not the area of the tube. So when you determine the forced of drag, you will have to write it as CD into A projected into half rho V infinity square and the A projected would be D into L and not pi D into L; that will be wrong, whereas here this area is the actual heat transfer area and hence the area is pi DL.

Now that brings us essentially to the end of the study of forced convection. Let us see what we have done. We looked at some basic situation.

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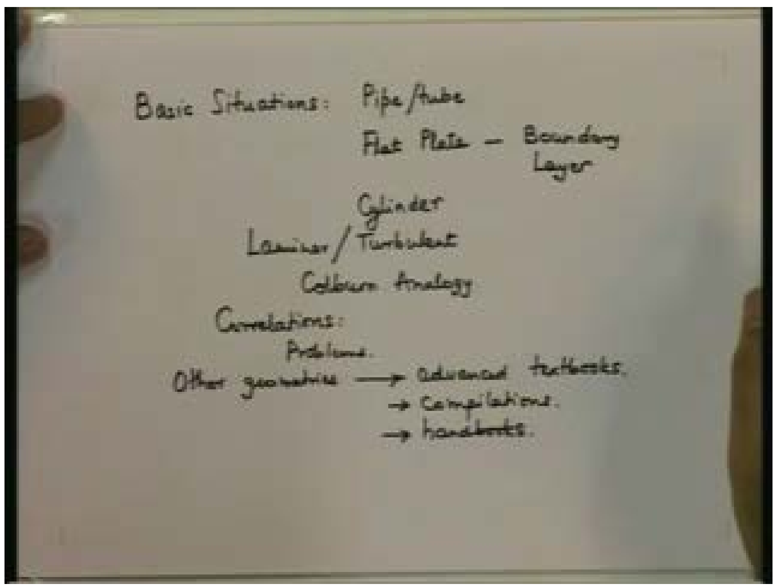


The situations we looked at was flow through a pipe or a tube, then we looked at flow over a flat plate for which we looked at the boundary layer approximation. Then we looked at flow across a cylinder; the pipe flow was a situation of internal flow, a flat plate and cylinder flow was a situation of external flow. In either case we looked at laminar flows and turbulent flows. We briefly looked the Colburn's analogy using which we could convert many of the correlations for friction factors or skin friction or drag to the appropriate correlation for heat transfer. Then we looked at a number of correlations while doing this both for laminar flow, for turbulent flow, average Nusselt number, local Nusselt number and we solved a number of problems as illustrative examples.

The situations considered here are basic situations; in actual practice, the geometries are different, the flow conditions are different and appropriate correlations are available for a large number of situations of practical interest, of industrial interest. So for other geometries and other flow situations, we will have to look up advanced text books or compilations or hand books and you will not always but quite often find a correlation or a graph which will suit your requirements. Any time you use this, remember what we had mentioned earlier; check that you are using it under the appropriate conditions, check that it is for laminar or turbulent flow as required, your Reynolds number is within the range, Prandtl number is within range and the properties are evaluated at the appropriate reference temperature. It will usually be the mean bulk temperature for pipe flow or local bulk temperature for local heat transfer in pipe flow or it would usually be the mean film temperature in case of external flow but there are situations where the correlation will specify that the properties be evaluated at a temperature other than the standard mean temperature for property evaluation. You will have to take care of that. Now with this, we come to the end of our study of forced convection. In the next lecture, we will begin our study on free convection.