Heat and Mass Transfer
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Lecture No. 20
Forced Convection-3

Welcome back to this third lecture on forced convection. Near the end of last lecture we had looked at some simple correlations for fully developed heat transfer during laminar flow in a circular tube, also correlations for heat transfer during thermally developing flow. Let us now take some illustrative examples; the first example we look at is this.

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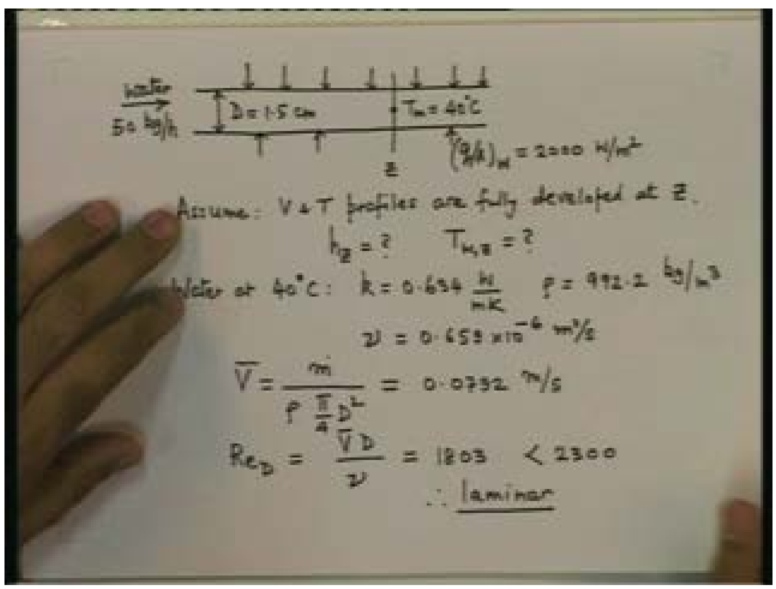

Let us say we have a tube, uniform smooth tube with internal diameter D of 1.5 centimeters. Water flows through that tube at a rate of 50 kilogram per hour; the tube is heated uniformly at a uniform heat flux of 2000 watt per square meter at some location z. The mean bulk, mean temperature of water is 40 degrees C. We will assume that the velocity and temperature profiles are fully developed at this location and we have to determine what is the local heat transfer coefficient at z and we also have to determine what is the wall temperature at z.

First the fluid is water; the local bulk mean temperature is 40 degrees C so we have to determine properties of water at 40 degrees C. This turn out to be conductivity .634 watt per meter Kelvin, density of 992.2 kg per meter cube and kinematic viscosity of 0.659 into 10 raise to minus 6 meter square per second. To determine Reynolds number we have to determine the mean velocity; this will be the flow rate divided by density divided by the flow area pi by 4 D squared. If you substitute the flow rate 50 divided by 3600 kilograms per second density as determined, the diameter of .015 meter, you will get this velocity to be 0.0792 just about 8 centimeters per second. Next we determine the Reynolds number which is V bar in to D divided by nu and you will get this thing substituting the values of V bar D and nu to be 1803. This is less than 2000; or say 2300. Hence flow is laminar - this is the first step.

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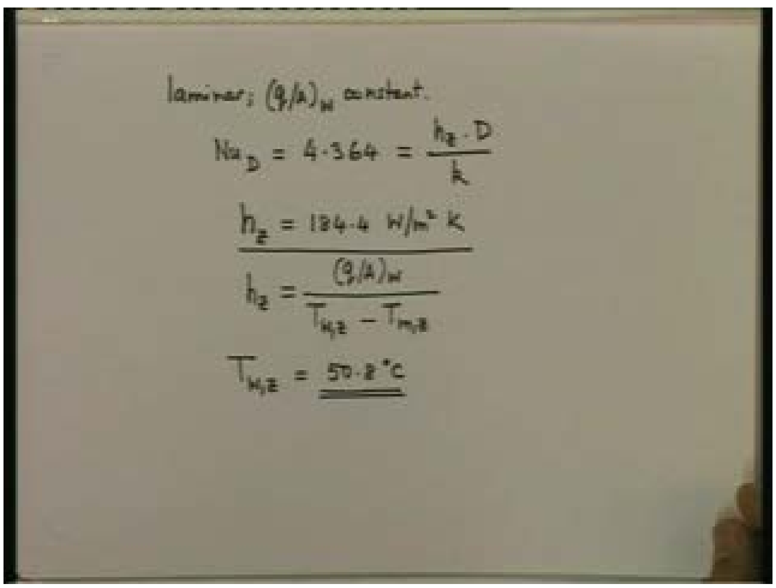

Now we have laminar flow, we have heat flux uniform, we have already assumed fully developed velocity and temperature profiles and hence we can use the result for the fully developed constant heat flux laminar flow situation in a circular tube which will give us the local Nusselt number out there as 4.364. The Nusselt number by definition is local heat transfer coefficient into the diameter D divided by the thermal conductivity k. So we know the Nusselt number, we already know the diameter; we have already determined the thermal conductivity. So the local heat transfer coefficient turns out to be 184.4 watt per meter squared per Kelvin; that is the first conclusion.

Now, the heat transfer coefficient itself is the heat flux; it is uniform so we don't have to use the subscript z divided by the wall temperature at that location minus the mean temperature at that location. So we know the heat transfer coefficient, the wall heat flux is specified 2000 watt per meter squared, T_{mz} is specified 40 degrees C so only unknown in this equation is T_{wz} and if you calculate that out, it turns out be 50.8degrees C. Now, let us look at some other problem - slightly different; let us modify the situation.

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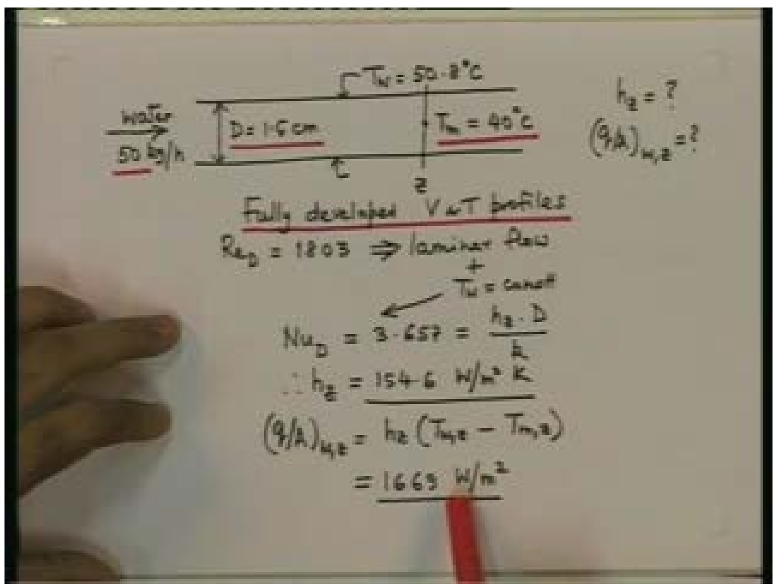

Let us say we have a very similar tube, in fact a very similar situation - same diameter, water flowing at 50 kg per hour. But let us say now that the wall temperature is maintained at 50.8 degrees C uniformly here as well as here and at some location z - actual location z - T_m happens to be 40 degrees C. Again assume fully developed velocity and temperature profiles and we have to determine the heat transfer coefficient at that location and the wall heat flux at that location.

Now, notice that compared to the previous problem, the diameter remains the same, the flow rate remains the same and the temperature at which the properties are to be determined also remains the same hence the Reynolds number also remains the same. So we have Reynolds number of 183 that means it is laminar flow and laminar flow plus T wall is some constant value gives us the situation where the local Nusselt number based on the diameter is 3.657 this will now be the nu hz into D by k. D and k are now different from the previous illustrative example.

Nusselt number is lower hence our heat transfer coefficient will also be lower and that turns out be 1504.6 watt per meter squared Kelvin and the heat flux at the wall at that z is calculated as heat transfer coefficient into wall temperature at that location minus the mean temperature of the fluid at that location and this turns out to be 1669 watt per square meter. So notice that uniform heat flux boundary condition gave us a slightly higher heat transfer coefficient a uniform temperature boundary condition will give us a lower heat transfer coefficient and hence a lower heat flux

Again I would like to bring your attention to this assumption that it is a fully developed velocity and temperature profile in this case as well as in the earlier case. Suppose the velocity or in particular the temperature profile was not fully developed then what will happen? Remember that in the thermal entry length, the heat transfer coefficients are higher than in the fully developed situation hence under the assumption of fully developed velocity temperature profile, we have obtained perhaps the lowest possible heat transfer coefficient in either the first case or the second case.

Hence, in the case we have obtained because the heat transfer coefficient was perhaps the lowest and estimate for the highest possible temperature of the wall if the heat coefficient were higher that would be lower than the value of computed and in the second case we have determined perhaps a lowest possible value for the heat flux because during the development region thermal entry length the heat transfer coefficient will be higher and hence the heat flux will also be higher.

So, remember that the fully developed situation acts as a limiting situation for the entry length problem. Now let us look at a third problem

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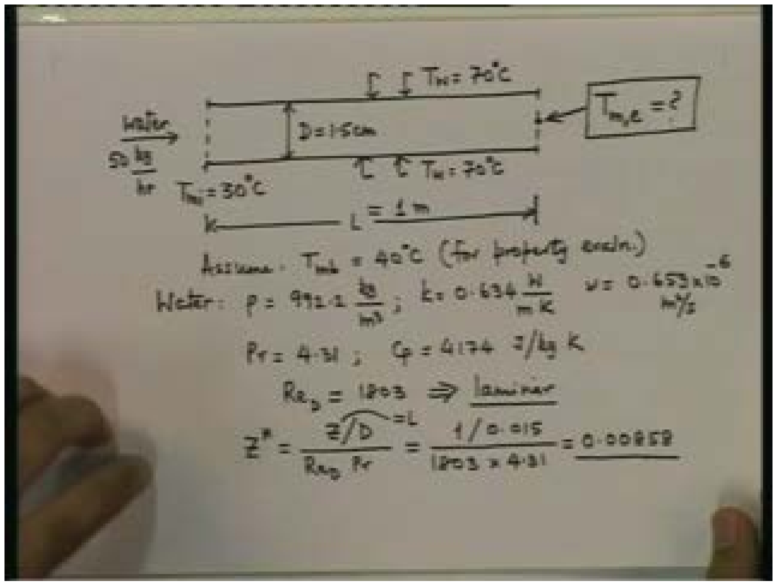

Again we have the same tube and a similar flow situation - water flows at a rate of 50 kg per hour through a tube of diameter 1.5 centimeter. The inlet temperature of water T_{mi} is 30 degrees Centigrade, the length of the tube is 1 meter. We have a fixed length of tube; the wall of the tube is maintained at a uniform temperature of 70 degrees C. Because the wall is at a higher temperature than the inlet temperature of water, water will get heated up and the exit temperature will be somewhere between 30 and 70 degrees C. Our problem is to determine what is the mean bulk temperature at the exit.

Now this is a problem where we will have to consider the average heat transfer coefficient over this length and relate the answer to that. Now for determining the average heat transfer coefficient over the length we will have to use properties at the bulk mean temperature; that is the inlet temperature plus the exit temperature by 2 that would be the bulk mean temperature. Now the exit temperature is not known so we will have to estimate the bulk mean temperature to start with; later on we will check this estimate.

See the exit temperature will never exceed 70. Depending on the length whether it is short or long - if it is too short, it will be near 35-40, if it is long it will be 60-65. Let us assume just as a guess that let the exit temperature be 50; we are not estimating directly the exit temperature but suppose it were 50 then the bulk mean temperature for property evaluation would be 40 degrees C.

We have, we already have listed with us properties at 40 degrees C so that is a good assumption to start. So let us assume that T mean bulk is 40 degrees C; for property evaluation again, I will list out the properties for water rho 992.2 kg per meter q, k 0.634 watt per meter Kelvin, nu 0.659 into 10 raise to minus 6 meter square per second, Prandtl number equal to 4.31 that is dimension less, C_p equals 4174 joules per kilogram Kelvin.
We have the same flow rate, same diameter, same properties, so the Reynolds number turns out to be 1803 which implies that the flow is laminar. Now we have to begin heating from z equal to 0 so this is a thermal entry length situation. Let us calculate the dimensionless value of z. z star we know is z by D divided by Re_D in to Prandtl number; the total length in this case z would be equal to L, 1 meter divided by 0.015 meter divided by Reynolds number 1803 into Prandtl number 4.31 - this turns out to be 0.00858.

Now, we look at the correlations which we saw yesterday for thermal entry length. We notice that this particular value of z star, it is marked within the range of the correlation; it is not large and beyond the correlation where we will say that look it is almost fully developed.

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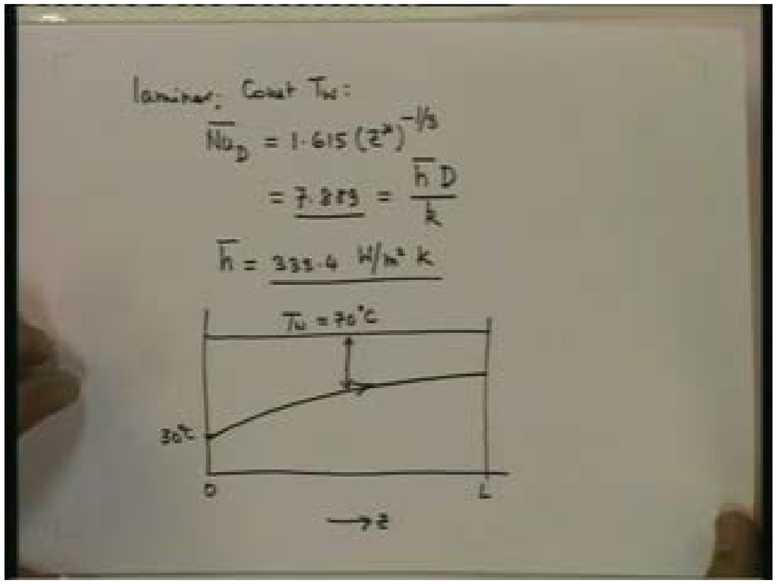

So, we will have to use the appropriate correlation. Since it is laminar and it is a situation of constant wall temperature, we have this correlation. This is the average Nusselt number; over the length L which is represented by z star is 1.615 z star raised to minus 1 by 3 - this turns out to be7.889. If it were fully developed this would be 3.634 so this is significantly higher than that, not quite twice but definitely more. Why more than twice? Now this Nusselt number is nothing but the mean heat transfer coefficient or the average heat transfer coefficient over that length multiplied by D divided by k. We know the diameter, we know the conductivity so h bar turns out to be 3303.4 watt per meter squared Kelvin. Notice that this is higher than what we obtained for the fully developed situation.

Now, we know the area of the tube pi into D into L; everything is known. We now know the average heat transfer coefficient so we should be able determine the heat flow rate from the wall to the fluid because heat flow rate from the wall to the fluid would be heat transfer coefficient into area into temperature difference between the wall and fluid. However because it is a constant heat flux, constant wall temperature situation, we have a uniform wall temperature; this is the, say the length, this is 70 degrees C. The fluid enters at 30 degrees C, it will heat apply like this, not in a straight line but in a curvilinear fashion initially it will heat up fast then it will heat up slow. Consequently it is not easy to directly write an expression for the mean temperature difference between the wall and the fluid because it varies from point to point. Let us do the following exercise to determine the mean temperature difference.

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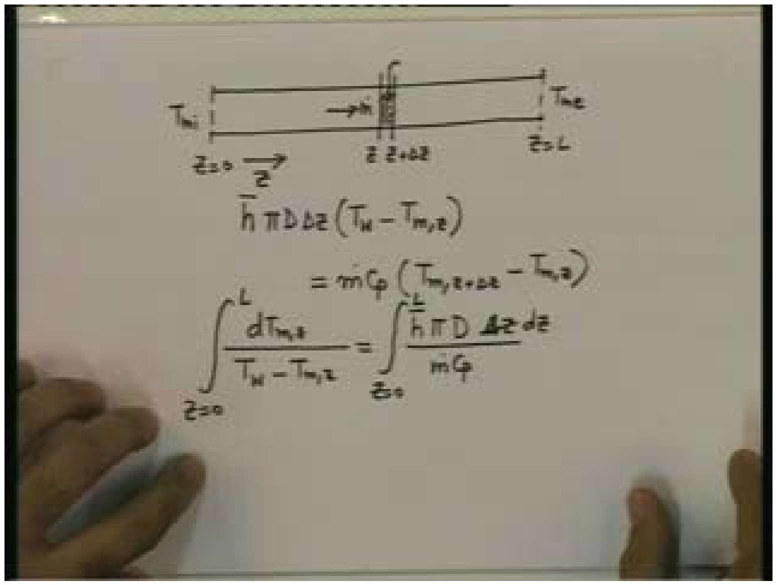

Let us say that this is the tube, this is, this is the inlet section, this is the exit section. This is z equal to 0 where the temperature is T_{mi}; this is the exit section temperature T_{me}. z equals L, z increases like this. Let us take a small slice and consider it as a control volume. In this control volume thickness delta z so one wall is at z, another wall is at z plus delta z. We will now apply to this the first law of thermodynamics. Flow takes place at a rate of m dot so the heat absorbed must equal the outflow of enthalpy.

How much is the heat transfer from the wall to the fluid? That will be the mean heat transfer coefficient; once having calculated the mean heat transfer coefficient, we assume that that is the heat transfer coefficient applicable everywhere. Multiplied by the area of heat transfer in this slice, that will be pi D delta z multiplied by the local temperature difference that is the wall temperature. We need not use the subscript z minus the local mean temperature T_m or T_m at z and this should equal m dot into enthalpy out minus enthalpy in and that would be equal to m dot into C_p of the fluid into T_m at z plus delta z minus T_m at z. Transposing terms notice that this can be written down as dT mz; we will get dT mz divided by T wall minus T_m z equal to h bar pi D delta z divided m dot C_p - this delta z can be considered a small slice dz. Now we integrate either side from z equal to 0 to L.

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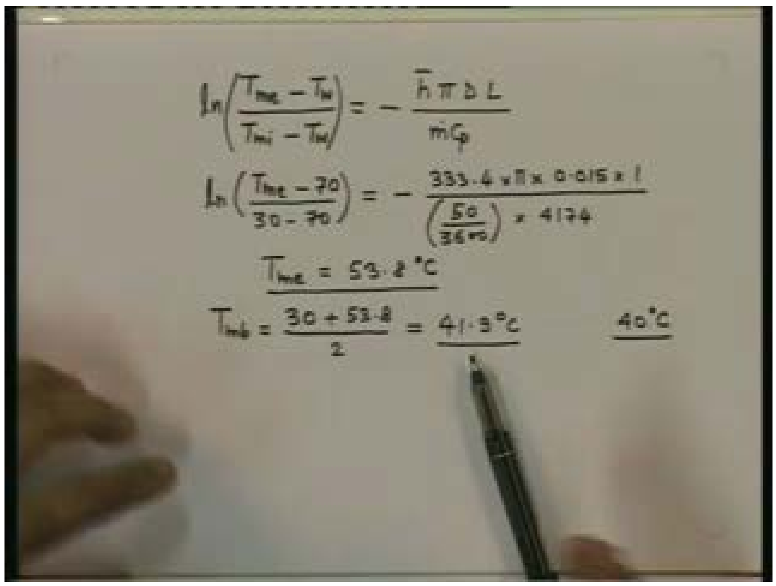

The resulting equation is - Log of T_m exit minus T wall divided by T_m inlet minus T wall equals minus h bar pi DL divided by m dot C_p. Now, notice that in this expression, we have the values of everything except T_{me}; wall temperature is known, inlet temperature of the fluid is known, heat transfer coefficient has been calculated DL, m dot are specified, C_p has been determined. So if you substitute you will get ln T_{me} minus wall temperature of 70, 30 minus 70 will be equal to minus h bar that was 334 multiplied by pi multiplied by .015 multiplied by L that is 1 meter m dot which is 50 divided by 3600 kg per second multiplied by C_p which is 4174.

Solving this for T_{me} notice that the right hand side is negative and that means this argument of the logarithm will be a number less than 1 and that means the numerator will be smaller in magnitude than the denominator. The denominator has a magnitude 40 so the numerator will have a magnitude less than 40 so T_{me} will be higher than 30; it will be somewhere on the side of 70, higher than but lower than 70. And if you calculate T_{me} that turns out be 53.8 degree C.

Now, we started off by having assumed the bulk mean temperature; we needed that assumption to determine the properties. We had assumed 40 degrees C; let us check that assumption. The bulk mean temperature now turns out to be inlet bulk temperature plus exit bulk temperature by 2; average of inlet and exit - this turns out to be 41.9 degrees C, we had assumed 40 degrees C. It is different - higher by about 2 degrees C properties; will be slightly different. If insists one can now recalculate the properties at say 41.9 degrees C go through the same exercise and maybe you will get a slightly improved value of T_{me}.

If this assumption were to differ from this calculated value by very significant amount say 10 degree, 15 degree then it is definitely worth redoing the calculation with the new corrected value of the bulk mean temperature because the properties would change reasonably significantly. With a temperature difference of about 2 degrees C they are unlikely to change but anyway it may be a good exercise for you to check that out. After having studied laminar flow through a tube, let us now look at turbulent flow heat transfer in a tube or a pipe - circular one.

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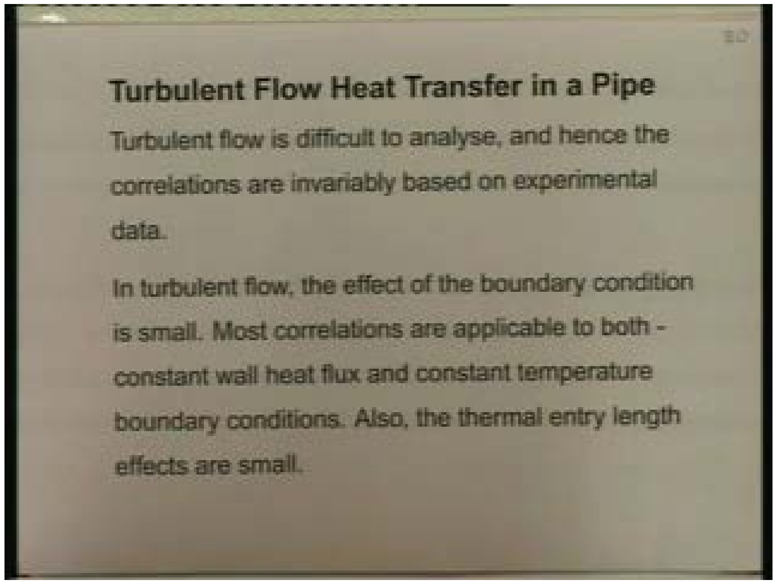

Unlike laminar flow because of the eddies form during turbulence and turbulent mixing, turbulent flow is invariably difficult to analyze; much more difficult than laminar flow and hence we obtain information for turbulent flow essentially from experiments and all the correlations or almost all correlations which we have for turbulent flow are based on experimental data. In a way turbulent flow is slightly simpler because in turbulent flow the effect of the boundary conditions are small.

We notice that for fully developed boundary condition in laminar, fully developed profiles in laminar flow, we had significantly different values of the Nusselt number depending on whether the boundary condition is that of uniform heat flux or the boundary condition is that of uniform wall temperature. In turbulent flow, because of the mixing of all lamina these differences reduces significantly.

You will hardly ever find a correlation in turbulent flow specifically for say uniform wall temperature boundary condition or specifically for uniform wall heat flux boundary condition; almost always a single correlation is applicable to both. Again, because of turbulent mixing, the effect of the entry length is small during turbulent flow; laminar entry lengths tend to be larger, they tend to increase with Reynolds number. Turbulent entry lengths are usually smaller and they do not have a significant effect of Reynolds number on them.

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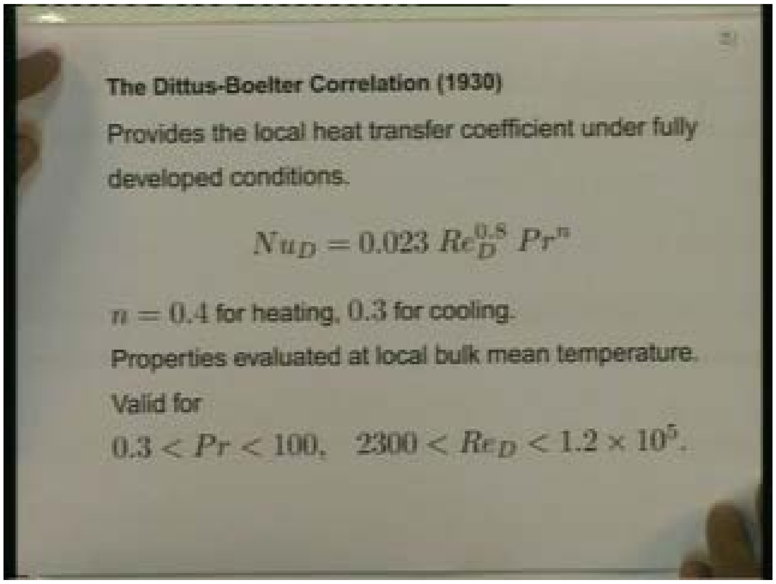

For turbulent flow, heat transfer in a pipe, perhaps the famous historically important is the celebrated Dittus Boelter correlation more than 75 years old but it is still being used because it is simple, straight forward and works reasonably well. The Dittus Boelter correlation provides the local heat transfer coefficient under fully developed conditions and the equation is reasonably simple. The Nusselt number is a function of Reynolds number and Prandtl number and is equal to.023 into Reynolds number raise to .8 Prandtl number raise to some exponent n where the exponent n is .4 for heating and .3 for cooling so heating data correlates well with n equal to .4, cooling data correlates well with n equal to .3.

For this correlation we will have to determine properties which are evaluated at local bulk mean temperature to determine the local heat transfer coefficient. But if you are considering an average heat transfer coefficient over a length, you must use the bulk mean temperature over that length. It is reasonably wide in its validity - Prandtl number going from .3 right up to 100 - so water is included, air is included and some not very high viscosity oils are also included. And Reynolds number in the turbulent zone from 2300 to something like 100 and 20000. Of course, the Dittus Boelter correlation is pretty old; we are attracted to it because of its simplicity.

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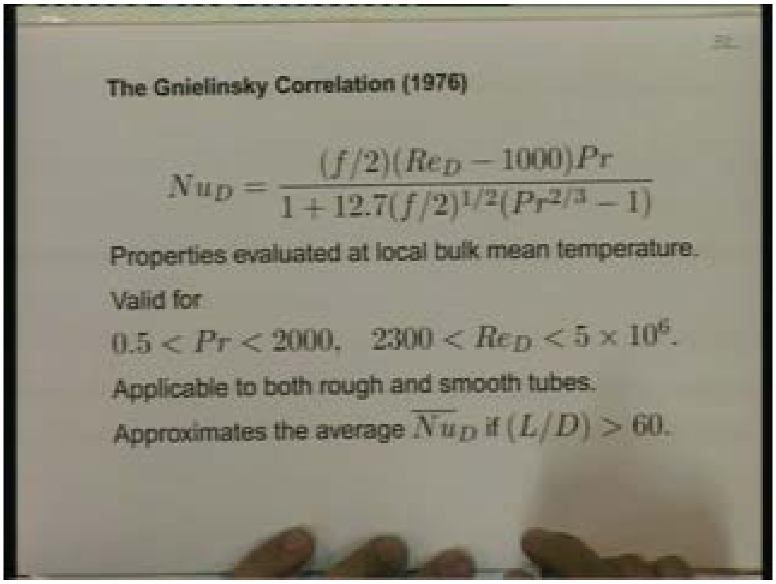

The current recommendation for the most suitable correlation is the Gnielinsky correlation; it is also about 25-30 years old. Notice that here the Nusselt number is provided as a function of not only the Reynolds number and the Prandtl number but also the friction factor for flow through that tube and the friction factor itself will have a dependents on Reynolds number so it has reasonably complicated dependents on Reynolds number as well as Prandtl number. This also provides the local value of the heat transfer coefficient when the velocity and temperature profiles are fully developed.
Again like the Dittus Boelter correlation, the properties are evaluated at the local bulk mean temperature. It has a slightly wider validity - Prandtl number going from .5 to 2000 - so some reasonably high viscosity oils are also included. The Reynolds number going from 2300 to 5 million - this also is a higher range. Depending on the smoothness or the roughness of the pipe, this can be applied by using appropriate correlation for f; for the smooth pipe, you use a correlation for f which pertains to smooth pipe. For rough pipe the

effect of epsilon by D or e by D can be included using the Chen's correlation. So the effect of smoothness or roughness of the pipe is included through the effect on f.

Let us take an illustrative example based on the Gnielinsky correlation.

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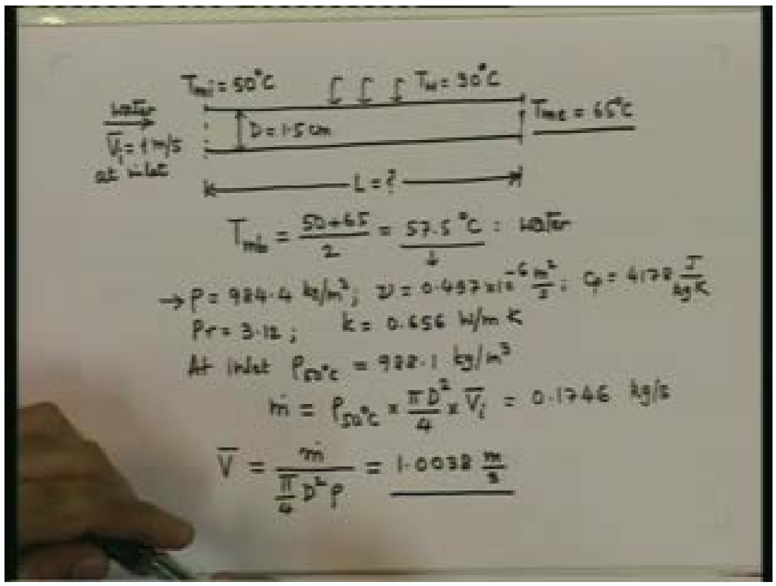

You are now going to look at the same old pipe with which we were playing around. We have a tube of some length L which we are going to determine here. It is given that the diameter is 1.5 centimeter, the wall temperature is maintained everywhere at 90 degrees C. The inlet temperature is 50 degrees C; we want the exit temperature to be 65 degrees C. So this is a design problem where we are not given the length of the pipe; we are given the requirement that the exit temperature should be 65 degrees C and we have to determine the length of the pipe.

The fluid flowing through this is water and the velocity is 1 meter per second at inlet. One thing is straight forward here; we know the inlet temperature, we know the outlet temperature. We have to determine the overall length so we are talking of a situation where we will have to determine the average heat transfer coefficient.

We know the inlet temperature and outlet temperature so we can determine the bulk mean temperature 50 plus 65 by 2 which is 57.5 degrees C and at this for water, the following properties will be interpolated out. So at 57.5 degrees C the properties of water are density 984.4 kg per meter cube, kinematic viscosity .497 into 10 raise to minus 6 meter square per second, C_p 4178 joule per kilogram Kelvin, Prandtl number 3.12, conductivity .656 watt per meter Kelvin. Now we need to know the velocity at the bulk mean temperature where the density is 984.4, what is specified is velocity at inlet. So we need to determine at inlet, density will, density at 50 degrees C and this turns out to be 988.1 kg per meter cube.

Using this density and the velocity at inlet, let me call this V bar. We get m dot is rho at 50 degrees C multiplied by pi D squared by 4, area multiplied by the mean velocity at inlet - this turns out to be 0.1746 kilogram per second and based on this if we calculate velocity at the bulk mean properties that is at a density of9eighty 4.4 this will turn out to be m dot which is calculated divided pi by 4 D squared in to rho where rho is9eighty 4.4
We will get a slightly higher velocity which is 1.0038 meters per second, it is this velocity which determine the mean properties that we will be using to determine the Reynolds number.

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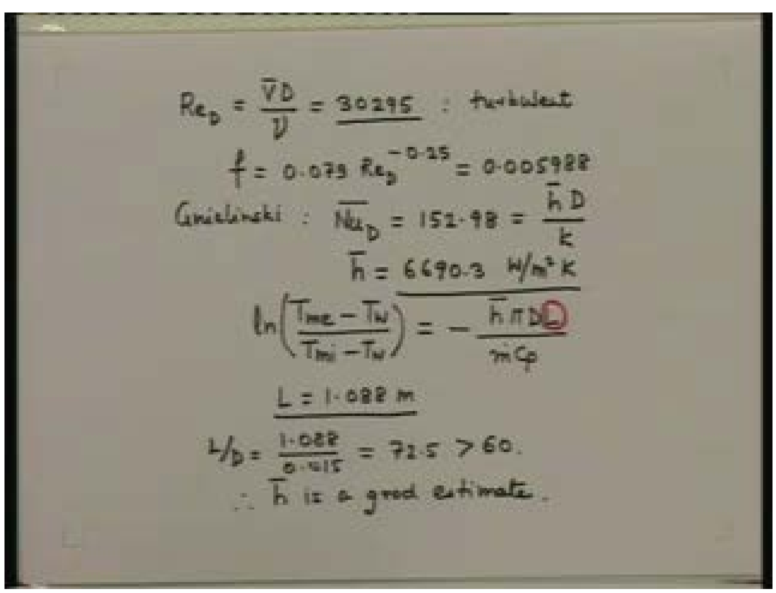

The Reynolds number is now calculated as V bar D by nu that turns out to be 30295 which is well in the turbulent zone. Prandtl number is already greater than .5 so in principle the Gnielinsky correlation should be applicable. For that, first we have to determine the friction factor assuming the tube to be smooth; there is no specification of any roughness. We will use the smooth tube correlation .079 Re_D raise to .25 which is the recommended correlation for smooth tubes to be used with the Gnielinsky correlation; we will get .00598.

If you use the Gnielinsky correlation you will get the Nusselt number which because we have determined it at average properties, the average Nusselt number which turns out be 152.98 which is h bar D by k from which we get h bar equal to 6690.3 watt per meter squared Kelvin, see the high value a few thousands for this. Now we have our relation for a uniform wall temperature situation - log of T_{me} minus T wall divided T_{mi} minus T wall equal to minus h bar pi DL by m dot C_p.

Now we have been specified T_{me}, T_{mi}, T_w; left hand is known, we have determined h bar, we know D and L, we know D, we know m dot, we have determined C_p, L happens to be the unknowns. So solving this equation for L, we get L to be 1.088 meters nearly 1.9 centimeter. Remember that for the average, the local heat transfer coefficient to be used for the average value, we need L by D to be greater than 60.

If we determine L by D, this turns out to be 1.088 divided by 0.015. This turns out to be something like 72.5 which is greater than 60; hence h bar is a good estimate that means the effect of entry length is small. With this we come to the end of this lecture and with this we complete the basic study of a forced convection heat transfer through a tube. In the next lecture we begin a study of heat transfer from a flat plate exposed to a free stream.