Heat and Mass Transfer
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Lecture No. 19
Forced Convection - 2

Welcome back to the second lecture in forced convection.

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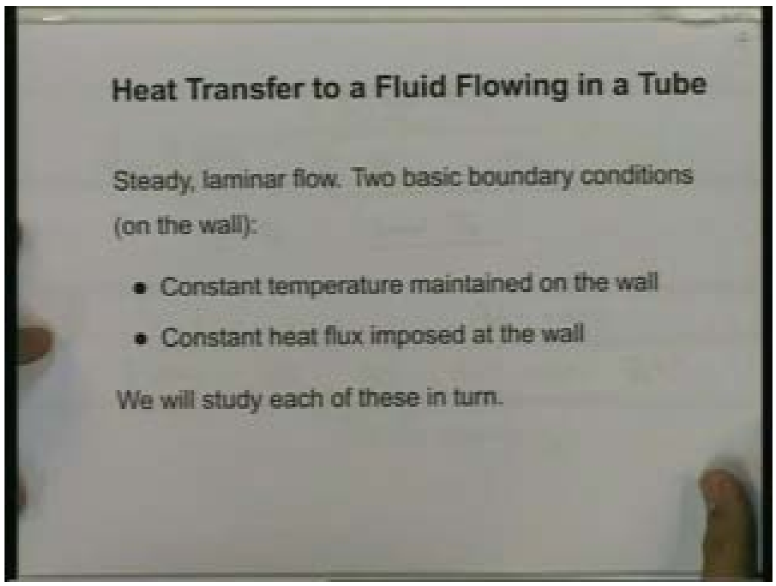

Near the end for the previous lecture, we were about to study heat transfer to a fluid flowing in a tube and we said we would look at the constant temperature boundary condition and the constant heat flux boundary condition on the wall.

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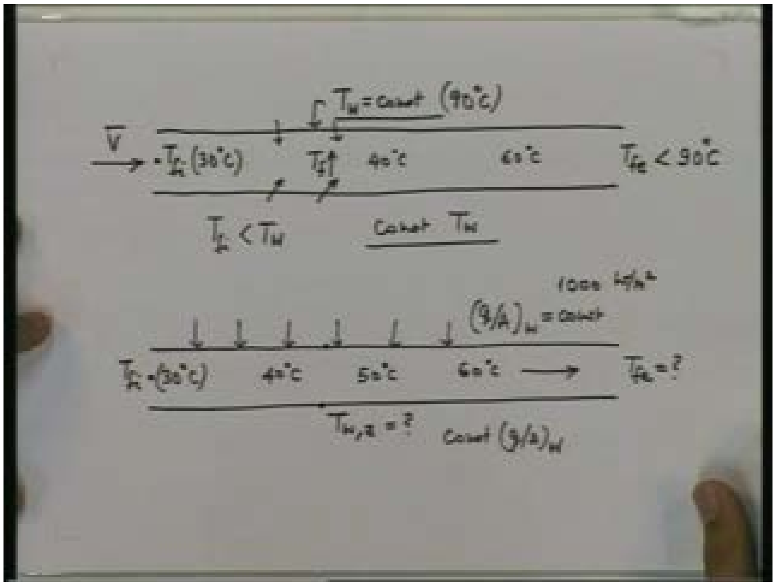

We had sketched briefly what was expected in the 2 cases; now let us look at the phenomenon and exactly what happens in some detail.

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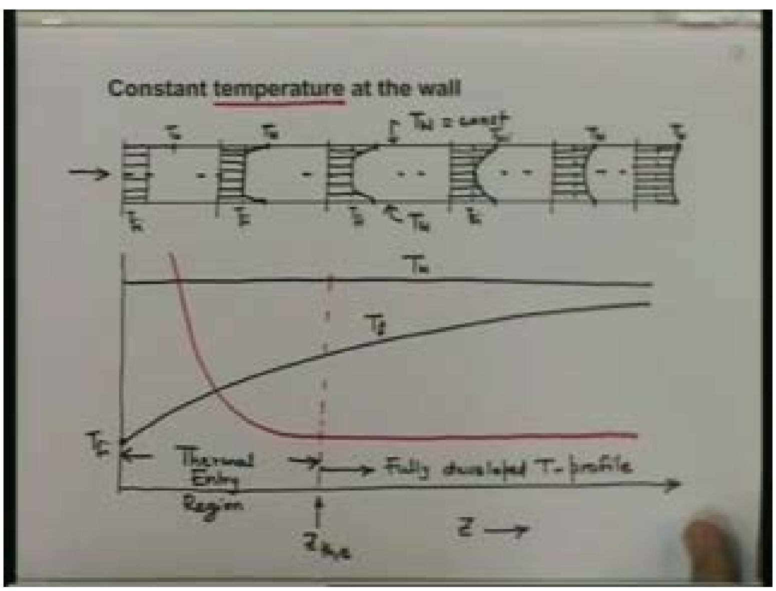

First let us consider the situation of constant temperature at the valve; let me sketch a tube and let us say - of course I should also show a centerline - the wall is maintained at T_w, some fixed value. And let us say that the fluid enters at some uniform temperature

T_{fi}; let me say this is the temperature profile at the inlet T_{fi}. Since the wall temperature is higher than this let us say the wall temperature is represented by T_w. Now we are going to study what happens as the fluid gets heated.

Let us draw a few sections. If there were no heating the temperature profile would remain what it was - equal to T_{fi}. Suppose the wall were insulated, this would have been the temperature profile but the wall is not insulated, it is a temperature higher than the fluid inlet temperature. So there will be some heat transfer from the wall to the fluid, heat transfer will continue as the fluid flows.

After a short distance, the fluid layer near the wall would have reached the wall temperature but the effect of the wall temperature would not have penetrated significantly into the fluid. The wall temperature here, the fluid temperature over much of the cross section would remain equal to T_{fi}. After some more distance, a larger layer near the wall would have been affected by the hot wall and we will get a profile perhaps something like this. The flat region here - temperature continues to be almost equal to T_{fi} - would have narrowed down.

After some distance may be the effect could have been complete all over the diameter of the tube, may be just at the center the lamina would remain at a temperature of T_{fi}. Beyond that the even this layer would get heated; we will get a profile like this and still further down if this is the wall temperature it would come much nearer the wall temperature. So the profile which was initially flat would go on developing, the flatness in the middle would reduce, wall effect will increase and then after some distance - after long distance - everything would tend to come to T_w; the effect of T_i would almost be lost.

Let us plot some parameter against distance from entry Z; let me first plot T_{wall} which is easy to plot - T_{wall} is constant. Let us plot the fluid temperature; we assume that at the inlet it was lower than the wall temperature. So this is T_{fi} and as fluid gets heated up the mean temperature of the fluid, it would go on increasing after a large length, it would come almost equal to T_w but over a finite length it will increase but never really reach T_w. So this is the fluid temperature, mean temperature of the fluid, as it gets heated up.

If we calculate the heat transfer coefficient notice that initially the gradient near the wall will be large, it will go on reducing as we proceed. Similarly the temperature difference between the wall - the mean temperature of the fluid will go on also reducing. Initially it would be almost equal to T_w minus T_{fi} later on it would be a small fraction of that. Combining these 2 it turns out that the heat transfer coefficient, only I should use a different color for that starts off as large values and then becomes almost constant after some distance.

This particular zone where the heat transfer coefficient is a function of z is known as the thermal entrance region and this is, this z is known as the thermal entry length. Beyond that although the temperature of the fluid keeps on increasing the heat transfer coefficient doesn't significantly change. So from this point onwards is a zone of fully developed temperature profile.

We will look at what is exactly meant by a fully developed temperature profile slightly later; we will define it properly. Now after looking at a situation where we have constant temperature at the wall, let us look at the other situation where we have a constant heat flux at the wall.

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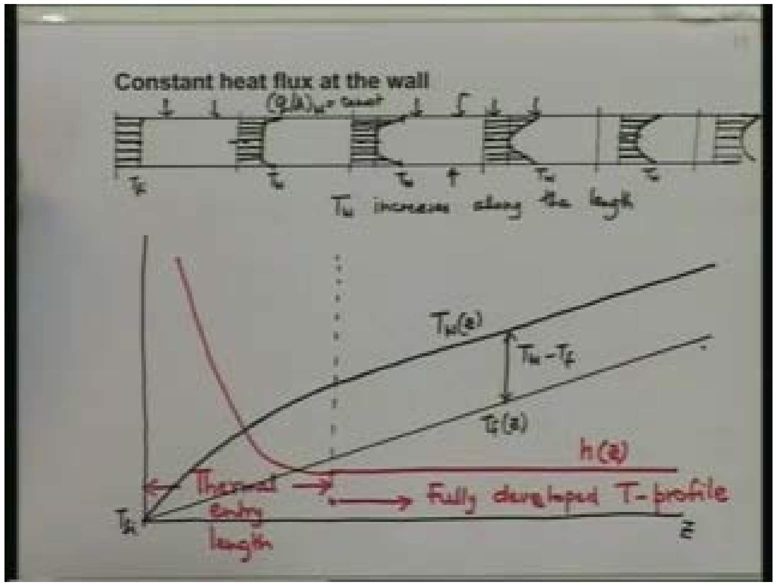

Again let me sketch a tube but now on the wall we have imposed all over right from the beginning, a uniform heat flux all over the circumference; this is the center line. Let us again say that the temperature profile at the entry is flat and is given by T_{fi} and now again let us plot and look at the temperature profile as it develops. If there were no heat flux, the temperature everywhere would equal T_{fi} but now there is a heat flux so the fluid gets heated up and first the lamina of the fluid near the wall would get heated up.

So, we will have a profile something like this. Notice that the heat flux at the wall would dictate the temperature gradient at the wall Dt by Dr because k into Dt by Dr would be equal to heat flux in magnitude by Fourier's law. So after some distance we will have a flat profile in the center and rising to the wall this would now be wall temperature at this location. As we progress, more and more lamina will get heated and we will have a temperature profile like this.

The gradient at the wall will remain the same; the center position portion which would be unaffected more or less would remain flat but that would narrow down; T wall would naturally be higher now. You will reach a stage where perhaps the penetration of the wall effect is through and through. The wall temperature here would still be higher and after that the gradient at the wall will remain the same but even the center temperature would go on increasing. So we notice that the wall temperature increases along the length in the direction of flow.

Again let us sketch the fluid temperature profile, the wall temperature profile and the heat transfer coefficient qualitatively. Now notice that we have a constant heat flux from the wall so over a given length a fixed amount of heat is being added and by using the first law you will be able to determine that over fixed lengths, the temperature would rise by fixed amount or in mathematical terms, the fluid, mean fluid temperature would increase linearly from the inlet along the length of the tube.

So, let me plot that; let me emphasize it is linear by strictly drawing it linear. So this is T_{fi} and this is T_f as a function of z. What happens to the wall temperature? Notice that initially the wall temperature would be near the fluid inlet temperature so the wall temperature would also start from here, it would increase and then after sometime it would keep on increasing but parallel to this and after some distance it will become parallel to this. The gradient at the wall is fixed; even the temperature difference between the wall temperature and the mean fluid temperature would be constant.

You will notice that the wall temperature goes on increasing faster than the fluid temperature up to a certain distance, beyond that the temperature difference between the wall and the fluid will remain constant. This is the temperature difference this is T wall minus T fluid. What happens to the heat transfer coefficient? The heat flux is constant so the heat transfer coefficient is inversely proportional to this temperature difference between the wall and the fluid.

In this zone where the temperature difference doesn’t change the heat transfer coefficient plotted as a function z would be a uniform value whereas here it will start decreasing from very high values because here the temperature difference is pretty small but the heat flux is uniform. So you start with high values of heat transfer coefficient; as the fluid continues to flow it reduces and then remains constant. Again the zone here is the thermal entry length and the zone from this point onwards is the zone of fully developed temperature profile.

Now we have qualitatively mentioned a fully developed temperature profile in this case as well as in the earlier constant wall temperature case. It is time for us now to look at a quantitative definition of the fully developed temperature profile.

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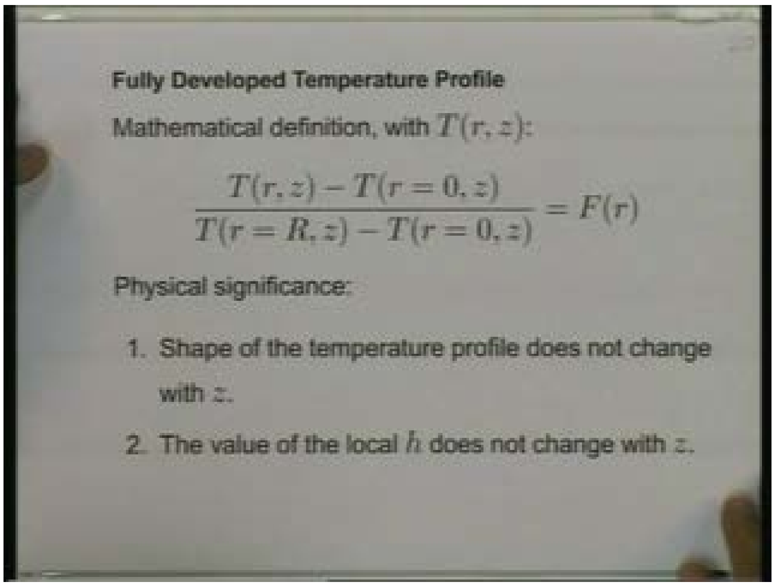

This is a mathematical definition and what it says is this – take a ratio of temperature differences 1 at any location z; take the temperature at the wall, subtract from that the temperature at the center that is in the denominator. In the numerator, you have temperature at some radius r, subtract from that the temperature at the center. So here we have the temperature at some r measured with respect to the center temperature. Here is the wall temperature measured with respect to the center temperature. You take the ratio of these 2 since this temperature is a function of r as well of that of z. In general, this could be a function of both r as well as z however we say that if this ratio happens to be a function only of r, then we have a fully developed temperature profile.

What is the physical significance of the fully developed temperature profile? We know that in case of the constant heat flux case, the temperature profile maintains its shape but keeps on shifting to higher and higher values. In the case of the uniform wall temperature case, we have a temperature profile which sort of asymptotically approaches the flat temperature profile but when it is fully developed the physical significance is that the shape of the temperature profile - the qualitative shape - does not change along the flow length that is along z. And the second one is what we have already seen is that the value of the local heat transfer coefficient also does not change with z; this is more important for us.

Now with this background, let us solve the governing equations of force convection for the case of a fully developed temperature profile, fully developed velocity profile and laminar flow. Now, if we have a fully developed velocity profile in laminar flow, we know what the velocity profile is; that is a standard problem all of us have solved in fluid mechanics.

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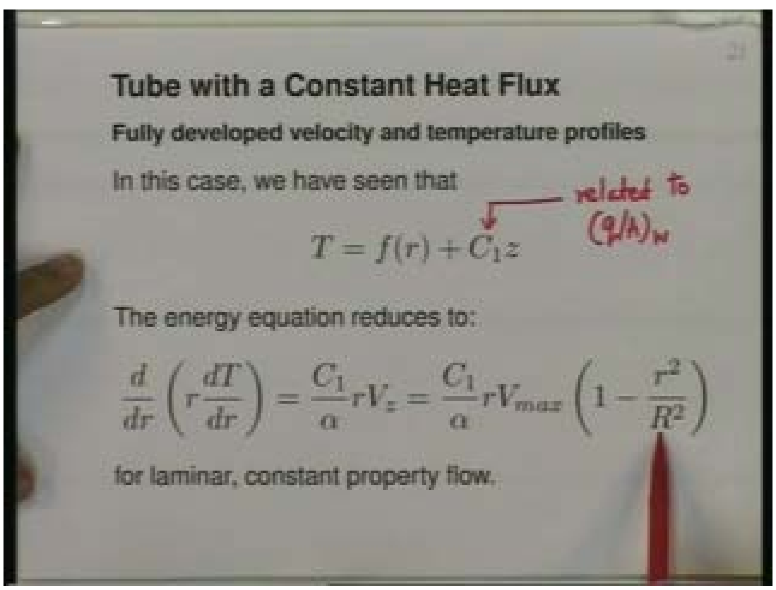

Now we have constant heat flux and this indicates that the mean temperature of the fluid will go on increasing linearly with z which is the distance along the flow. So we can say that the temperature in the fully developed case would be some constant into z plus some function of r and we can even determine this constant; this is related to the heat flux on the wall, also to the mass flow rate of the fluid and it is C_P.

Using this that it varies linearly with z and has a component which is an additive component which is a function of r, the energy equation reduces to this equation. This C_1 exists here and the V_z component of the velocity also enters the equation. And since we know from our fluid mechanic study that V_z in case of fully developed laminar flow is V_{max} into 1 minus r squared by r square, I think this is an equation all of us know. We have an equation, a differential equation for temperature in terms of r containing the radius constant C_1 which is related to the heat flux alpha and the radius also, the radius of the tube. This is a second order ordinary differential equation so we will need 2 boundary conditions. The 2 boundary conditions for this tube with a constant heat flux r as follows.

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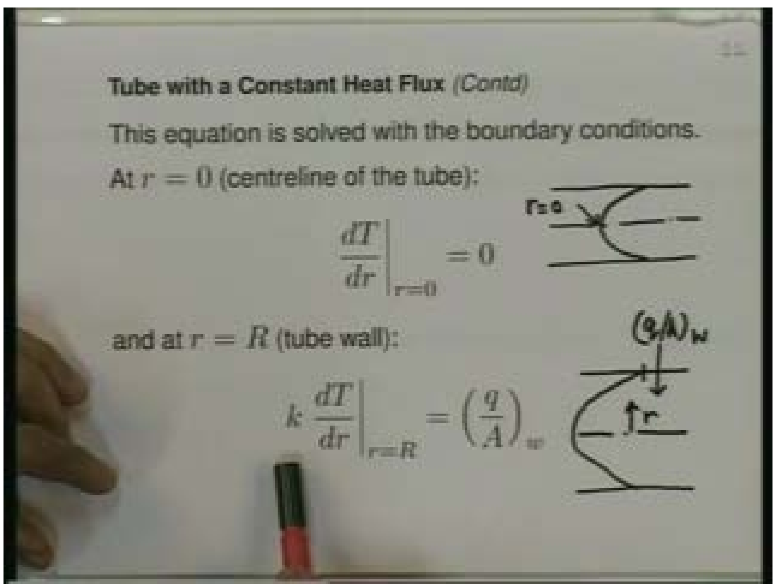

One is at the center length; we have seen that whenever the flow is fully developed or even otherwise the temperature profile is symmetric with respect to r equal to 0 and we use that fact to create a boundary condition at r equal to 0. That is the centerline of the tube which is dT by dr at r equal to 0 if 0. So the temperature profile at the center is locally flat that is what it means.

The Fourier's law gives us the condition for the temperature profile at the wall. If this is the temperature profile at the wall, there is a heat flux; specify q by A_w - it is in the negative r direction. So by Fourier's law, conductivity into temperature gradient at the wall would equal the heat flux from the wall into the fluid because of the heat flux in the negative r direction; the negative sign in the Fourier's law is taken care off. So this gives us the temperature profile, sorry, the boundary condition at the wall.

So, we have a second order ordinary differential equation and 2 boundary conditions - one at r equal to 0 and one at r equal to r - and these two we can use to obtain a solution. The general solution is - I will close this for the time being – so that this solution is clear.

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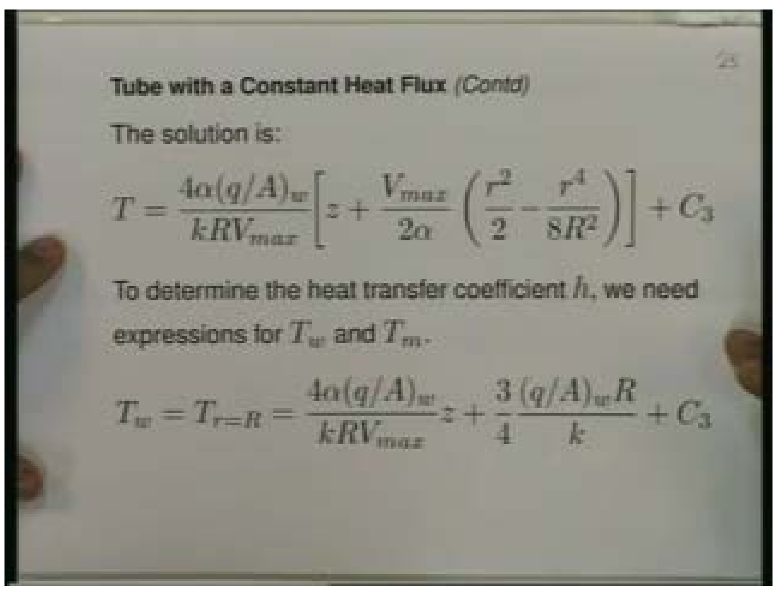

Notice that it depends on the heat flux; there is a term which says part of the temperature is linearly increasing with z. So this is the C_1 into z term, this is the effect of the radius r so this is that f of r which we talked about earlier. You will notice that since alpha equals k by rho C_P the term containing z does not really depend on k because alpha also contains a k in it. So this term will be independent of the conductivity but it will depend on C_P; it will depend on V_{max} whereas the second term which is a function of r does not depend on V_{max}. This V_{max} cancels out, alpha also cancels out so it does not depend on V_{max}. So it does not depend on the low rate, it does not depend on alpha, so it does not depend on rho or C_P but it will depend on k. So we have one term which is proportional to z, which depends on V_{max} and CP and we have another term which is a function only of r which does not depend on the rho CP V_{max} it depends only on k.

Now this is a general solution for temperature as a function of z and as a function of r but what we are really interested in is to determine the heat transfer coefficient and the heat transfer coefficient is the heat flux at the wall divided by the temperature difference between wall and the mean temperature of the fluid. So we have now determined T_w and T_m from this expression; now naturally T_w would be a function of z. T_m would be function of z but in the fully developed temperature profile case we have seen that we expect T_w minus T_m to be independent of z.

The wall temperature can be obtained by substituting for the radius small r the value of the radius of the tube, inner radius of the tube. So you put small r equal to capital R and you get temperature at the wall; if you do that you will get the temperature at the wall in this form. Again you will notice there is a term containing z and that is a term containing capital R; small r does not have a place here because we are evaluating this at small r equal to capital R. So this is the variation with z and this is the local variation with respect to r.

Now, we have to determine the mean temperature of the flow rate and here we have to be careful. The mean temperature is determined using this definition and let us look at what this definition means.

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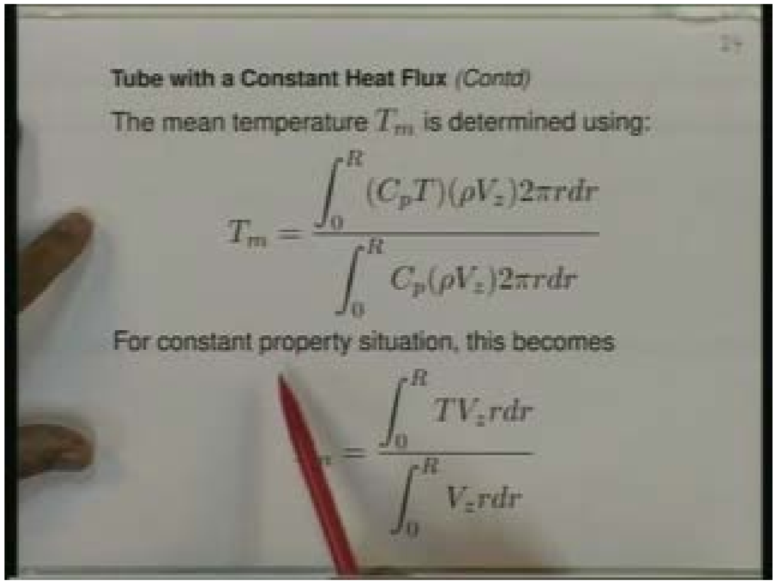

Notice in the numerator - the numerator is CpT which is a representation of the enthalpy into rho V_z into pi; rdr is a area element. So rho V_z into pi rdr is the mass flow rate flowing through a small radial annulus at r width dr. So this is something like m dot into enthalpy so this is the actual enthalpy outflow at that particular cross section whereas if you multiply the denominator with the left hand side it indicates now that T_m is the mean temperature at which we can say that the enthalpy outflow occurs. So m dot into Cp into T_m would represent the actual enthalpy outflow because this is the actual enthalpy outflow and this is the mean temperature which represents that.

Now, this is the expression to explain what a mean temperature is; we have a constant property situation and in this situation this simplifies. C_p is constant so it comes out of the integral sign in the numerator and denominator gets canceled out; density is also assumed constant so that also gets canceled out, also 2 pi is a common factor in the numerator and denominator. So the mean temperature - the final expression - turns out to be integral from 0 to capital R T V_z rdr divided by integral 0 2 capital R V_z rdr.

Now, we have for this temperature which is a function of r and V_z which is a function of r expressions; V_z is nothing but V_{max} into 1 minus r square by r square and for the temperature, we substitute this equation where the temperature is again a function of r. When you do that you will get an expression for T_m which looks like this.

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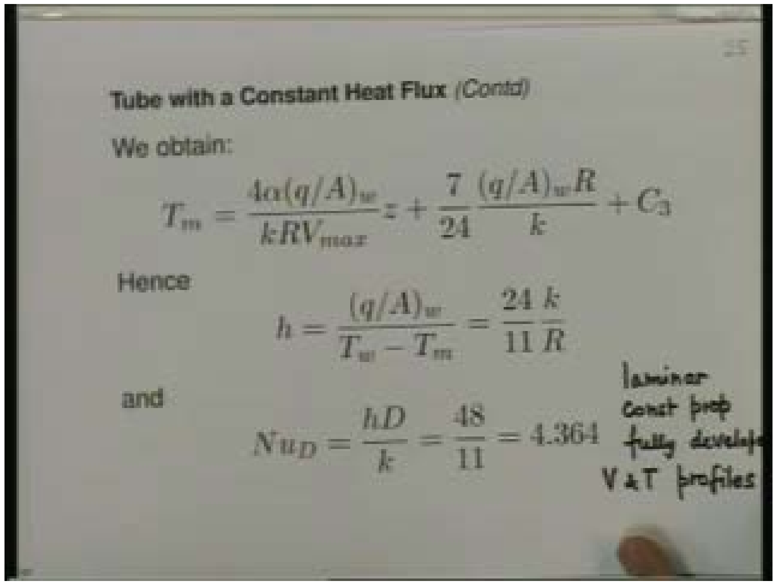

In characteristics, it is very similar to the expression for the wall temperature. Let me get the expression for the wall temperature; expression for the wall temperature is here. So this is the wall temperature expression here; this is the mean temperature expression.

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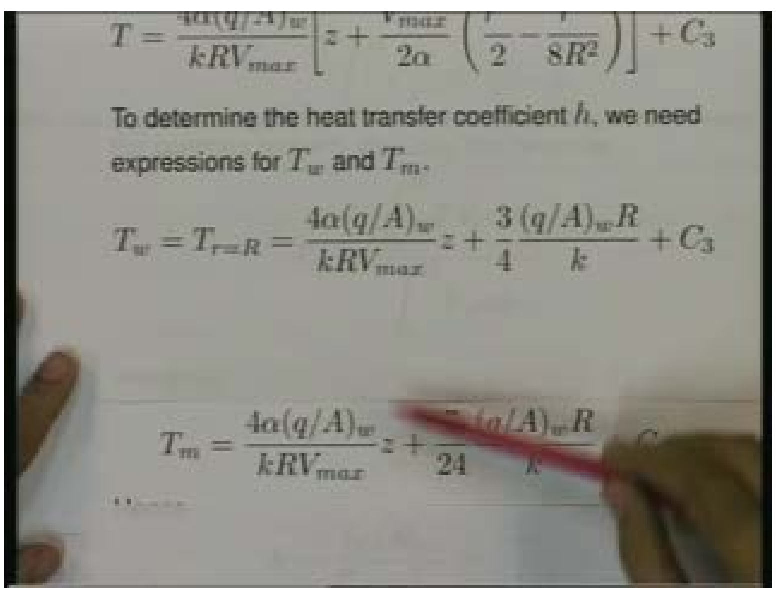

Notice that the first term which contains z is the same because this increases linearly with z. This also increases linearly with z and slope is the same; the coefficient of z would be the slow rate of rise of temperature. This C_3 also is a common term, the difference exists only in these 2 terms so when you subtract from the wall temperature the mean temperature we will get an expression which we substitute in the heat transfer coefficient definition to get the heat transfer which is by definition the heat flux at the wall divided by wall temperature minus the mean temperature to be equal to 24 k by R.

Notice that there is no effect of z; C_3 also drops out from T_w minus T_n, there is a common factor q by A at the wall. So, all that remains is some number and conductivity and the radius. We have said earlier that we would like to express the heat transfer coefficient in the form of the dimensionless number and hence if we calculate the Nusselt number in this case which is h into diameter. For Nusselt number, the typical characteristic length used is the diameter divided by k so Nusselt number based on diameter equals hD by k - that turns out to be 48 by 11 which numerically is 4.364.

So, if you have a circular tube with constant heat flux the fluid is assumed to have uniform properties, the flow is laminar and when the temperature profile as well as the velocity profile is fully developed the Nusselt number is 4.364. Whenever you remember this, you should also remember the conditions under which it is valid. The conditions for this are laminar flow, constant properties and fully developed V and T profiles. This is where the problem which we started to solve - determine the Nusselt number for fully developed flow, laminar, uniform heat flux – ends.

Let us look at the constant wall temperature case; in this case an analytical solution can be obtained but the algebra in calculus is not as simple and straightforward as that for a constant wall temperature case. If you look up some advanced on convective heat transfer you will find this derivation there. All that we need to note is the result and notice that the result is similar to that for constant wall temperature case, constant heat flux case. There the Nusselt number was 4.364; now we have a lower Nusselt number 3.657.

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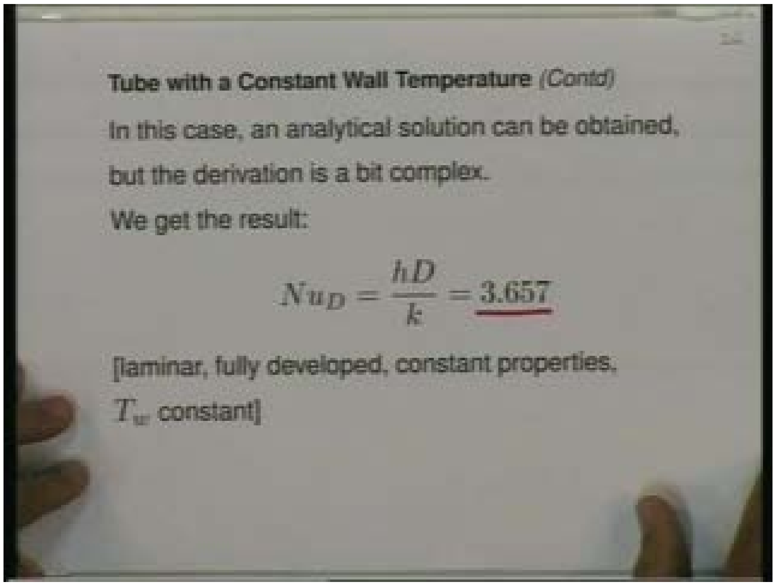

Again notice that this is for a circular tube in which the flow is laminar. Fully developed fluid can be assumed to have constant properties and the wall temperature is constant. By fully developed we mean both fully developed velocity profile as well as fully developed temperature profile.

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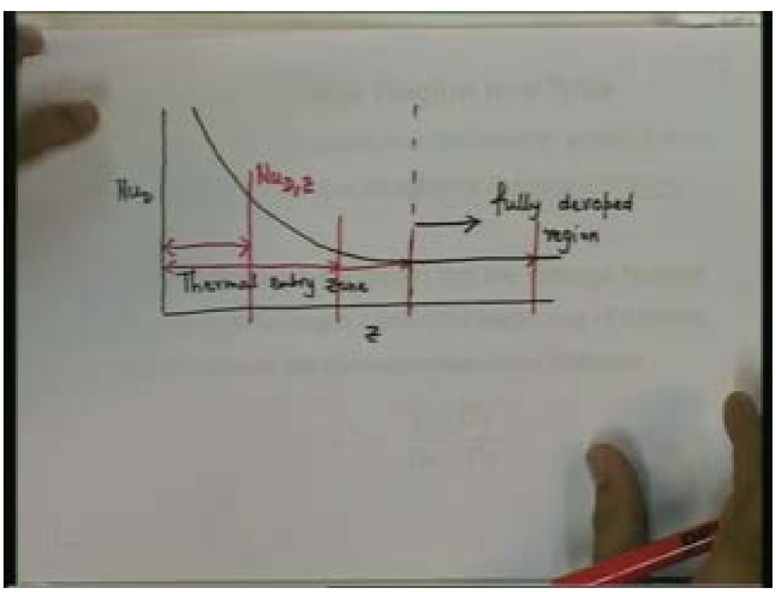

Now let us go back; we said that when we had a developing temperature profile if you plot the Nusselt number which is proportional to the heat transfer coefficient along z, you will see there is a thermal entry length in which the heat transfer coefficient will reduce and beyond that it will essentially remain constant. We have obtained solution in this zone; what happens in this zone in the zone of thermal entry zone.

Let us look at the solutions which have been obtained in thermal entrance region; it is assumed that the velocity profile is fully developed so before the heating begins either at a uniform heat flux or at a uniform wall temperature, the fluid has travelled through the tube for some distance and the velocity profile is already fully developed. Then the temperature profile starts developing so we have a situation of thermal entrance zone and in this case it has been shown that the average Nusselt number over length z.

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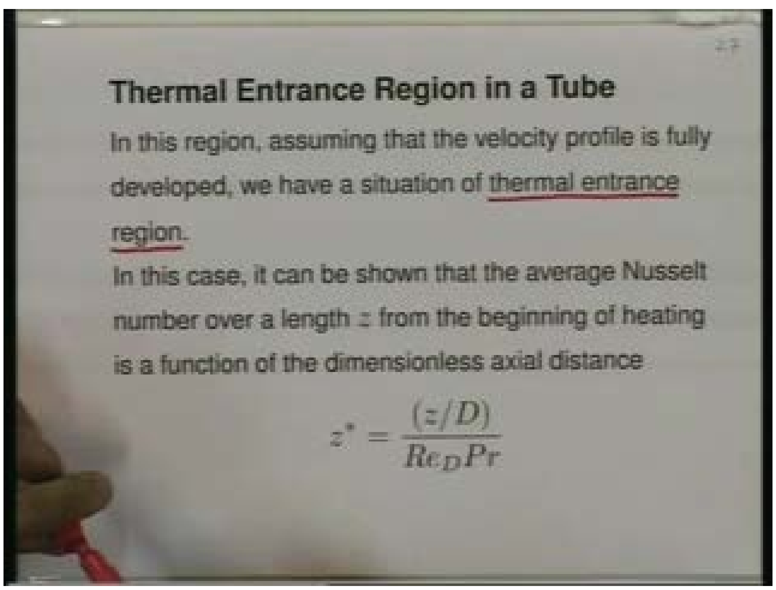

See if this is the local Nusselt number, what we want to know is - say over this length - what is the average Nusselt number? Then over this length, what is the average Nusselt number? The average Nusselt number will go on changing till the end of the thermal entry zone. It will go on changing for sometime in this region but beyond that the effect of the thermal entry length will be too small for us to worry about this zone of higher Nusselt numbers.

So, it turns out that both the average Nusselt number over a length z from the beginning of heating or even the local Nusselt number at a given z is a function of the dimensionless axial distance. See, in the fully developed zone, we notice that it is not function of anything; it is a constant but in the thermal entry region it is a function of the distance but the distance as represented by a dimensionless axial distance called z star. And that is nothing but z divided by D which itself is a dimensionless number being a ratio of 2 lengths divided by Reynolds number based on diameter and Prandtl number.

Let us look at the solutions which can be put in terms of this correlations; we have correlations for the constant wall heat flux case and then we have correlations for the constant wall temperature case.

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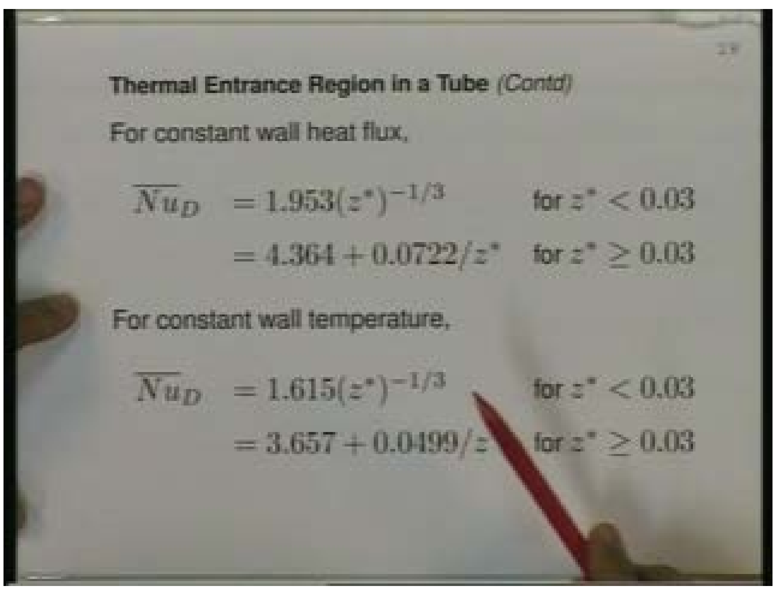

There are 2 zones of z where we have 2 different forms of the correlation but you should notice the following. The average Nusselt number based on diameter over the length z from the beginning of thermal entry to some location z is a function only of z star which is z divided by D divided by Reynolds number into Prandtl number. Whichever thing you look for - z less than point z star, less than .3 or greater than .03 - the average Nusselt number keeps on decreasing with z star. So as you go along the length, the average Nusselt number will go down and down but it will not reduce to 0.

Notice that as z star becomes larger and larger this Nusselt number for constant wall heat flux merges or approaches the fully developed value of the Nusselt number. The value of the Nusselt number for the fully developed temperature profile in case of constant wall temperature again we have a similar set of results. Here also we will find that the Nusselt number keeps on reducing with z star.

Let us say common characteristic again as z star becomes larger and larger; the Nusselt number - the average Nusselt number, average Nusselt number - over that large length will keep on approaching or will asymptotically approach the average Nusselt number the local Nusselt number for the fully developed temperature profile under that constant wall temperature case. You will notice that for a given a value of z star, that is at for a given situation, for a given length, the average Nusselt number and hence the average heat transfer coefficient which is obtained in case of a constant wall heat flux situation will always be higher than that obtained in a constant wall temperature case.

After sometime we will solve an illustrative example based on these correlations. Now apart from these correlations which give you average Nusselt number over a thermal entry length.

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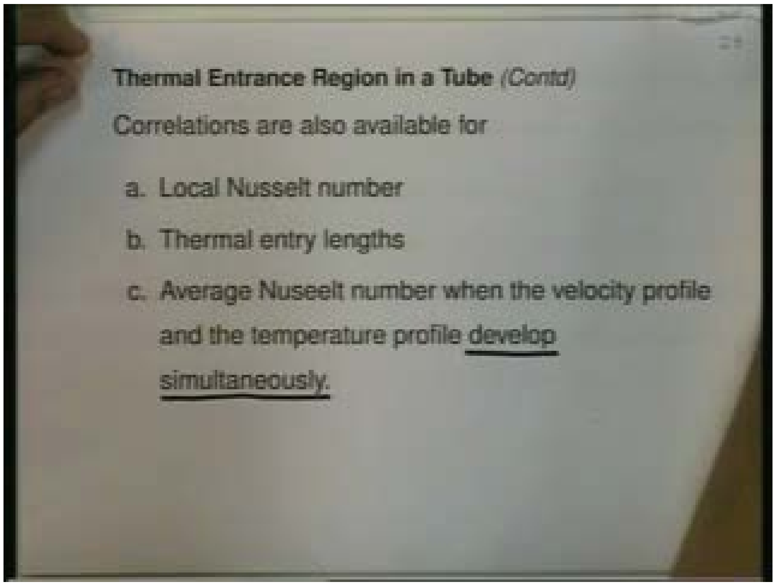

There are textbooks and handbooks and compilations which provide correlations for the following. The local average Nusselt number - we have seen that; we looked for the average Nusselt number over z but a correlation for the local Nusselt number as function of z are also available. Then we notice that after certain distance, the local Nusselt number does not really change with z star. So this is the value of z star at which the thermal entry zone ends and we have correlations available for thermal entry lengths also. Now these correlations which we looked at are for a situation where the velocity profile is fully developed but the temperature profile is developing and we said that this means that before heating begins - heating of any kind begins - there is a length of tube through which the fluid has been flowing. So the velocity profile is fully developed but it is also possible that the fluid enters a tube with a flat velocity profile say from a stagnation chamber and almost immediately starts getting heated so in that case we have a developing temperature profile simultaneously with a developing velocity profile. So this is a situation where we have simultaneous development of temperature profile and velocity profile; so both of them develop simultaneously and correlations are available slightly more complex in books than the average correlation average Nusselt number correlations but we will find these correlations in textbooks.

For design of heat exchange equipment usually we will need the average Nusselt numbers. That is why I have exposed you to the average Nusselt number correlation; if you really want to study local effects you must look at correlations for local heat transfer coefficients. Now it is time to solve some problems but before solving problems remember that as preliminary.

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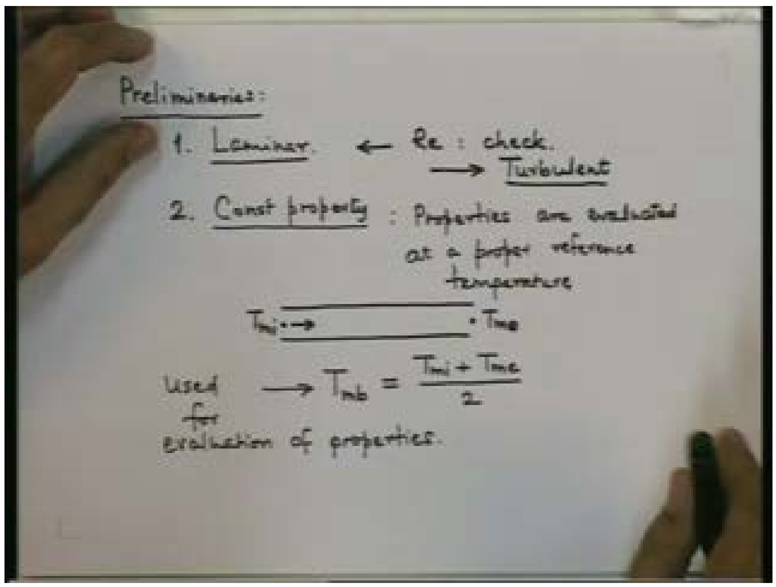

We have to do the following. We have looked at correlation for laminar flow; later we will look at correlations for turbulent flow. So we have to check using Reynolds number whether the flow is laminar or not; if it is not laminar it will be turbulent and a different set of correlations will have to be used. So this laminar turbulent thing we will have to resolve. Second - actual flow it is like water and air have their properties which are may be minor, may be major functions of temperature. So our constant property data or constant property correlations are not exactly applicable but they work very well when properties are evaluated at a proper reference temperature.

For example, if there is a fluid flowing in a tube; T_{mi} is the mean temperature at the inlet, T_{me} is the mean temperature at the exit. Quite often the properties are evaluated at the mean bulk temperature which is the arithmetic average of inlet and exit temperature. This temperature is used for evaluation of properties; this is what is usually done. However, the authors of the correlation may specify a different temperature for property evaluation. It may be a modification of this or emetic mean bulk temperature, wall temperature, take the average of that.

So, whenever you look at a correlation in a book or handbook, read around. Find out the conditions under which it is applicable and the mean temperature at which properties are to be evaluated. We stop here today. In the next lecture, before going to be turbulent flow, we will solve some problems using the information and correlations which we have studied so far.