Heat and Mass Transfer
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Lecture No. 18
Forced Convection-1

Welcome. We now begin our study of forced convection which is the next topic. Perhaps, as you already know, forced convection is the presence of fluid flow along with heat transfer and this fluid flow is caused by some external agency like a pump or blower. Typical situations for forced convection would be flow through a duct or a set of ducts when there is a blower or a pump.

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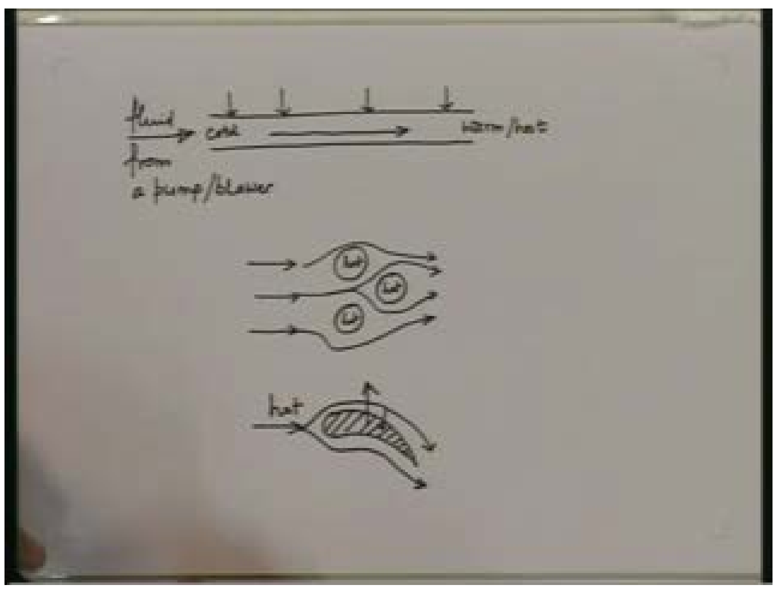

A cold fluid may be entering the duct, the duct is heated either because it is exposed to a hot fluid on the other side, may be to a flame or may be an electric heater is wound on it. It picks up heat and goes out as warm or hot fluid; this is a typical, very common forced convection situation. Another situation is one in which you have a number of tubes as in a heat exchanger. I am showing a cross section and fluid is forced to flow over them. Tubes contain hot fluid so their surface is at a higher temperature than that of the fluid;

the fluid passes over and through the tube banks becoming warmer - this is another forced convection situation.

A third forced convection situation not much different from this is a gas turbine blade, hot fluid from the combustion chamber flows around the blade. The blade may be stationary if it is a straighter blade, the blade may be rotating if it is a rotator blade and these hot gases expose this blade material to very high temperature and there is a heat transfer from the head hot gases to the blade material. There are many other situations but these are the 3 typical situations which are very common. What will we study in this set of lectures on forced convection? This is a typical list of topics.

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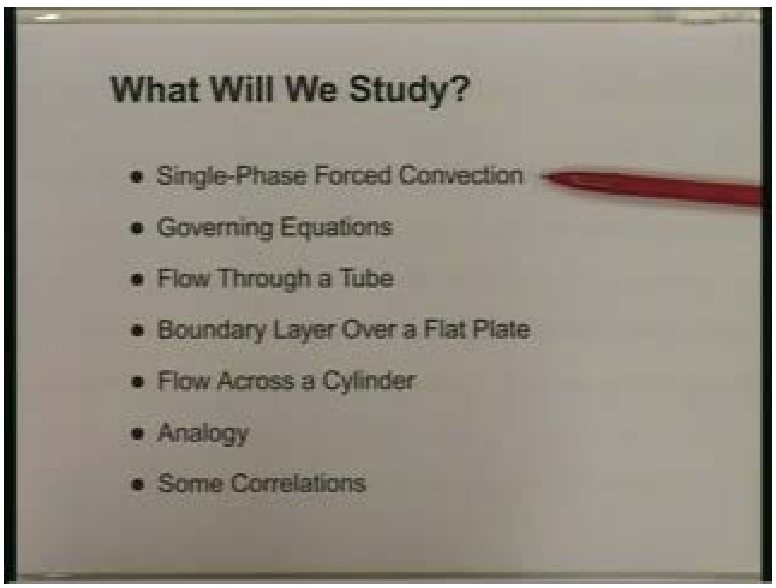

First we will restrict ourselves in this set of lectures to single phase forced convection; that means we are not considering situations where liquid evaporates or boils in to a vapor nor a situation where a vapor will condense in to a liquid nor a situation where a liquid will freeze and form solid particles like water freezing in to ice these are 2 phase situations and we will keep away from them at least for the time being.

After some initial discussion, we will look at some basic governing equations for forced convection. We will look at their rather simplified forms because in the general case the governing equations for forced convection tend to be very complicated. Then we will look at 3 typical, very common situations; one is flow through a tube essentially a case of internal flow, then boundary layer over a flat plate - a basic case of external flow - and flow across a cylinder which also is a basic case of external flow. Whenever appropriate we will be looking at the analogy principles, analogy between fluid flow and heat transfer and we will make ourselves conversant with some correlations for these situations and may be for some slightly different situations. Now we will look at the basic situation in forced convection.

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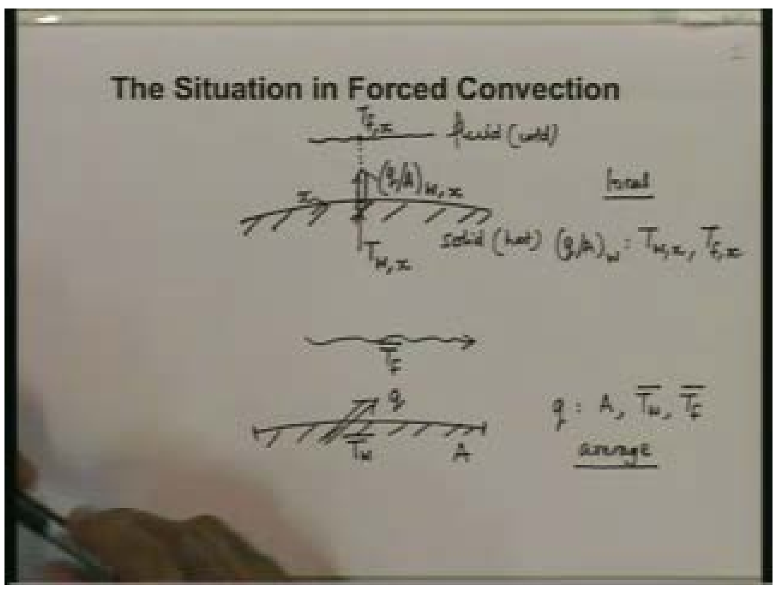

In forced convection, invariably we have a surface which is exposed to a fluid - may be liquid or gas. For the sake of illustration, let us assume that the surface is hot and the fluid is cold compared to each other. Let us take a point on the surface; let us say that the temperature at that point on the wall of the solid is T_w. It would vary from location to location if this is the position on the wall, it could be a function of x. Similarly the fluid flowing across this at a reasonable distance away from the solid so as not to directly feel the presence of the hot solid let me a temperature T_f.

The fluid will be getting heated up so this temperature could also vary along x because the fluid is cooler than the solid, there will be a flow of heat from the solid to the fluid by convection. Now let me say that the heat flux at that point is q by A at the wall and again this may be varying from one point to another point so let me say this is q by A wall at that particularly x. Our aim in forced convection is to determine a relation between the heat flux from the wall to the fluid and related to the temperature of the wall at that point and temperature of the fluid at that point; this is a local situation.

We may not be interested in local variation of temperature, local variation of heat flux so may be in a gross way in an way let us say that I have a surface. Let us say the surface area is A, the mean temperature of the surface is T_w bar. There is fluid flowing, the mean temperature of the fluid which is exposed to the surface is T_f bar and let us say that the flow of heat from the solid to the fluid is q over this area A. Here we would like to relate q to the area A, the temperature T_w bar and the temperature T_f bar whereas this is the average case, whereas here we would like to relate heat flux at the wall to temperature of the wall at that point and temperature of the fluid at that point. Naturally this will not be related only to this; it will be related to the geometry, the flow rate of the fluid, the local velocity of the fluid and properties of the fluid.

We have already seen in the introductory lectures that whenever there is a process of convection, a solid exposed to a heat fluid and heat transfer from the solid to the fluid by convection, we define a parameter as the heat transfer coefficient.

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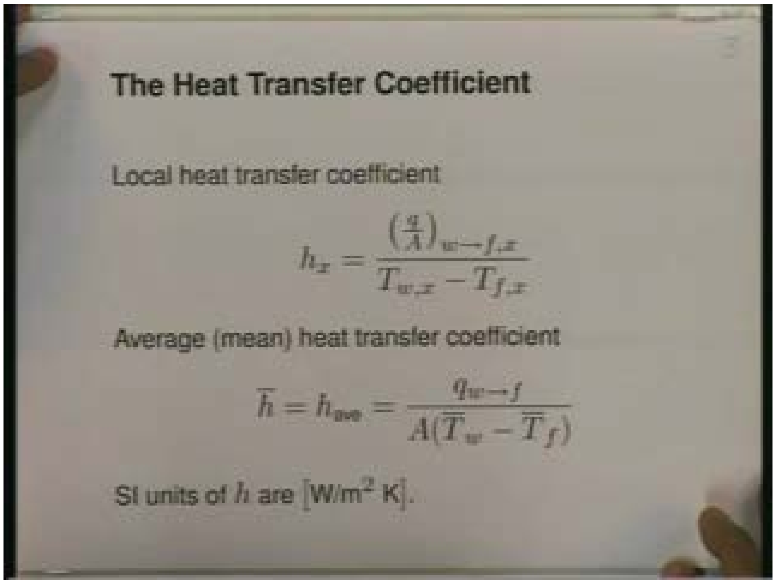

Considering the 2 situations which we looked at just now - the local situation and the average situation we can define the local heat transfer heat coefficient and the average or the mean heat transfer coefficient. The local heat transfer coefficient may vary from 1 position to another so we generally use a subscript x denoting the position for h. h_x is the local heat transfer coefficient which is a ratio of the wall heat flux from the wall to the fluid at that location divided by the temperature of the wall at that location minus temperature of the fluid at that location.

Notice that the heat transfer coefficient is always defined as a positive so if you have heat flux from the wall to the fluid in the denominator, you should have wall temperature minus the fluid temperature. If you use a heat flux from the fluid to the wall which may be convenient if the fluid is warmer than the wall, then you will have heat flux from the wall to the fluid in the numerator and in the denominator you will have the fluid temperature minus the wall temperature. On an average or mean basis, you can determine the average heat transfer coefficient or a mean heat transfer coefficient over an area A.

So, here we have defined the average heat transfer coefficient denoted often as h bar or h_{ave} as the heat flow rate from the wall to the fluid divided by the area. So q wall to fluid divided by area gives us something like the average heat flux; then that thing is divided by the mean temperature of the wall minus the mean temperature of the fluid. We should remember that the SI units for the heat transfer coefficients are Watt per meter squared Kelvin.

Heat transfer coefficients hardly ever go below a few Watt per meter squared Kelvin so we never have any reason for using mill watt or micro watt. Large heat transfer coefficients do occur; a few thousand Watt per meter squared or even a few hundred thousand Watt per meter squared are not uncommon in some situations. So if need be we can use kilowatt per meter squared Kelvin but somehow it turns out that Watt per meter squared Kelvin is the most common unit.

Now, those of and almost all of us are exposed to fluid mechanics and we know that we tend to correlate all our data in terms of dimensionless numbers. So, even the data for heat transfer in forced convection is finally correlated in terms of some dimensionless numbers.

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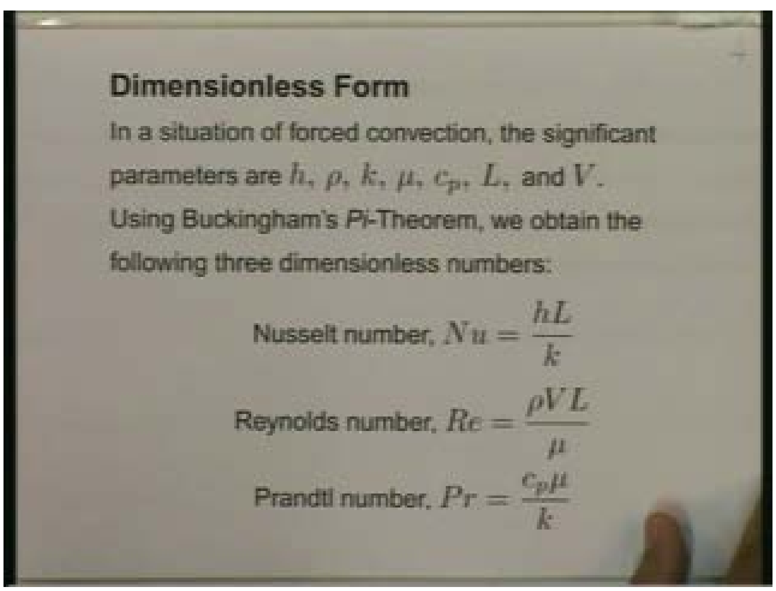

In a situation of forced convection the significant parameters turn out to be the heat transfer coefficient, properties of the fluid namely density, conductivity, viscosity; here I have used dynamic viscosity, specific heat. Some geometric parameters I have just included; 1 L as a reference length and the flow parameter - the velocity of the fluid.
If you use Buckingham's pi theorem, we obtain the following 3 dimensionless numbers. One turns out to be h into L by k, another row VL by mu and the third cp mu by k. This row VL by mu, well, we already know that as Reynolds number so there is nothing new about it. So the Reynolds number represents the flow properties of the fluid, flow situation of the fluid. The Nusselt number hL by k you know similar to the bio-number; we remember in the bio-number, we have the conductivity of the solid; here we have the conductivity of the fluid. This Nusselt number is a dimensionless version of the heat transfer coefficient so any heat transfer coefficient information or data is finally converted in to dimensionless form in the form of the Nusselt number.

The third dimensionless number, the second number specific to convective heat transfer is the Prandtl number and it is interesting to know that the Prandtl number c_p mu by k is a function only of the properties of the fluid - specific heat, dynamic viscosity and thermal conductivity. We already know the significance of the Reynolds number from our fluid dynamic study. Let us look at the significance of the Nusselt number and the Prandtl number.

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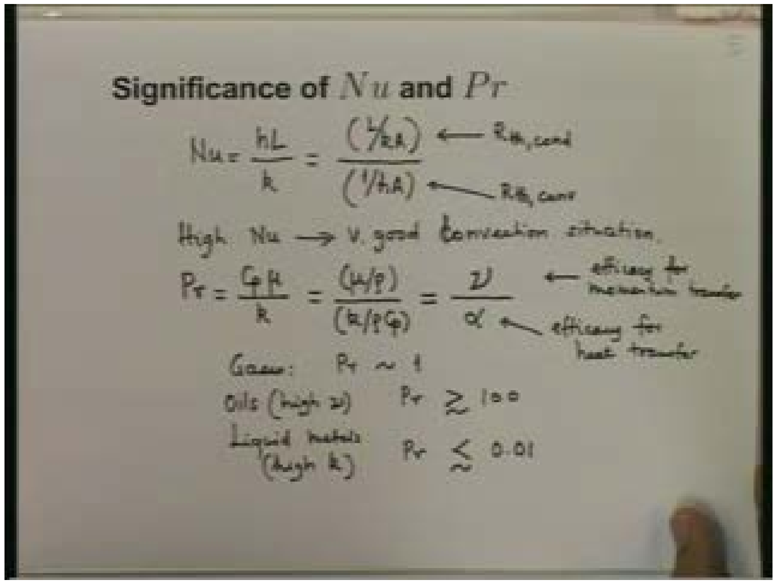

We know that the Nusselt number is hL by k. Let us consider A area to be the area across which heat transfer takes place and we can rewrite this as L by kA in the numerator and 1 over hA in the denominator. Now notice that the numerator represents the thermal resistance for a conduction situation in particular; if we consider a slab of thickness L area A and conductivity k, a slab of fluid in which only conduction takes place of thickness L, this will be the thermal resistance for conduction. This, well, there is a heat transfer coefficient so this is the, for the, in a similar situation the thermal resistance for convection. So the Nusselt number represents the relative importance of the conduction, thermal resistance and the convective thermal resistance.

In a situation where forced convection effects are significant, we expect the convective heat transfer coefficient to be high and hence the resistance to convection R_{th}, convection which goes in to the denominator to be low meaning we will have a high Nusselt number so high Nusselt number represents very good convection situation. So if we have a situation of forced convection, a better situation, a more effective forced convection effects would mean that the Nusselt number is high.

Prandtl number – it is special in the sense that it is a function only of 3 properties and we can rewrite this as mu by row divided by k by row c_p. We know that mu by row is the kinematic viscosity, usual represented - nylon nu and k by row c_p is the thermal diffusivity alpha. Notice that both have units of something like meter square per second area per unit time and what does the numerator represent? A high nu means a reasonably large viscosity. So this represents the efficacy of the fluid for momentum transfer; a viscous fluid will have a high value of nu.

alpha on the other hand, notice that larger the conductivity means larger the alpha; heat will be transferred faster by conduction. Lower the c_p means larger the alpha, lower the c_p means for the same amount of heat gain, the temperature will rise significantly so the alpha represents efficacy of the fluid for heat transfer. So Prandtl number in a sense represents the ease at which a fluid is able to transfer momentum or shear forces to the ease by which the fluid can transfer heat.

Naturally, we expect a fluid which has a high viscosity to have a Prandtl number; a fluid which has a conductivity will have a low Prandtl number. Since the Prandtl number is a property of the fluid and not of the flow situation, one should notice that for a large number of gases Prandtl number is of the order of 1. Air at ambient temperature and pressure as roughly .7, you take oil, say lubricating oil, engine oil of a car. Here we have high viscosity; Prandtl number is of the order of or greater than 100. Prandtl numbers of 300, 500 are not uncommon, particularly for low temperature oil.

If you take mercury, a liquid metal, low viscosity, high conductivity, any other liquid metal as liquid sodium liquid potassium, situations of high conductivity, comparatively low specific heats, comparatively low viscosities - here the Prandtl number is of the order of or less that .01. So you will get a range of Prandtl numbers depending on the fluid and its state, pressure and temperature going from something like .001 at one extreme with liquid metals and something like almost of the order of 1000 for viscous oil. After undertaking this basic discussion on forced convection and the heat transfer coefficient, Nusselt number and Prandtl number, let us start looking at the governing equations for forced convection.

The forced convection situation is characterized by a flow of fluid which already exists; there is a also a heat transfer so conductivity will play a part. Process of conduction will also be involved so the governing equations pertaining to forced convection are the governing equations for fluid flow plus the governing equation for transfer of energy.

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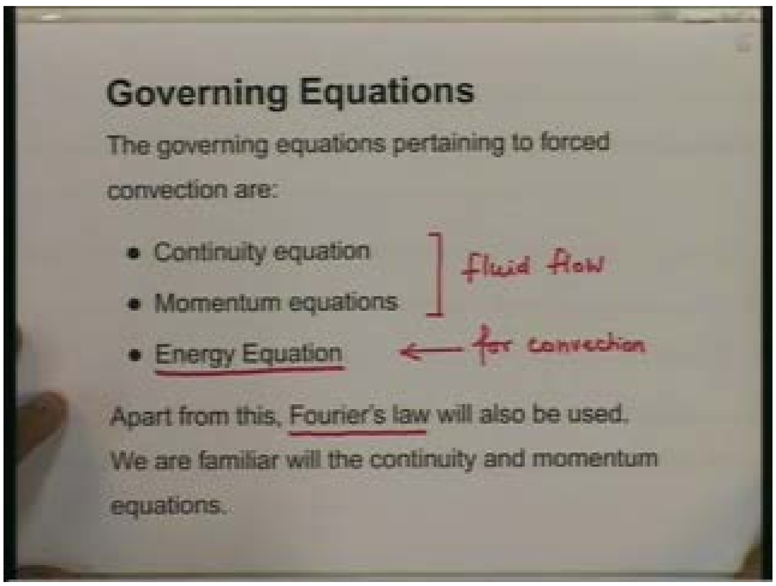

So, we will have definitely the continuity equation and the momentum equation - these 2 equations pertain to fluid flow. But now there is a heat transfer, heat is a form of energy so the energy equation is required for heat transfer for convection and as we have already seen in the study of conduction, the energy equation by itself is not sufficient because it provides relations between heat fluxes. We have to relate heat fluxes to temperatures and temperature gradients by using the Fourier's law. So the equations for fluid flow, we will use continuity equation and momentum equations; the additional equations used are the energy equations, energy equation and the Fourier's law of heat conduction. Since we have studied the continuity equation and the momentum equations already, let us look at the energy equation.

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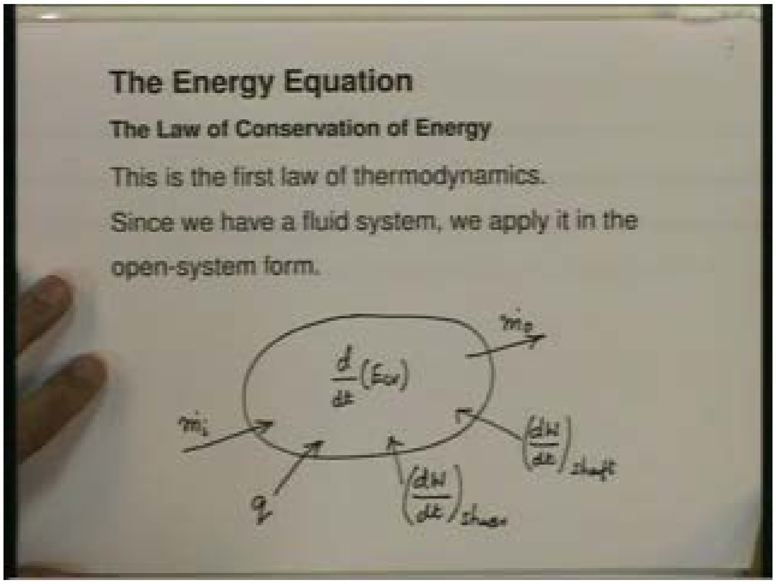

The energy equation represents the law of conservation of energy - the first law of thermodynamics. Now in forced convection, we have a fluid system; something is coming in to any control volume, we consider something is going out of the control volume so we apply it in the open system form. So the general open system would be a control volume; some mass flow rate would be going out, some mass flow rate would be coming in. There will be heat transfer to the system, there will be shear work, there could be shaft work and even the energy of the system could be changing with time. The first law of thermodynamics relates all these quantities to each other. In the introductory part, the first few lectures we have seen this reasonably general form of the first law of thermodynamics for a control volume.

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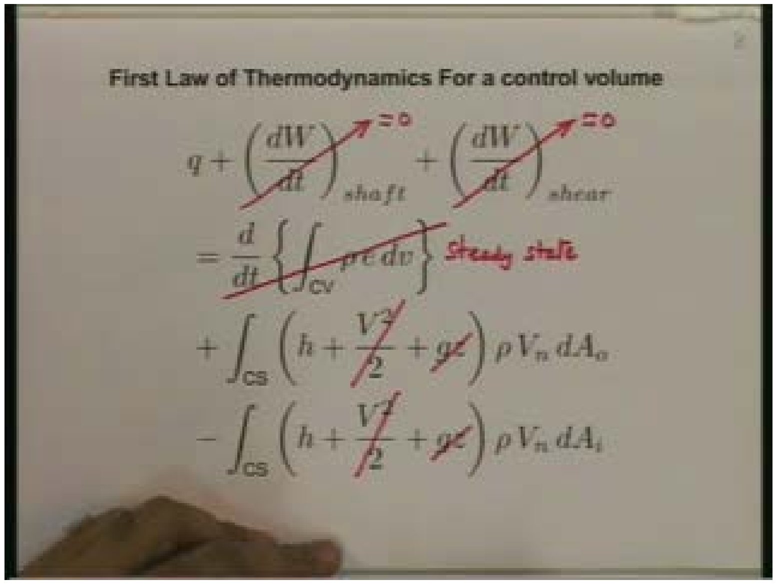

We have heat transfer term, we have the shaft work term, we have the shear work term, we have the energy storage term and this is the net outflow of energy by convection, net inflow of energy by convection and we have the kinetic energy term, the enthalpy term and the potential energy term. In a typical, a general forced convection situation many of this terms would be nonexistent or negligible; we can get rid of them.

For example, we will assume that steady state exists so if you have steady state this term will go away. We will assume that it is a pure heat transfer situation, there is no work being done; this term would be significant. In case of turbines, compressors, pumps it is not significant in case of boilers, condensers, heat exchanger and so on, so this will be 0. We will also neglect shear work in a large number of heat transfer situations; the shear work term doesn’t exists, it is absolutely negligible. It becomes significant only in case of very high speed gas flows high MAC number almost hypersonic flows so this also is negligible. Then we will also assume that the potential energy terms are negligible and the kinetic energy terms are also negligible. So you will notice that the general form of the energy equation will end up with some very gross major simplification and this simplification leads to the simplified form which is directly pertinent to forced convection heat transfer.

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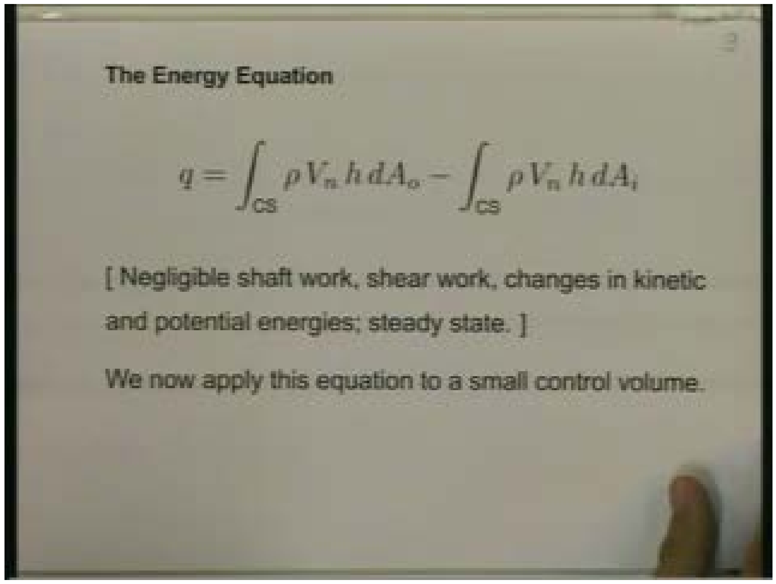

And you will notice that we have the heat transfer to the control volume and on the right hand side we have the total net outflow of enthalpy. This is the outflow of enthalpy from parts of the controlled surface where the fluid goes out; we have denoted it with this subscript o. And this is the inflow of enthalpy over the part of the control surface where the fluid is coming in as denoted by the subscript i. And notice the assumptions - negligible shaft work, negligible shear work, negligible changes in kinetic and potential energy and also steady state.

Now, our aim is to finally derive a differential equation for forced convection just the way we derived earlier a differential equation for conduction. We do that by applying this equation to a small control volume; first we will apply it to a control volume and model the q term, next we will model the enthalpy outflow and inflow terms.

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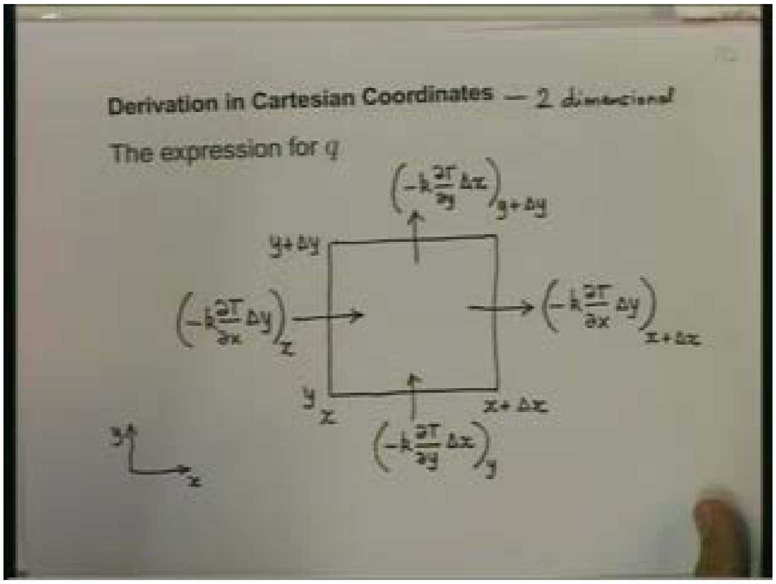

To obtain an expression for q, let us consider Cartesian coordinates and let us also consider a two dimensional situation. Let us say we have, this is the x direction and this is the y direction and let us say in our fluid domain, let me a draw control volume. I am drawing it large so that I can mark out the heat flow rates. Let us say the extent is from x to x plus delta x and from y to y plus delta y; just for convenience you can take even take x minus delta x by 2 to x plus delta x by 2. We will have heat flowing in by the process of conduction from here and here and heat flowing out like this. I am showing heat flow rates in the positive x and positive y direction.

Now, let us use the Fourier's law of conduction to write expression for these 4 heat flow rates by conduction. This heat flow rate would be minus k dT by dx that is gradient in the x direction. We are looking at the flow heat in by conduction in the x direction multiplied by area which will be delta y in to 1. Let us say we are considering volume which is a unit thick, perpendicular to the plane of the paper so this delta y. A similar term will occur here minus k dT by dx into delta y; the difference is this term is evaluated at x location on the x coordinate, this term is evaluated not at x but at x plus delta x.
Similarly this heat flow from the lower edge will be minus k. We are looking at heat flow in the y direction so dT by dy partial multiplied by area which will be delta x in to 1.

Similarly on the other side, we will have a very similar term minus k dT by dy in to delta x into y. Again this term is evaluated at the bottom end of the control volume which is at y and this term is evaluated at the opposite end of the control volume which is y plus delta y. Notice that when we evaluate this term and this term y doesn't change so let us look at x and x plus delta x. Similarly when we evaluate these 2 terms, they are at some fixed value of x, y only changes. We now write these down together as the q term in our first law of thermodynamics.

Notice that the 4 terms shown in the previous figure are all represented here but since q is inflow, these 2 terms will contribute positively.

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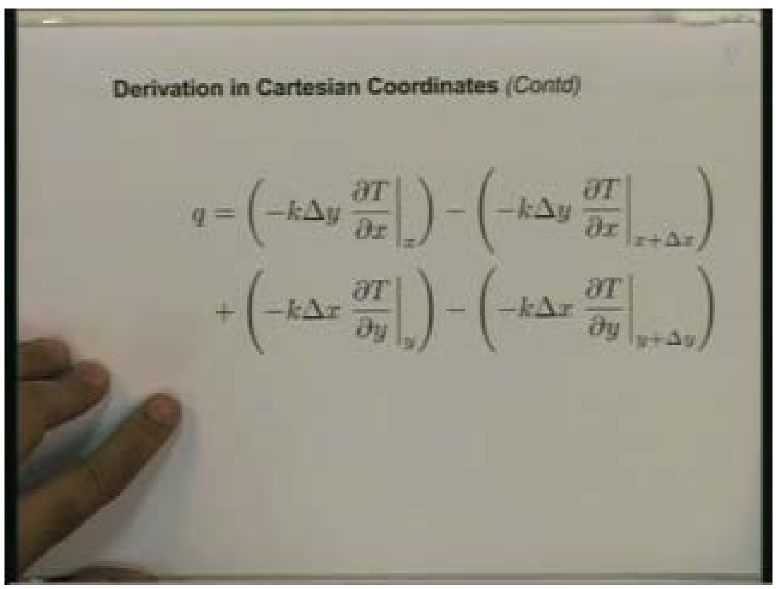

These 2 terms would effectively be subtracted because the arrows move out of the control volume so we have the 2 inflow terms and 2 outflow terms with a negative sign. Now we go back, we have modeled our q term; it is now necessary for us to model the enthalpy outflow terms.

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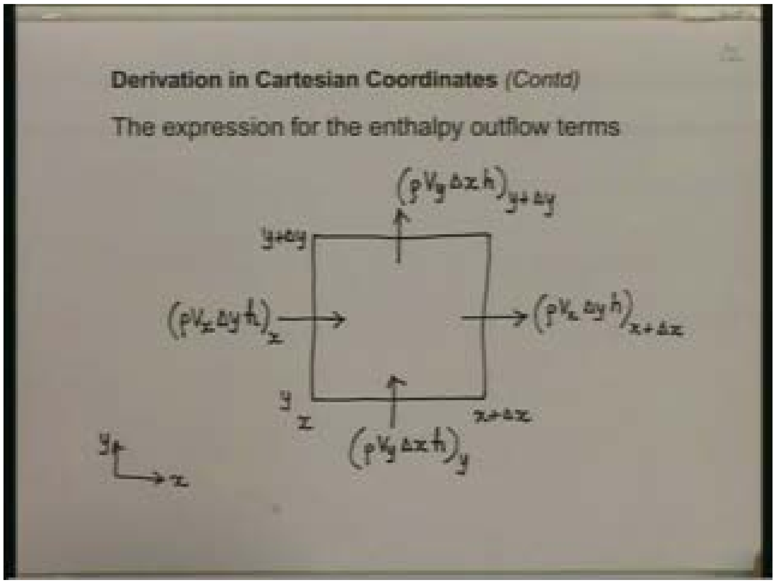

Again to model the enthalpy outflow terms, we have the same coordinate system x and y and let me re-sketch that control volume whose extent was from x to x plus delta x and y to y plus delta y. Now mass flow is in and out and along with the mass, enthalpy will be convicted so there is a mass flow like this, some mass flow is out like this, there is a mass inflow like this, there is a mass outflow like this.

Let us look at this term, the mass flow here would be row into V_x into area which will be delta y in to 1 and the enthalpy which is being carried by that is h. So the enthalpy inflow through this face is row V_x delta y in to h, similarly there will be enthalpy outflow here row V_x delta y into h. The difference this is at locating x at the left edge of the control volume, this is at location x plus delta x which is the right edge of the control volume. Let us look at these 2 enthalpy flows, the mass flow in here will be row into V_y. We are now looking at the flow in the y direction; area will be delta x in to 1. it carriers with it enthalpy h and a similar term here row V_y delta x h. But the difference - this term is evaluated at y the lower edge of the control volume, this term is evaluated at y plus delta y which is the upper edge of the control volume.

We combine these 4 terms - remember what we want is enthalpy outflow; we have 2 outflows here and 2 inflows. So we will have 2 terms contributing additively and 2 terms contributing in a negative way so this is what we get.

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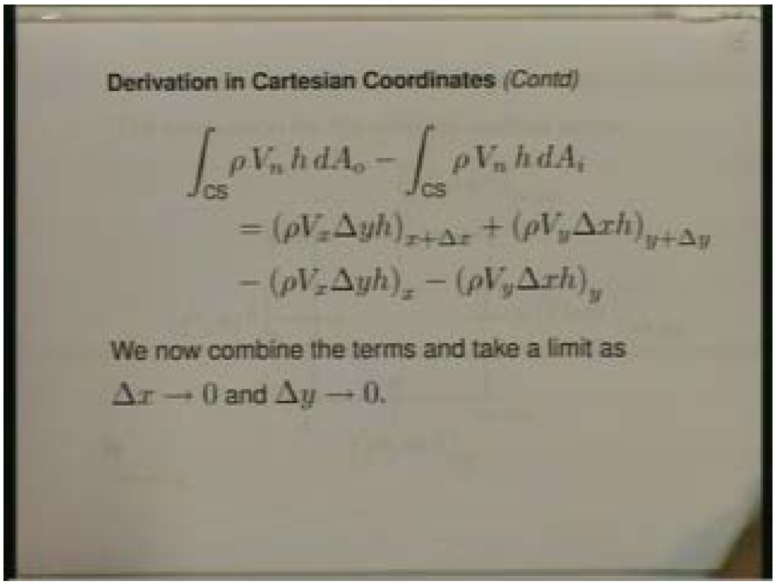

The enthalpy outflow terms, these are the sum of the outflowing streams, this is the sum over incoming streams. We have 2 outflowing streams and 2 incoming streams so these 4 terms together represent these 2 terms. Now we have an expression for the q term, we have an expression for the other 2 terms in the first law of thermodynamics. We will now combine these terms and the resulting equation we will try to convert into a differential equation by taking the limit as delta x tends to 0 and delta y tends to 0. When you do that, this is what you will end up with.

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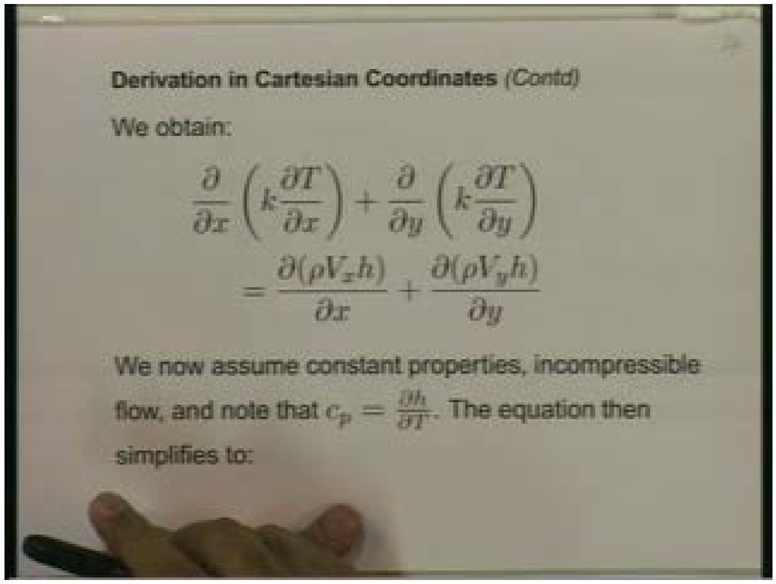

On the left hand side of this equation, we have terms containing conductivity and temperature gradients both in the x direction and y direction; this is the final form of the q term of our first law of thermodynamics. On the right hand side, we have terms containing V_x, V_y and h. These are the enthalpy outflow terms - the final form in the differential equation of the enthalpy outflow terms. You should immediately notice that if V_x and V_y are both 0, there is no fluid flow taking place; then what we have obtained is nothing but the conduction equation, the right hand side becomes 0.

Now although this is a proper energy equation it is not directly useful because we have temperature and we have enthalpy. So we relate enthalpy to temperature by noticing that the derivative of enthalpy with temperature is defined as the property c_p - specific heat at constant pressure. What we also do is we expand these terms and once we expand these terms and assume constant properties and incompressible flow, we will find out that a number of these terms get canceled because they satisfy the continuity equation. So what we do is expand these terms, use the continuity equation for simplification and replace h in terms of c_p and T. When we do that, this is what you will get.

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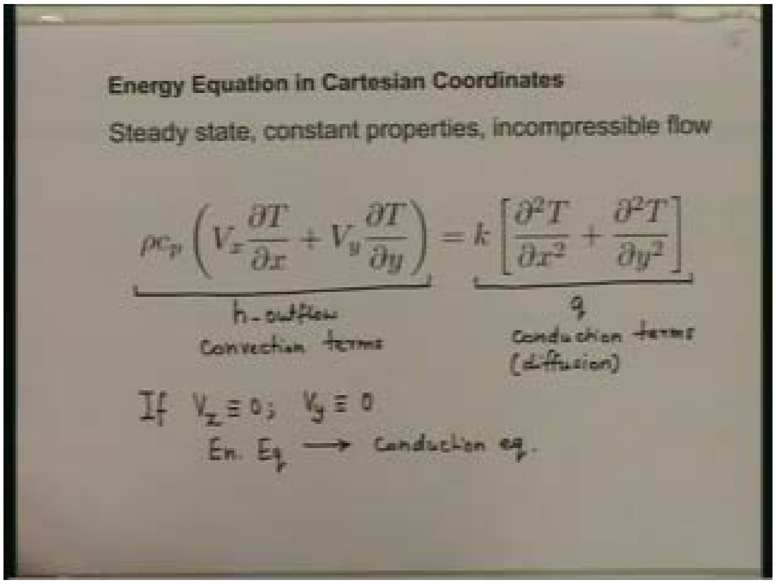

I have written it in a form which you will generally find in text book; so you have the enthalpy outflow terms on the left hand side and the net heat inflow terms on the right hand side. So these are the terms representing enthalpy outflow and these are the terms representing the net heat inflow. Quite often, these are known as the convection terms and these are known as the conduction terms, sometimes also called diffusion terms. Again you will notice that if V_x is 0 everywhere, V_y is 0 everywhere, that this equation energy equation reduces to conduction equation.

A very common situation in forced convection heat transfer is flow through a duct, particularly circular ducts. I think flow through a circular duct is one of the most common, overwhelmingly common forced convection heat transfer coefficient. And the Cartesian coordinate system is not a useful system for, through a circular duct, to analyze flow through a circular duct which we will do slightly later. We convert the energy equation from Cartesian coordinate to cylindrical polar coordinates.

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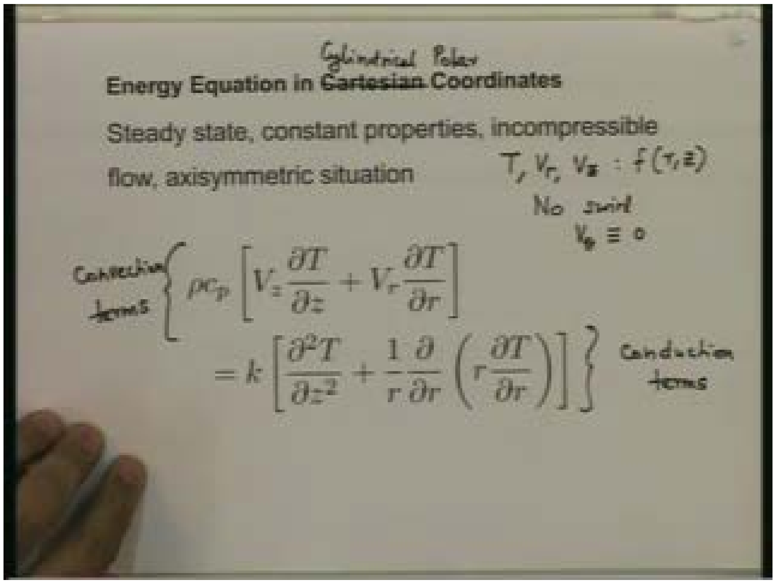

So again, remember that this is a simplified version which is valid for the steady state constant property situation, flow situation and axisymmetric situation. It also assumes that temperature V_r and V_z, these are functions of only the radius and the axial coordinate z and there is no swirl and that means v theta is 0 everywhere. In case of a swirl situation, you will have to consider non-zero V theta and appropriate terms. You will enter not only in the momentum equations and continuity equation but even in the energy equation. Again in this energy equation, notice that on the left hand side here we have the convection terms and here we have the conduction terms.

After this basic derivation of the governing equation, the most important governing equation being the energy equation for situations for forced convection, we now move to analyze and look at 3 typical situations. The first is heat transfer to a fluid flowing in a tube.

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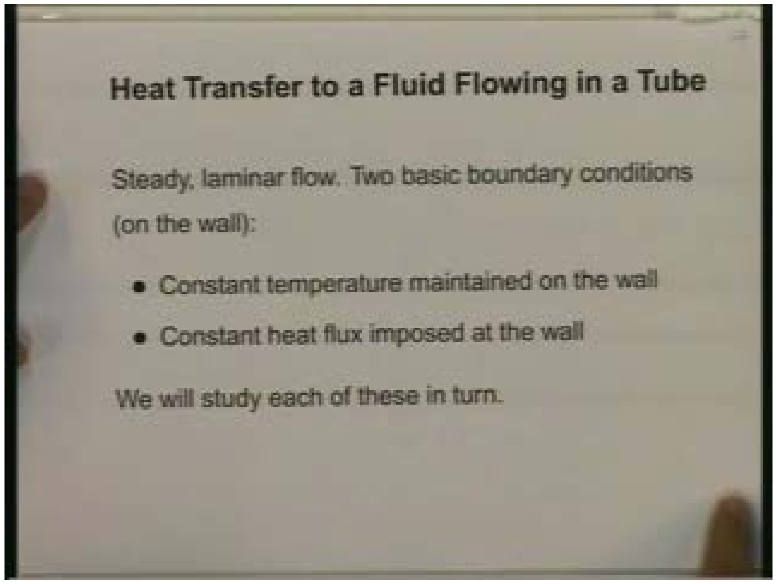

We will consider steady laminar flow and we will consider 2 situations represented by 2 basic boundary considerations - these are thermal boundary conditions on the wall. One is if the wall is maintained at a constant temperature and the other when the wall is exposed to a constant heat flux. This happens and it is a good approximation if the wall is directly heated by resistivity or the wall has a heating coil wound over it. We will study each of these in terms but let me sketch and show you what we mean by the 2 situations.

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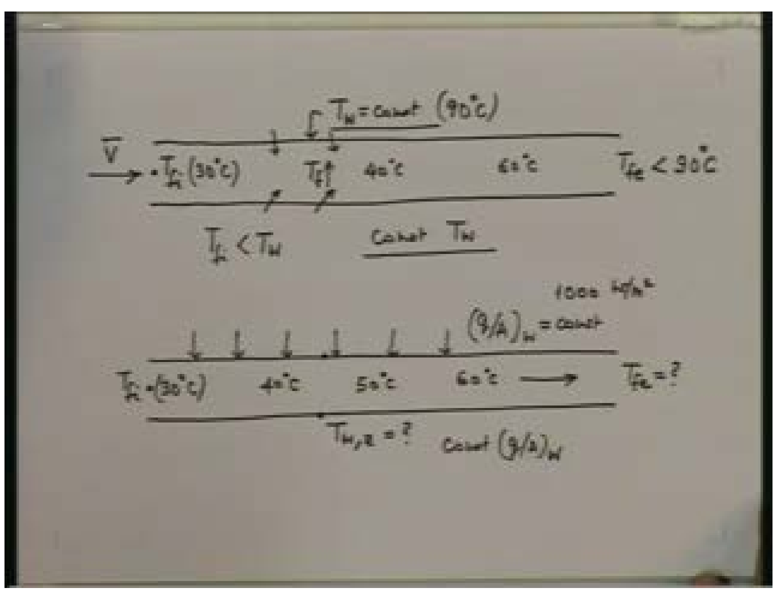

The first one, a constant wall temperature situation the temperature here, will be T wall is constant. This is the inner surface of the tube through which the fluid is flowing and let the inlet temperature of the fluid be T_{fi} and let us assume for the time being T_{fi} is less than T wall, let us consider this case. Let us say T wall is something like 90 degrees C and T_{fi} is something like 30 degrees C. What will happen? The wall is warm, is warmer than the fluid.

So, there will be a heat flow from the wall to the fluid and this T_f will go on rising as the flow proceeds from 30 degrees C it will go to 40 degrees C to may be 60 degrees C but in any case but in any case T_{fe} exit temperature will always be less than 90 degrees C because the fluid will never be able to heat up beyond the wall temperature. This is the situation of constant T wall; the other situation is that of a uniform heat flux on the wall q by A at the wall is maintained at some constant value say 1000 watt per meter square. So, we don't talk about the wall temperature; it is not specified. Let us say the fluid enters at a temperature T_{fi}, let us say 30 degrees C. What happens is it gain heat from the wall at a uniform rate; since we have a steady flow this will go on rising almost uniformly. So, you will end up with 40 degrees C, 50 degrees C, 60 degrees C - it will go on rising. What is the exit temperature? There is no limit; it will have to be calculated based on the area the amount of heat flow. The wall temperature will also need to be calculated; it will vary along z, we will have to determine what it is. Exit temperature will also have to be determined; here temperature will also have to be determined along the fluid flow and the exit temperature but the wall temperature is maintained - this is the constant wall heat flux situation. In this lecture, we stop here; in the next lecture, we will consider in detail first the problem of heating with a constant wall heat flux, then the problem of heating with a constant wall temperature.