Heat and Mass Transfer
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Lecture No. 16
Review of Fluid Mechanics -1

Hello, my name is Gaitonde and it is now my task to continue with the lectures on heat and mass transfer. Professor Sukhatme, the first instructor in these series of lectures and whose lectures you have been watching and listening to all these days, will be back after this mini-series of my lectures. So far we have studied conduction and radiation; it is now time for us to begin the study of convection - both forced convection as well as free convection. Before we study convection, it is necessary for us to review fluid mechanics.

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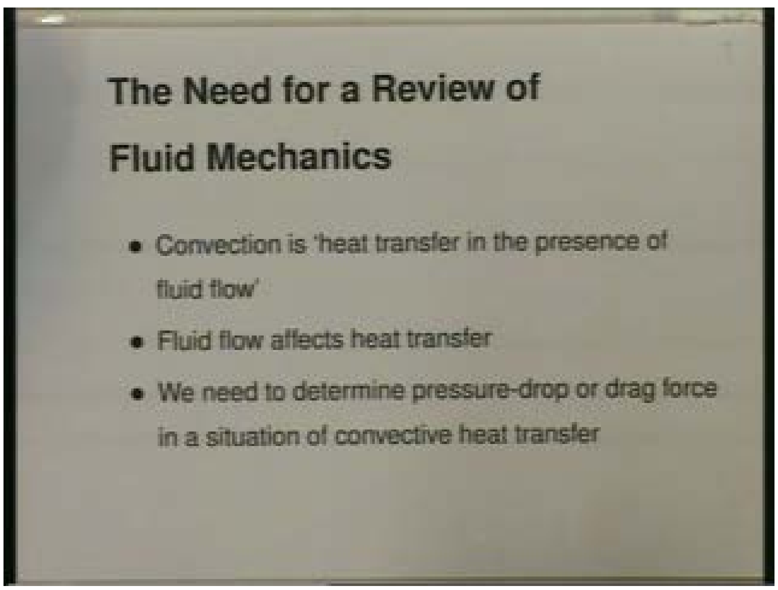

This is because convection is nothing but heat transfer in the presence of fluid flow and because of this fluid flow affects the process of heat transfer. Also in any heat transfer situation, we will have to determine if there is a fluid flow, the pressure drop of the fluid as it flows through a duct or a passage or the drag on a surface or an immerged body if that body is immersed in a fluid and is looksing or gaining heat from the fluid. All of you are expected to have undergone a course or have studied the subject of fluid mechanics; however, in the next few lectures, we will go through a quick review of fluid mechanics.

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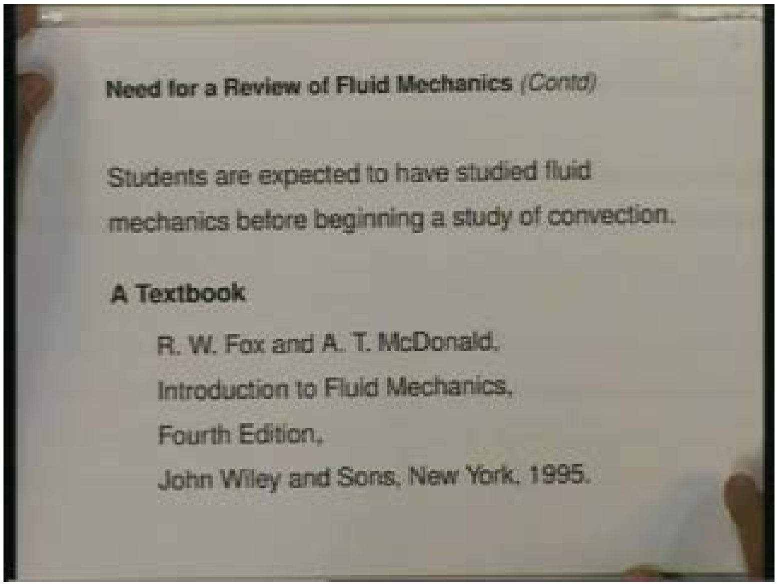

If you want to study fluid mechanics on your own, a reasonably nice text book is by Fox and McDonald. There are many other equivalent text books available – one is by Streeter and Wiley; we have another by Youvan and there are a number of others.

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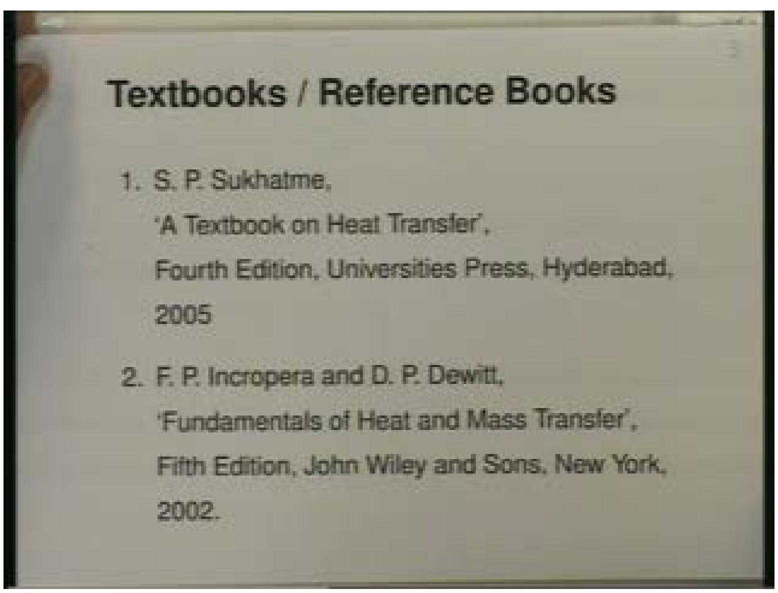

In the very first lecture professor Sukhatme would have indicated these 2 text books or reference books for this particular series of lectures and in these 2 books or any other book on heat transfer, you will find a chapter reviewing the principles of fluid mechanics or at least a few sections undertaking such a review and you may wish to go through that particular portion of these books.

What are we supposed to be familiar with? We are not going to undertake a full fresh study of fluid mechanics but we are expected to be familiar with - before we undertake a study of convection - the following topics.

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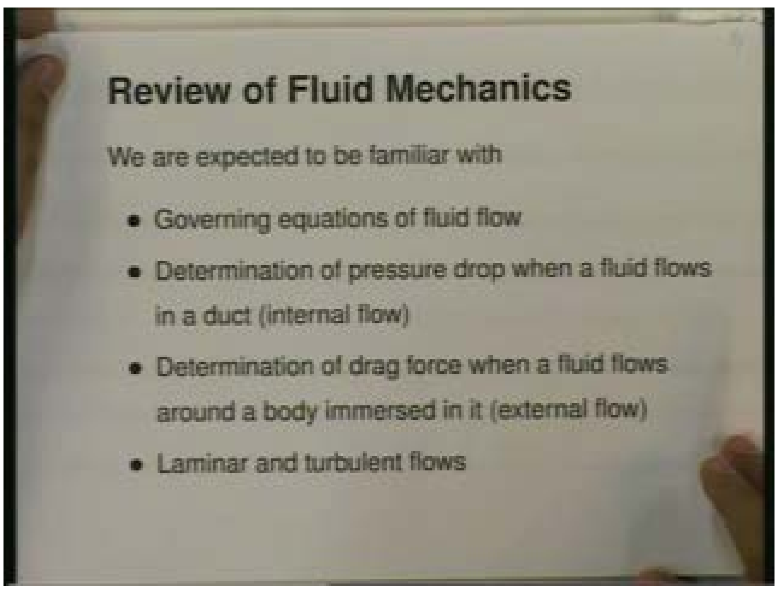

We must have some clear idea of the governing equations of fluid flow, then we must be able to determine the pressure drop when a fluid flows in a duct - these are the situations of internal flow. Then in situations of external flow where a body is immersed in a fluid, we should be able to determine the drag force when fluid flows around such a body and of course fluid mechanics does not go without the terminology of laminar and turbulent flows. So, we must appreciate the difference between laminar and turbulent flows because the characteristics of these 2 type of flows are different and hence the difference will manifest itself in convective heat transfer also. Let us begin our study with the governing equations of fluid flow.

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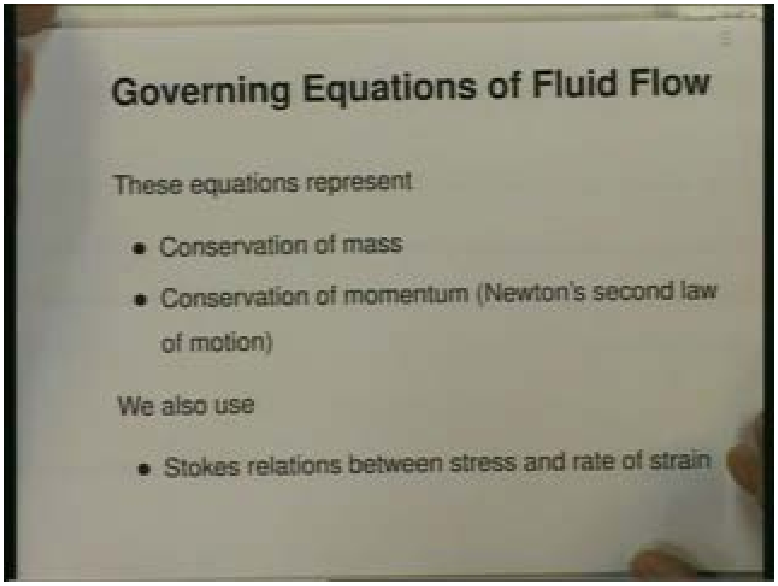

In conduction, the major governing equation was the equation of conservation of mass. There was no flow hence the equation of conservation of mass and equation of conservation of momentum was more or less trivially satisfied and we had to study convection essentially using the equation of conservation of energy whereas for fluid flow the governing equations represent entities related to flow. We have to have an equation representing conservation of mass and another equation or set of equations representing conservation of momentum. Conservation of momentum is nothing but Newton's second law of motion applied to an open system through which fluid flows in and fluid flows out.

Now, just the way we couldn't solve problems of conduction purely by using the governing equation of conservation of energy, we needed a subsidiary equation which was the Fourier's law of heat conduction, similarly these 2 equations - conservation of mass and conservation of momentum - by themselves are not sufficient to solve fluid flow problems. We also need a subsidiary set of equations which are known as Stokes relations between stress and rate of strain in a fluid and when we use Stokes relations, it automatically means that we are restricting ourselves to the domain of Newtonian fluids. When we apply conservation of mass and conservation of momentum along with the Stokes relations to the flow of fluid, the resulting differential equations are known as the Navier Stokes equations.

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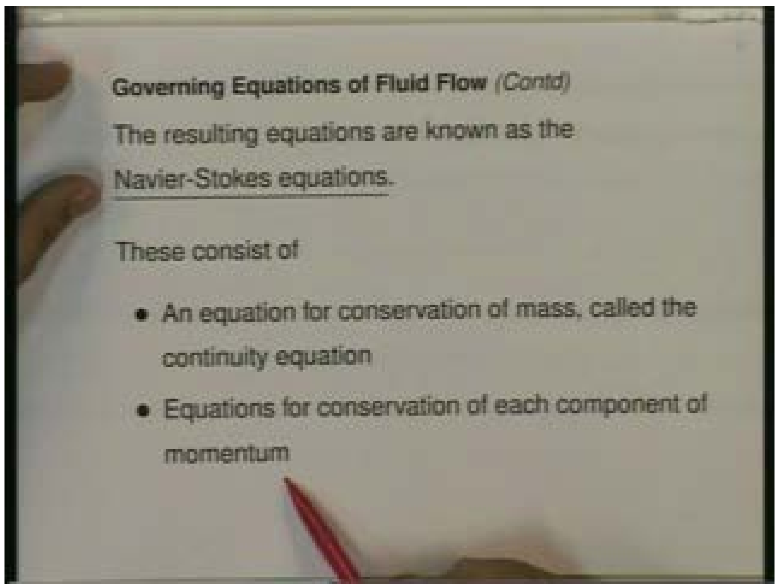

These equations consist of an equation for conservation of mass - this equation is usually called the continuity equation. We also have a set of equations for conservation of each component of momentum - these equations are known as the momentum equations. In a one dimensional flow, there will be a single component of momentum which will be significant and we will have one momentum equation. In the general 3 dimensional flow case, there will be 3 components of momentum and we will have to write down conservation equations for each of these 3 components of momentum. So in the general 3 dimensional case, we will have one equation - continuity equation - representing conservation of mass and 3 momentum equations each representing conservation of momentum in the x direction, in the y direction and the z direction represented. We will quickly look at some simple version of the Navier Stokes equations.

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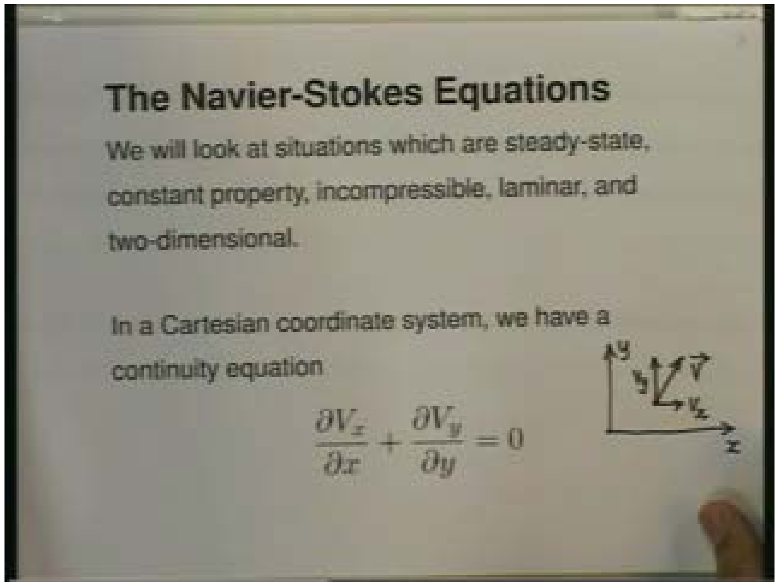

We will assume that apart from the fluid being Newtonian, that is, Stokes relations are assumed to be applicable. We will look at situations which are steady state, constant property, incompressible, laminar and two dimensional. Two dimensional means any property, any velocity component and pressure will vary only in say x and y direction or r and theta direction or r and z direction. Steady state means nothing changes with time; constant property means properties like density, viscosity, etcetera are not functions of pressure and temperature; incompressible means that there is no link between density and pressure - even if pressure varies density does not change with pressure.

Let us first look at a situation in the Cartesian coordinates system where let us say we have - this is the x direction and this is the y direction. At any point there will be velocity component V_x in the x direction and V_y in the y direction; if you want to show the resultant velocity, the resultant velocity V will be a vector. The continuity equation representing conservation of mass in this simplified case looks as follows. Partial derivative of V_x with respect to x plus partial derivative of V_y with respect to y is 0.

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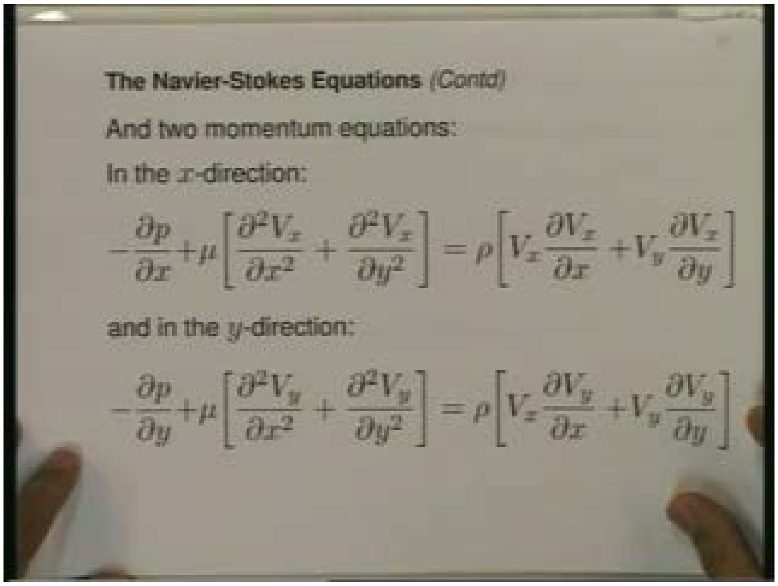

Since we are looking at a two dimensional case, we will have 2 momentum equations - one representing conservation of momentum in the x direction and another representing conservation of momentum in the y direction. These are the 2 equation - the x momentum conservation equation and the y momentum conservation equation. The left hand side represents the net force acting on an element of fluid and the right hand side represents the net outflow of momentum from that small element.

Notice that nothing changes with time so a term containing say dV_x by dt does not enter in these equations. On the left hand side, the first term represents the force because of variation of pressure in the x direction, the second term whatever is in square brackets multiplied by the viscosity mu represent the net viscous force on a small element of fluid. In the right hand side, row into these derivative terms in square brackets represents the net outflow of momentum from that small element of fluid. The equation in the y direction is very similar; you will notice that the variation of pressure in the x direction occurs in the x direction momentum equation and the variation of pressure with respect to y occurs in the y direction momentum equation. On the right hand side you will notice that here, you have derivatives of V_x with respect to x and y and here you have derivatives of V_y with respect to x and y.

The Navier Stokes equations can also be written down in many other coordinate systems but they go on becoming more and more complex as we go to cylindrical, polar and spherical polar coordinate systems.

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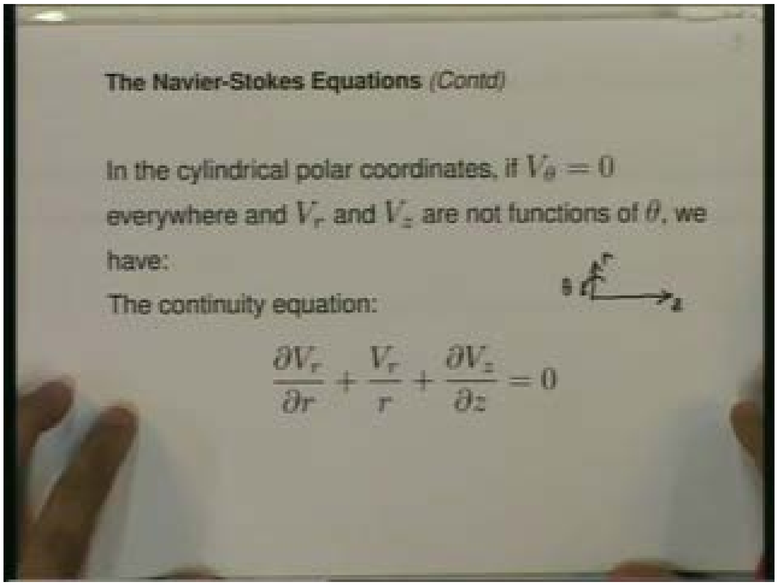

Here is simplified situation in cylindrical polar coordinate where we assume that the velocity in the theta direction is 0; that means there is no swirl. If this is the z direction, this is the r direction, then we are assuming that in the theta direction there is no V theta so there is no swirl. Also we assume that V_r and V_z - the velocity component in the r direction V_r and the velocity component in the z direction V_z - are not functions of theta; that means we are assuming an asymmetric situation. The simplified form of Navier Stokes equations for this asymmetric situation without swirl and with all the other assumptions like steady state, incompressible, laminar, etcetera, the continuity equation reduces to this form.

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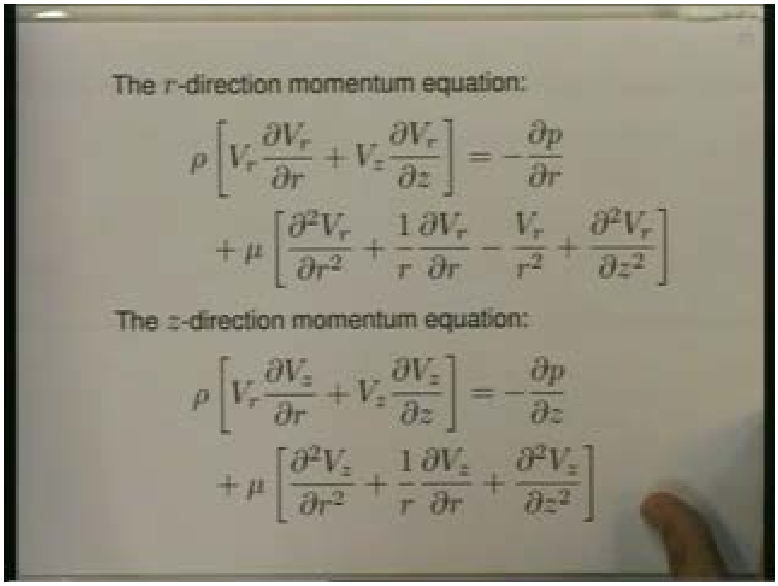

The momentum equations look a bit more complex but the structure remains the same. We have the r direction momentum equation and we have the z direction momentum equation; in each of these equations, we have the momentum outflow terms in the r direction and in the z direction. We have the right hand side which represents the force on that fluid element we have the pressure term in the r direction pressure term in the z direction and these are the viscous terms in the r direction and the viscous terms in the z direction.

So, you will notice that even in simple two dimensional laminar flow situation, we have 3 equations - a momentum equation, sorry, a continuity equation and 2 momentum equations to be solved for the 3 unknowns V_r, V theta and p for the cylindrical polar coordinates and V_x, V_y and p for the Cartesian coordinate systems.

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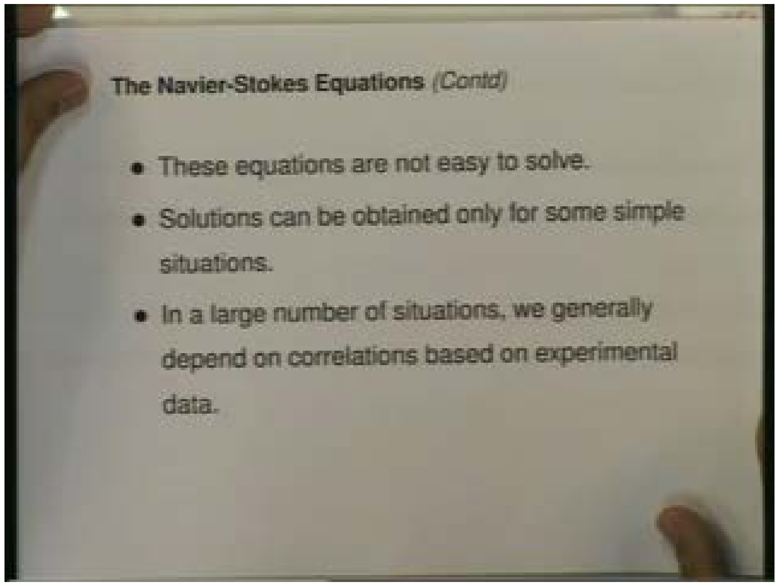

These equations are not easy to solve; solutions can be obtained only for some very simple situations. For example one solution which can be obtained is for laminar fully developed flow in a circular pipe and because solutions cannot be obtained for a large number of situations of practical importance, we generally depend on experimental data. This data is collected, correlated in terms of dimensionless variables and we use those resulting correlations. If you are interested in the solution of Navier Stokes equation, the solution procedures you will have to refer to some book on basic fluid mechanics and fluid dynamics. We now move on to the next topic which is internal flow

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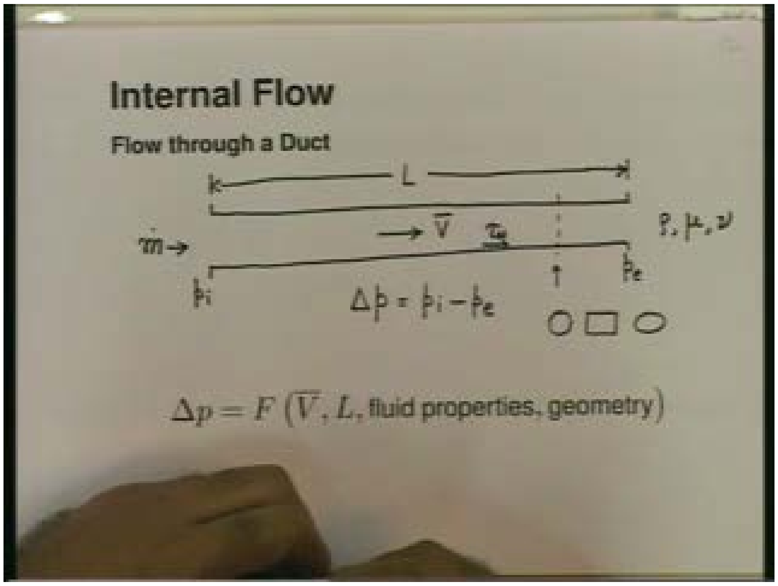

The internal flow represents flow through a duct of some cross section. A simplest situation would be a duct representing a tube or a pipe or a channel through which a fluid flows with some mean velocity say V bar. There is a flow of mass and hence we will have properties for this fluid say density, viscosity; sometimes we may even use the kinematic viscosity which is nothing but the dynamic viscosity divided by the density. Let the length of the tube be L, let the pressure at the inlet be p_i, let that at the exit be p_e. Because of the viscous action of the fluid, the exit pressure will be lower than the inlet pressure; there will be a pressure drop represented by delta p which is nothing but p_i minus p_e.

The duct may have some cross section, the cross section maybe uniform or the cross section could be circular - a very common case. It could be square or rectangular; it could even be some odd shape like an oval. The fluid mechanic problem here is to determine the pressure drop as a function of velocity, length, properties of the fluid and the geometry of the duct - maybe the diameter here or the width and breadth here or, if it is an elliptical flow, the major axis and the minor axis.

At the gross level we will be interested in the pressure drop but because of the viscous action there will be a shear force acting on the wall. We may call it tau wall; in fact we can relate the shear force on the wall to the pressure drop through some simple geometric relation. In such a case of internal flow, we define a term called the friction factor.

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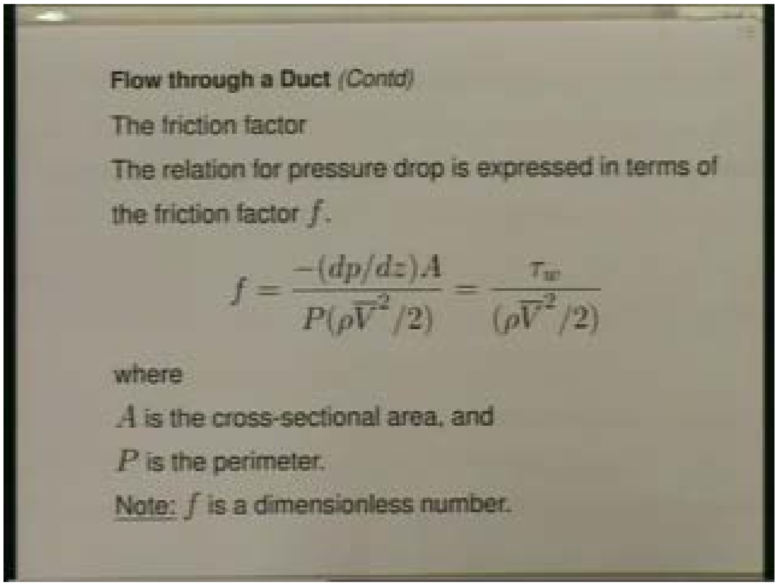

Friction factor relates the pressure drop to the other properties of flow and velocity and also to geometry. The friction factor is defined either in terms of the pressure gradient - it is the partial derivative of pressure with respect to the direction of flow. The negative sign indicates that we expect the pressure to drop in the direction of flow so dp by dz will be a negative number; the friction factor with this negative sign becomes a positive number. This is the area of cross section, this is the perimeter of a cross section and V bar is the mean velocity of flow. So row V bar square by 2 is what is sometimes called the velocity head.

The friction factor can also be shown to be equal to the shear stress on the wall tau w divided by row V bar squared by 2. In any flow situation - internal flow situation - we will almost always be interested in knowing what is friction factor and we should remember that the friction factor is a dimensionless number. There is something about the friction factor which all of us should know.

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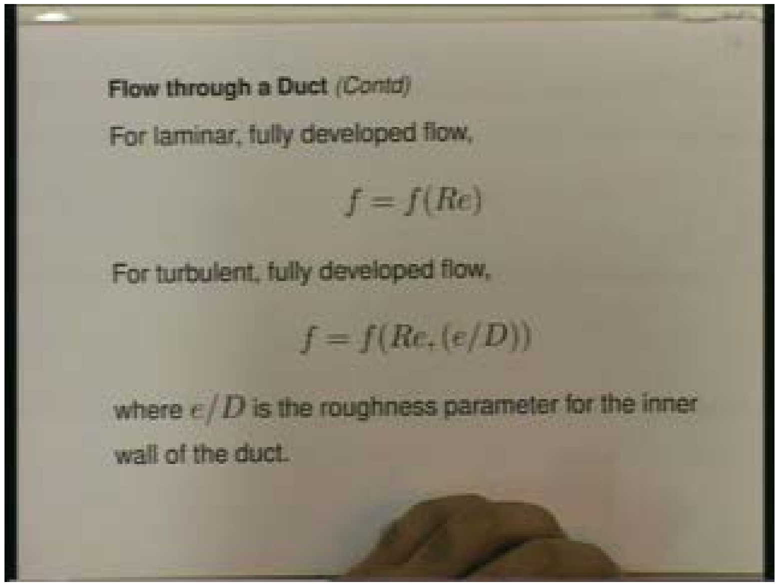

If the flow is laminar and fully developed, that means there are no entrance effects; then the friction factor is a function only of Reynolds number - the function maybe different for different geometry but there is no other effect. Similarly for turbulent fully developed flow in a duct, the friction factor is a function of Reynolds number and e by D which is the wall roughness parameter sometimes known as the relative roughness in text books on fluid mechanics. Let us look at fully developed flow in a circular pipe.

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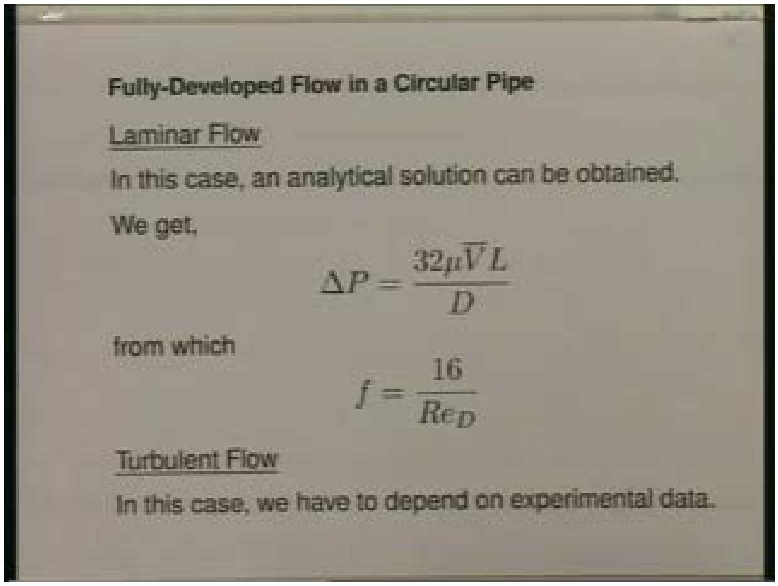

This is one of those rare situations where an analytical solution of the Navier Stokes equations can be obtained. The analytical solution indicates that the pressure drop over a length L is related to the length L, the diameter D, the mean velocity V bar and the dynamic viscosity mu by this simple equation. And if we use the definition of the friction factor, we will notice that it is related only to the Reynolds number by this simple relation - 16 divided by the Reynolds number Re_D; subscript D indicates that the Reynolds number is based on the diameter.

For turbulent flow we have to depend on experimental data because the Navier Stokes equations cannot be solved exactly for turbulent flow.

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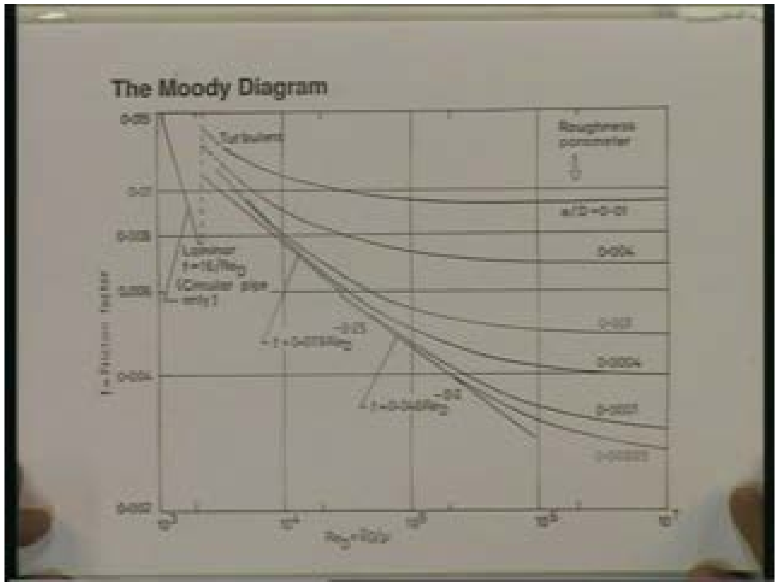

In fact, flow through a circular duct is of so much important, importance in industry that large amount of data as been obtained and it is put together in a famous chart; all of us should know the Moody diagram. The Moody diagram is a log graph - on the y axis we have the friction factor and on the x axis we have the Reynolds number. The Moody diagram is a relation between friction factor and Reynolds number with the roughness parameter as the third parameter for flow through circular ducts.

Notice that when the flow is laminar, Reynolds number less than 2000, the laminar relation F equal to 16 by Re_D applies; that is represented by this straight line here. Beyond 2000 roughly till about 4000 or 5000, Reynolds number is the zone of transition and beyond that we have the zone of turbulent flow where the friction factor will be a function of Reynolds number as well as the roughness parameter. And you will notice that for a given roughness parameter, as Reynolds number increases the friction factor decreases whereas for a given Reynolds number, as the roughness parameter increases the friction factor increases.

In many situations particularly situations pertaining to heat transfer, we have smooth tubes with roughness parameters almost 0 triple 0 5 or 4 0s 5 and so on. And for such tubes over some small ranges correlations are available between a Reynolds number of about 4000 to approximately 30000. We have this correlation, beyond that we have this correlation. Notice that these two correlation are for smooth tubes hence the friction factor is only a function of Reynolds number; the roughness parameter does not appear in these equations. Even this data for almost smooth tubes to rough tubes in turbulent flow has been correlated by Chen.

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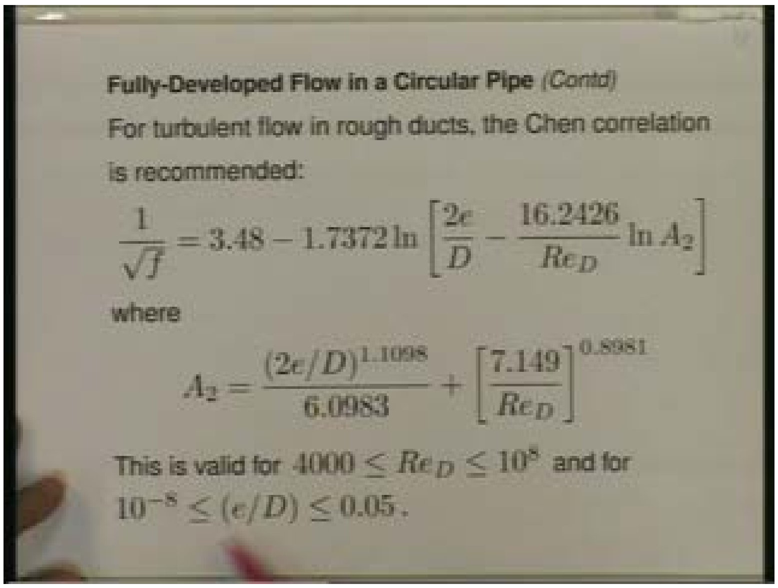

And we have a reasonably complete correlation here where the friction factor is related to e by D, the roughness parameter, Reynolds number and another intermediate term A_2 where A_2 also you will notice is a function of Reynolds number as well as the roughness parameter. So overall we have a relation between friction factor, roughness parameter and Reynolds number and this equation is valid over a reasonably wide range of roughness parameters as well as Reynolds number.

Now we are in a position to solve a simple problem so that we rekindle our ideas of fluid flow through to ducts. The problem which we are going to look at is as follows.

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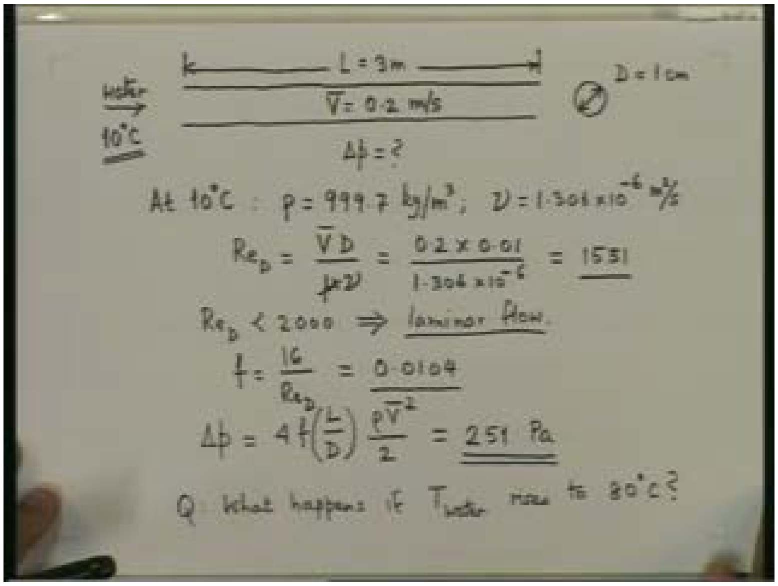

We have a straight duct with a circular cross section of length 3 meters; the cross section is circular and the diameter is 1 centimeter. Water flows through it and it is assumed that the temperature of the water is uniform at 10 degrees C; the flow rate is such that the mean velocity is .2 meters per second. We have to determine the pressure drop over this length. We will assume fully developed flow, the process will be, we will determine properties at 10 degrees C, then we will determine the Reynolds number. Looking at the Reynolds number, we will decide whether the flow is laminar or turbulent. Depending on the decision, we will use a suitable equation or a correlation to determine the friction factor and from the friction factor and the mean velocity and density we will then determine the pressure drop.

First let us look at the properties - at 10 degrees C for water, we can read of the following properties from any property table. Density is 979.7 kg per meter cube. We need either than dynamic viscosity and kinematic viscosity; here I have used the kinematic viscosity which is 1.306 into 10 raise to minus 6 meter square per second. Let us first calculate the Reynolds number. The Reynolds number is based on the diameter in terms of kinematic viscosity; it will be V bar D by the kinematic viscosity.

Substituting .2 meters per second, diameter of .01 meter divided by 1.306 in to 10 raise to minus 6 - this gives us 1531. Now this means that the Reynolds number based on the diameter is less than 2000 and this implies laminar flow. Now since the flow is laminar and since we have assumed fully developed flow we can use this simple relation f equal 16 by Re_D. To determine the friction factor, substitution of Re D equal to 1531 gives us a friction factor which is 0.0104.

So first, we have determined the Reynolds number, then we have determined that the flow is laminar, then we have determined the friction factor and the pressure drop is now calculated as delta p equal to 4 f L by D in to row V bar square by 2. We have calculated the friction factor, we know the length 3 meters, diameter 1 centimeter, density has been read off, V bar is .2 meters per second. If you substitute these numbers, you will get the answer to be 251 Pascal.

Now let us modify this problem here. We had water at 10 degrees C; now the question is what happens if temperature of water rises to 80 degrees C. Instead of cold water, almost chilled water, real hot water at 80 degrees C is now made to flow through pipe. We will assume that the velocity remains the same. What will change are essentially properties and this is what will happen.

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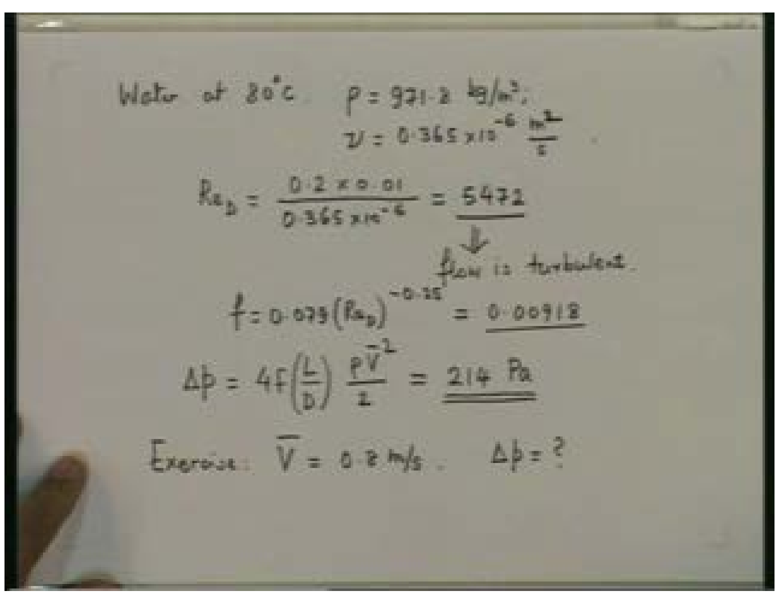

We have now water at 80 degrees C; the density reduces slightly - 971.8 kg per meter cube - but the kinematic viscosity reduces reasonably significantly, almost by a factor of 3. We will have to recalculate the Reynolds number. Mean velocity is .2 meters per second diameter is .01 meter divided by the kinematic viscosity at 80 degrees C and this turns out to be 5472. This is definitely larger than 2000; it is even greater then 5 000 and this implies that the flow is turbulent. Now if we assume that the flow is fully developed and that the tube is smooth, then we are somewhere here on the Moody diagram and we will use this correlation which is applicable for smooth tubes in this zone of Reynolds number. And using that correlation which is f equal to 0.079 Re_D raised to minus .25, substituting for this new value of Reynolds number we will get our friction factor as 0.00918 and the pressure drop will now be 4 f into L by D into row V bar squared by 2.

Compared to the first case, we have the density which is slightly different and the friction factor which also lower and the resulting value of pressure drop is 214 Pascal. So what has happened is because of the rise of temperature from 10 degrees to 80 degrees C and mainly because of the reduction in the kinematic viscosity. The Reynolds number has increased the flow, has become turbulent; the friction factor has decreased slightly and the pressure drop which was earlier 251 Pascal has reduced to 214 Pascal. I will leave it to you as an exercise to determine what happens if the velocity - average velocity - instead of being .2 meters per second increases and say becomes .8 meters per second either at 10 degrees C or at 80 degrees C.

We have now looked at circular ducts. In practice, in a large number of heat transfer situations, we use ducts with non-circular cross sections.

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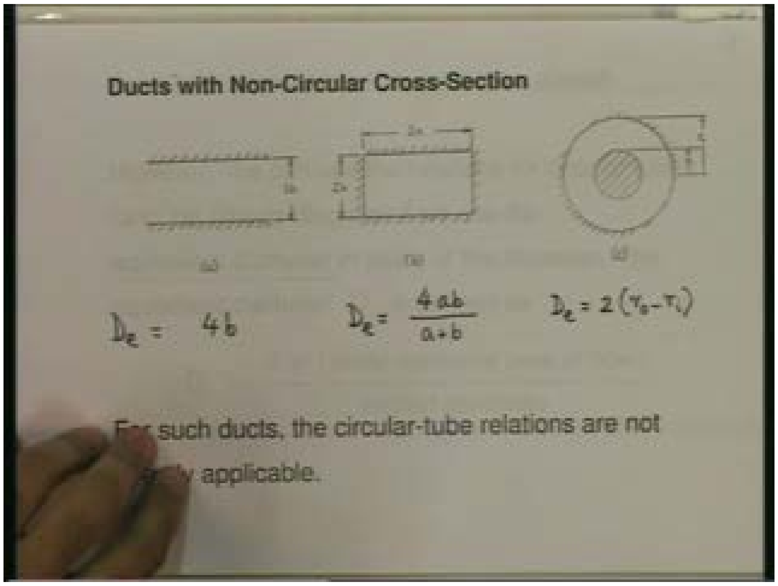

We may have a rectangle so long that we may consider it to be a pair of parallel plates or we may have rectangle of sides 2 a by 2 b or we may have a zone of flow which is bounded by 2 circles and annular zone of flow. So it is possible that this is a tube inside another tube and this is the zone of, annular zone of flow; inner tube will have radius r_i, the outer tube will have an radius r_o. This is a common situation in double pipe heat exchangers and this is an approximation of a situation which is reasonably common in car radiators.

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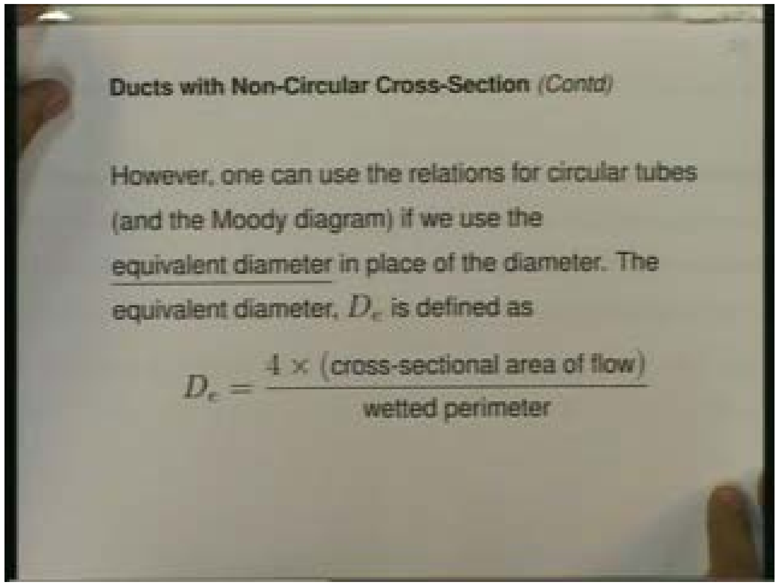

Of course, for such ducts circular tube relations are not directly applicable and experimental data shows that in case of such ducts, one can use the relations for circular tubes and hence one can use the information available in the Moody diagram. If we use the equivalent diameter in place of the diameter which occurs in the Reynolds number, the equivalent diameter D_e sometimes also known as the hydraulic diameter or the hydraulic mean diameter is defined by a simple relation. It is 4 times the cross sectional area of flow divided by the wetted perimeter.

Notice that if you substitute for the cross sectional area of flow and the wetted perimeter the relations for a circular tube, you will get the equivalent diameter equal to the actual diameter of that circular tube. But for noncircular cross sections, you will have equivalent diameter based on the geometry. For example if you consider this to be 2 long parallel plates, the equivalent diameter here will turn out be 4 b. If you have a rectangular channel of sides 2 a by 2 b, area of this channel will be 4 ab; the perimeter will be 4 a plus 4 b and hence in this case the equivalent diameter will be 4 ab divided by a plus b. And in this particular case, the area is that bounded by the outer circle minus that bounded by the inner circle whereas the wetted perimeter is the circumference of the outer circle plus the circumference of the inner circle and if you substitute these relations you will get this to be equivalent diameter to be twice the difference in the radii of the 2 circles.

So in this particular case, in case of noncircular geometries, all that we do is after having determined the mean flow velocity based on the actual area of flow and in the calculation of the Reynolds number and in the calculation of pressure drop where we need that L by D, we use the equivalent diameter and the equivalent diameter is based on the cross sectional area of flow and the wetted perimeter. Once you determine the equivalent diameter or the hydraulic diameter D_e, calculate the Reynolds number, determine whether the flow is laminar or turbulent and use the Moody diagram for determination of the friction factor. So that brings us to the end of this lecture. We will continue with a review of fluid mechanics in the next lecture.