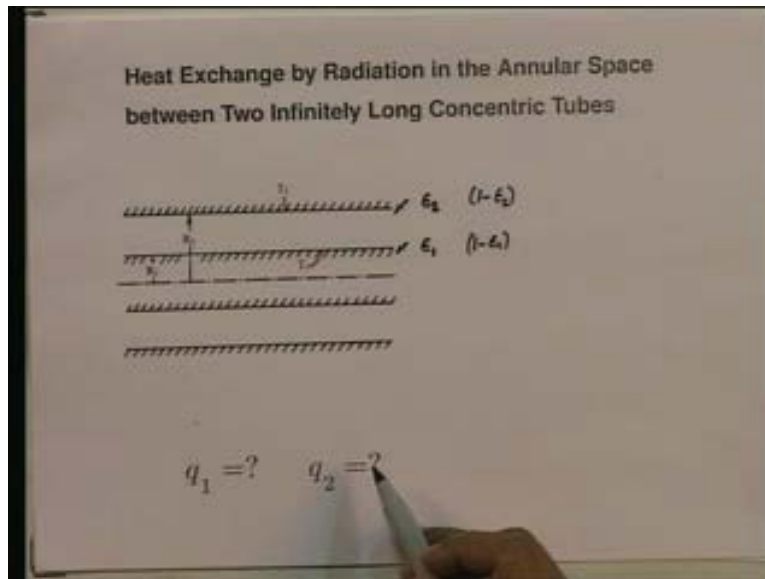


**Heat and Mass Transfer**  
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**Lecture No. 15**  
**Thermal Radiation-6**

In the previous lecture, we calculated or we rather we derived an expression for the heat exchange by radiation between 2 infinite parallel plates which are maintained at different temperatures  $T_1$  and  $T_2$ . We got the expression for the heat exchange rate and we today we want to do a similar derivation but we would like to calculate the heat exchanged by radiation now in the annular space between 2 infinitely long concentric tubes. So once again, it is a two surface enclosure - the outer surface of the inner tube is one surface the inner surface of the outer tube is the other surface. These two surface make up a two surface enclosure.

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Looking at the sketch, the outer surface of the inner tube is this one here and it is maintained at a temperature  $T_1$ , it is maintained at temperature  $T_1$ . The inner surface of the outer tube is this surface and it is maintained at a temperature  $T_2$ . The outer surface of the inner tube has a radius  $r_1$  and the inner surface of the outer tube has a radius  $r_2$  and the

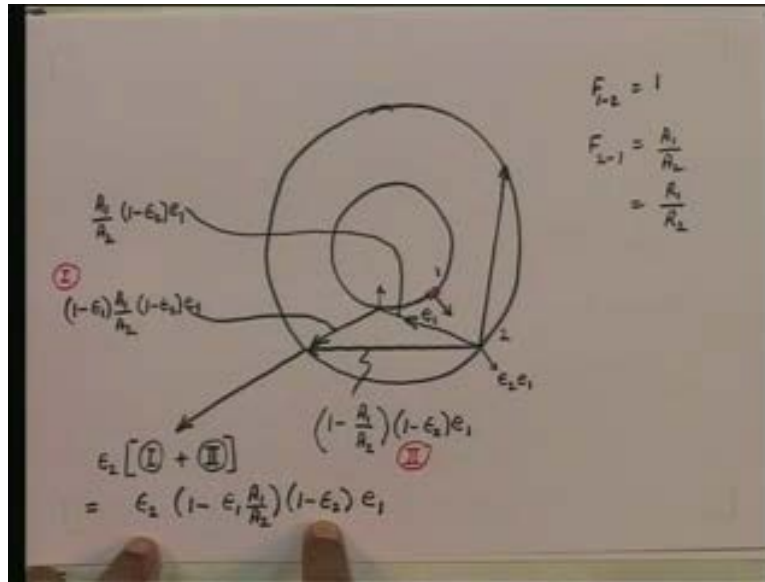
space, the annular space is the radial distance between  $r_2$  and  $r_1$  - that is the annular space in which the heat exchange by radiation takes place.

Let us assume as in the previous case that the surfaces are diffused gray; let us assume that the surfaces are diffused gray and that the emissivities are  $\epsilon_1$  and  $\epsilon_2$  as in the previous case. Since Kirchoff's law will hold true, it follows that the absorptivities are also  $\epsilon_1$  and  $\epsilon_2$ . Also we will assume that the surfaces are opaque, therefore the reflectivities are  $1 - \epsilon_1$  and  $1 - \epsilon_2$  respectively; as in the previous case we will make the same assumption. So the only difference really is that we have a situation now in which we have, we have changed the geometry of the situation now - that is the only change really that we have got.

So with this geometry, we would like to know what is  $q_1$  and what is  $q_2$  -  $q_1$  is the rate at which heat is supplied let us say per unit length at say, supplied per unit length from the surface  $1$  or per meter length.  $q_2$  the rate at which heat is supplied for meter length at the surface  $2$  if  $T_1$  is let us say greater than  $T_2$ , it follows that  $q_1$  will be a positive number,  $q_2$  will be a negative number and  $q_1$  will be equal to minus  $q_2$  as in the previous case. If  $T_1$  is less than  $T_2$ , the situation will be reversed.

Now again we are going to consider radiation emitted by one surface and which goes back and forth in the annular space and is successively absorbed and reflected at the surface which it encounters - that is how we are going to proceed. Once we get terms for these quantities, we will like in the previous generate a geometric progression and sum it up. So let us proceed, let me draw a sketch of the 2 tubes and the annular space in an end elevation so that I can show the rays going back and forth.

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Let us say that this is, let us say this is the radius  $r_1$  and this is the radius  $r_2$  and this is the annular space in which the heat exchange by radiation is taking place between the surface<sub>1</sub> and the surface 2. Consider now the radiation emitted from some element on the surface<sub>1</sub>; let us say I have some element on the surface<sub>1</sub> and let us say we consider radiation emitted from it. The emissive power of surface 1 is  $e_1$  so let us say symbolically that  $e_1$  is what is emitted;  $e_1$  so many watts per meter squared is what is emitted from this surface.  $e_1$  will be given by  $\epsilon_1 \sigma T_1^4$  by the Stefan Boltzmann law.

Now this arrow is symbolic; mind you radiation from this elementary surface is really going off in all directions so the arrow is simply symbolic showing radiation is being emitted from here and ultimately all of it has to reach the surface 2. Mind you this is the case in which the surface 2 completely surrounds the surface 1 therefore  $F_{1 \rightarrow 2}$  is going to be 1. All the radiation from any element on 1 has to reach 2 so  $F_{1 \rightarrow 2}$  is 1 and also the reciprocal relation tells us that  $F_{2 \rightarrow 1}$  is going to be  $A_1/A_2$  which is nothing but  $r_1/r_2$  because we have 2 infinitely long concentric tubes. So this is a situation in which  $F_{1 \rightarrow 2}$  is equal to 1 and  $F_{2 \rightarrow 1}$  is equal to  $A_1/A_2$  which is nothing but  $r_1/r_2$  rather  $2\pi r_1 L$  upon  $2\pi r_2 L$ , the  $2\pi L$  will cancel out so I will get  $r_1/r_2$ .

So, now let us go back; let us consider the radiation emitted from this element.  $e_1$  watts per meter squared are emitted when all this radiation emitted in all directions of a hemisphere reaches the surface 2, the amount that is absorbed here will be  $\epsilon_2 e_1$ ; so many watts per meter square will be absorbed and a certain amount will be reflected which will again be reflected in all directions like this. Out of this reflected amount, reflected amount will be  $1 - \epsilon_2$  into  $e_1$ ; out of this reflected amount a certain will go back to the surface 1 and a certain amount will go back straight to the surface 2 because the  $F_{2 \text{ to } 1}$  is  $A_1$  upon  $A_2$ . So what is the amount that goes directly to the surface 2 itself from 2? Well the amount that goes to surface 2 would directly, will be the amount that goes to surface 2 directly.

Let me draw it in this, this point here rather than that arrow, just let me show it here. The amount that goes to surface 2 directly will be  $1 - \epsilon_2$ , this quantity is going to be  $1 - \epsilon_2$  multiplied by  $A_1$  upon  $A_2$  multiplied by  $1 - \epsilon_2 e_1$ . That is the quantity that is going to be because the amount that is absorbed is  $\epsilon_2 e_1$ , so the amount that is reflected is  $1 - \epsilon_2 e_1$  and out of that, the amount that goes directly to surface 2 will be  $1 - \epsilon_2$  by  $A_1$  by  $A_2$  into  $1 - \epsilon_2 e_1$ .

The rest goes back to surface 1 and what is the quantity going to surface 1? Well, let me show it by an arrow; the quantity that is going to surface 1 is the difference. So that is going to be, let me show that here the quantity going to the surface 1 itself will be  $A_1$  upon  $A_2$  into  $1 - \epsilon_2$  into  $e_1$  - that is the quantity that goes to the surface 1. Out of that, the quantity that is going to be absorbed in the surface 1 will be, the quantities that is absorbed in the surface 1 will be that multiplied by  $\epsilon_1$  and the quantity that is reflected back to surface 2 will be the difference.

So, it will be  $1 - \epsilon_1$  multiplied by  $A_1$  upon  $A_2$  multiplied by  $1 - \epsilon_2$  into  $e_1$  and out of that the quantity that is absorbed will be this quantity which I have put down here multiplied by  $\epsilon_1$ . So if I multiplied this quantity which I have got here I will call this as  $R_1$ . I will, I will use this as  $R_1$  and I call this is  $R_2$ . What is, here  $R_1$  is the quantity that is coming to the surface 2 and after deflection from

surface 1 and this expression is the one that is coming directly to surface 2. If I multiply both of these by epsilon 2 that is the absorptivity of surface 2, I will get this quantity which is finally absorbed at the surface 2.

So, I am going to get the arrow, the big arrow showing the amount absorbed will be epsilon 2 in the bracket 1 plus 2, that is what going to be the quantity. And if I work that out, if I work out that expression, I am going to get epsilon 2; if I work out that expression I am going to get epsilon 2 into 1 minus epsilon 1 A<sub>1</sub> upon A<sub>2</sub> multiplied by 1 minus epsilon 2 multiplied by e<sub>1</sub>. Let me just check again 1 minus epsilon 2 e<sub>1</sub> that is here and if I multiply this I will get 1 minus epsilon 1 epsilon 2 and the whole thing is to be multiplied by epsilon 2. Am I right? Let me just, let me check that, that is okay.

So, this is the expression I will get for the quantity that will come back to surface 2 and absorbed by the surface. Now this is the first set of absorptions and reflections; I can go on doing this and generate terms like this one.

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Rate at which radiation emitted by inner tube 1  
and absorbed by outer tube 2

$$= A_1 e_1 \left[ \epsilon_2 + \left(1 - \epsilon_1 \frac{A_1}{A_2}\right) (1 - \epsilon_2) \epsilon_2 \right. \\ \left. + \left(1 - \epsilon_1 \frac{A_1}{A_2}\right)^2 (1 - \epsilon_2)^2 \epsilon_2 \right. \\ \left. + \dots \dots \dots \right]$$

$$= A_1 e_1 \epsilon_2 \frac{1}{1 - \left(1 - \epsilon_1 \frac{A_1}{A_2}\right) (1 - \epsilon_2)}$$

Therefore I can say having done this, I can say the following - I can say rate at which, rate at which radiation emitted by inner tube 1 and absorbed and absorbed by outer tube 2

is equal to, going to take them term by term now, I take  $A_1 e_1$  outside because that is the common. I will take  $A_1 e_1$  outside  $A_1$  is the area of the surface 1 and  $e_1$  is the emissive power, so I am going to take that outside. I will get the terms inside to be - the first term is  $\epsilon_2$ , the next term will be  $1 - \epsilon_1 \frac{A_1}{A_2}$  multiplied by  $1 - \epsilon_2$  multiplied of course by  $\epsilon_2$  plus the next term is going to be, which we haven't derived but you can do on your own,  $1 - \epsilon_1 \frac{A_1}{A_2}$  the whole squared  $1 - \epsilon_2$  the whole squared multiplied by  $\epsilon_2$  plus so on; that is what we are going to get. So we derive the first 2 terms mind you; the first term was  $A_1 e_1 \epsilon_2$ .

Let me go back to the sketch, the first terms which we derived was this one,  $A_1$  we didn't write down, that was the area of the inner tube into  $\epsilon_1$  that was the first term and the second terms we derived was this one for, what is absorbed by the surface 2? That is  $\epsilon_2$  into  $1 - \epsilon_1 \frac{A_1}{A_2}$  into  $1 - \epsilon_2$   $e_1$ ; that was the second term. This is now very easy to sum up, we can take  $\epsilon_2$  also outside. So we get  $A_1 e_1 \epsilon_2$  and inside the bracket we will get a geometric progression and if you sum up that geometric progression you will get 1 upon, it is a very simple geometric progression,  $1$  upon  $1 - 1 - \epsilon_1 \frac{A_1}{A_2}$  into  $1 - \epsilon_2$ .

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$$= \frac{A_1 \epsilon_1 \epsilon_2}{\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}}$$

Similarly, rate at which radiation emitted by the outer tube 2 is absorbed by the inner tube 1

$$= \frac{A_1 \epsilon_1 \epsilon_2}{\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}}$$

$$\therefore q_{12} = \frac{A_1 (\epsilon_2 \epsilon_1 - \epsilon_1 \epsilon_2)}{\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}}$$

That is the summation of the geometric progression, the infinite series that we have and if we clean up this expression a little, we will get - this is further equal to  $A_1 \epsilon_1 \epsilon_2$  divided by, divided by  $\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}$  - that is what we will get. This you can you now do one your own - the summing up of the geometrical progression; that is the expression we will get. It is the rate at which radiation emitted by the inner tube is absorbed by the outer tube.

Now, in exactly the same wave do the reverse calculation; calculate the rate at which radiation emitted by the outer tube is absorbed by the inner tube. Similarly I am not going to derive this now; I am saying similarly rate at which, rate at which radiation emitted by the outer tube, by the outer tube 2 is absorbed by the inner tube 1, by the inner tube 1 equals; now this derivation I am not doing. It will come out to be  $A_1 \epsilon_1 \epsilon_2$  divided by, if one does the derivation you will get the following and I would like you to do this on your own as an exercise, multiplied by  $\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}$  - that is what you will get.

Therefore, now the  $q_1$  to  $2$  that is the heat exchanged between the two surfaces 1 and 2, the net heat exchange between the two surfaces 1 and 2 is given by, therefore  $q_1$  to  $2$  is

equal to  $A_1$  - the difference of the 2. So we will say  $A_1$  into bracket  $\epsilon_2 E_1$  minus  $\epsilon_1 E_2$ , the difference of the 2 terms divided by  $\epsilon_1 A_1$  upon  $A_2$  plus  $\epsilon_2$  minus  $\epsilon_1$   $\epsilon_2 A_1$  upon  $A_2$  - that is the net heat exchanged rate between the two surfaces.

We are proceeding really exactly in the same way that we did for infinite parallel plates with the difference that the shape factor in one direction is 1,  $F_{1 \rightarrow 2}$  is 1,  $F_{2 \rightarrow 1}$  is  $A_1$  upon  $A_2$ .

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$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

Since only two surfaces are involved,

$$q_{12} = q_1 = -q_2 = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

That is the only difference that we have in this case therefore further if now we write for  $E_1$ , I write  $\epsilon_1 \sigma$  into  $T_1$  to the power of 4 and for  $E_2$  I write  $\epsilon_2 \sigma$  into  $T_2$  to the power of 4. Then I will get  $q_{1 \rightarrow 2}$  is equal to  $\sigma A_1 T_1^4$  minus  $T_2$  to the power 4 divided by  $1$  by  $\epsilon_1$  plus  $A_1$  by  $A_2$  A plus  $A_1$  upon  $A_2$  into  $1$  upon  $\epsilon_2$  minus 1. Now only two surfaces are involved; it is only a two surface enclosure so obviously  $q_{12}$  is also  $q_1$  and it is also minus  $q_2$ .

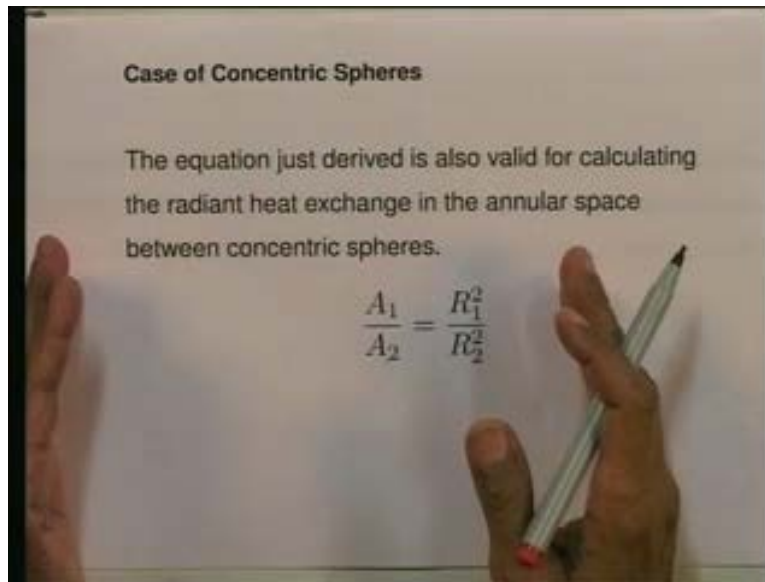
We have argued this before so it is the same argument again. Two surfaces are involved; the heat exchanged rate between the two surfaces is also the rate at which heat is supplied



as each of those surfaces. So  $q_{12}$  is equal to  $q_1$  is equal to minus  $q_2$  therefore  $q_1$  and  $q_2$ ,  $q_1$  equal to minus  $q_2$  is nothing but the expression which we have that is namely sigma into  $A_1 T_1$  to the power 4 minus  $T_2$  to the power 4 divided by, divided by  $1 + \frac{A_1}{A_2} \frac{1 - \epsilon_2}{\epsilon_1}$  and this is a rather well known expression which I would like you to note. This particular expression that we have derived is a rather well known expression; as I will show you in a moment it has wider applicability just for the situation for which we have derived it and therefore it is rather an important expression -  $q_1$  equal to minus  $q_2$  is equal to sigma  $A_1 T_1^4 - T_2^4$  the power 4 the whole thing divided by  $1 + \frac{A_1}{A_2} \frac{1 - \epsilon_2}{\epsilon_1}$  minus 1.

Now, let us consider a different situation; let us say the, considering the annular space between 2 tubes 2 long tubes concentric tubes, let us say we consider the annular space between concentric spheres, radius of the inner sphere being of radius  $r_1$  and the outer sphere being of radius  $r_2$ . Let us consider the situation we have 2 concentric spheres and the inner sphere of surface 1 is at a temperature  $T_1$ , the outer sphere - the hollow space of the outer sphere - is at a temperature  $T_2$ , the hollow of a surface I mean and the heat exchange by radiation is taking place in the annular space between the 2 spheres. We want to find out the rate of that heat exchange. In this case - the case of the 2 sphere - you will agree with me  $F_{1 \rightarrow 2}$  is again 1,  $F_{1 \rightarrow 2}$  is 1 and  $F_{2 \rightarrow 1}$  is  $\frac{A_1}{A_2}$  where  $\frac{A_1}{A_2}$  is nothing but  $\frac{R_1^2}{R_2^2}$ .

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In the previous case, in the cylinders case  $A_1$  upon  $A_2$  was  $R_1$  upon  $R_2$ . Now  $A_1$  upon  $A_2$  will be  $R_1$  square by  $R_2$  square. Now if I proceed to derive an expression for the heat exchange by the, between the two surfaces and the, derive an expression for  $q_1$  or  $q_2$ , you will agree with me that since  $F_{1 \text{ to } 2}$  is 1 and  $F_{2 \text{ to } 1}$  is  $A_1$  upon  $A_2$ , I am going to get exactly the same expression that we derived earlier.

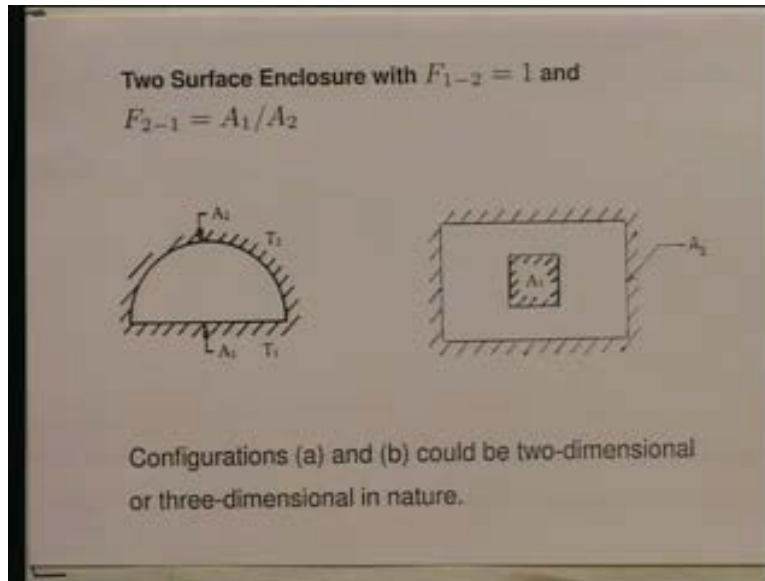
So, for the case of concentric spheres, if I want to derive an expression for the radiant heat exchange in the annular space between concentric spheres, the expression will still be for concentric spheres also. The expression will still be  $q_1$  equal to minus  $q_2$  equal to sigma  $A_1$  into  $T_1$  to the power 4 minus  $T_2$  to the power 4 divided by 1 by epsilon 1 plus  $A_1$  by  $A_2$  into 1 by epsilon 2 minus 1. So this expression is also valid for calculating the radiant heat exchange in the annular space between 2 concentric spheres 1 and 2.

Why? For that matter, why do the spheres have to be concentric or why do the 2 cylindrical tubes have to be concentric? They could even be eccentric because even when they are eccentric  $F_{1 \text{ to } 2}$  is going to be 1 and  $F_{2 \text{ to } 1}$  is going to be  $A_1$  upon  $A_2$ . So even if the 2 tubes are eccentric or even if the 2 spheres are eccentric the relation that you have derived is still valid. The only thing to remember is if they are tubes,  $A_1$  upon  $A_2$

will be  $R_1$  upon  $R_2$ , if they are spheres  $A_1$  upon  $A_2$  will be  $R_1$  squared upon  $R_2$  squared; that is the only difference.

Now let us go further, let us carry our argument still further. Why do we in fact need to think only of cylinders and spheres?

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Suppose I have general situation - a two surfaces enclosure is  $F_{1-2}$  equal to 1 and  $F_{2-1}$  equal to  $A_1$  upon  $A_2$ . I have, any general situation like this could be two dimensional or it could be three dimensional. So long as I satisfy this condition  $F_{1-2}$  equal to 1 and  $F_{2-1}$  equal to  $A_1$  upon  $A_2$ , then the expression which we have derived is valid. Now look at the examples like this for instance. Here is a semi circular tube, a semi circular space let us say. The semi circular part we will call as surface 2, the flat part will call as surface 1 and let us say surface 1 is at a temperature  $T_1$ , the surface 2 is at a temperature  $T_2$ ; this could be a long long semi circular space like this.

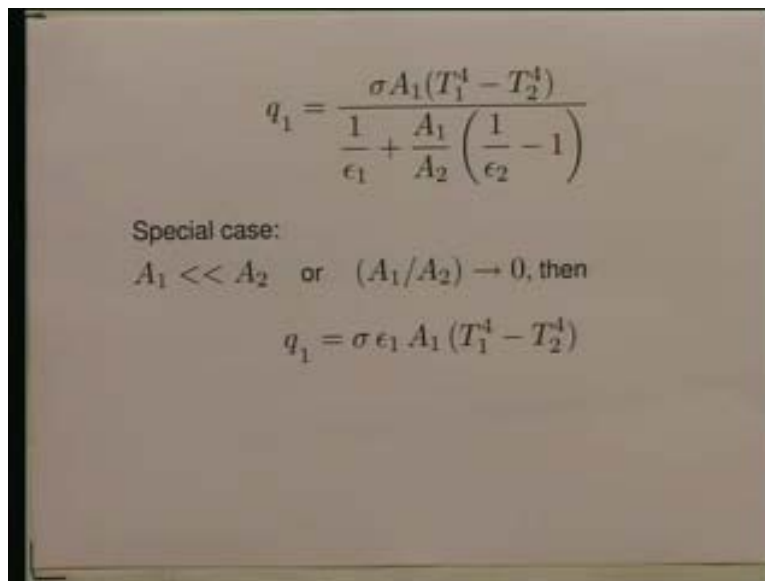
Now in this case  $F_{1-2}$  is 1 because all the radiation from 1, from the flat surface 1 has to go to 2 and  $F_{2-1}$  is going to be  $A_1$  upon  $A_2$  therefore the expression we just derived is valid. Let us go to this case, here is a case where we have let us say 2 square or 2

rectangular tubes - long square or rectangular tubes 1 inside the other. Here also  $A_1$  F 1 to 2 is 1, F 2 to 1 is  $A_1$  upon  $A_2$  and the relation which we derived earlier is true. This need not be 2 dimensional, could be a 3dimensional situation; for instance  $A_1$  could be a cube inside this room and  $A_2$  could be the walls, the 6 walls of this room, that case also F 1 to 2 is 1, F 2 to 1 is equal to  $A_1$  upon  $A_2$ .

So, for a variety of situations, although we derived this expression which I derived a movement ago for the annular space between 2 cylindrical tubes, although we derived it for that specific situation, keep in mind that this relation is valid whenever we have a two dimensional or a three dimensional situation involved in such a manner that F 1 to 2 is 1 and F 2 to 1 is  $A_1$  upon  $A_2$ . In that case, this expression holds true.

So, this expression has very wide validity and that is why I said it is a rather useful expression and we will need it for solving many types of problems. So let us now in fact look at some problems but before I that do that let me take a special case of this situation.

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$$q_1 = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

Special case:  
 $A_1 \ll A_2$  or  $(A_1/A_2) \rightarrow 0$ , then

$$q_1 = \sigma \epsilon_1 A_1 (T_1^4 - T_2^4)$$

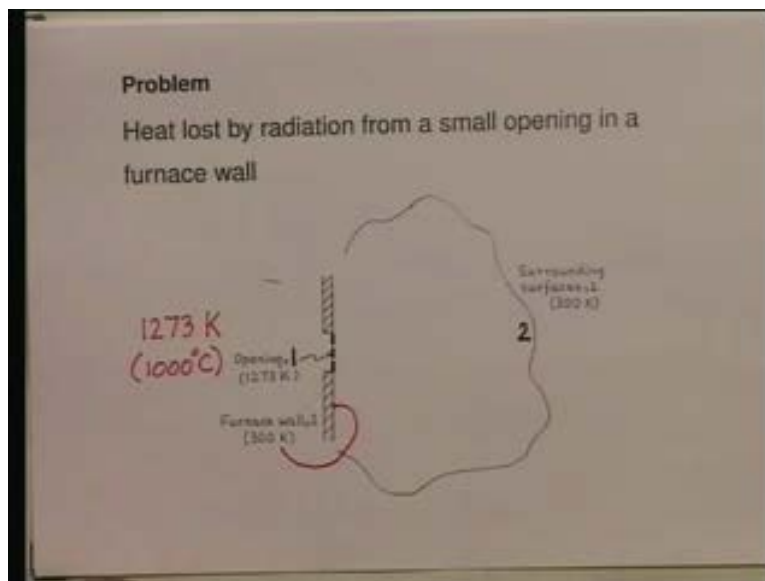
So we have got this expression. Now,  $q_1$  is equal to sigma  $A_1$   $T_1$  to the power 4 1 upon epsilon 1 plus  $A_1$  by  $A_2$  1 by epsilon 2 minus 1, this is our general expression valid

whenever  $F_{1 \rightarrow 2}$  is 1 and  $F_{2 \rightarrow 1}$  is  $A_1$  upon  $A_2$ . Let us take a special case of this; let us say  $A_1$ , the area  $A_1$  that is the surface 1 is very small compared to 2; that is  $A_1$  by  $A_2$  tends to 0. In that case, the denominator will simplify because  $A_1$  by  $A_2$  will become equal to 0. So the expression will simplify to the form  $q_1$  is equal to  $\sigma \epsilon_1 A_1 T_1^4 - T_2^4$ . So we will get this simpler expression if  $A_1$  by  $A_2$  tends to 0. Keep that in mind.

So, now we have derived a general expression for a variety of geometries though we started off only the, with the case of the annular space between 2 tubes; we extended it to the annular space between 2 spheres. We said the spheres could be eccentric, the tubes could be eccentric and then we said in fact it could be applied to any situation where one object completely surrounds another so that one surface completely surrounds another; so that  $F_{1 \rightarrow 2}$  is 1 and  $F_{2 \rightarrow 1}$  is  $A_1$  upon  $A_2$  and then finally we took up the special case inside; if  $A_1$  by  $A_2$  tends to 0 we get the simpler expression.

Now we are going to solve a problem or two. The problem we are going to take up first is the following one which occurs in practice quite often. Let us say I have a furnace on the left hand side here.

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We would like to calculate through a small opening in this furnace, this is an opening in the furnace, what is the heat being lost by radiation through that small opening? So out here, the shaded thing is the wall of a furnace; on the left hand side is the furnace like this and it is at a temperature of 1000 degrees centigrade. There is a small opening and we would like to know how much heat is being lost by radiation through the small opening in this furnace wall.

Let us say all the surrounding surfaces to outside the furnace  $r$  at a temperature of 300 Kelvin and the furnace wall itself on the outside is at 300 Kelvin. Let us say that the furnace wall is at 300 Kelvin here and of course all the surroundings are at 100, 300 Kelvin and this is my opening through which radiation is being, heat is being lost by radiation. I would like to know how much is being lost because inside here the temperature is 1273 Kelvin that is 1000 degrees centigrade. It is a furnace inside here at 1000 degrees centigrade, so how much energy radiant energy is being lost through this opening?

Now, we are going to solve the problem as follows – we are going to idealize the situation by making a two surface enclosure. So we are going to say close this opening with this dotted line that I am showing here, close this opening with this dotted line which I am showing here and close this opening with some imaginary surface and let us call that as surface 1. And let us say all the others, that is outside of the furnace wall that is here and the surroundings everything else is at, is at 300 Kelvin; so let us call that as surface 2.

So, let us say this is my enclosure as I am showing it here; this is my enclosure in which the opening is surface 1, the small opening is surface 1 and everything else that it sees once the radiation comes out is surface 2. So obviously this is a situation in which  $A_1$  upon  $A_2$  is going to be tending to 0; it is a situation in which  $F_{1 \text{ to } 2}$  is going to be 1, it is a situation in which  $F_{2 \text{ to } 1}$  is going to be  $A_1$  upon  $A_2$ .

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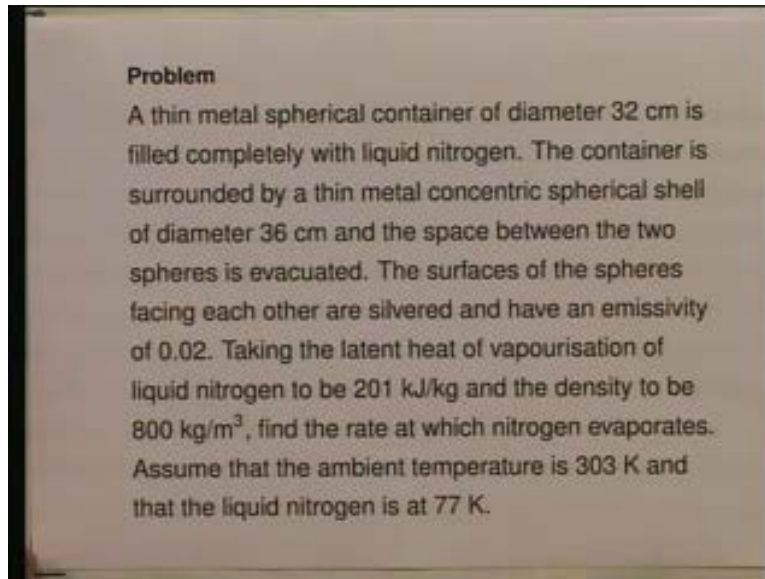
$$\begin{aligned} q_1 &= \sigma A_1 \epsilon_1 (T_1^4 - T_2^4) \\ &= 5.670 \times 10^{-8} \times \pi \times 0.01^2 \times 1 \\ &\quad (1273^4 - 300^4) \\ &= 46.6 \text{ W} \end{aligned}$$

Therefore for this particular problem, for this particular problem which we have idealized as follows, for this problem we can say  $q_1$  is equal to, the expression which I just had showed you a moment ago -  $\sigma A_1 \epsilon_1 T_1^4 - T_2^4$  - that is the rate at which heat must be supplied at the surface<sub>1</sub> if surface<sub>1</sub> is to be at 1000 degree centigrade that is 1273 Kelvin and surface is to be at 300 Kelvin. So that is equal to 5.670 into 10 to the minus 8. That is the Stefan Boltzmann constant multiplied by the opening, we are told to take the opening to be to be 2 centimeters in diameter. I will put that down - opening 2 centimeter diameter.

So,  $A_1$  will be pi into .01 the whole square pi r squared - that is the area multiplied by the emissivity of the surface; that is we can treat it to be a black surface. So we will say it is 1 and  $T_1$  to the power 4 would be 1273 to the power 4 minus 300 to the power 4. So we imagine that the opening is plugged with an imaginary surface and that imaginary surface is at 1273 Kelvin and is a black surface. Now that is a good assumption because the opening inside is really looks like a black surface from outside; so that is a good assumption. So, we get 1273 to the power 4 minus 300 to the power 4 and that comes out to be equal to 46 .6 watts. So in this case, this is the rate at which heat is being lost by radiation from this opening in the, in a furnace wall.

Now let us do one more problem; problem we want to do next is the following. I am going to read it out slowly so that you can take it down; the problem is the following.

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A thin metal spherical container of diameter 32 centimeters is filled completely with liquid nitrogen. I repeat a thin metal spherical container diameter 32 centimeters is filled completely with liquid nitrogen full stop. The container is surrounded by a thin metal concentric spherical shell of diameter 36 centimeters and the space between the 2 spheres is evacuated, and the space between the 2 spheres is evacuated. The surfaces of the spheres facing each other are silvered and have an emissivity of .02. I repeat the surfaces of the spheres facing each other are silvered and have epsilon of .02.

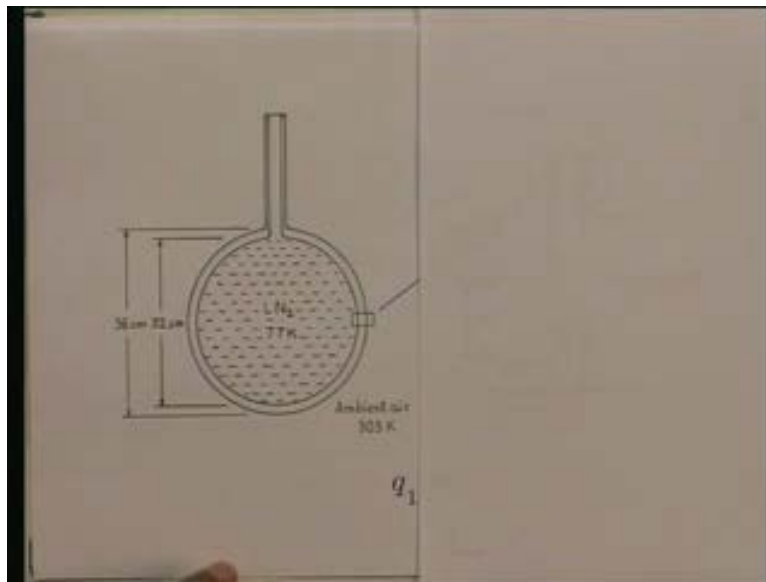
Taking the latent heat of vaporization, taking the latent heat of vaporisation of liquid nitrogen to be 201 kilojoules per kilogram and the density to be 800 kilograms per meter cubed, find the rate at which nitrogen evaporates. Let me repeat that sentence - taking the latent heat of vaporisation of liquid nitrogen to be 201 kilojoules per kilogram and the density to be 800 kilograms per meter cubed, find the rate at which nitrogen evaporates. Assume that the ambient temperature is 303 Kelvin and that the liquid nitrogen is at 77



Kelvin. I repeat the last sentence - assume that the ambient temperature is 303 Kelvin and that the liquid nitrogen is at 77 Kelvin.

Now let us picture this situation; I want you to focus only on the left hand side of this figure for the movement, not on the right hand side so that you see the geometry involved.

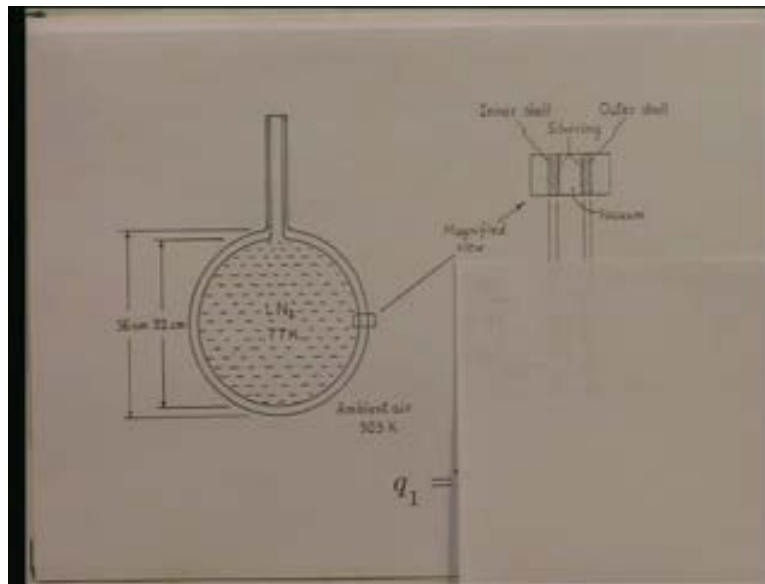
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The inner circle is the spherical container of diameter 32 centimeters surrounded by another spherical shell of diameter 36 centimeters and there is a neck here through which the liquid nitrogen is poured or taken out. This is all filled with liquid nitrogen so cryogenic liquids like liquid nitrogen are stored in spherical containers.

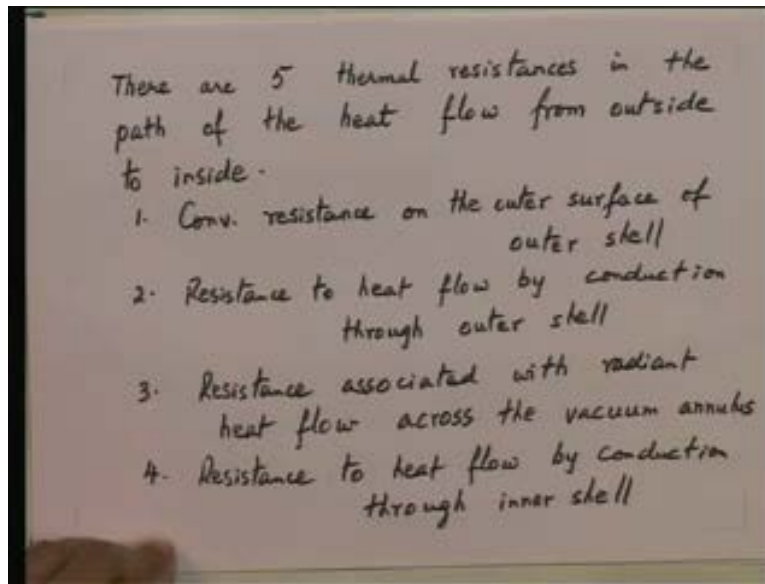
The sphere has the advantage of providing the smallest area for a given volume and so it is advantageous to have a spherical shape. You want to after all reduce the amount of heat flowing in which will make the liquid nitrogen evaporate. You are told the ambient air is at 303 and the liquid nitrogen inside is at 77 K. Now let me, the rectangle that is drawn here; it is expanded and let us magnify and let us look at the magnified view here so that you see the whole picture now.

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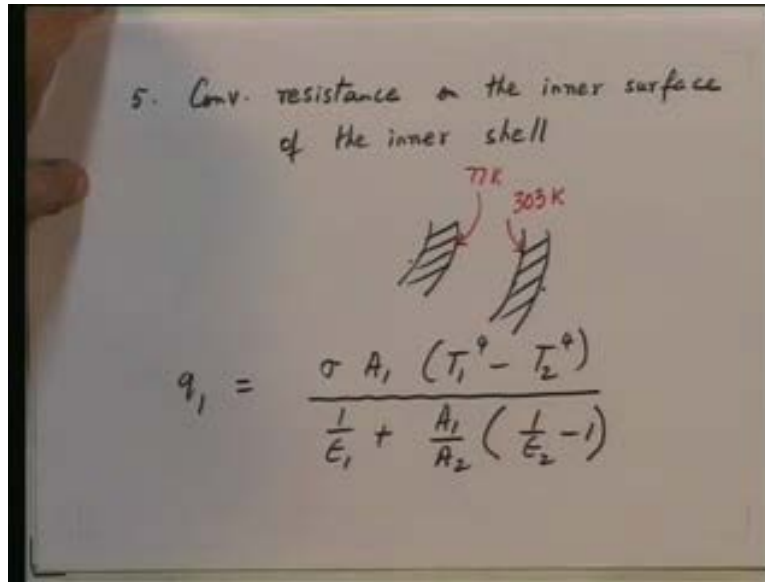
What you are seeing now is the inner shell and this is the thickness of the metallic shell and the hach, the zigzag line indicates that it is silvered. Similarly this is the outer shell, this is the thickness and the zigzag line indicates that it is silvered; that space between the 2 is a vacuum. So this is the inner shell, this is the outer shell, the space between the 2 is a vacuum and the two surfaces facing each other are silvered and have an emissivity of .02. Now when heat flows it will flow from outside to inside obviously. The outside temperature is 303 Kelvin, the inside temperature is 77 Kelvin so heat is going to flow from outside to inside. Now when heat flows, I think it is obvious to you that the, it will encounter a large number of thermal resistances and what are these? There are 5 thermal resistances in the path of the heat flow from outside to inside

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What are they? There are 5 thermal resistances in the path of the heat flow from outside to inside; what are these? Number<sub>1</sub> - convective resistance; convective resistance on the outer surface of the outer shell, on the outer surface of outer shell, that is the first resistance. Then heat flows by conduction through the outer shell so resistance to heat flow by conduction through outer shell - that is the second resistance. Third resistance associated, resistance associated with radiant heat flux - radiant heat flow, the radiant heat flow across the annulus across the vacuum annulus. That is the third resistance.

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Then the 4th is again resistance to heat flow by conduction, resistance to heat flow by conduction through inner shell and the last resistance will be the convective resistance, convective resistance on the inner surface of the inner shell. These are the 5 thermal resistances. Now out of these, the dominant resistance in this case is going to be the resistance associated with radiant flow across the annulus. Number 3 - that is going to be the dominant resistance. That is the purpose of having that vessel in which you are storing the liquid nitrogen. You have a vacuum between in the annular space, you silver those surfaces so you try to reduce the radiant heat flow rate; that is your dominant resistance.

Therefore, what we are going to do is we are going to say this is so dominant a resistance that the other resistances are insignificant. So, although I have a situation like this, across the annular space starting from the outside, a resistance here, a resistance by conduction here, then the radiant heat flow dominant resistance, then a resistance by a conduction here, then a convective resistance, I say the dominant resistance is in this vacuum and therefore I can assume that the temperature - if it is the dominant resistance - I can take this temperature to be 303 and these other temperature at the inner surface to be 77 Kelvin.

I will say since it is the dominant resistance all the temperature drop really takes place across this dominant resistance and therefore take the temperature of the outer surface of the inner shell to be 77 and the temperature of the inner surface of the outer shell to be 303. If I do that then it is just a substitution into the expression we have derived. We will get  $q_1$  is equal to, now I am substituting  $\sigma A_1 T_1^4$  to the power 4 minus  $T_2$  to the power 4 the whole thing divided by  $1/\epsilon_1 + A_1/A_2 (1/\epsilon_2 - 1)$  - just substitute into this expression now, all the data is given. So if you substitute you will get 5.67; let me just show that now.

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$$q_1 = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

$$= \frac{5.670 \times 10^{-8} \times 4\pi \times 0.16^2 \times (77^4 - 303^4)}{\left( \frac{1}{0.02} \right) + \left( \frac{16}{18} \right)^2 \left( \frac{1}{0.02} - 1 \right)}$$

$$= -1.73 \text{ W}$$

$$\text{Evaporation rate} = \frac{1.73 \times 3600 \times 24}{201 \times 1000} = 0.74 \text{ kg/day}$$

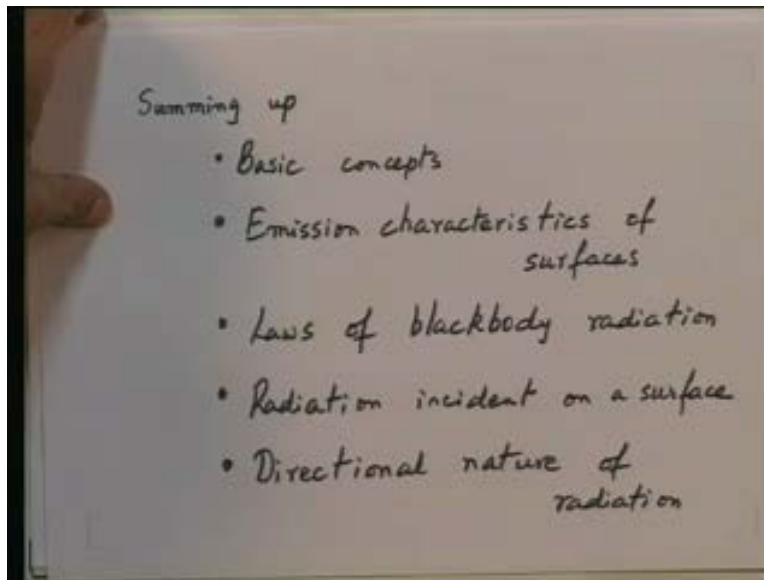
Substituting you will get - 5.6 7 into 10 to the minus 8 to 4 pi into .16 square that is  $A_1$ ; 77 to the power 4 minus 303 to the power 4 divided by 1 upon .02 plus 16 by 18 the whole square - that is  $A_1$  upon  $A_2$ . 1 upon .02 minus 1 which is minus 1.73 watts and if I want the evaporation rate, I find out how much evaporates during the whole day and divided by the latent heat and I get .74 kilograms per day.

Now, I want you on your own to show that the resistances which we ignored, that is the convective resistances and the 2 convective resistances and the 2 conductive resistances are in fact insignificant. We have studied conduction; you know the values of  $h$  that you

would get in convection, you should be able to show that in fact they are insignificant and that the temperature drops across those would be insignificant compared to the temperature drop that we encounter in the annular space.

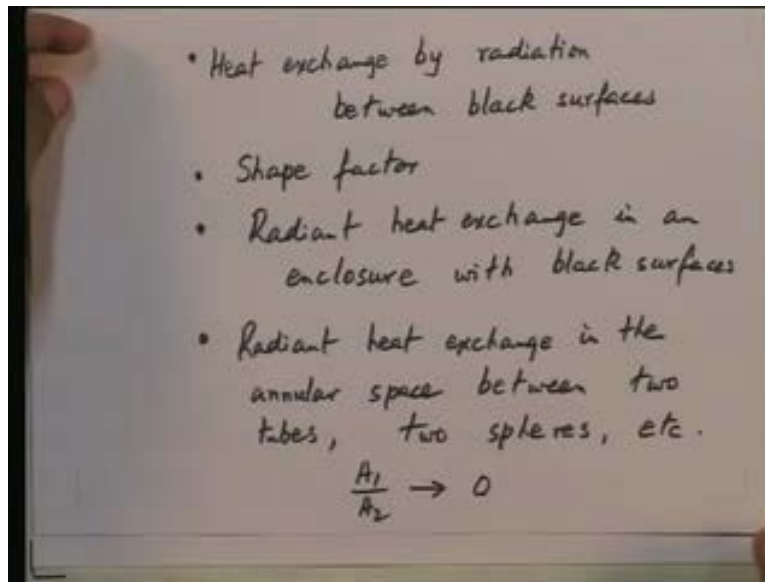
Now, we have come to the end of what we wanted to do in radiation, let me just sum up the, sum up the topics that we have covered in radiation.

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We have covered first basic concepts, then we have covered emission characteristics of surfaces, then we have covered the laws of black body radiation. Then we have talked about radiation incident on a surface, what happens to it - it is absorbed, it is reflected, etcetera - and defined terms like absorptivity, reflectivity, etcetera. Then we have talked about the directional nature of radiation and defined terms like solid angle and intensity of radiation.

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Having done all this, we then calculated, derived expressions for heat exchange by radiation between black surfaces, heat exchange by radiation between black surfaces, in the process defined an important terms called the shape factor; then went on to radiant heat exchange in an enclosure, in an enclosure with black surfaces and finally we talked about radiant heat exchange in the annular space between cylindrical, between 2 tubes, 2 spheres, etcetera and we took the special case finally today of  $A_1$  upon  $A_2$ . The last thing which we looked, that was the special case when  $A_1$  upon  $A_2$  tends to 0.

Now, we have come to the end of what we wanted to do with radiation and we will now be turning our attention to the third mode convection, studying first a little fluid mechanics and then studying convection.