Heat And Mass Transfer
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Lecture No. 14
Thermal Radiation-5

So far we have calculated the radiant heat exchange between black surfaces - 2 black surfaces at 2 different temperatures - and in the process of deriving expressions for the radiant heat exchange between 2 black surfaces, we have defined a term called as the shape factor. Now today we would like to extend these ideas to first calculate radiant heat exchange rates in an enclosure consisting of a number of black surfaces. Consider the following enclosure as shown here.

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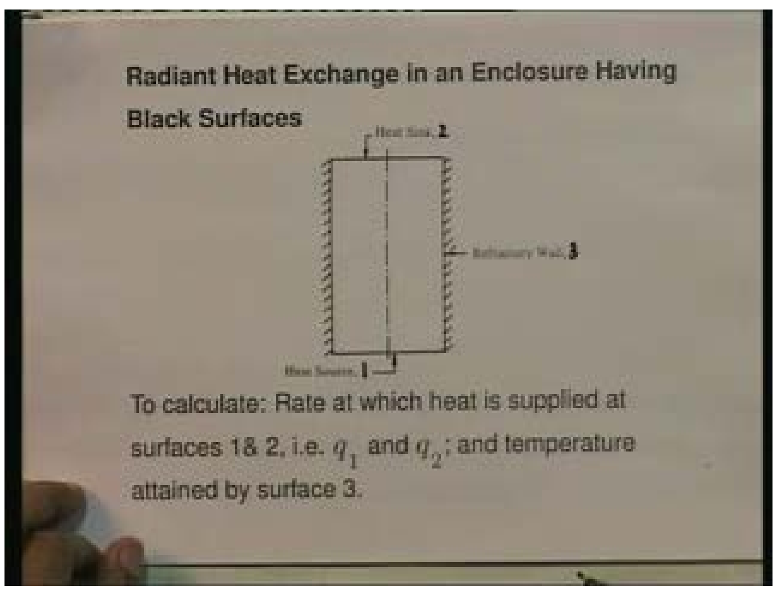

This is A_3 surface enclosure; it could be, it is shown as a cylindrical enclosure but it could be any 3 surface enclosure. The surface 1 is this surface, this surface; surface 2 is this surface and surface 3 is the round surface, the curved surface of the cylinder. Now we have told in this case that the surface 1 is at a temperature T_1, the surface 2 is at a temperature T_2 - is at a temperature means is maintained at a temperature T_2 and surface

3 is an, in perfectly insulated wall, the curved surface is perfectly insulated. It has refractive material all round and is perfectly insulated and we call that a surface 3.

We assume arbitrarily that surface 1 is at a higher temperature than surface 2. It could be any way; it just, I am taking, making just an assumption. Therefore, obviously the surface 1 is going to be the surface at which heat is going to be supplied and surface 2 is going to be the heat surface at which heat is going to be received because it will be at a lower temperature. So T_1 is the temperature of the surface 1 and T_2 temperature of surface 2. q_1 is the rate at which heat is supplied at surface 1 and q_2 the rate at which heat supplied at surface 2. We would like to find in this case the values of q_1 and q_2 and also find what is the temperature attained by the wall 3 - the curved wall 3.

Now again repeat - although I have shown a cylindrical enclosure, it could by any 3 surface enclosure; it doesn't have to a cylinder at all. So let me again just put that down what is specified and what is to be found in this particular case.

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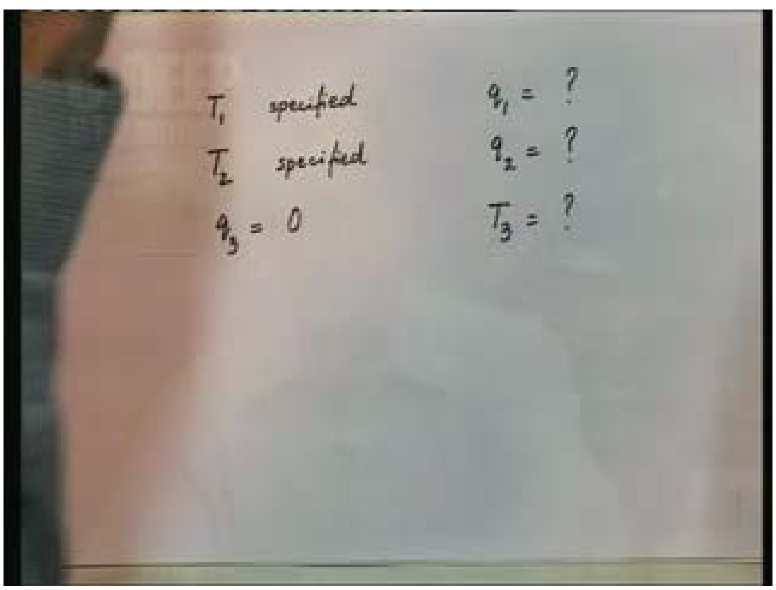

T_1 is specified, the temperature T_1 is specified; surface 1 is maintained at this temperature T_1. T_2 is specified and it is specified that at the surface 3 we have very good insulation. Therefore it is specified that q_3 - the rate at which heat flows at the surface 3 - would be 0 because it is perfectly insulated. These are all specified quantities; what are we asked to find? We are asked to find q_1 - the rate at which heat is supplied at the surface 1. It orders that it should remain at the temperature T_1, q_2 rated which heat is supplied at the surface 2 in order that it should remain at the temperature 2 and T_3 the temperature attained by the surface 3 so that it is, it satisfies the condition of being a perfectly insulated surface and this is the problem as is posed.

All the surfaces are black; I repeat it is A_3 surface enclosure with the all surfaces black. Now let us use the expression which we have derived for heat exchange between black surfaces. In order to derive the q, well expressions for q_1 q_2 and T_3, let us use the expression which we have derived the earlier for heat exchange rate between two black surfaces.

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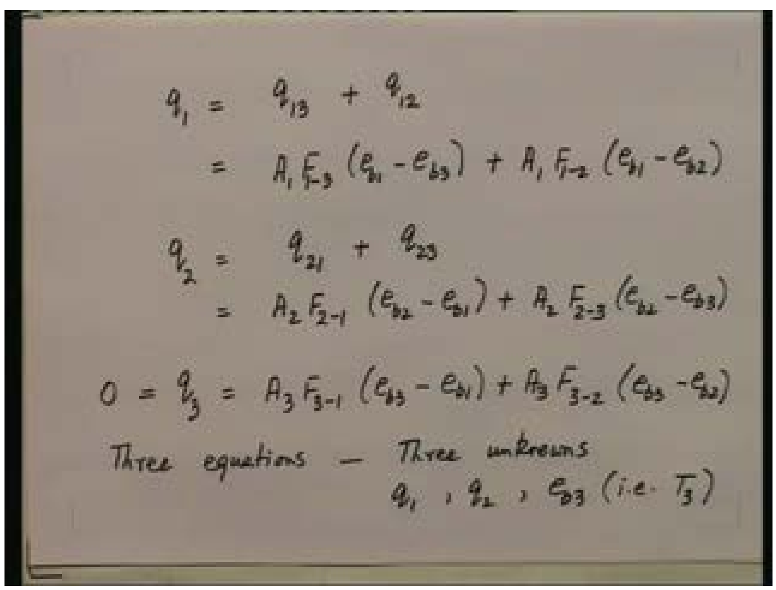

q_1 the rate at which heat is supplied at surface 1 is obviously equal to q_{13} plus q_{12} and that in turn what is q_{13}? q_{13} is nothing but A_1 F $_{13}$ F $_{13}$ e_{b1} minus e_{b3} - this is the expression which we have derived earlier - plus q_{12} is nothing but A_1 F $_{12}$ e_{b1} minus e_{b2}. I am writing e_{b1} e_{b2} and e_{b3} instead of writing everywhere sigma T_1 to the power 4 or sigma T_2 to the power of 4 or sigma T_3 to the power of 4; that is really what e_{b1} e_{b2} and e_{b3} represent. Instead of keeping on writing the bigger expression, I will write e_{b1} e_{b2} e_{b3}; at the end I will substitute the Stefan Boltzmann law.

So, the rate at which I supply a heat at the surface 1 must be equal to the heat exchange rate between 1 and 3 and the heat exchange rate between 1 and 2. Similarly q_2 must be equal to q_2 1 plus q_{23} which is equal to A_2 F 2 2 1 e_{b2} minus e_{b1} plus A_2 F_{23} e_{b2} minus e_{b3} and finally for the surface 3 I can say q_3 is equal to A_3 F_3 1 e_{b3} minus e_{b1} plus A_3 F_3 2 e_{b3} minus e_{b2}. And we know that q3 is equal to 0; so I have 3 equations now: the first, this equation, the second of third equation. I have 3 equations and 3 unknowns; what are my 3 unknowns? q_1, q_2 and e_{b3}. e_{b3} is nothing but, e_{b3} is nothing but T_3; instead of writing sigma T_3 to the power of 4, I am writing e_{b3}.

So, I have got three linear equations in 3 unknown; solve for them and I will get the quantities q_1 q_2 and e_{b3}. So it is a very straight forward problem which we have. If you solve these equations, we will get now, I am skipping a few simple algebraic steps and giving the solution.

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$$q_1 = -q_2$$

$$= (e_{b1} - e_{b2})\left[A_1F_{1-2} + \frac{A_1F_{1-3}F_{3-2}}{F_{3-2}+F_{3-1}}\right]$$

$$T_3 = \left[\frac{F_{3-2}T_2^4 + F_{3-1}T_1^4}{F_{3-2}+F_{3-1}}\right]^{\frac{1}{4}}$$

I am saying if you solve those 3 equations for the 3 unknowns q_1 q_2 and e_{b3} that is T_3, we will get q_1 is equal to minus q_2 which is equal to e_{b1} minus e_{b2} in to A_1 F_{12} plus A_1 F_{13} F_3 2 divided by F_{32} plus F_{31} and the third unknown T will be equal to F_3 2 T_2 to the power of 4 plus F_3 1 T_1 to the power 4 divided F_{32} F_{31} the whole thing within square bracket to the power 1 by 4 - this is the solution to the problem, this is what we are looking for. So you can see 3 surface enclosure where all the surfaces are black and the temperature is specified or the quantity q is specified at that surface. Then we get as many equations, there are surfaces for the enclosure and we solve for the required number of unknowns; that is all that it really comes to.

Now in this particular case, as I said this is the general situation that we have solved for. Suppose I have a 3 surface enclosure in which F 1 to 1 is equal to F 2 to 2 is equal to 0

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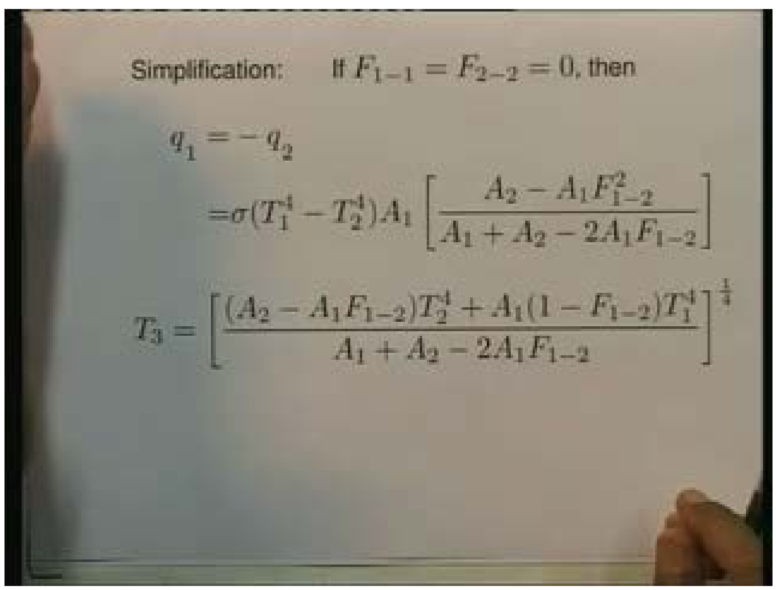

That means I have a three surface enclosure like the one I showed at the beginning. That means the cylindrical enclosure which I showed - let me show it again. Suppose I had such an enclosure in which one is the flat surface, 2 is a flat surface therefore F 1 to 1 and F 2 to 2 are automatically 0.

Let us say that is the enclosure that I have, then one can in fact show quite easily that expressions which we derived for q_1 and q_2 will simplify to the following form. And this again is a matter of just little algebra so I am skipping a few lines here I would like you to proceed from the earlier solution which I displayed again. This is the earlier solution; in this solution, assume in this solution, add the extra knowledge that F 1 to 1 and F 2 to 2 are 0 and then show that this solution reduces to this solution now.

The simplified solution q_1 equal to minus q_2 equal to sigma into T_1 to the power of 4 minus T_2 to the power of 4 A_1 within square bracket A_2 minus A_1 F 1 to 2 the whole square divided by A_1 plus A_2 minus 2 A_1 F 1 to 2. Notice know the only unknown is A F 1 to 2; all other shape factors are eliminated because they are related in some way to F 1 to 2 through the enclosure relationship which we mentioned earlier.

We can show further that T_3 will come out to be equal to A_2 minus A_1 F 1 to 2 the whole thing multiplied by T_2 to the power of 4 plus A_1 into bracket 1 minus F 1 to 2 close a bracket T_1 to the power of 4 divided by A_1 plus A_2 minus 2 A_1 F 1 to 2 the whole thing 2 to the power of 1 upon 4. So we get the simplified expressions if we have an enclosure in which F 1 to 1 is equal to F 2 to 2 is equal to 0.

So, we have a general situation which we have solve for; this is the more specific situation in which F 1 to 1 is equal to F 2 to 2 is equal the 0. Now in order to just clear up ideas, let us do the following problem. I am going to solve the following problem just to, it is a really just a substitution; let us say we have an enclosure like this.

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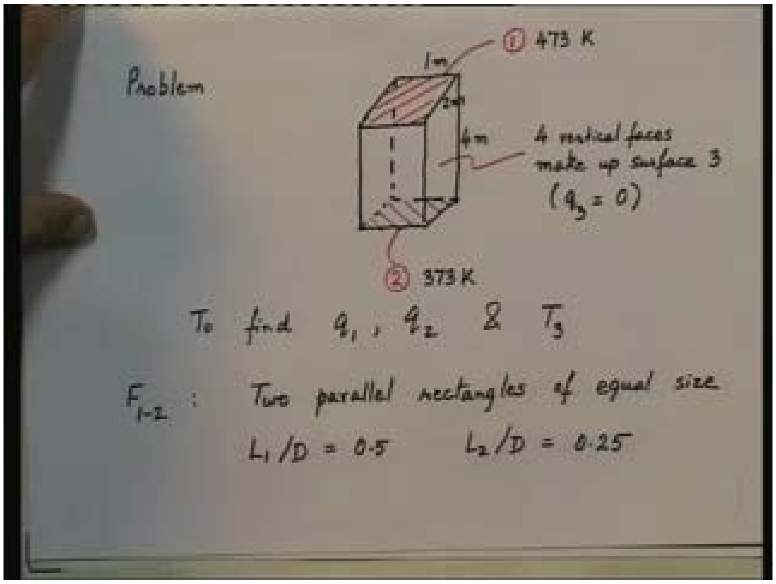

Let us say we have a rectangular enclosure like this - this is the rectangular enclosure that we have and in this rectangular enclosure you are told that the sides are 1 meter by 2 meters; the sides of the horizontal faces are 1 meter by 2 meter and the vertical edges are all 4 meters. You are further told, let us say that the top surface, the horizontal face at the top, this one is the surface 1. This is the surface 1 maintained at a temperature of 200 degrees centigrade, 473 K. Let us say the horizontal face at the top, the 1 by 2 meter face at the top which is maintained at a temperature of 473 K is this one now. The bottom face which I will shade again this 1, this is surface 2 this is surface 2 and this is at a 100 degree centigrade that is 373 K and you are told that the 4 vertical faces make up surface 3. The 4 vertical faces that is these - the 4 vertical faces make up surface 3, 4 vertical faces make up.

That is the 3 surface enclosure now because we are specifying the top horizontal face is surface 1, the bottom horizontal face is surface 2 and all the 4 faces together which are vertical make up what we call as a surface 3 and that is perfectly insulated. Therefore for those 4 faces q_3 is equal to 0 - perfectly insulated surfaces. To find q_1, q_2 and the temperature attained by the surface 3 that is T_3; so it is a direct substitution into the expression that we just derived and the first thing we will need to know of course is the shape factor F 1 to 2 between the top face, the top horizontal face and the bottom horizontal face of this rectangular enclosure. So let us first find out F 1 to 2.

Recall in the previous lecture, we had given we had shown a figure in which the values of a F 1 to 2 are plotted. So what do we need for that figure? We have a situation of 2 parallel rectangles; this is a situation of 2 parallel rectangles of equal size. In order to read of the value F 1 to 2 from the available graphs, we need the values of L_1 by D and L_2 by D and in this case L_1 by D will be .5; it is 2 by 4 and L_2 by D will be 1 by 4 that is .25. So now go to the graphs.

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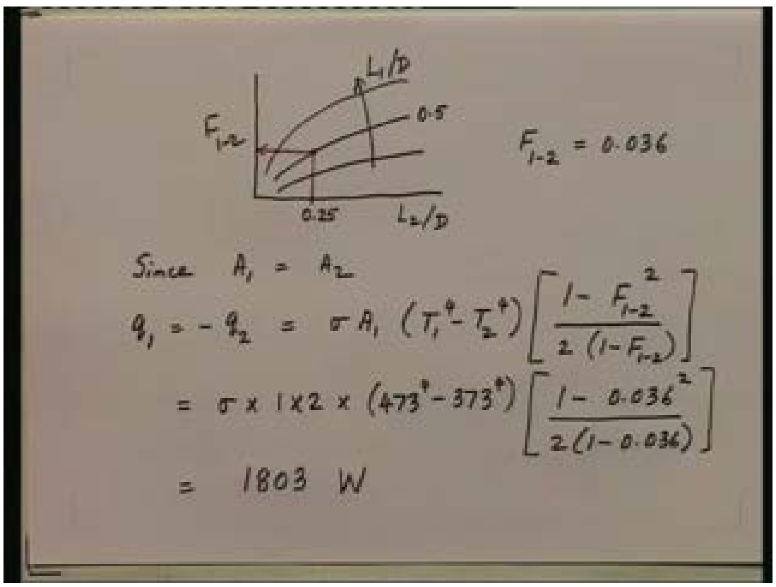

The graphs are - if you go to the available graphs, they are, give the value of F 1 to 2 plotted. All we have is a plot like this F 1 to 2 is plotted against L_2 by D and there are different graphs like this for different values of L_1 by D. So in this particular case, look up the value of h, go to the value of .25 here for L_2 by D. Go up vertically to the graph corresponding to L_1 by D equal to .5 then go horizontally and this is our value of F 1 to 2. And in this particular case, we will get, if you look up the graphs which I want you to do on your own. Go to any text book, any of the references that I have given you and you will read off the value F 1 to 2 in this case is to be .036.

Now, let us substitute into the expression that we have for q_1 and q_2; we have also in this particular case A_1 is equal to A_2. So since A_1 is equal to A_2, our expression for q_1 equal to minus q_2 in this particular case becomes sigma A_1 T_1 to the power 4 minus T_2 to the power 4 within square bracket 1 minus F 1 to 2 the whole squared divided by 2 into 1 minus F 1 to 2. The earlier expression that I have given you in that expression - put A_1 equal to A_2 and you will get this for q_1 equal to minus q_2 equal to this. And note I haven't mentioned it earlier but note q_1 is equal to minus q_2; it has to be all right why because there are 3 surfaces making up the enclosure.

In a steady state q_1 plus q_2 plus q3 must be equal to 0 - the first law thermodynamics requires that. Net rate at which heat enters through all the 3 faces must be equal to 0. q3 is equal to 0 in this case of, q_1 must be equal to minus q_2. Now in this particular case substitute now the given numbers; that means substitute the value of sigma into A_1 which is 1 by 2 in to 473 to the power 4 minus 373 to the power 4. The whole thing into 1 minus .036 the whole squared 2 multiplied by 1 minus .036. And if you calculate the value, you will get 1803 watts. So the rate at which you supply heat at the surface 1 will be 1803 watts.

The rate at which you extract heat at the surface 2 is 1803 watts and that is what maintains the steady state in that enclosure and further if you calculate T_3 in this case, you will get T_3 is equal to again making use of the fact that A_3 A A_1 is equal to A_2, T_3 becomes T_1 to the power 4 plus T_2 to the power 4 divided by 2 the whole thing to the power of one fourth.

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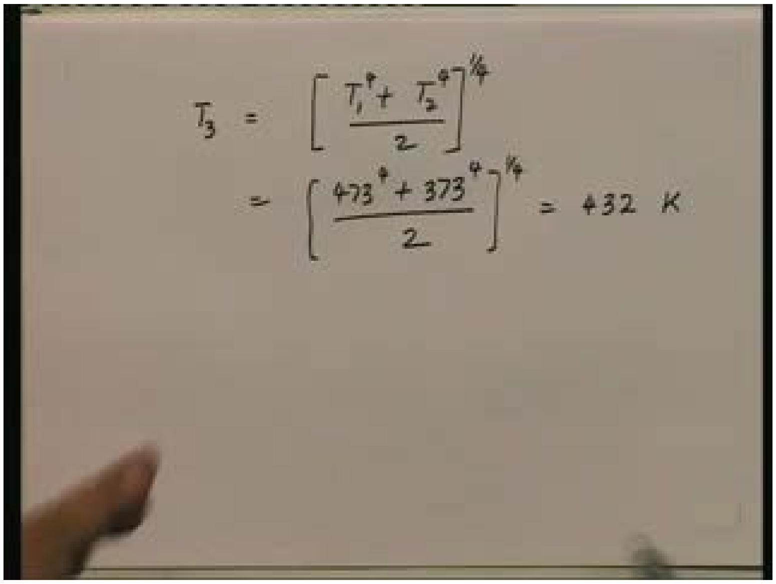

So it is nothing but 473 to the power 4 plus 373 to the power of 4 divided by 2, the thing to 1 4th which is equal to 432 K. T_3 must be a temperature between T_1 and T_2; quite obviously surface 3 has to attain a temperature somewhere between the 2 extremes. 1 is the surface at which we are supplying heat, 2 is the surface at which we are extracting heat; surface 3 must acquire a temperature in between - within this case comes out to be 432 K. So that is the way we would use the results that we have got for the expression that we have just derived.

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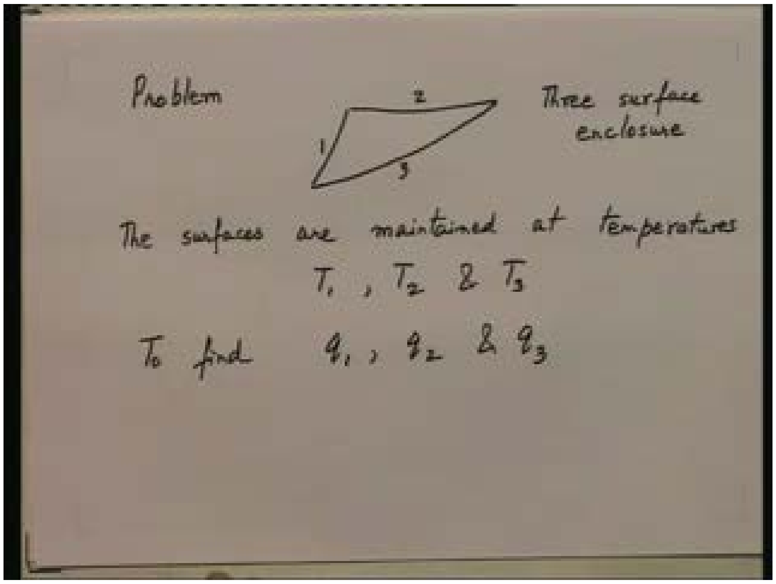

Now let us go on; let us say, let me just change the problem a little and say, suppose I have some 3 surface enclosure; I am just drawing some arbitrary 3 surface enclosure like. Suppose I have a 3 surface enclosure of a given geometry so I can get the values of all the shape factors in that enclosure and suppose the surfaces are maintained, suppose the surfaces are maintained at temperatures, at temperatures T_1, T_2 and T_3. The 3 surfaces are maintained at temperatures T_1, T_2 and T_3. Calculate to find q_1, q_2 and q3; do this one your own, there is nothing to it. It is just the same earlier 3 expressions that we had but you know now in this case the values of eb_1 eb_2 eb_3, so you can directly from 3 equations which we have put down earlier get the values of q_1 q_2 and q_3.

So, the point I want to make is when you have for any one of the surfaces, you need to specify either the rate at which heat is supplied at the surface or the temperature at the surface, either the q for the surface or the T for the surface and you can find if one is the specified, the other can be found out. So keep that always in mind; be it any 3 surface enclosure, if I specify either q or T for it, the other quantity which is not specified can be determined. That is basically what we can do with the help of the 3 equations for the 3 surfaces.

Once I have said that for 3 surfaces it is very obvious to you that I can extend these ideas to an N surface enclosure. I could have a 4 surfaces in which case I will get 4 equations, I could have 5 surface enclosure in which case I will have 5 equations. And in each case ask you self - is q specified or is T specified? Depending upon that one is the known, the other is the unknown and we solve for as many unknowns, as many q which are unknowns or as many T which are unknowns given the other quantities which are given to, given the other values which are known.

So, now basically we have got with a theory for calculating heat exchange rates in a black surface enclosure. We have really derived expression for A_3 surface enclosure but I think it is obvious to you that extending these ideas to a 4 surface or 5 surface enclosure presents no real problems. The only thing we must able to calculate is the shape factors for this enclosure. Whatever be the enclosure, you must be able to get the appropriate shape factors; then you can put down a set of equation, then get the values of q or T for each surface of the enclosure.

Now, we want to move on to enclosures which have gray surfaces rather diffused gray surfaces. Now we want to go on to enclosures which have not black surfaces but diffused gray surfaces and what we are going to do first is

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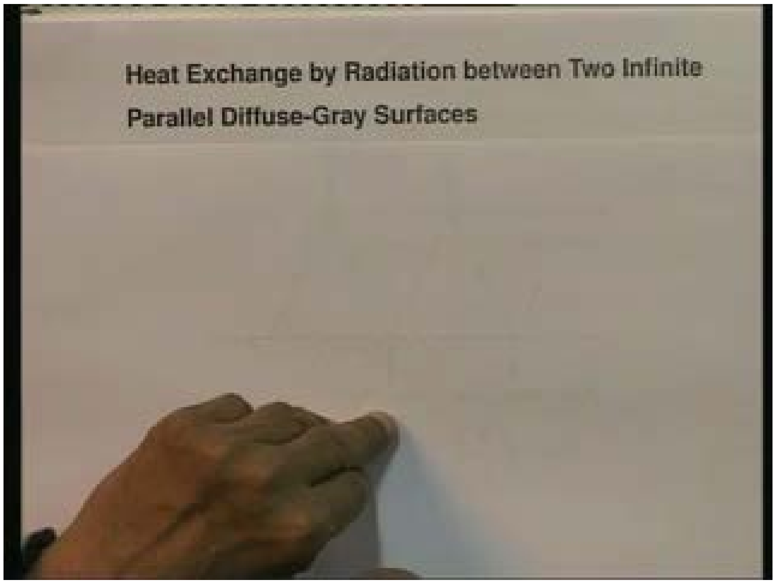

We are going to calculate heat exchange rates by radiation in an enclosure which consists of only 2 surfaces. What are the 2 surfaces, 2 infinite parallel diffused gray surfaces we are going to consider such an enclosure? An enclosure which has 2 surfaces and the 2 surfaces are 2 infinite parallel surfaces which are diffused gray. We have taken a two surface enclosure because it is the simplest to handle first of all and then we will go on to a few more complicated cases later.

So, the first thing we are going to look at now is heat exchange rates by radiation between 2 infinite parallel diffused gray surfaces that is effectively a two surface enclosure made up of 2 infinite parallel surfaces. Now let us, let me just sketch the surfaces.

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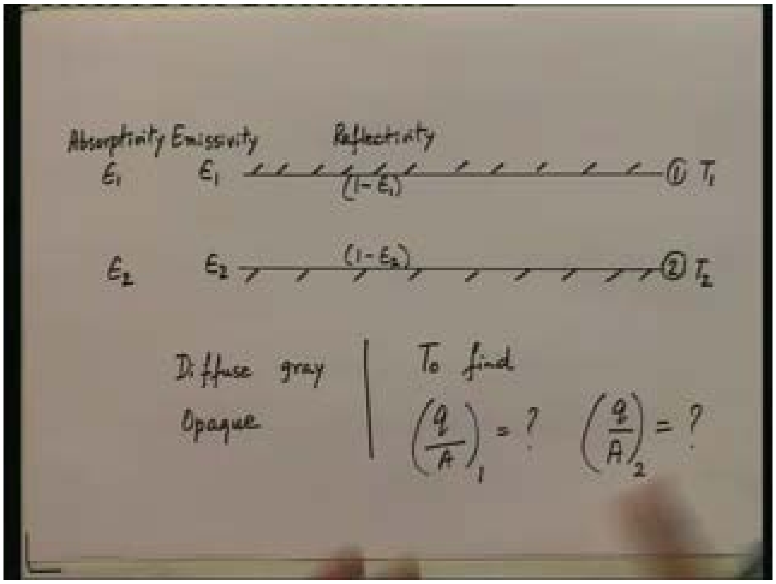

Let us see - these are 2, this 1 surface, this is another surface. These are 2 diffused gray surfaces, infinite surfaces parallel to each other and obviously they make up a two surface enclosure because they are infinite in extent. I will call this as the surface 1 and I will call this as the surface 2 and let us say that the surface 1 is maintained at temperature T_1, the surface 2 at temperature T_2. These are diffused gray surfaces and further let us say they are opaque, further let us say they are opaque surfaces. Let the emissivity of these 2 surfaces, let the emissivities of these 2 surfaces be epsilon 1 and epsilon 2.

It follows from Kirchoff's laws since they are diffused gray surfaces that the absorptivity is also epsilon 1 and epsilon 2; emissivity is equal to absorptivity because they are diffused gray surface and opaque. If they are opaque, it follows that the reflectivity of the surfaces will be 1 minus epsilon 1 and 1 minus epsilon 2, so let me write that also. The reflectivity of these 2 surfaces, this 1 will be 1 minus epsilon 1, this 1 will be 1 minus epsilon 2 because they are opaque therefore absorptivity plus reflectivity must be equal the 1. So, this is the characterization of the surface - a two surface enclosure made up of 2 infinite parallel plates.

Diffused gray surfaces, opaque surfaces maintained at temperatures T_1 and T_2, emissivity and absorptivities are epsilon 1 and epsilon 2, reflectivities are therefore 1 minus epsilon and 1 epsilon 2. We are required to find, to find the rate at which heat is supplied at the surface 1 or the heat flux is supplied at the surface 1 and rate at which heat is supplied at the surface 2 - q by A_1, q by A_2, heat flux supplied at surface 1, heat flux supplied at surface 2.

Obviously, I need notes, repeat what I have said earlier - q by A_1 must be equal to minus q by A_2 that is obvious. So I will be getting both these, the values for both expressions, for both these simultaneously. So we want to find out expressions for q by A_1 - rate at which heat is supplied at the surface 1 and at the surface 2. Now let us do the following in order to derive this expression; let us consider the radiation going back and forth between these 2 infinite surfaces. We are going to consider radiation going back and forth between these 2 surfaces.

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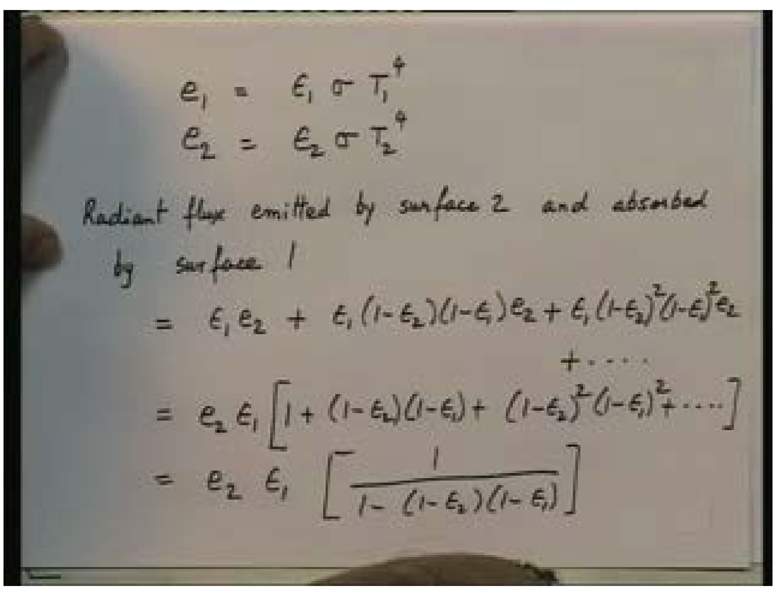

Before I do that, let me first of all state from the Stefan Boltzmann law, the emissive power of surface 1 is epsilon sigma T_1 to the power 4 and the emissive power of surface 2 is epsilon 2 sigma T_2 to the power of 4. So I keep on writing e_1 and e_2 for some time;

right at the end I will substitute these expressions for e_1 and e_2. And as I said, moment ago, let me repeat I am going to analyze what happens to the radiation which is emitted by 1 surface. What is going to happen is any radiation emitted by say the surface 1 or the surface 2 will go to the other surface, get partly absorbed and partly reflected, will come back to the original surface where it will get partly absorbed, partly reflected, will again go back to the first, next surface partly absorbed, partly reflected.

So, you will have successive reflections and absorptions as the radiation is bounced back and forth. Any radiation emitted from 1 surface bounces back and forth and is successively absorbed and reflected. We want to analyze in this manner. So, let us now look at the surface 1 and see how the radiation is absorbed and reflected.

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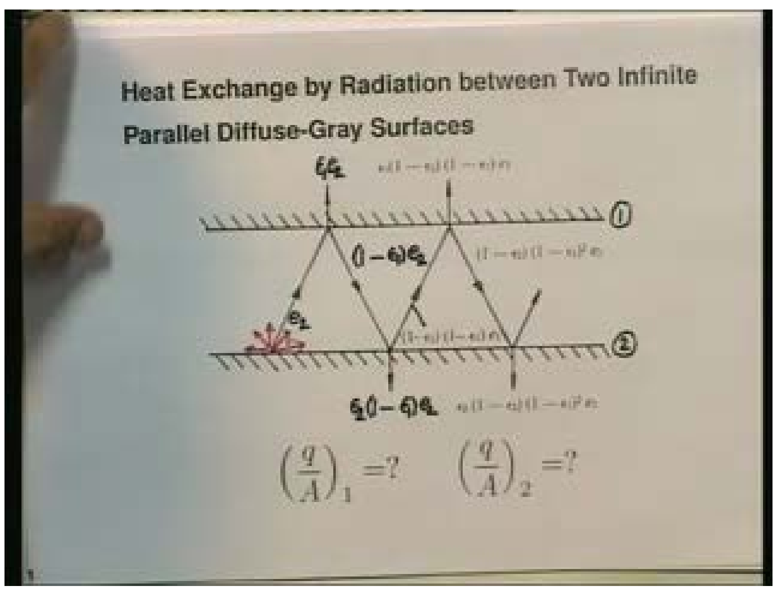

e_2 is the emissive power of surface 2 so many watts per meter squared let us say is being emitted from this surface 2; this is the surface 2. e_2 is its emissive power now I am showing one arrow here but this arrow is symbolic. Actually this radiation emitted from the surface 2 from this element here is, this radiation is really going off in all directions because it is a diffused gray surface. So radiation is really going in all directions like this, all of it ultimately has to reach the surface 1 because 1 and 2 are infinite in extent but I am just showing one arrow to say, to indicate symbolically radiation emitted by the surface 2 and what is its flux? Its flux is e_2 so let me draw it in bold letters here - e_2 so many watts per meter square.

This e_2 when it ultimately reaches the surface 1 not by this arrow but in any, whichever direction, it goes ultimately hits the surface 1. It is going to get absorbed; how much is going to get absorbed? What is going to get absorbed is epsilon1, e_2 – that is the quantity that is going to get absorbed. What is going to get reflected is 1 minus epsilon 1 e_2 – that is the quantity that is going to get reflected. Again out of this reflected amount, a certain amount will come back and get absorbed in the surface 2. What will that be? That will be epsilon 2 into 1 minus epsilon 1 e_2. That is the quantity that will come, come back to the surface 2 and be absorbed by. The balance will get reflected that will be1 minus epsilon 2 into epsilon 1 into e_2 that is this quantity and so on.

So, as you can see - as the radiation goes back and forth, we will have a certain amount absorbed, reflected, absorbed, reflected, absorbed, reflected and so on. So now you say to yourself the radiant flux - if you take an algebraic sum of all this - you can say radiant flux emitted by the surface 2 and absorbed by the surface 1; if you ask yourself what is that amount? Radiant flux emitted by 2, emitted by the surface 2 and absorbed by surface 1 will be equal to the first term, will be epsilon 1 e2 - this is the first quantity. e_2 is emitted, epsilon 1 into e_2 is absorb by the surface 1 now again comes back to surface 2, is reflected from surface 2, again get absorbed plus the next quantity is going to be epsilon 1, 1 minus epsilon 2, 1 minus epsilon 1 e_2 plus the next quantity is going to be epsilon 1, 1 minus epsilon 2 the whole squared 1 minus epsilon 1 the whole square in to e_2 plus.

That is how we are going to get an infinite geometric progression series which is equal to let me just put it properly this is equal to e2 i will take that outside that is a common factor i will take epsilon 1 outside that is common and i will geT_1 plus 1 minus epsilon 2 1 minus epsilon 1 plus 1 minus epsilon 2 the whole squared 1 minus epsilon 1 the whole squared plus so on and if I sum this geometric progression within the square bracket, I will simply get e2 epsilon 1 1 upon 1 minus 1 minus epsilon 2 1 minus epsilon 1.

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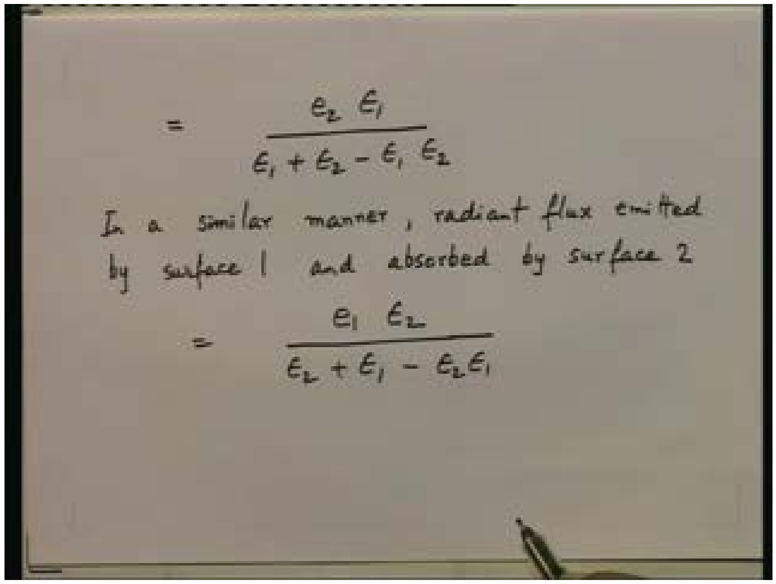

This is the radiant flux emitted by surface 2 and absorbed by surface 1 and I can simplify that expression a little further and say this further equal to, this is further equal to, on simplifying e_2 epsilon 1 divided by epsilon 1 plus epsilon 2 minus epsilon 1 epsilon 2 that is what it will simplify to; which you can check yourself. So this is the radiant flux emitted by surface 2 and absorbed by surface 1.

Now in a similar manner, in a similar manner we can find the radiant flux, radiant flux emitted by surface 1 and absorbed by surface 2. If you do that you will get e_1 epsilon 2 divided by epsilon 2 plus epsilon 1 minus epsilon 2 epsilon 1 - that is what you will get. All that I have done is I have replaced in the previous expression which we derived - I have replaced 2 by 1 and 1 by 2. And I will get this; I don’t need to really derive it and go through a geometric progression.

So, I have now 2 expressions - the radiant flux emitted by surface 2 and absorbed by 1, the radiant flux emitted by 1 and absorbed by surface 2. I take the difference of the 2 and that is the radiant heat flux exchanged between surface 1 and surface 2. So we will get the radiant heat flux exchanged between surface 1 and surface 2 will be given by - let us put that down now.

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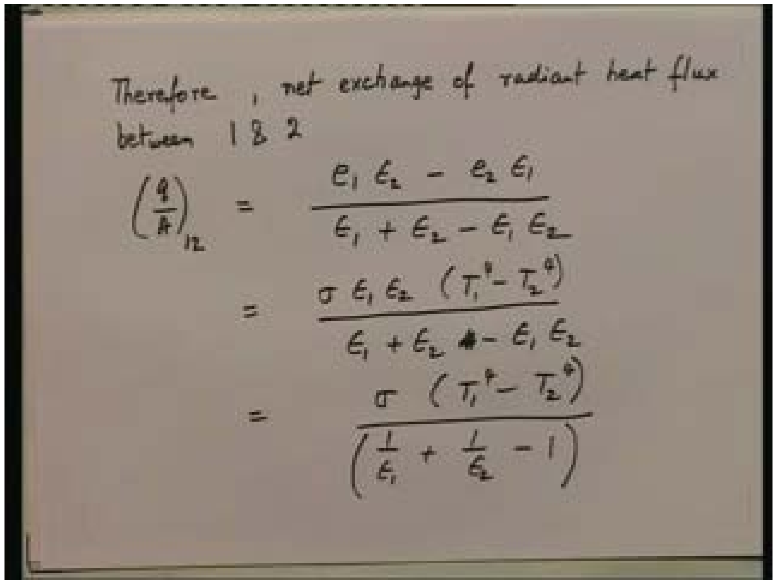

Therefore, the radiant heat flux exchange, net exchange of radiant heat flux, net exchange of radiant heat flux between 1 and 2, between the surfaces 1 and 2 which is nothing but q by A_1 to $_2$ will be equal to the difference of the 2 expressions we had earlier. So I am to going to get e_1 epsilon 2 minus e_2 epsilon 1 the whole thing divided by epsilon 1 plus epsilon 2 minus epsilon 1 epsilon 2. And now if I write the full expressions for e_1 and e_2 that is epsilon 1 into sigma into T_1 to the power 4 and epsilon 2 into sigma into T_2 to the power of 4. I will get this is equal to sigma epsilon 1 epsilon 2 T_1 to the power 4 minus T_2 to the power 4 divided by epsilon 1 plus epsilon 2 minus epsilon 1 epsilon 2 which I can write further as sigma into T_1 to the power of 4 minus T_2 to the power 4 divided by 1 by epsilon 1 plus 1 by epsilon 2 minus 1. This is the expression that we get for q the heat flux - net radiant heat flux exchanged between the surface 1 and the surface 2.

Now, there are 2, only surfaces are in this enclosure so if there are only 2 surfaces, the net radiant heat flux exchanged between the 2 surfaces is equal in magnitude to the net radiant heat flux leaving either of the 2 surfaces. So in this particular case, since there are only 2 surfaces involved q by A_{12} must be equal to q by A_1 which in term must be equal to q minus q by A_2.

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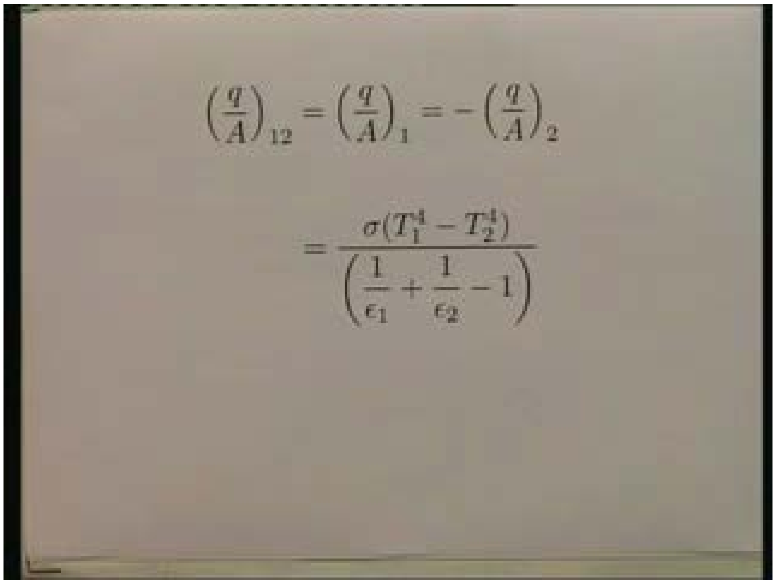

And that in terms is equal to sigma T_1 to the power of 4 minus T_2 to the power of 4 divided 1 by epsilon 1 plus 1 plus epsilon 2 minus 1. So this is the expression that we are looking for the heat supplied or the heat flux supplied at the surface 1 or at the surface 2 and this is a rather well known expression; you should know this - anyone studying radiation has to know this expression - the radiant heat flux exchanged between 2 diffused gray surfaces constituting a system of infinite parallel plates.

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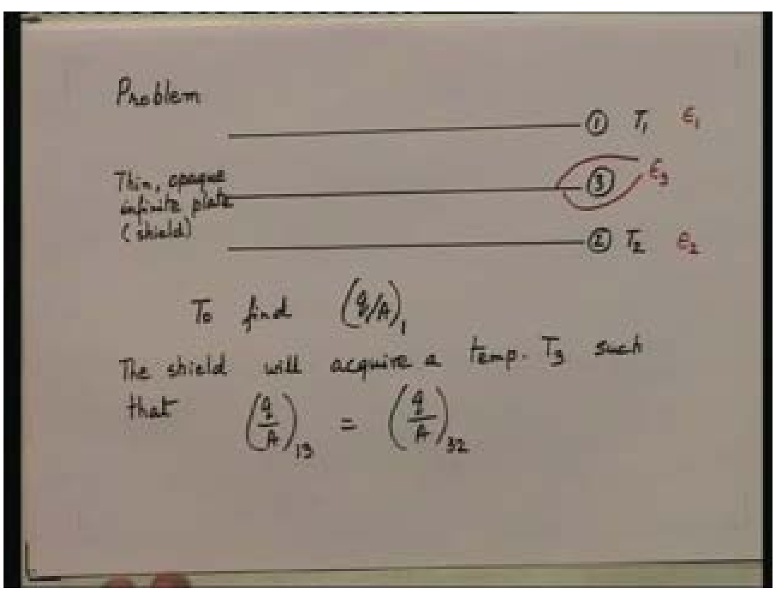

Now we want to look at a problem; we are going to look at the following problem, we will say, suppose I have a surface 1 and a surface 2 like, this is surface 1 and this is the surface 2. Surface 1 is maintained at a temperature T_1, surface 2 is maintained at temperature T_2. They are a system of infinite parallel plates; we know the expression for heat flux exchange between these 2 surfaces. By the, this is the radiant heat flux exchange so in all this we have assumed that there is a vacuum between these 2 infinite parallel plates; we have not looked at any convective heat transfer. So keep in mind we are doing all these calculation as if there is a vacuum in the space between the infinite parallel plates or there is a vacuum in the black surface, 3 surface enclosure which we considered earlier.

Now in this system, let us say we insert a third plate like this in between. Let us say I have inserted between these two a third plate 3 which is a thin opaque infinite, a thin opaque infinite plate. Let us say I insert between 1 and 2 a thin infinite opaque plate 3 and let us say that its emissivity on both surfaces is e_3 - it is a gray surface diffused gray. Surface 1 and 2 are diffused gray of course so their emissivities are epsilon 1 and epsilon 2. So in between, you insert a thin opaque infinite plate which has, which is also diffused gray and has an emissivity epsilon 3.

Now, what is to find - q by A_1; that is the problem, this thin opaque infinite plate incidentally is often referred to as a shield. It is something between 1 and 2 so it is like a shield. Now it is quite obvious that the shield will acquire a temperature T_3; the shield will acquire a temperature T_3 such that q by A_1 will be equal q by A_1 to 3 will be equal to q by A_3 to 2 - that is what is going to happen in the steady state. If I put a shield in between, 3 will acquire a temperature T_3 such that the net heat exchanged between 1 and 3 is equal to the net heat exchange, radiant heat flux exchange between 3 and 2. Now we can put down expressions for these quantities; we just derived them.

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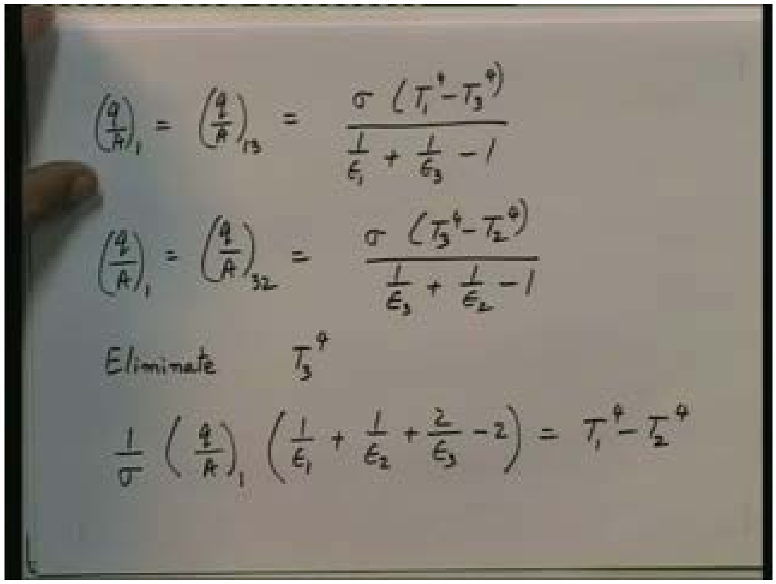

What is q by A_1 to 3? q by A_{13} is equal to sigma; the expression that we have just derived T_1 to the power of 4 minus T_3 to the power of 4 divided by 1 by epsilon on 1 plus 1 by epsilon 3 minus 1 and q by A_2 is equal to sigma T_3 to the power of 4 minus T_2 to the power of 4 divided by 1 by epsilon 3 plus 1 by epsilon 2 minus 1 and q by A_{13} and q by A_2 are equal and what are they equal to? They are nothing but equal to q by A_1 which is what we want to solve for. So we have got two equations now which we have to solve for q by A_1 and for T_3. You got two equations which you want to solve primarily for q by A_1 but we could also get the value of T_3. So since we are interested in q by A_1 eliminate T_3 to the power of 4.

Now, I am skipping a few steps; that means you will have to write this as T_1 to the power of 4 minus T_3 to the power of 4 equal to something; and this equation you will have to write as T_3 to the power of 4 minus T_2 to the power of 4 equal to something. Then add that two expressions in which case you will eliminate T_3 and you will get - I will just put down the final expression. You will get 1 upon sigma q by A_1 into 1 by epsilon 1 plus 1 by epsilon 2 plus 2 by epsilon 3 minus 2 is equal to T_1 to the power of 4 minus T_2 to the power of 4. That is what you will get or finally you want an expression for q by A_1, you will get q by A_1 is equal to in this case

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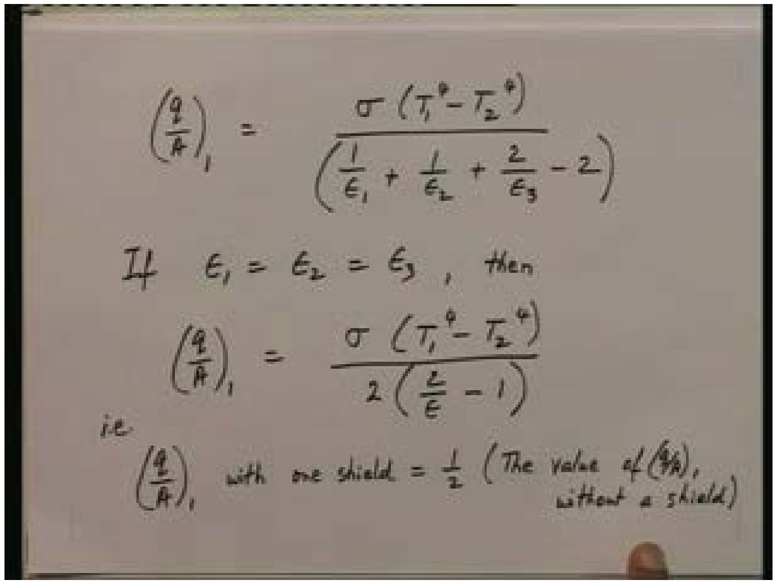

q by A_1 is equal to sigma into T_1 to the power of 4 minus T_2 to the power of 4 divided by 1 by epsilon 1 plus 1 by epsilon 2 plus 2 by epsilon 3 minus 2. That is the final expression for the heat flux - the rate at which heat is supplied at the surface 1 or this also equal to minus q by A_2.

Now, consider the special case if epsilon 1 equal to epsilon 2 equal to epsilon 3. In this case then q by A_1 is equal to sigma T_1 to the power of 4 minus T_2 to the power of 4, 2 times 2 by epsilon minus 1 - that is what you will get. What you have got really is if the epsilons are all equal; what in effect you are saying is q by A_1 - that is what you are saying is - q by A_1 with one shield is equal to half into the value of q by A_1 without a shield. That is what you are getting with one shield, the value of q by A_1 is reduced by half 1 by 2 extend this on your own now.

Suppose I have 2 shields instead of 1, what result will you get? Show that you will get 1 by 3 instead of 1 by 2; if have n shields you should get 1 upon n plus 1 into the value of q by A_1 without a shield. So a shield, one shield reduces the value by half, 2 shields reduce the value by 1 by 3, to what you would get to the value you would get without a shield. So that is the special case if of course epsilons are all equal.