**Heat and Mass Transfer**
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**Lecture No. 13**
**Thermal Radiation – 4**

Today we are going to first look at radiant heat exchange between 2 finite sized black surfaces.

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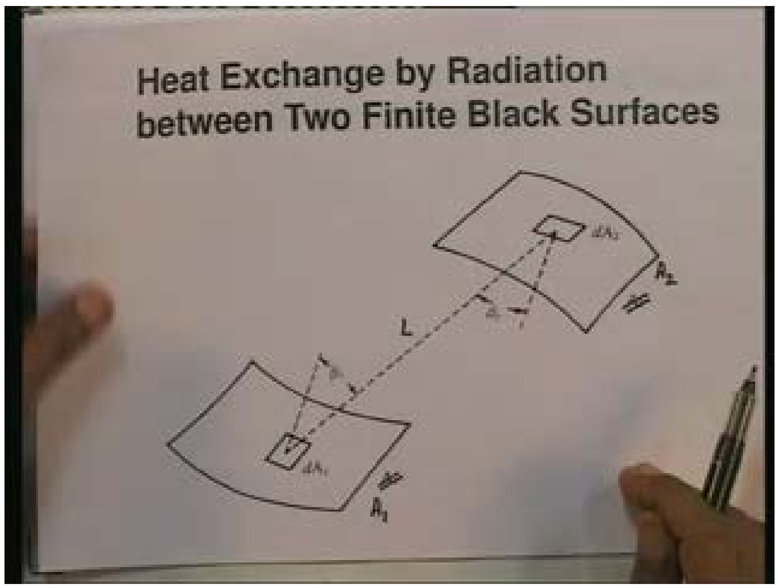


Last time you will recall that we had, when we stopped, we had derived the expression for heat exchange, rate of heat exchange between 2 differential areas $dA_1$ and $dA_2$ at temperatures $T_1$ and $T_2$ and the expression which we had derived was the following.

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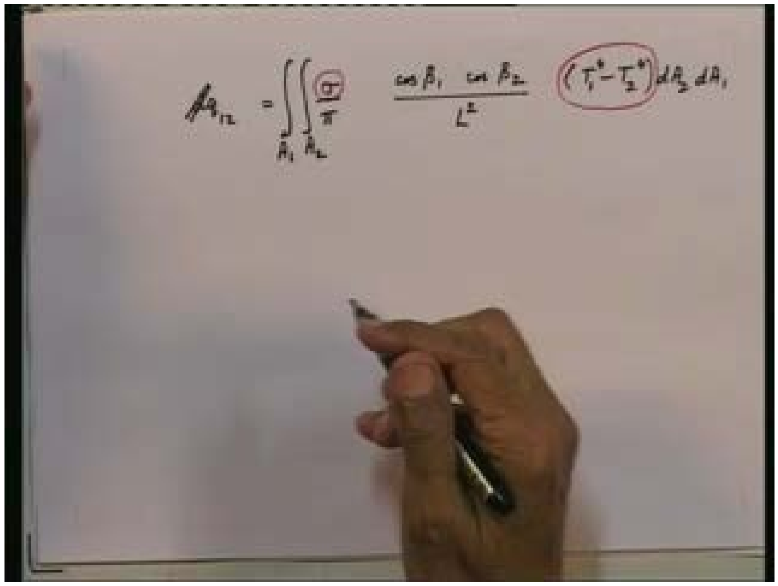

Let me first repeat that the expression which we had derived last time for heat exchange between 2 differential areas $dA_1$ $dA_2$ was dq 1 to 2 is equal to sigma by pi cosine beta 1 cosine beta 2 divided by L squared multiplied by T 1 to the power of 4 minus $T_2$ to the power of 4 $dA_2$ $dA_1$ - that was the expression which we derived now, that is where we stopped, we got this expression. Now we want to extend this expression to the situation where we have 2 finite sized black surfaces maintained at uniform temperatures $T_1$ and $T_2$; that is what we want to do.

So, let us go back, see the sketch again; these are the 2 surfaces - this is surface A with an area $A_1$; this is the surface with the area $A_2$. Let met write this again in big letters, it is rather small here. This is $A_1$, this is the finite area $A_2$; they are both black and $A_1$ is maintained at a temperature $T_1$, $A_2$ is maintained at temperature $T_2$. We would like to know what is the net radiant heat exchange rate between $A_1$ and $A_2$.

Now, suppose I take some differential area $dA_1$ on $A_1$and I take some other differential area $dA_2$ on $A_2$, join the 2 areas by this line and let the length be L like last time; then we know the expression for heat exchange between $dA_1$ and $dA_2$, so let us extend that now.

Obviously, if we want to know what is the value for 2 finite size surfaces and we know the expression for 2 differential areas, it follows that all we have to do is to integrate the expression which we have for differential areas and we will have to integrate over the area $A_1$ and we will have to integrate also over the area $A_2$ which means the expression which I have put down here for $dq_1$ to $_2$.

If I want $q_1$ to $_{2,}$ let me cancel out the d on this side; if I want $q_1$ to $_2$ that is the heat exchanged by radiation between A and A2, I will integrate this expression twice over the area $A_1$ and $A_2$, that is what we will do and that is our expression for $q_1$ to $_2$. Mow $T_1$ minus, $T_1$ is a constant, $T_2$ is a constant so $T_1$ to the power 4 minus $T_2$ to the power of 4 is a constant. I can take it outside the integral; let me also take sigma outside then take, so if I take both these outside the integral which is permissible then I will end up with the expression, if I do that then I will end with the expression and let me show that I will get as is written here.

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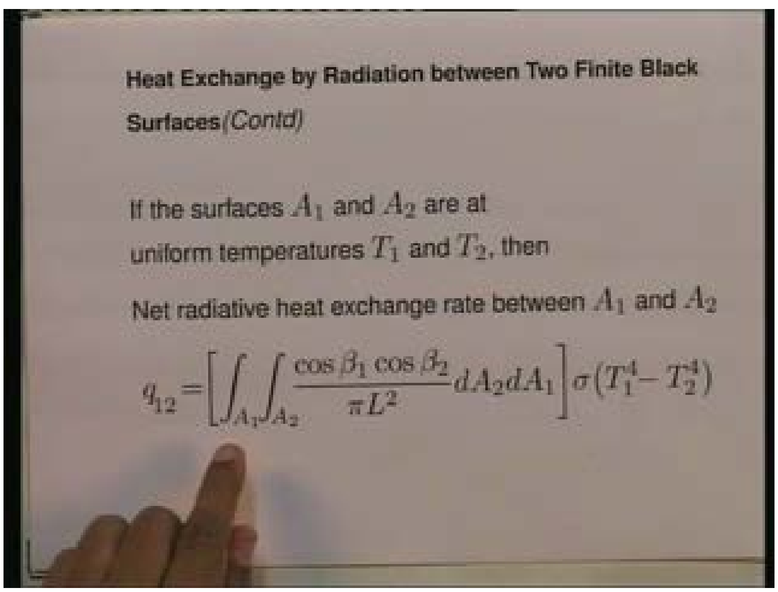


If the surfaces A and $A_2$ are at uniform temperatures $T_1$ and T2, then net radiative heat exchange rate between $A_1A_2$ $q_1$to$_2$ becomes the following - I have just taken sigma and $T_1$ to the power of 4 minus $T_2$ to the power 4 outside the integrals because they are constants and the quantity within square bracket is the integral which I will have to perform first over the area $A_2$ and then over $A_1$ or vice versa; it does not matter. Now let us again like in the earlier situation define a shape factor; this expression is fine but let us define now and put it in terms of a shape factor.

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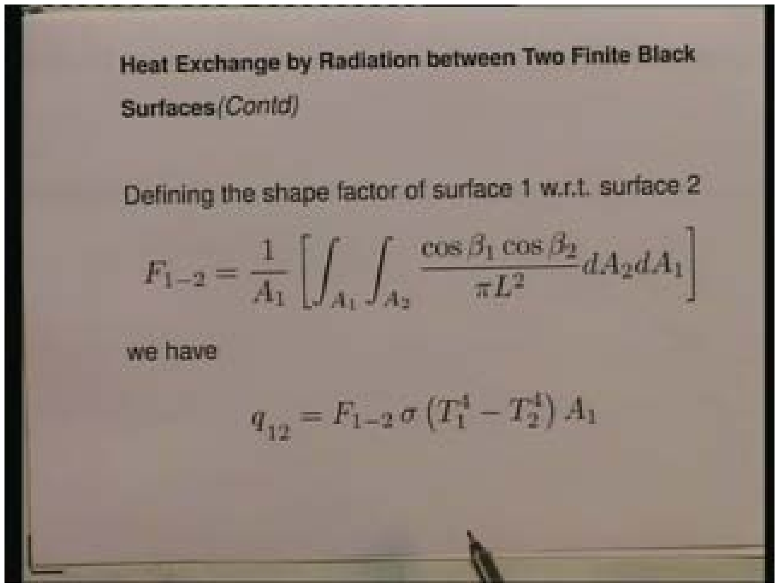


So, we will say, defining a shape factor defining the shape factor of surface 1 with respect to surface 2; let me repeat that defining the shape factor of surface 1with respect to surface 2 $F_1$ to $_2$ defining it as 1 upon $A_1$ the quantity which I had earlier within square bracket all that integral $A_1$ integral over $A_2$ cosine $\text{beta}_1$ cosine $\text{beta}_2$ upon pi L square $dA_2dA_1$. If you use this definition of shape factor, our expression for $q_1$ to $_2$ becomes $q_1$ to $_2$ is equal to $F_1$ to 2 sigma $T_1$ to the power of 4 minus T to the power of 4 into $A_1$ multiplied by $A_1$. Now notice again how we have defined $F_1$ to $_2$. $F_1$ to $_2$ is defined, $F_1$ to $_2$ is the shape factor of 1 with respect to 2, surface 1 with respect to 2 and we have defined it in such a manner that it represents the fraction of the radiation emitted from 1 which is intercepted by 2.

Note that the definition is not just some arbitrary definition; we have defined it in that manner so that if I take the first part of this $F_1$ to $_2$ sigma $T_1$ to the power 4 into $A_1$, you can see sigma $T_1$ to the power of 4 into $A_1$ represents the total amount per unit time emitted from the surface 1. If I multiply by $F_1$ to 2, I get the amount that goes from 1 towards 2 is intercepted by 2 and because 2 is black is also absorbed by 2. So again I am defining shape factor with that physical meaning in mind; it is not some arbitrary definition with an integral in it. So we could also define a reverse shape factor.

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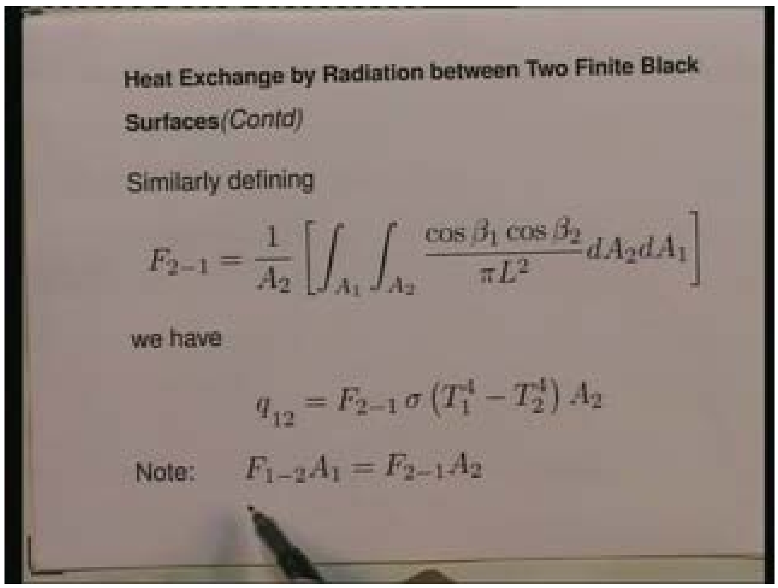


I could define a shape factor in the reverse direction that is from surface 2 to surface 1, I could say F 2 to 1 in exactly in the same fashion in the reverse direction I could say. $F_2$ to $_{1,}$ $F_2$ to $_1$ is the shape factor of 2 with respect to 1, is equal to 1 upon $A_2$ the integral over $A_1$, the integral over $A_2$ cosine $\text{beta}_1$ cosine $\text{beta}_2$ pi L square $dA_2dA_1$ the same quantity within this square bracket that we had earlier. And we have therefore, we could say $q_1$ to $_2$ equal to $F_2$ to $_1$ sigma $T_1$ to the power of 4 minus $T_2$ to the power of 4 multiplied by $A_2$ and we should note again like we did earlier that the 2 shape factors $F_1$ to $_2$ and $F_2$ to $_1$ are related by a reciprocal relation - $F_1$ to $A_1$ is equal to $F_2$ to $_1$ $A_2$. There is a reciprocal relation relating the 2 shape factors, there are 2 independent quantities.

So, we have expressed the quantity $q_1$ to $_2$, we have expressed the quantity $q_1$ to $_2$ which represents the net radiative heat exchange rate between 1 and 2 in 3 ways. First we have expressed it without a shape factor; that was the first expression which we got simply by putting at 2 integrals on the expression for $dq_1$ to $_2$. Secondly, we have expressed it in terms of $F_1$ to $_2$ and said $q_{12}$ equal to $F_1$ to $_2$ sigma $T_1$ to the power of 4 minus $T_2$ the power of 4 multiplied by $A_1$. We have expressed it also in the reverse manner by saying $q_1$ to $_2$ is equal to $F_2$ to $_1$sigma $T_1$ to the power of 4 minus $T_2$ the power of 4 into $A_2$.

We got 3 different ways all equivalent ways of, are the same quantity. Now we want to move ead so we have considered 2 situations. What are they? We have considered heat exchange rate between 2 differential areas $dA_1$and $dA_2$; we have considered heat exchange rate between 2 finite sized areas $A_1$ and $A_2$.

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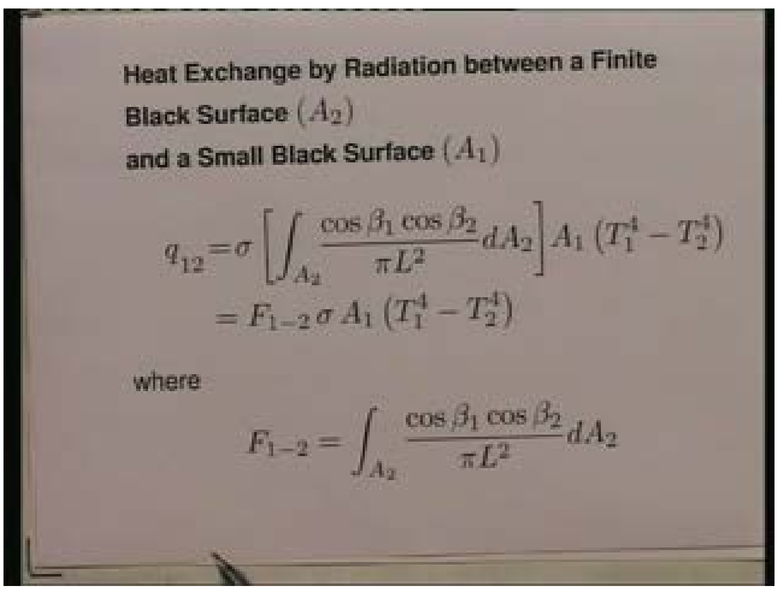


Now we say; suppose we have a situation in between the 2, that means we want to consider heat exchange by radiation between a finite black surface that is one area is big and one is small - small means like a differential area, a small black surface $A_1$ which is like $dA_{1.}$ Suppose we have this situation - a small area and a large area - and we want to calculate heat exchange rate between them, heat exchange by radiation between them if they are maintained at 2 temperatures T and $T_2$. So that is the next situation - the third situation - which is in between the first 2 situations that we have considered and this is rather easy to derive; it should not be a problem.

Let us go back again; so let me again repeat - we are deriving the heat exchange by radiation between a finite black surface $A_2$ and a small black surface $A_{1;}$ that is what we are doing now. $A_1$ can be treated like $dA_1$ - keep that in mind because it is small. So to derive it, go back to our initial expression for $dq_1$ to $_2$; what was our expression?

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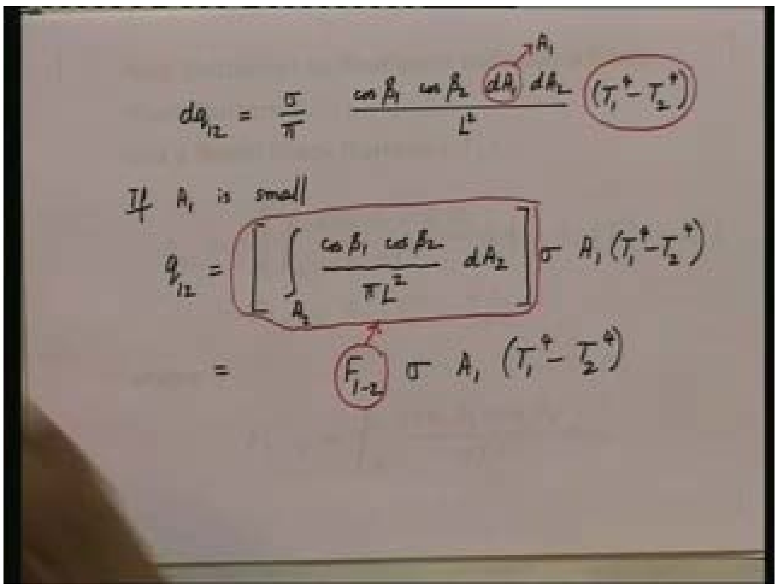


$dq_{12}$ between 2 differential areas sigma by pi cosine beta$_1$ cosine beta$_2$ $dA_1dA_2$ divided by, divided by L squared multiplied by $T_1$ to the power of 4 minus $T_2$ to the power for 4 - that is how our expression was, heat exchange by radiation between 2 differential areas. Now, I mean we want to modify this expression when 1 area is small and 1 is large; the small area is $A_1$ and we can treat it like $dA_1$ - that means I can simply replace $dA_1$ by $A_1$ because it is, it is a small area.

Once again $T_1$ is a constant $T_2$ is a constant so I can take this expression outside the integral sign; so what do we get now? If $A_1$ is small, if $A_1$ is small, I need not perform the integration over $A_1$ and treat $dA_1$ as just $A_1$ and I will simply get $q_1$to$_2$ in this case, equal the quantity within square bracket will be integral over $A_2$. I have to perform an, only an integral over $A_2$ cosine $beta_1$ cosine $beta_2$ divided by pi L squared $dA_2$ the whole thing multiplied by sigma $A_1T_1$ to the power of 4 minus $T_2$ the power of 4 - that is what we will get as the expression for $q_{12}$.

So, all we have done is substituted $A_1$ for $dA_1$ because $A_1$ is small - equivalent to a differential area - taken $T_1$ to the power of 4 minus $T_2$ to the power of 4 outside because it is a uniform there; a uniform temperatures are there so that expression is a constant and the rest we have put within the square bracket obviously is a shape factor so I can say this is equal to $F_1to_2$ sigma $A_1T_1$ to the power of 4 minus $T_2$ to the power of 4. So again now I have an expression for the heat exchange by radiation between 2, between a small surface $A_1$ and a large surface $A_2$ and the expression is given here.

I can use the first expression which I am showing just now or I can use the second expression which is in terms of a shape factor - shape factor $F_1to_2$ being nothing but the quantity within the square bracket. So the shape factor $F_1to_2$ is nothing but the quantity within the square bracket and again it has the same physical meaning; it stands for the fraction of the radiation emitted by 1 which goes towards 2, is intercepted and is intercepted by 2 that is the meaning of the shape factor.

So, you have got 3 expressions for the heat exchange, net heat exchange rate between surfaces, between black surfaces; the surfaces may be differential that means they may be very small surfaces $dA_1$and $dA_2$ - small relative to the distance between them. That is what we mean when we say small or they may be 2 finite sized surfaces $A_1$ and $A_2$, maybe flat, maybe curved. We have put no restrictions on them but at uniform temperatures $T_1$ and $T_2$ and the third situation is - one of the surfaces maybe small, 1 maybe large and we have an, for each of these we have expressions for $q_1to_2$ or in the first situation $dq_1to_2$ because they are differential areas.

Now in all these expressions that we have derived, notice the importance of the quantity which we have defined as shape factor. It keeps recovering all the time and it will therefore be useful for us to spend a little time talking about the shape factor and certain relations between shape factors. So we will spend a little time now talking about that before we move on to doing more heat exchange calculations by radiation. We have done some calculations, we have got expressions for 3 situations of heat exchange by radiation between black surfaces. But I want to spend a little time talking about shape factor, how to calculate it, relations between shape factors before we again move ead to doing calculations for heat exchange by radiation for other situations.

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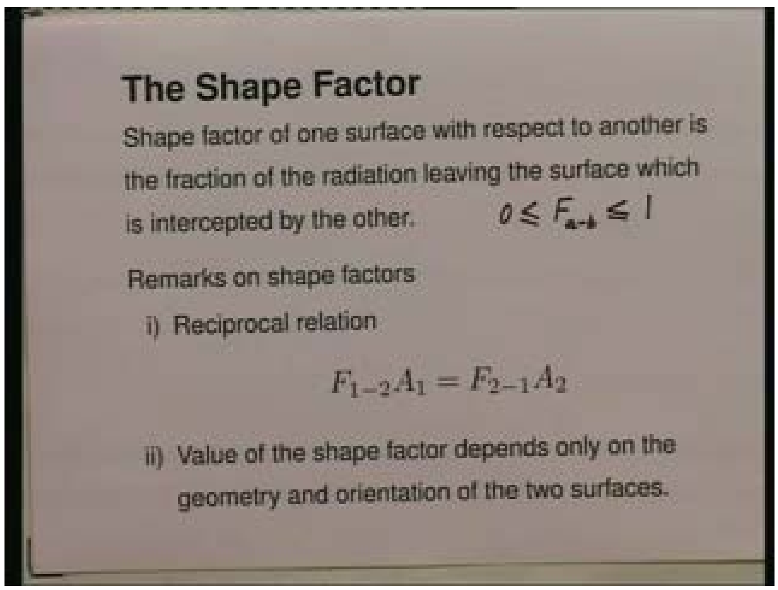


Now let me again define shape factor I have done it earlier but I repeat, the shape factor, it is written here, the shape factor of one surface with respect to another is the fraction of the radiation leaving the surface which is intercepted by the other - that is our definition of shape factor. By definition, shape factor must be between 0 and 1, a number between 0 and 1; so let me write that here. By definition, shape factor F whatever it be between 2 surfaces a and b must be between 0 and 1 - it is it is bounded always by definition.

Now, let us make some remarks on shape factor; the first remark which I have made earlier and I am repeating is that between the shape factors of two surfaces 1 and 2, the shape factor of 1 with respect to 2 is called $F_1$ to$_2$, the shape factor of 2 with respect to 1 is called $F_2$ to$_1$. Between these 2 shape factors, by definition, we have the relationship $F_1$to$_2$ $A_1$ is equal to $F_2$ to$_1$ $A_2$. We have this definition, by definition we have this relation, this is called the reciprocal relation. Now it is an obvious relation because it follows from the definition but the real reason why I am mentioning it is that, to point out the utility of this reciprocal relation.

The relationship, the reciprocal relationship is particularly useful when one of the shape factors either $F_1$ to$_2$ or $F_2$ to$_1$ is unity, then it is particularly useful. Now when does this happen?

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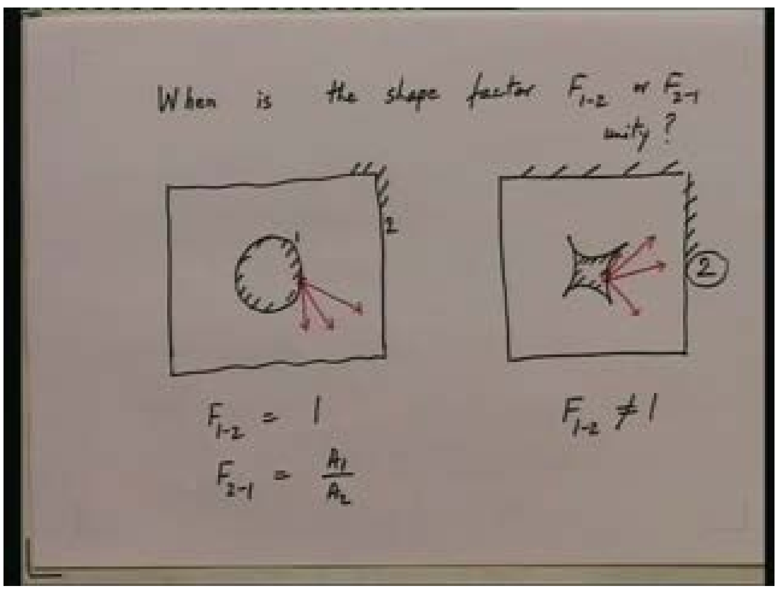


When is the shape factor $F_1$ to $_2$ or $F_2$ to $_1$ equal to unity, when? If you ask the question, you obviously if all the radiation emitted by 1 by the surface 1 is intercepted by 2 and one of it goes back to 1, mind you then the shape factor of 1 with the respected 2 is 1 – unity. Or the reverse F 2 to 1 will be unity when all the radiation emitted by 2 is intercepted by 1, it goes nowhere else. It doesn't even go to 2 and when does this happen? This will happen when the surface 2 of, I am talking now of F 1 to 2, F 1 to 2 will be unity when the surface 2 completely surrounds 1 and the surface 1 is completely convex.

Let me given an example; suppose I have a sphere. This is a sphere and let us say sphere or a spherical shell, it doesn't matter and it is at a, I call it surface 1 and let us say this sphere is inside this room. It is a sphere of any size inside this room so let me draw this room here; this is a rectangular room so let us say this is the room. It has it is a rectangular room with 6 faces on this paper, I am drawing only 4 but remember it has 6 faces so I will call this as the surface 2 the room. Now consider radiation emitted from the surface of 1 - surface 1 is spherical, it is a sphere. Radiation emitted from any element on surface 1 anywhere, consider that radiation emitted from the surface anywhere on surface 1 has to first hit 2, it has no choice therefore in this case F 1 to 2 is equal to 1.

But mind you, suppose just to illustrate to you, 2 surrounds 1 but that is not enough; no radiation from 1 should go back to 1 itself. Consider for instance that the object inside that is 1 is not a sphere but a shape something like this, let us say this is 2, this is the surface 2 still which is this room. But consider that surface 1 instead of being convex as a spherical shape is, suppose it had some shape like this. Instead of being a sphere which is inside, let us say it has some shape like this. Now let us say this is surface 2; now you can see if I take an element on this surface, radiation going out from this element, any element like this some of it will go and hit the surface 1 itself.

So, if the surface of 1 is concave in some place, not everywhere, at any place then part of the radiation from 1 will come back to 1 itself in which case F 1 to 2 will not be equal to 1. So the 2 conditions for F 1 to 2 to be 1 or 2 should completely surround 1 so that all the radiation from 1 can go towards 2 and no other surface. Secondly, 1 itself should not have a concavity so that radiation from some part of 1 goes to 1 itself, it must all go towards 2.

Now these are situations, so the first 1 is a situation in which F 1 to 2 is 1. Now in such a case if I ask you what is F 2 to 1, you will straight away say for this case F 2 to 1 is equal to $A_1$ by $A_2$ multiplied by 1; that is $A_1$ by $A_2$. I don't have to use the double integral for F2 to 1; if you see in, from the definition F 2 to 1 will have a double integral equation which is not a double integral expression which is not easy to solve. But using the reciprocal relation, I can straight away tell what is F 2 to 1 and say simply F 2 to 1 is equal to $A_1$ upon $A_2$; so you see the benefits of using the reciprocal relation in some cases. So that is the first remark that we want to make about shape factor.

The second remark which I want to make about shape factor and which is rather important is the following.

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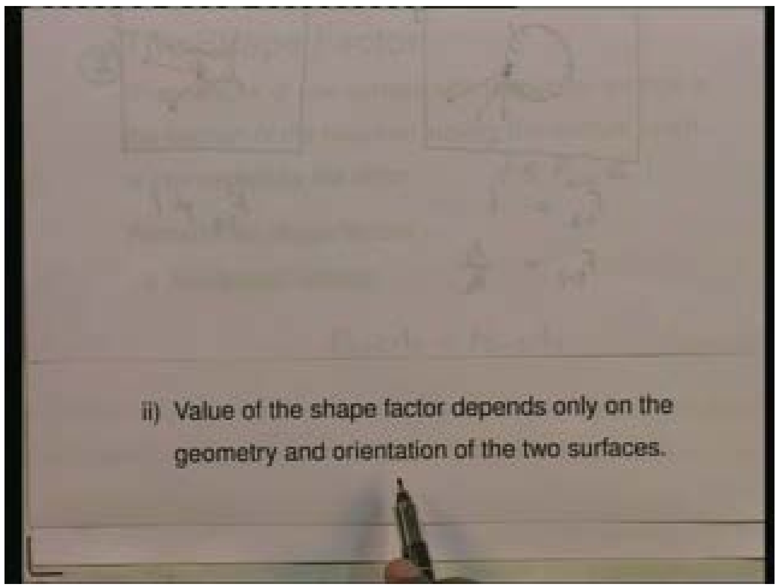


Let me just show that the second remark which I want to make is the shape factor, the value of the shape factor depends only on the geometry and orientation of the two surfaces. You will notice when we put the definition, we took temperature outside the definition; there is no $T_1$ or $T_2$ in that definition. So shape factor does not depend on the temperatures of the surfaces; it depends strictly on the geometry. What are the shapes, what are sizes of the surfaces involved, what are the relative orientations, how are they orientated with respect to each other - that is what is shape factor depends on.

So, because of this, because of this definition I can do shape factor calculations in advance for a variety of shapes and situations encountered in practice; I don't need to know their temperatures for calculating radiant heat exchange. If, there are geometries encountered in practice, frequently encountered in practice; for those geometries, for those shapes, sizes, orientation of the surfaces, I can do shape factor calculations and I have those available in the form of values which I can use anytime for doing radiant heat exchange calculations and that is what people do.

For a variety of situations, for a variety of common situation which occur in practice, we have with us shape factors calculated by other people and we use those for doing our calculation. We don't sit down and try to evaluate double integrals most of the time.

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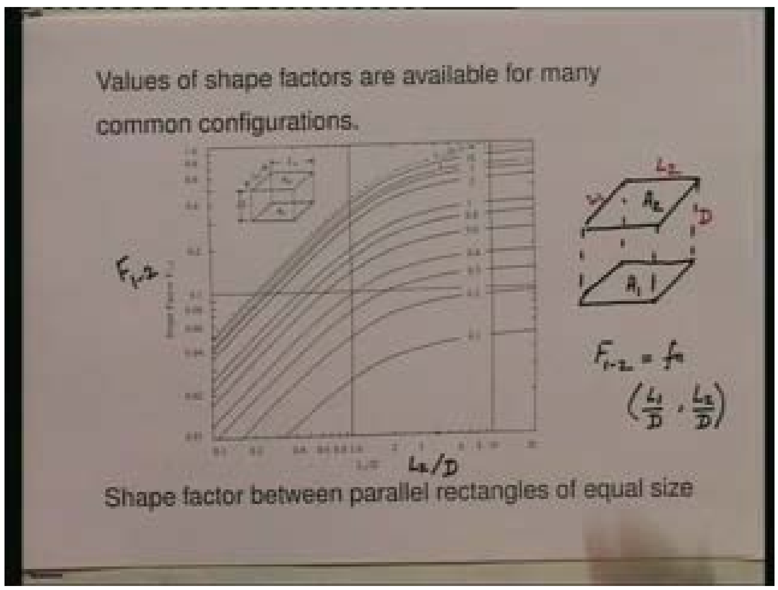


So, I will say values of shape factors are available for many common configurations. Now this figure it is a little small so let me explain it in a figure which shows the shape factor between 2 parallel rectangles of equal size, the shape factor between 2 parallel rectangles of equal size. Since it is not so clear, let me draw it again. 1 parallel rectangle could be this and the other parallel rectangle of equal size which is below is this. I call this the surface 1 $A_1$ and this 2 $A_2$; $A_1$ and $A_2$ are 2 rectangles of equal size parallel to each other and opposite to each other like the floor and the ceiling of this room these are 2 parallel rectangles of equal size facing each other or the 2 walls of this room, the 2 side walls which I am pointing. These 2 are 2 rectangles of equal size facing each other or the other 2 walls, one behind me and one in front of me; they are also like that.

So, any 2 walls - the floor and the ceiling are 2 rectangles, 2 parallel rectangles of equal size facing each other; the shape factor for that has been calculated and that is what is given in this graph which is given here. The shape factor F 1 to 2 here, F 1 to 2 is plotted, is the value from the red of from the y axis and it is given as a function of F 1 to 2 is a function of 2 parameters L 1 by D and $L_2$ by D that is how it is given. $L_1$ and $L_2$ are the sides of the rectangles; this is $L_1$, this is $L_2$ are the sides of the rectangles and D is the distance between the 2 rectangles, D is the distance between 2 rectangles. So the shape factor F 1 to 2 is given as a function of $L_1$ by D and $L_2$ by D - that is what we have in this graph.

So, anytime you have this situation, in practice all you have to do is: for the given situation calculate the 2 parameters $L_1$ by D, $L_2$ by D, go to this graph $L_2$ by D is on the x axis, this is $L_2$ by D and $L_1$ by D is the parameter which is varying between each graph, which is drawn here - this is the parameter $L_1$ by D. So in this case, this is the graph for $L_1$ by D equal $L_1$ by $D_2$, $L_1$ by $D_3$ the final one which is dotted is $L_1$ by D equal to infinity so all values of $L_1$ by D from .1 to infinity are covered in this graph. Similarly, L tow by D is covered from .1 to twenty so all situations in practice are covered in this graph. So for a given situation in practice, calculate $L_1$ by D; calculate $L_2$ by D. Go to this graph $L_2$ by D, go to the appropriate curve for $L_1$ by D, read off the value of F 1 to 2 - that is what we will be, you have to do.

Now, this is just one as an example I showed this. This is one example of shape factor calculations which have been done by somebody else which are available to us; in for usage whenever we have to do radiant heat exchange calculations and such graphs or equations corresponding to these graphs are available for a very wide variety of common configuration; a very large number they are available.

Let me show you one more example so that you get the idea; here is one more example now.

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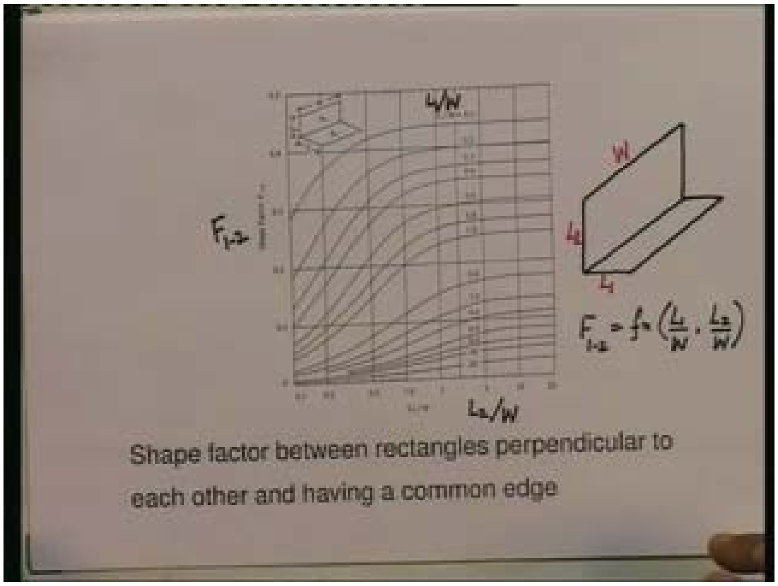


Here is a shape factor between rectangles which are perpendicular to each other and have a common edge something like this. It is drawn in small here; it is rather small here so let me draw it again. These are 2 rectangles like this - this is one rectangle, this is another rectangle. This is, they have a common edge and they are perpendicular to each other and the dimensions are $L_1$, and $L_2$ and W. So one of rectangles is $L_1$ into W, the other one is $L_2$ into W and they are at right angles to each other; we would like to know the shape factor.

Now for this case also shape factor has been calculated. It is available to us and what we have is - on the y axis here we have F 1 to 2, on the x axis here we have L 2 by W and the different graphs which are drawn here are for varying values of $L_1$ by W. So given a particular situation - now say this could be the floor of this room and one of the side walls or the ceiling of this room and one of the side walls, they are two situations of 2 rectangles right angles to each other with a common edge could be anyone of these. Now given this situation, calculate if you have to do a radiant heat exchange calculation.

Calculate $L_2$ by W, calculate $L_1$ by W and then from this graph read off the value of F 1 to 2. So F 1 to 2 is a function of $L_1$ by W and $L_2$ by W; so again this is very common configuration and somebody has done this integral for us and got, we have got the expression for us to use anytime we need it.

And like this as I said there are literally hundreds of situations for which shape factor calculations have been done and the values are available either in the form of graphs, equations, tables whatever it is, they are available to us to use anytime we have to do radiant heat exchange calculations. So this is the second remark - the value of shape factor depends only on geometry; it has been calculated for a wide variety of situations and configurations which are commonly used in practice and these are available in text books, hand books for us to readily use.

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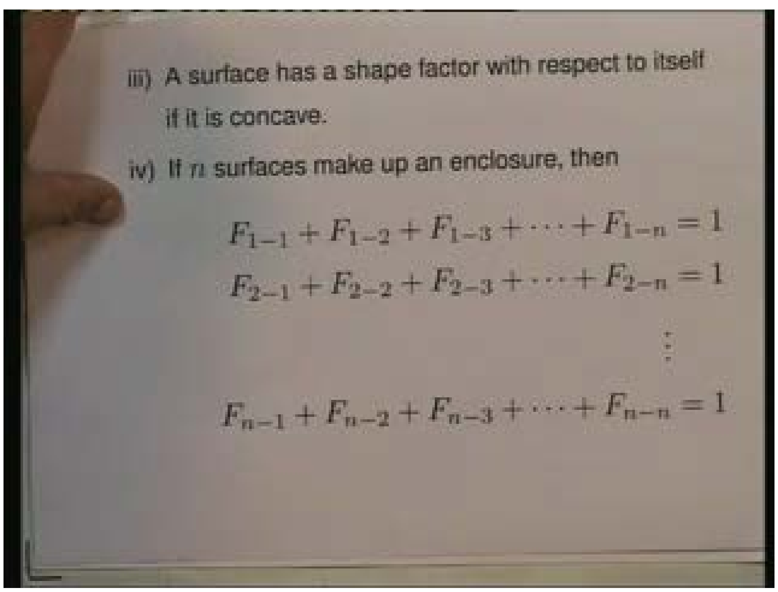


The third remark which I want to make is the following which I have made earlier and I am repeating, I am saying a surface has a shape factor with respect to itself. If it is concave, this is a remark which I made a little earlier but let me repeat it - what I mean is the following.

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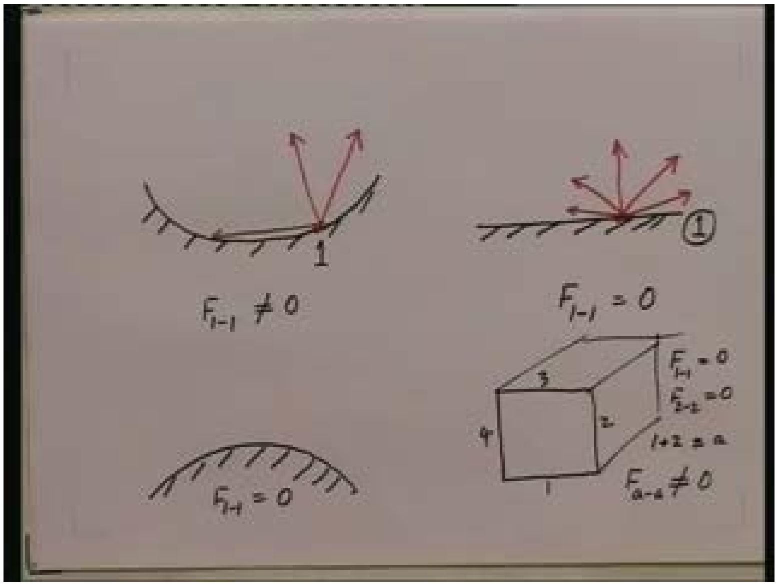


Suppose I have a surface like this - a concave surface like this surface 1; this is my surface 1, it is concave like this, emitting radiation. It is obvious now if I take some element on this surface, then some of the radiation from it is emitted; radiation is emitted in all directions of a hemisphere. But some of the radiation emitted is obviously going to come back to the surface itself so this is a case because of the surface being concave; it follows that F 1 to 1 is not equal to 0.

On the other hand, if the surface 1 is say flat; suppose this is the surface 1 and now if I consider radiation emitted from any element, elementary area on this surface, you will agree with me that radiation emitted from surface 1 anywhere on surface 1 will go out like this and will not be intercepted by 1; so this is a case where F 1 to 1 will be equal to 0 or let us take the case when surface 1 is let us say convex like this; then also F 1 to 1 quite obviously equal to 0.

Just to clarify, the surface 1, it may be flat, the surface 1 maybe curved, the surface one maybe consisting of 2 flat surfaces together that also maybe surface 1. For instance, let us go back to this room; here is our room. Let me draw it in 2 dimension, this is my room which I am sitting. If I want to do radiant heat exchange calculation and want to treat all the surfaces, the 6 surfaces of this room as being independent then I will call them as 1 2 3 4 and the 2 surfaces which I can't show here which in 3 dimension will be seen like this as 5 6 so I will have 6 surfaces.

Now if, treat the 6 surfaces as surfaces independent of each other; F 1 to 1 , F 2 to 2 , $F_3$ to $_3$ all for each of the surfaces will be 0. But let us say surface 1and 2 are at the same temperature so for purposes of calculation radiant heat exchange, I can treat 1 and 2 as 1 surface. Then it follows that 1 and 2 - now surface 1 and 2 if I treat it as 1 surface call it as say surface a - it follows that for this surface a which consists of 1 plus 2 together 2 rectangles together as the common edge, it follows for this case that the shape factor of surface a with respect to itself will be non-zero.

So, for this case now - let me write it here - for this case F 1 to1 was 0 if 1 is only a flat surface, F 2 to 2 will be 0. But if 1 and 2, the 2 flat surfaces are treated as 1 surface together and I call this as a that is 1 plus 2 I call as equivalent to the surface a, some surface a, then F a to a is non-zero. So a surface maybe flat, a surface maybe curved, a surface maybe a combination of 2 flat surfaces, 2 curved surfaces, one flat surface and a curved surface, whatever it is, it is up to me to decide depending upon the radiant heat exchange calculation that I have to do. And for that situation once I have defined what are my surfaces then I have to see whether F 1 to1 or F a to a is 0 or non-zero - this remarkable surface having a shape factor with respect to itself.

Now let us go to one more relation which is important and that is the following.

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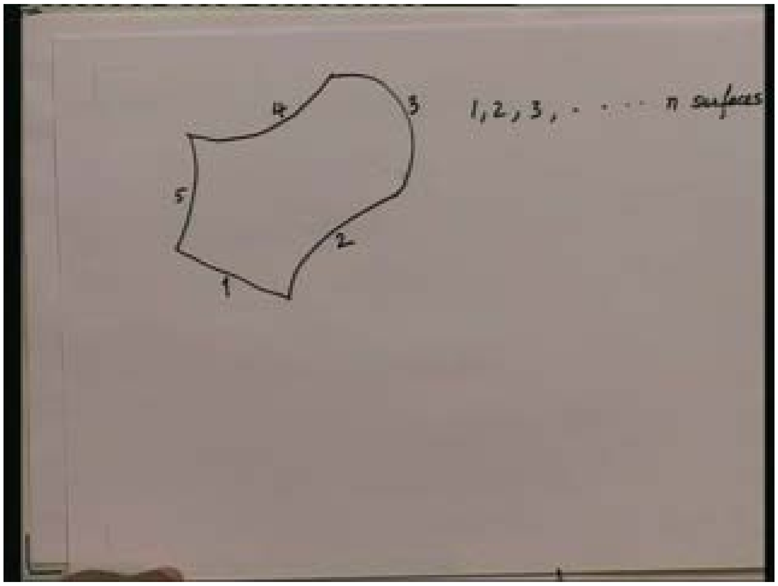


Suppose I have an enclosure made up of n surfaces, let us say I have some enclosure and just to make things, you know show that they are massy, I have drawn one as a flat surface - surface 1, surface 2 I have drawn as a convex surface. Surface 3 I have drawn as a concave surface, then surface 4 again I have drawn as a convex and again as a convex surface I have a drawn a 5 surface enclosure in 2 dimensions here. Let us say theses 5 surfaces make up an enclosure; I could think in general terms of an n surface enclosure making up an enclosure. So the n surfaces maybe flat, they maybe curved; the curved surfaces maybe convex or concave, it doesn’t matter what.

Now if these n surfaces - n surfaces 1 2 3 up to n surfaces - if they make up an enclosure, then it follows that all the radiation emitted from any one surface has to go to all the other surfaces that is surfaces 2, 3, 4, etcetera up to n and to the surfaces 1 itself if F 1 to1 is non-zero. So I can state that as follows - I can say for an n surface enclosure, let me write that now; let me show that.

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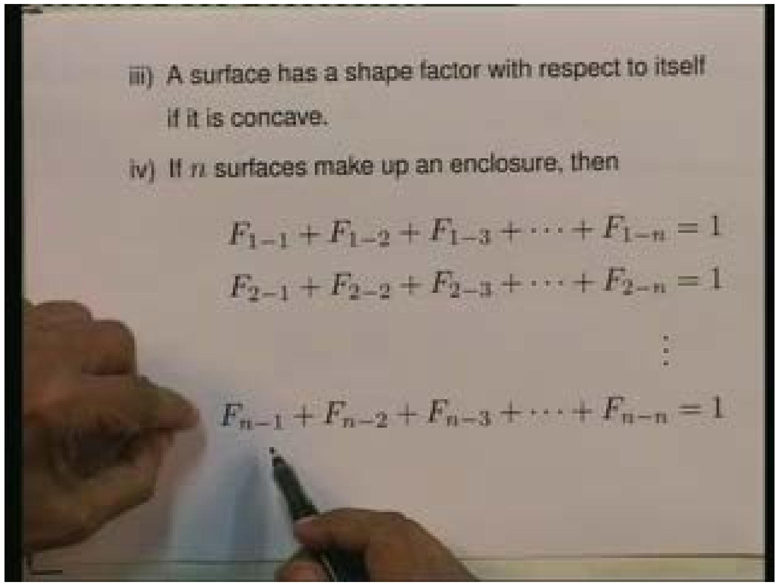


For an n surface enclosure F1 to 1 plus F1 to 2 plus F1 to 3 plus dot dot dot F1 to n equal to 1 because all the radiation emitted from 1 has to go to either to 1 or to 2 3 up to n all that must add up to1 and if 1 is a flat surface or if 1 is convex, then this F1 to1 will be automatically, we will put a 0. Similarly, consider the radiation emitted from surface 2; it follows that F 2 to 1 plus F 1 to 2 plus F 2 to 3 plus F 2 to n is equal to 1 and so on. If I consider the $n^{th}$ surface of this enclosure, Fn to1 plus Fn to 2 plus Fn to 3 dot dot dot Fn to n is equal to 1. So I can write down n such equations for the n surfaces making up this enclosure; so these are relations between all the shape factors when surfaces make up an enclosure and we will use these, we can use these to calculate one shape factor from another - that is the advantage. The last remark which I want to make is the following

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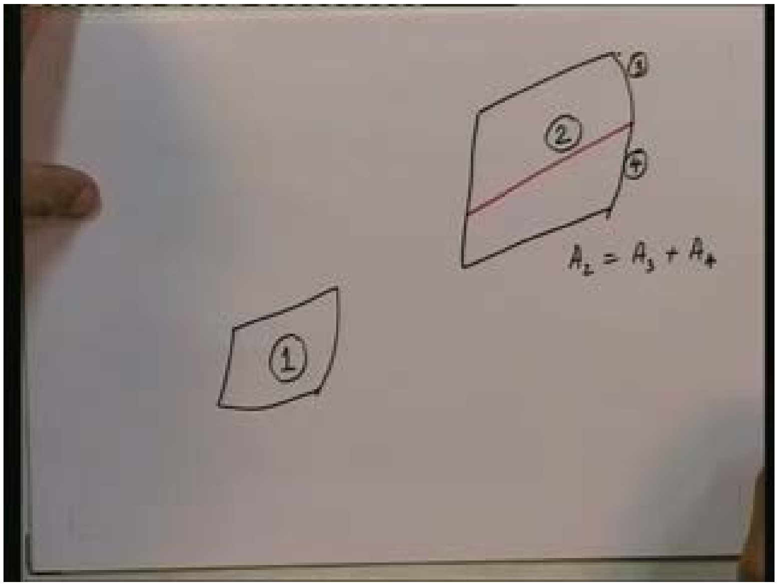


Suppose I have two surfaces - 1 and 2, let us say I have a surface 1, some arbitrary surface, maybe curved or may not be curved. Let us say I have a surface 1 like this and let us say I have some surface 2 like this arbitrarily located in space - 2 finite sized surfaces - and let us say I break up the surface 2 into 2 parts like this. I have broken it up in to 2 parts, I call this as surface 3, this is surface 4. That means $A_2$ is equal to $A_3$ plus $A_4$ - the two surfaces 3 and 4 make up the surface 2. Now radiation going from 1 towards 2, the fraction of radiation going from 1 towards 2 is F1 to 2; it follows that since 3 and 4 make up to that, F1 to 2 must be equal to F1 to 3 plus F1 to 4, it follows, isn't it? So let me show that in writing.

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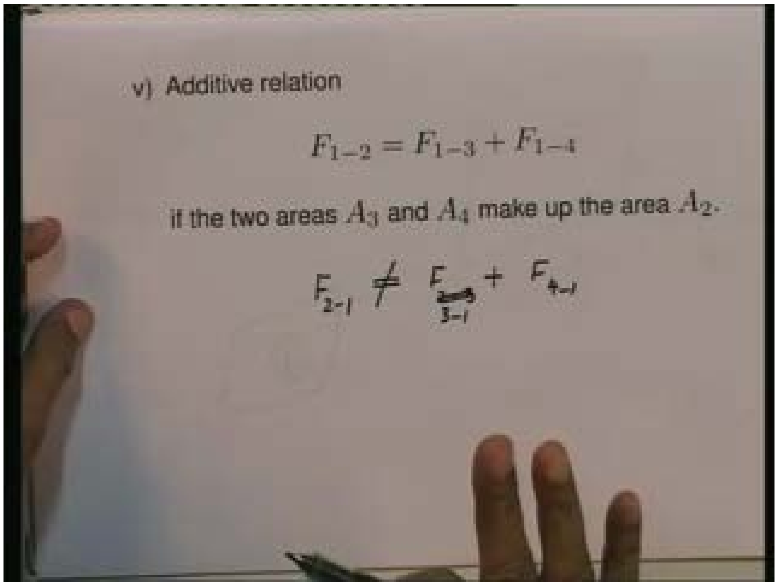


It follows that F1 to 2 must be equal to F1 to 3 plus F1 to 4 if the 2 areas $A_3$ and $A_4$ make up the area $A_2$ but note the reverse is not true. Many people sometimes will say F 2 to1, F 3 to1 and F 4 to 1 - these are the reverse shape factors in the reverse direction. This F 2 to1 is not equal to F 3 to1 plus F 4 to1; this from the definition itself. You can see that this is not true, F 2 to1 is not equal to F 3 to1 plus F 4 to1; please don't make this mistake. The additive relation which we can use is F1 to 2 is equal to F1 to 3 plus F1 to 4 and this follows from the physical meaning of shape factor.

Radiation going from to1 intercepted by 2 if 2 consists of 3 and 4, radiation going from1 towards 3 and from1 towards 4 - the sum of the 2 fractions must be equal to the first fraction F1 to 2. So keep this in mind - the first relation is valid, the second is not. The first one is called the additive relation so these are various relations between shape factors that we have given and the advantage will be obvious from various situations which we have to consider.

Now, we will do some problems in which we calculate shape factors for a variety of situations so that you get used to using these relations. I am going to do some problems; so the first problem I am going to do is following. Let us do the following problem.

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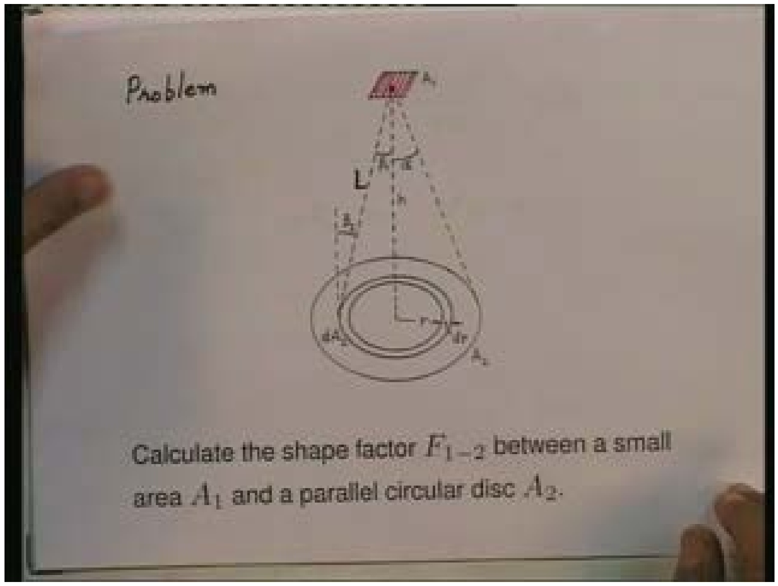


Calculate the shape factor F 1 to 2 between a small area $A_1$ and a parallel circular disc $A_2$. Here is the small area $a_1$; let me just shade it in red, it is a small area, differential area $A_1$ and parallel to it a circular disc $A_2$ is like this. So we have, let us say this table, on this table I have a circular disc. At the centre of the disc, let me draw a perpendicular in the centre of this disc and go up a certain distance h. At that distance h, consider a small area $A_1$ which is like a differential area parallel to the plane of this table; that is what we are talking about.

We would like to calculate F 1 to 2; we are going to use obviously the formula which we derived for shape factor between a small area $A_1$ and a large area $A_2$, we are going to use that formula. So what is the formula? Let me just put that down

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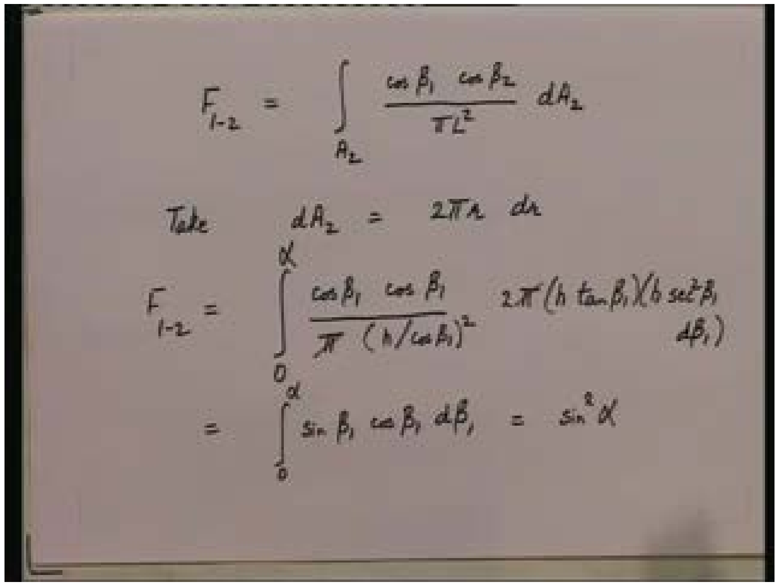


For such a situation, we know F1 to 2 for the case; this case, this is a small area F1 to 2. If you go back in your notes, you will see it is nothing but the integral over $A_2$ cosine beta$_1$ cosine beta$_2$ divided by pi L squared $dA_2$ - that is our expression. Now let me go back to the sketch; here we must take our $dA_2$ in this case, we have symmetry, we want the radiation coming from 1 which falls within this $c_1$. So since we have symmetry for $dA_2$, I can take a ring of thickness dr at some radius r. We have got symmetry all over; I don't have to take $dA_2$ as r d theta dr in cylindrical, in polar coordinates; I can take ring of radius at a radius r of thickness dr. So my $dA_2$ will be 2 pi r dr. Let us take that as our $dA_2$, so take $dA_2$ to be, let me write that down - take $dA_2$ because of symmetry, we will take $dA_2$ to be 2 pi r dr and therefore I can now perform the integration.

I say the distance between the 2 surface is h and the semi vertex angle, let me again show the sketch, I will say that the distance between the 2 surface is h the semi vertex angle subtended by this disc circular disc at $A_1$ is alpha. Now using our nomenclature if I join $A_1$ to $dA_2$ the distance between the 2 will be the capital L which we have in our formula; this will be capital L and the angle made by this line joining $A_1$ to $dA_2$ and the normal; then 2 normals will be parallel to each other because the 2 surfaces are parallel to each other and the angles, this will be $beta_1$ here and this will be $beta_2$ here and they will be equal in this case.

So, let us substitute into our expression; now we will get F1 to 2, we are going to express everything in terms of the angle as the variable $beta_1$. So we will get F1 to 2 is equal to cosine $beta_1$ the second $beta_2$ equal to $beta_1$ so I can write it also as cosine $beta_1$ divided by pi. Now instead $L_I$ we will write L as h - the distance between the 2 surfaces divided by cosine of $beta_1$ the whole squared because beta1 is my variable and then for 2, for $dA_2$ I will write 2 pi for r I will write h tangent beta1 that is r and for dr I get by differentiating h tangent $beta_1$, I will get h secant squared $beta_1$ d $beta_1$ - that is my expression.

I will have to integrate this from 0 till the semi vertex angle which is alpha; so I will integrate this from 0 to alpha that is over the whole area then I will cover the whole area of the disc. If I do that now, I am skipping the details here you can do the simplification. You can see that the cosine $beta_1$s will cancel out quite a bit, the pis will cancel out quite a bit, etcetera but all this - it is very easy to show that this will reduce to nothing but the integral 0 to alpha sine $beta_1$ cos $beta_1$ d $beta_1$. Very simple; you can do that in yourself and if I perform the integration and put in the limits this will reduce to sine squared alpha. So this is an example of the calculation of shape factor between a small area $A_1$ and a parallel disc $A_2$; it is a very simple example just to illustrate how to pick an area $dA_2$ and to do the integral. Now let us go on to another situation; I would like to calculate the shape factor using the graphs which I showed you earlier. So let us pick another problem. We are going to do the following.

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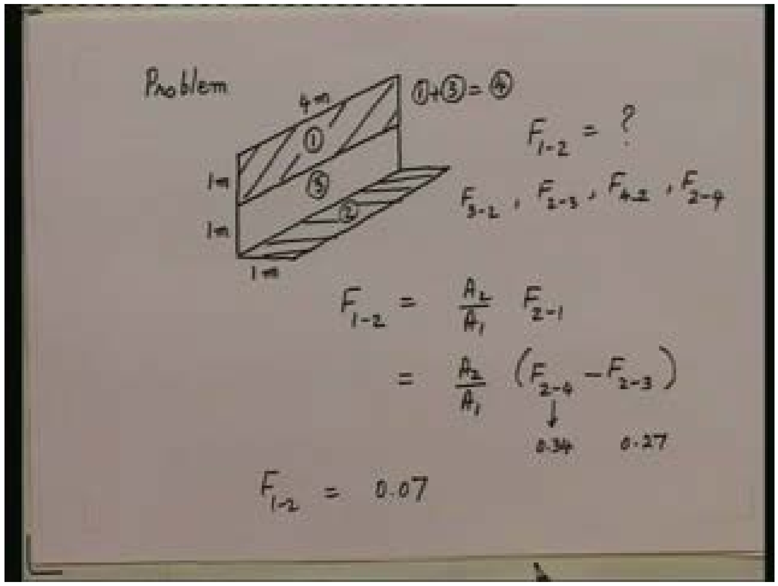


I will just state the problem today; problem is the following. We have 2 rectangles like these, this is one rectangle, surface 1 and this is the second rectangle, surface 2. These are 2 rectangles F 1 to 2 dimensions are given, this is 1 meter this is 1 meter, this is 4 meters and this is also 1 meter. So these are 2 rectangles - 1 and 2; they are at right angles to each other but they don't have a common edge. We would like to get F 1 to 2. Now mind you, I cannot read this off directly from the graph which I had earlier for 2 rectangles with a common edge. In that case, in that situation, remember we had the common edge situation here; we don't have a common edge – F 1 to 2 are at right angles but don't have a common edge.

Suppose I call this area, this rectangle as surface 3 and I call 1 and 3 together, 1 plus 3 together as 4. Now what can I read from the graph, graphs which I showed you earlier; we can from those graphs get F 3 to 2, this is known or 0 F 2 to 3; similarly we can get F 4 to 2 or F 2 to 4; these are available to us. So I need to express F 1 to 2 in terms of these quantities and that is what we can do very easily; we can say F 1 to 2 is equal to A 2 by A1, F 2 to1 here is my reciprocal relation which from the additive relation is nothing but A 2 by A1 F 2 to 4 minus F 2 to 3.

So I use first the reciprocal relation, then the additive relation and when I do that I have expressed F 2 to F 1 to 2 in terms of F 2 to 4 and F 2 to 3 which I can now read offf from the graphs which have calculation, which have been done earlier which I showed to you. So from that read offf these 2 values for this case; I am telling you what if you read them off? You will get the first one to be .34. The second one will come out to be .27 and A 2 by A 1 is obviously one in this case so you will get F1 to 2 is equal to .07.

I want you to do this yourself; that means for bit these cases for F 2 to $_4$ and F 2 to $_3$ calculate L by w and L 2 by w then go to those graphs, read off the values of the 2 shape factors F 2 to $_4$ and F 2 to $_3$ and then you will get F 1 to 2 equal to .07. So I am leaving that last part for you to do; to illustrate the whole idea is to illustrate how those graphs can be used for calculating shape factors for other situations not corresponding exactly to the situation of those graphs. Now we will stop here today.