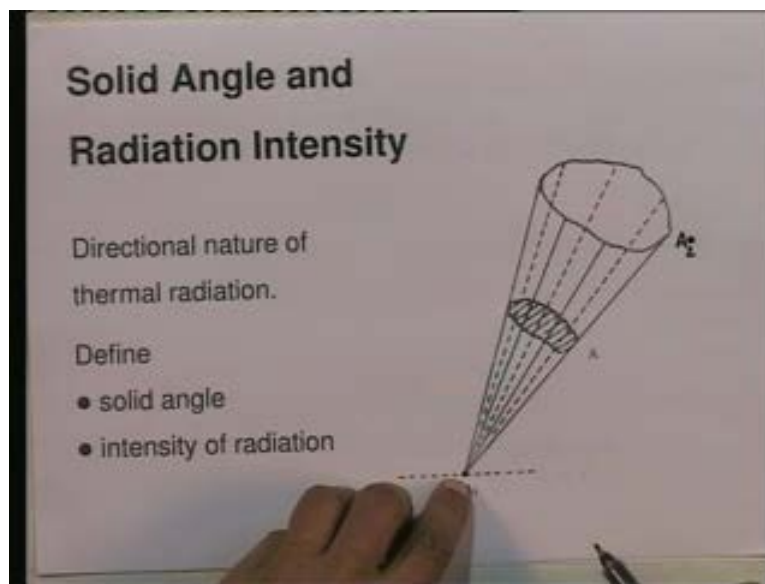


Heat and Mass Transfer
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Lecture No. 12
Thermal Radiation-3

We now turn our attention to the directional nature of thermal radiation and in that context we are going to define 2 terms - solid angle and radiation intensity.

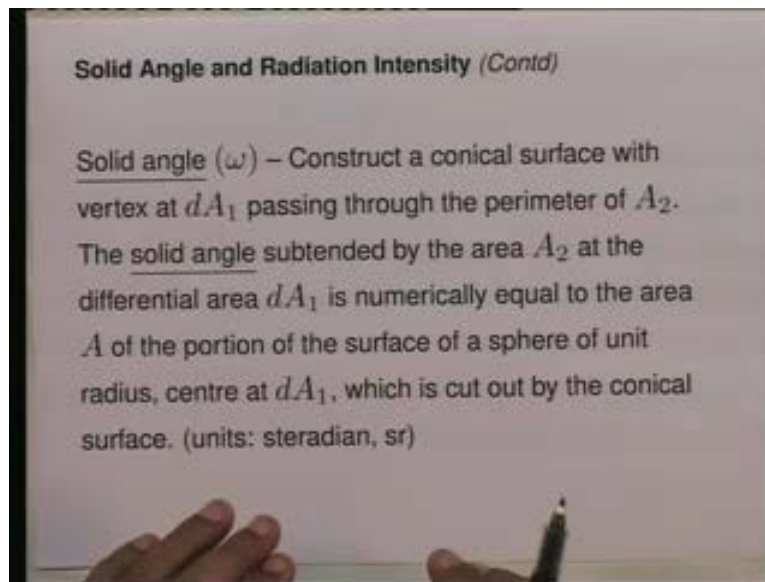
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So, let us define these 2 terms – first solid angle and then intensity of radiation. Now consider some area A_2 which is drawn here - any arbitrary area, it may be flat, may be curved; doesn't matter - some area A_2 . We would like to find out the solid angle subtended by A_2 at the point dA_1 . This is the area A_2 ; let me write it in bigger letters - A_2 and we would like to find out the solid angle subtended by A_2 at dA_1 at a differential area dA_1 . So what we will do is the following - join the perimeter of A_2 to the point dA_1 . So you will get a c_1 as is being seen here; some kind of a c_1 will be obtained. We will get a c_1 like this; then with center at dA_1 draw a sphere of unit radius, draw a sphere of unit radius with center at dA_1 .

Now the c_1 which we have drawn earlier will intersect this sphere of unit radius and let us say that on the surface of this sphere of unit radius, the c_1 cuts out an area A ; let me repeat - let us say the c_1 cuts out an area A on the sphere of unit radius which we have drawn with center at dA_1 . The solid angle subtended by A_2 at dA_1 is numerically equal to A ; the solid angle subtended by the area A_2 at the point dA_1 is numerically equal to the area A . So let me now read out the definition which I have described to you - the definition of solid angle.

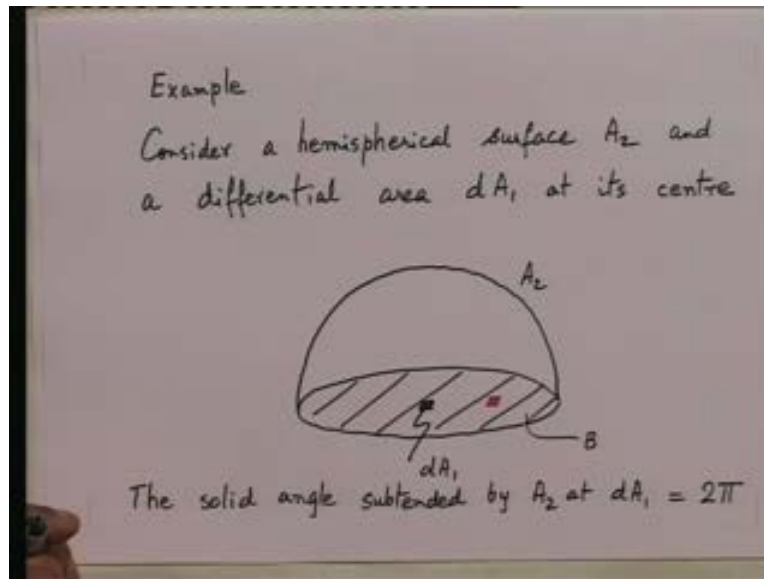
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We use the symbol omega for it and say construct a conical surface with vertex at dA_1 passing through the perimeter of A_2 . What I said earlier; the solid angle subtended by the area A_2 at the differential area dA_1 that is, that the point dA_1 is numerically equal to the area A of the portion of the surface of a sphere of unit radius centre at dA_1 which is cut out by the conical surface. This is the definition which I have described to you and in 2 dimensions the unit of an angle is radians, in 3 dimensions the unit of an angle is steradian. So keep that in mind; so this is how we define the term solid angle.

Now, let us take a, just to illustrate ideas, let us take the example of a, let us say I take the example of a hemisphere.

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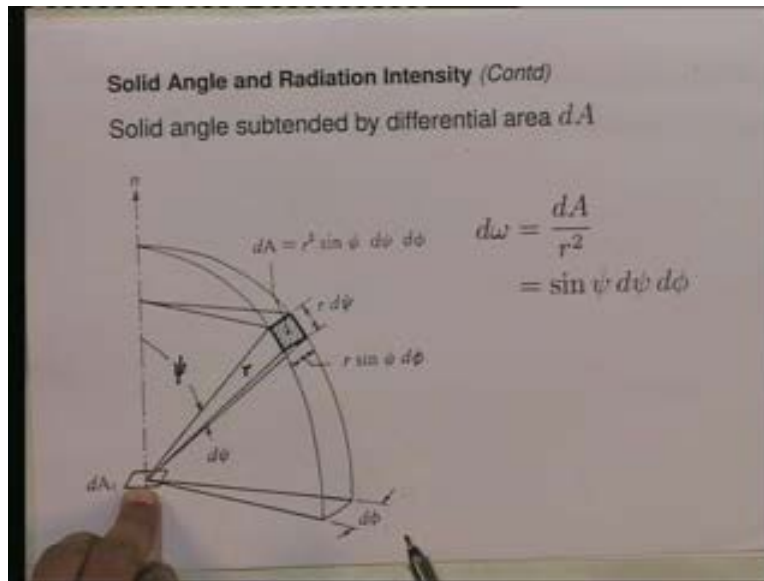


Take the example of a hemisphere; consider a hemispherical surface, consider a hemispherical surface A_2 and a differential area - an elementary area dA_1 at its centre. Let me draw that; suppose now I have a, let us say this is a hemispherical surface and this is the hemispherical surface which I will call as A_2 and the plane surface which is closing it - let us say, let us say the plane surface closing it at the bottom we will call as B and at its centre here, let us say this is dA_1 . So A_2 is a hemispherical bowl, dA_1 is at the centre of that hemispherical bowl. What is the solid angle subtended by A_2 at dA_1 ? Now it is quite obvious that if I were to draw a sphere of unit radius with centre at dA_1 then the area cut out on that sphere of unit radius is going to be 2π by the c_1 which is found by joining the perimeter of A_2 to dA_1 . Therefore the solid angle subtended, the solid angle subtended by A_2 at dA_1 will be equal to 2π ; 2π will be the area cut out on the surface of a sphere of unit radius centre at dA_1 .

Now let me change the example little - I say suppose the area dA_1 or the point dA_1 instead of being located at the centre let us say it is located somewhere else on this plane surface B . Let us say this is dA_1 , let us say this is dA_1 ; now I ask you the question if this is dA_1 , what is the solid angle subtended by A_2 at this red dA_1 ? What is the solid angle subtended by A_2 , the hemispherical ball A_2 at this red differential point, differential area

dA_1 instead of being at the centre and you should be able to tell the answer yourself. Now let us go on; we would next like to derive an expression for the solid angle subtended by a differential area dA . Suppose I have some differential area dA which I have shown here.

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In spherical coordinates, suppose I have some differential area dA which I am showing here in a spherical coordinate system here like this. This is dA shown in a spherical coordinate system; I would like to know what is the solid angle subtended by dA in spherical coordinate system. Now this is a typical spherical coordinate system - r is one coordinate, the radius r from the centre to the area dA . ψ is the angle from the measure from the vertical which is called the zenith angle and ψ - this is ψ - and ϕ is the azimuth angle measured in a horizontal plane 0 to 2π ; ϕ varies from 0 to 2π . So 3 coordinates in a spherical coordinate system are r the radius vector, the radius ψ and ϕ radius, zenith angle, azimuth angle; now the differential area dA is $r^2 \sin \psi \, d\psi \, d\phi$. Very easy to do by simple trigonometry; I want show that to you.

So, the differential area dA is $r^2 \sin \psi \, d\psi \, d\phi$; now what is the solid angle subtended by dA at the centre? It is very easy to see - it is nothing; it is going to be

nothing but dA upon r squared because r is the radius and we want the area cut out on a sphere of unit radius. So dA upon r square will be the solid angle subtended by dA at the centre; so $d\omega$ that is the solid angle subtended by dA at the centre is equal to dA upon r square and we know that dA is r square $\sin \psi$ $d\psi$ $d\phi$. Therefore this is equal to $\sin \psi$ $d\psi$ $d\phi$; so we have a nice simple expression for a differential solid angle in spherical coordinates, a differential solid angle $d\omega$ in spherical coordinates is equal to $\sin \psi$ $d\psi$ $d\phi$ where ψ is the zenith angle and ϕ is the azimuth angle and we will use this shortly.

Now, let us define the second term which I mentioned; what was the second which I mentioned just a few minutes ago? The second term which I mentioned was intensity of radiation, radiation intensity or intensity of radiation.

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Solid Angle and Radiation Intensity (Contd)

Total intensity of radiation in a given direction (i)
 – Radiant flux passing in the specified direction per unit solid angle

$$i = \frac{de}{d\omega}$$

Therefore $e = \int i d\omega$

where the integration is carried out over all directions encompassed by a hemisphere.



The diagram shows a hemisphere with a flat base. Several arrows originate from the center of the base and point upwards, representing radiation directions. A small red-shaded area on the surface of the hemisphere is labeled 'de', representing a differential area element. A dashed line indicates the radius of the sphere.

To be more specific we will call it total intensity of radiation; total means summed over all wavelengths - the usual definition, total means summed overall wavelengths. The total intensity of radiation in a given direction i is equal to the radiant flux passing in the specified direction per unit solid angle, radiant flux passing in the specified direction per unit solid angle. What we mean is the following - suppose I have a surface like this and it

is emitting radiation in all directions like this in normal direction at an angle, all directions given by a hemisphere, emitting radiation in all directions given by hemisphere.

If I am taking any element on the surface, that element is emitting radiation in all direction of the hemisphere. I want the intensity in a particular direction so I say to myself - suppose I have some direction, some specific direction like this which I will draw in red. I want the intensity in this direction so in this direction, about this direction we, in which I am interested let us draw a small elementary solid angle $d\omega$ like this. Let us say this is an elementary solid angle $d\omega$ and let us say through this elementary solid angle $d\omega$ the radiation which is being emitted from this surface is d_e . Therefore i is nothing but $d_e / d\omega$; the radiant flux emitted per unit solid angle in a particular direction d_e upon $d\omega$.

Or, another way of looking at intensity of radiation is to say - it is that quantity which when integrated over all directions of a hemisphere will give me the emissive power of a surface. So let us put it in the reverse by here; therefore e is nothing but the integral $i d\omega$ where the integration is carried out over all directions encompassed by a hemisphere. So there are 2 ways of looking at I ; one is it is the radiant flux so many watts per meter square per unit solid angle in a given direction or it is that quantity, i is that quantity which when integrated over all directions encompassed by hemisphere will give me the emissive power of a surface.

And the word total implies that we are talking about something which is summed over all wavelengths; intensity means you are summing, you have to sum it over all directions to get the emissive power.

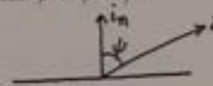
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Solid Angle and Radiation Intensity (Contd)

Thus
$$e = \int_0^{2\pi} \int_0^{\pi/2} i \sin \psi \, d\psi \, d\phi$$

For many surfaces

$$i = i_n \cos \psi \quad \text{— Lambert's law}$$



Such a surface is called a diffuse surface.

Substituting Lambert's law, we get

$$e = \int_0^{2\pi} \int_0^{\pi/2} i_n \cos \psi \sin \psi \, d\psi \, d\phi$$

$$= \pi i_n \quad \text{if the surface is diffuse}$$

Thus proceeding from the earlier expression for e is equal to integral $i \, d\omega$, now instead of $d\omega$ I will write $\sin \psi \, d\psi \, d\phi$ because that is my expression for the elementary solid angle; I will derive it earlier in spherical coordinates and if I want all directions of a hemisphere, integrate from 0 to $\pi/2$ to cover all the variation, the zenith angle and from 0 to 2π to get the variation of the azimuth angle. Both the 0 to $\pi/2$ covers the vertical in the, from the zenith vertical direction down to the horizontal plane and 0 to 2π covers the azimuth angle in a horizontal plane. So, all the directions of a hemisphere are covered when I do this integration. So I say e is equal to the double integral 0 to 2π , 0 to $\pi/2$ $i \sin \psi \, d\psi \, d\phi$ – that is our, one way also of looking at the relation between e and I ; e is the emissive power, i is the intensity of radiation.

Now, it so happens that for many surfaces, the intensity of radiation in any direction can be related to the intensity in the normal direction by a simple law which is called as Lambert's law and Lambert's law simply says i is equal to $i_n \cos \psi$ but we are saying in effect is the following. If I have a surface and if this is the normal direction to the surface, the intensity in the normal direction is i_n , the intensity in any direction other direction is i . We, and let us say that direction makes an angle, zenith angle ψ with the normal; then what we are saying is - Lambert's law simply says i the intensity in any

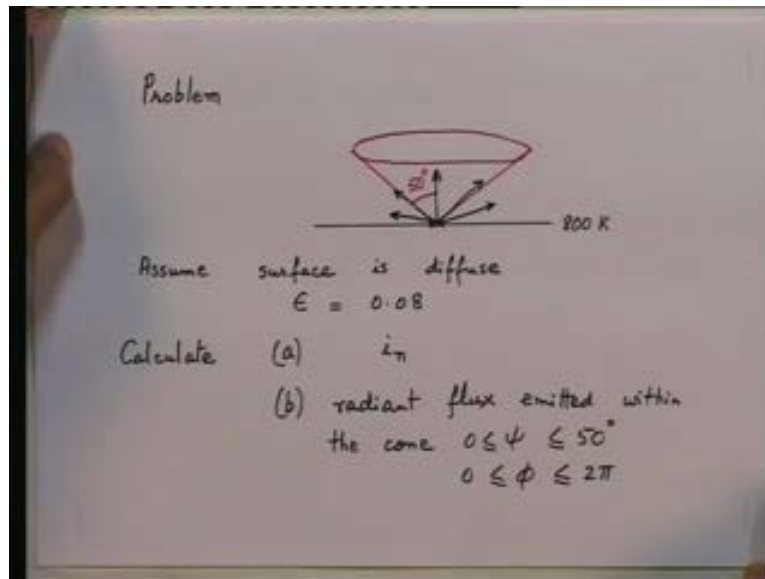
direction is equal to i_n the intensity in the normal direction multiplied by the cosine of ψ . Such a surface which is defined by Lambert's law which relates intensity in any direction to intensity in the normal direction, such a surface which says that Lambert's law is valid is called a diffused surface.

So, we say and many real surfaces to a large extent obey this law; therefore we will use it often as an idealization. Now suppose we substitute Lambert's law into the expression which we had earlier - e is equal to $\int_0^{2\pi} \int_0^{\pi/2} I \sin \psi \, d\psi \, d\phi$. Instead of i let us write i_n into the, into i_n into the cosine of ψ ; I think we forgotten something here. We should have had i_n into cosine of ψ , I have forgotten that here. i_n into cosine of ψ into sine of ψ uh into $d\psi \, d\phi$, now that is correct. If you perform this integration, take the i_n outside and perform then the double integral over ψ and ϕ ; it is very to show that you will get π . So the net result of the integration is simply to say e is equal to π times i_n .

We have got a nice easy relationship, simple relationship between the emissive power of a surface and the intensity in the normal direction. So let me repeat, first we had a general expression for e . e is equal to the double integral $I \sin \psi \, d\psi \, d\phi$; it is a general expression. Lambert's law relates the intensity in any direction to the normal intensity; substituting Lambert's law that is assuming that the surface is diffused and performing the integration we get e is equal to π times i_n . That means we are saying the intensity in the normal direction multiplied by π is equal to the emissive power of a surface if the surface is diffused. e is equal to π times i_n if the surface is diffused and we will make this assumption. It is made by most people; we will make this assumption.

So, we will always relate the emissive power to the normal intensity by this expression assuming Lambert's law to be valid. So now let us do a simple problem again just to illustrate ideas; let us do the following problem.

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Let us say, let us do the following problem, let us say I have a surface; some surface at, a surface at a temperature of 800 Kelvin. Take some element on that surface - some elementary area on the surface - and we know that it is going to be emitting radiation in all directions of a hemisphere like this. It is emitting radiation in all directions of a hemisphere; now let us assume the surface emits, assume surface emits radiation in a diffused manner or assume surface is diffused. And let us say that the emissive emissivity of the surface is .08; it is a surface of 800 k with an emissivity of .08, it emits in a diffused manner. Calculate first of all, calculate a, calculate the value of i_n - the intensity in the normal direction, b - radiant flux emitted within the cone, within the cone 0 less than ψ less than 50 degrees, 0 less than ϕ less than 2π .

So, we would like to know how much radiation is being, how much radiant flux is being emitted within the cone which makes, has a semi angle of 50 degrees. So let me draw that; let us say this is 50 degrees here, this is the 50 degree cone. So this is 50 degrees within this cone, within this cone, how much is the radiant flux going outwards? That is what we would like to know, not all of it but what is going out within the cone. We want to, whole amount that is going over the whole hemisphere that is obviously nothing but

the emissive power of the surface. We don't want that, we want only the amount that is within this cone making an angle of, semi angle of 50 degrees.

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From S-B law

$$e = \epsilon e_b = 0.08 \times 5.670 \times 10^{-8} \times 800^4$$

$$= 1857.9 \text{ W/m}^2$$

$$i_n = \frac{e}{\pi} = 591.4 \text{ W/m}^2\text{-sr} \leftarrow$$

Radiant flux in cone $0 \leq \psi \leq 50^\circ$

$$= \int_0^{2\pi} \int_0^{50^\circ} i_n \cos \psi \sin \psi \, d\psi \, d\phi$$

$$= 2\pi i_n \int_0^{50^\circ} \frac{\sin 2\psi}{2} \, d\psi = 1090.3 \text{ W/m}^2$$

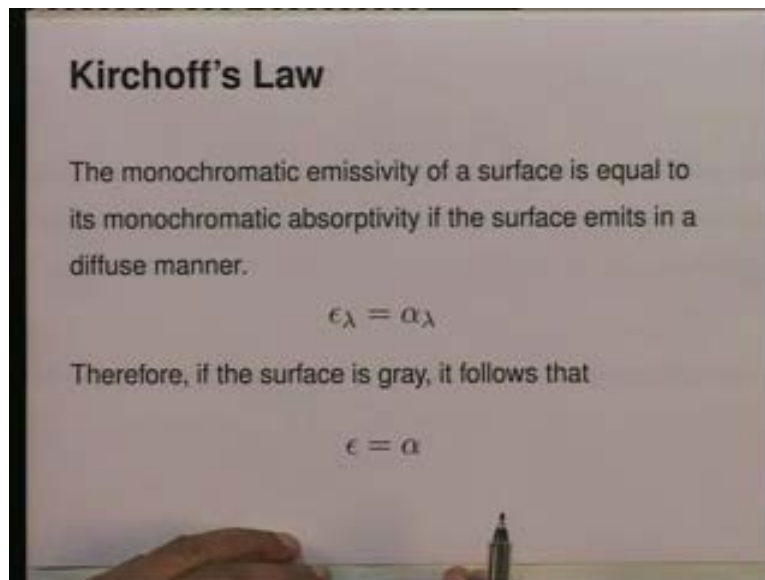
So let us do the problem; first of all from the Stefan Boltzmann law, from Stefan Boltzmann law, S B law, e is equal to ϵ times e_b is equal to .08 that is the emissivity multiplied by 5.670 into ten to the minus 8 multiplied by 800 that is degrees Kelvin to the power of 4. So that comes out to be 1857.9 so many watts per meter square. We want first of all the value of i_n ; we know very well that if it is a diffused surface - a surface which emits in a diffused manner - i_n is nothing but e divided by π . So that comes out to be 591.4 so many watts per meter square steradian so that is i_n . That is the first thing that we are looking for - the value of i_n . The second thing we are looking for is the radiant flux being emitted in the cone - 0 to 0 less than equal to ψ less than equal to 50 ; so radiant flux in cone 0 less than ψ less than 50 is equal to the double integral 0 to 2π that is integrating over the whole azimuth angle 0 to 50 that is the zenith angle $i_n \cos \psi \sin \psi \, d\psi \, d\phi$.

So, a note earlier - when we had integrated, we had integrated over the whole hemisphere. So we had 0 to π by 2 and 0 to 2π , now I want the radiant flux only in this

cone; so the first, the integral over ψ only from 0 to 50, the integral over azimuth angle is over the whole direction encompassing all horizontal plane going over the whole horizontal plane. Now this is an extremely trivial integral to do. All that we will get is, this is the equal to $2\pi i$ in the integral 0 to 50 sine 2 ψ by 2 d ψ and that comes out to be 1090.3 watts per meter squared. That is the second answer we are looking for; that is the radiant flux emitted within the cone which has a semi angle of 50 degrees.

So, this is an illustration of the formula that we just derived for intensity of a radiation and for solid angle. Now let us move on; we have defined 2 terms - solid angle and radiation intensity and in the process also defined what do you mean by diffused surface saying that it relates intensity in any direction to the intensity in the normal direction. Now we go on to another law which we are not going to prove but which is in fact a very important law and that law is Kirchoff's law.

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Kirchoff's law states that the monochromatic emissivity of a surface is equal to its monochromatic absorptivity if the surface emits in a diffused manner. Kirchoff's law is a very powerful statement relating the emission characteristics of a surface with the absorption characteristics of surface. Kirchoff's law I repeat is a very important law -

relates emission characteristics to absorption characteristics of a surface -and it states if the surface is diffused, epsilon lambda is equal to alpha lambda. If a surface, if emits in a diffused manner epsilon lambda is equal to alpha lambda; this is, we are not going to prove this law but I will ask you to take it as correct.

Now, a consequence of this is that if in addition to being emitting in a diffused manner, if I say, if the surface is also gray, if the surface is also gray it follows that epsilon is equal to alpha emissivity is equal to absorptivity. And this is quite easy to show; if epsilon lambda is equal to alpha lambda and the surface is gray then epsilon becomes equal to alpha. Let show, that is quite relatively easy to show. Let us for instance say take epsilon; let us take the quantity epsilon that is the emissivity of surface. What is epsilon equal to? epsilon is e upon e_b that is e is the emissive power of the surface, e_b is the emissive power of a black surface at the same temperature. e is nothing but the integral 0 to infinity; e lambda d lambda integration over all wavelengths upon e_b that is nothing but the integral 0 to infinity epsilon lambda e_b lambda d lambda upon e_b. Now epsilon lambda is a constant.

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$$\epsilon = \frac{e}{e_b} = \frac{\int_0^{\infty} e_{\lambda} d\lambda}{e_b} = \frac{\int_0^{\infty} \epsilon_{\lambda} e_{b\lambda} d\lambda}{e_b}$$

$$= \frac{\epsilon_{\lambda} \int_0^{\infty} e_{b\lambda} d\lambda}{e_b} = \epsilon_{\lambda} \frac{e_b}{e_b} = \epsilon_{\lambda}$$

Similarly it can be shown that

$$\alpha = \epsilon_{\lambda} \quad \text{if the surface is gray and diffuse}$$

$$\therefore \epsilon = \alpha$$

epsilon lambda is a constant; we have told that epsilon lambda is a constant; therefore this is nothing but, I can take epsilon lambda outside the integral. epsilon lambda 0 to infinity $e_b \lambda d \lambda$ upon e_b - that is nothing but epsilon lambda e_b upon e_b and that is nothing but a epsilon lambda. So the emissivity is equal to the monochromatic emissivity the moment I assume that the surface is gray.

Similarly the alpha, similarly, it can be shown that alpha the absorptivity is equal to the, also equal to epsilon lambda if the surface is gray and diffused. Therefore, epsilon is equal to alpha; I am not showing the second that alpha is equal to epsilon lambda but you can show it in the same way that we showed earlier for epsilon. Say alpha is equal to h upon h ; then go ahead and make h a a_s , describe it in terms of an integral and use the fact that alpha lambda is epsilon lambda which Kirchoff's law states and you can show alpha is nothing epsilon lambda. So therefore, in effect to we are saying - let me repeat again Kirchoff's law states epsilon lambda is equal to alpha lambda if the surface emits in a diffused manner. In addition if the surface is gray then it can be shown that epsilon is equal to alpha and that is a very powerful statement.

We will generally make the assumption that the surface is diffused and gray - a real surface that we are dealing is diffused and gray; so a surface for which both the equations that is epsilon lambda is equal to alpha lambda and epsilon equal to alpha, a surface for which both equations hold is called a diffused gray surface. We will assume that all real surfaces, all non black surfaces that we are dealing with are diffused, gray; we will make that assumption and therefore we will always assume epsilon equal to alpha and epsilon lambda equal to alpha lambda. It is a powerful law which will help us quite a lot in deriving formulae and simplifying our derivations as we go along.

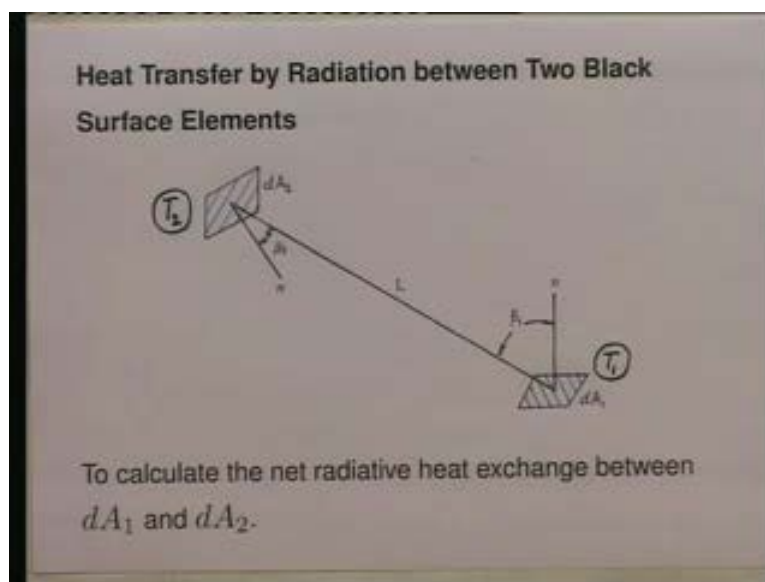
Now we have so far - let me again recapitulate for a moment - we have talked about the emission characteristics of surfaces and defined terms like emissive power, emissivity, monochromatic emissive power and monochromatic emissivity and we have defined the laws of black body radiation emissions from black bodies. Secondly, we have talked about the absorption characteristics of surfaces; defined quantities like absorptivity,

monochromatic absorptivity and similarly analogous quantities is for reflectivity and transmissivity.

Then, we have looked at the directional aspect of radiation today, defined quantities like solid angle and intensity of radiation that is radiation in a particular direction. We have related intensity in the normal direction to the emissive power of a surface by making the assumption that Lambert's law holds true that is the surface emits in a diffused manner. e_{λ} is, i_{λ} is equal to $e_{\lambda} \cos \psi$. Finally, we have stated Kirchoff's law; we stated that the monochromatic emissivity of a surface is equal to the monochromatic absorptivity if the surface emits in a diffused manner. We have gone on to show that if in addition the surface is gray then emissivity is equal to absorptivity.

So, now that we know how to deal with emissions from a surface and the absorption characteristics of surface, we are ready to talk about heat exchange by radiation between 2 surfaces at 2 different temperatures; now we are ready because a surface emits, a surface absorbs. So we are now in a position to calculate heat transfer by radiation between 2 surfaces. So let us do that now - we are going to study the heat transferred by radiation between 2 surface elements.

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So, to simplify matters, first instead of taking finite surfaces we will take 2 surface elements and to also simplify matters we will assume that they are black - that is our initial assumption mind you. Later on we may, we will be able to deal with other situations.

So, first now let us calculate the heat transfer by radiation between 2 black surface elements; let us say the black surface elements are dA_1 - this is one black surface element - and dA_2 . They are 2 black surface elements, 2 surface elements arbitrarily located somewhere in space. dA_1 is at a temperature T_1 , an absolute temperature T_1 and dA_2 is at an absolute temperature T_2 ; it is maintained, both of them are maintained at these absolute temperatures T_1 and T_2 . So obviously dA_1 is going to emit radiation by virtue of being at T_1 , dA_2 will be going to emit radiation by virtue of being at T_2 .

Some of the radiation emitted by dA_1 is going to go towards dA_2 and similarly some of the radiation emitted by dA_2 is going to go towards dA_1 . That radiation is going to be absorbed by dA_2 and dA_1 ; the heat transfer by radiation between the 2 elements is the difference between those 2 quantities. That is what is emitted by dA_1 going towards dA_2 and absorbed by dA_2 and what is emitted by dA_2 going towards dA_1 and absorbed by dA_1 . So to calculate the net radiative heat exchange between dA_1 and dA_2 - that is our problem.

Now let us do that; let us solve, get an expression. First of all let me consider the radiation emitted by dA_1 and going towards dA_2 .

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Intensity of radiation i emitted by dA_1 in the direction of $dA_2 = \left(\frac{\sigma T_1^4}{\pi}\right) \cos \beta_1$

Solid angle ($d\omega$) subtended by dA_2 at dA_1

$$= \frac{dA_2 \cos \beta_2}{L^2}$$

Rate at which radiation emitted by dA_1 flows towards $dA_2 = i \cdot d\omega \cdot dA_1$

$$= \left(\frac{\sigma T_1^4}{\pi} \cdot \cos \beta_1\right) \left(\frac{dA_2 \cos \beta_2}{L^2}\right) dA_1$$

So, I ask myself first, what is the intensity of radiation i emitted by dA_1 in the direction of dA_2 ? What is the intensity of radiation? Let us go back to the sketch for a moment; define some symbols before we put down this expression. Let me show a sketch again. These were the areas dA_1 and dA_2 at temperature T_1 and T_2 ; let us say the line joining these 2 areas, this is a line joining these 2 differential areas and let us say its length is L , is denoted by capital L and that the normal to dA_1 , the normal to dA_1 makes an angle β_1 with L . The normal to dA_2 makes an angle β_2 with L . So β_1 and β_2 are the angles made by the normals to dA_1 and dA_2 with the line joining dA_1 and dA_2 ; that is the nomenclature we will adopt.

Now let us go back with this nomenclature - intensity of radiation i emitted by dA_1 in the direction of dA_2 is going to be equal to, first of all what is the emissive power of dA_1 ? The emissive power of dA_1 , it is a black surface, so by this Stefan Boltzmann law, it is σT_1 to the power of 4 e_{b1} . If I divide this by π , I have got the intensity of radiation emitted by dA_1 in the normal direction. I want the intensity emitted by dA_1 in the direction of dA_2 so I will use Lambert's law and say multiply by cosine β_1 - that is the intensity in the direction 2.

Next, what is the solid angle $d\omega$ subtended by dA_2 at dA_1 ? It is nothing but, first of all take their area normal to the line joining that to that is normal to L; areas of dA_2 , projected area of dA_2 normal to L, it is going to be $dA_2 \cos \beta_2$ – that is the area of dA_2 normal to L. I want the solid angle so obviously I want to know how much will be cut out on a sphere of unit radius; the area of a sphere is $4\pi r^2$. So if I divide this by L^2 , then I get the area cut out on a sphere of unit radius; so this is $d\omega$.

Therefore, I know the intensity of radiation emitted by dA_1 in the direction of dA_2 ; I know the solid angle subtended by dA_2 at dA_1 . Therefore, rate at which radiation emitted by dA_1 flows towards dA_2 - that will be nothing but is equal to I the intensity multiplied by $d\omega$ multiplied by the area dA_1 . I is σT_1^4 to the power of 4 upon $\pi \cos \beta_1$; that is, $d\omega$ is $dA_2 \cos \beta_2$ upon L^2 and finally multiplied by dA_1 . That is the expression for the rate at which radiation emitted by dA_1 flows towards dA_2 , rate at which radiation emitted by dA_1 flows towards dA_2 is given by this expression. Let us clean it up a little; I can rewrite that as same expression.

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Handwritten mathematical derivation on a whiteboard:

$$= \left(\frac{\cos \beta_1 \cos \beta_2 dA_2}{\pi L^2} \right) \sigma T_1^4 dA_1$$

$$= dF_{1-2} \sigma T_1^4 dA_1 \quad \text{All this is absorbed by } dA_2$$

where $dF_{1-2} = \frac{\cos \beta_1 \cos \beta_2 dA_2}{\pi L^2}$

dF_{1-2} is called the shape factor of dA_1 w.r. to dA_2

Also referred to as view factor, angle factor, configuration factor

I can say that is equal to $\cos \beta_1 \cos \beta_2 dA_2$ divided by πL^2 the whole thing multiplied by $\sigma T_1^4 dA_1$; I have just rewritten the same expression in another manner and I am going to the quantity within the curved brackets. I

am going to indicate by the symbol $dF_{1 \rightarrow 2}$ and say that is equal to $dF_{1 \rightarrow 2} \sigma T_1$ to the power of 4 dA_1 – that is the rate at which radiation emitted by dA_1 flows towards dA_2 . And all this is absorbed by dA_2 , all this is absorbed by dA_2 ; why is that? Because dA_2 is a black surface, a black surface absorbs. So this is the expression where, let me indicate the symbol $dF_{1 \rightarrow 2}$ is equal to cosine β_1 cosine β_2 dA_2 divided by, divided by πL^2 . That is the quantity dF which we have defined as $dF_{1 \rightarrow 2}$.

$dF_{1 \rightarrow 2}$ is called the shape factor, is called the shape factor of dA_1 with respect to dA_2 ; $dF_{1 \rightarrow 2}$ is called the shape factor of dA_1 with respect to dA_2 . It is also referred to as the shape factor; let me underline that word. This is called the shape factor also referred to as, also referred to as the view factor, referred to as view factor; it is sometimes called as an angle factor, sometimes called as a configuration factor. The shape factor is also referenced to as a view factor, angle factor or a configuration factor.

Now what does this, what does it mean really? That is important; the shape factor if you go back to the definition here is the fraction of the radiant heat flow rate from dA_1 which is intercepted by dA_2 . It is, the shape factor is going to be a number between 0 and 1; it stands for that fraction of the radiant heat flow rate emitted from dA_1 which is intercepted by dA_2 . So dA_1 is like this - some elementary area emitting in all directions, dA_2 is somewhere here, so a certain amount of that emitted radiation goes towards dA_2 . The fraction out of the total amount that is emitted over whole hemisphere which goes from dA_1 towards dA_2 is called the shape factor of dA_1 with the respective dA_2 . So it is always a number between 0 and 1 – that is the physical meaning and you must never lose sight of that.

So, now that we have this, let us go on; we haven't yet got the final expression. We know what is the radiation emitted by dA_1 which is flows towards dA_2 and is absorbed by dA_2 so many watts.

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Similarly,
 rate at which radiation emitted by dA_2 flows towards dA_1 and is absorbed by dA_1

$$= \frac{\sigma}{\pi} \frac{\cos \beta_1 \cos \beta_2 dA_1 dA_2}{L^2} T_2^4$$

$$= dF_{2-1} \sigma T_2^4 dA_2$$

where $dF_{2-1} = \frac{\cos \beta_1 \cos \beta_2 dA_1}{\pi L^2}$

Similarly, let us do the reverse calculation now; similarly rate at which radiation emitted by dA_2 flows towards dA_1 and is absorbed by dA_1 because dA_1 is black and is absorbed by dA_1 . Similarly it can be shown this is equal to σ by π cosine β_1 cosine β_2 $dA_1 dA_2$ divided by, divided by L squared into T_2 to the power of 4 which we can write as dF_{2-1} ; again define a shape factor into σ into T_2 to the power of 4 into dA_2 . Note again shape factor dF_{2-1} where dF_{2-1} is equal to cosine β_1 cosine β_2 dA_1 divided by πL square divided by πL squared - that is the shape factor dF_{2-1} . So you got 2 expressions - rate at which radiation emitted by dA_1 flows towards dA_2 and is absorbed by dA_1 , we have a second expression - rate at which radiation emitted by dA_2 flows towards dA_1 and is absorbed dA_1 .

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$$\begin{aligned}
 & \text{Net radiative heat exchange rate } (dq_{12}) \text{ between } dA_1 \text{ and } dA_2 \\
 &= (\text{Rate at which radiation emitted by } dA_1 \text{ is absorbed by } dA_2) \\
 &\quad - (\text{Rate at which radiation emitted by } dA_2 \text{ is absorbed by } dA_1) \\
 &= \frac{\sigma \cos \beta_1 \cos \beta_2 dA_1 dA_2}{\pi L^2} (T_1^4 - T_2^4) \\
 &= dF_{1-2} \sigma (T_1^4 - T_2^4) dA_1 \\
 &= dF_{2-1} \sigma (T_1^4 - T_2^4) dA_2 \\
 &\text{Note that } dF_{1-2} dA_1 = dF_{2-1} dA_2
 \end{aligned}$$

Therefore, the net radiative exchange rate, the net radiative exchange rate which we will call as dq_{12} between dA_1 and dA_2 - this is what we are looking to calculate. The net radiative heat exchange rate, heat exchange rate dq_{12} between dA_1 and 2 is equal to rate at which radiation emitted by dA_1 is absorbed by dA_2 minus rate at which radiation emitted by dA_2 is absorbed by dA_1 . So that is equal to - put in both the expressions in sigma by pi cosine beta₁ cosine beta₂ $dA_1 dA_2$ upon L square into T_1 to the power of 4 minus T_2 to the power of 4 which is equal to, if I want to use the first shape factor dF_{1-2} it is equal to $dF_{1-2} \sigma T_1^4 - T_2^4$ into dA_1 or using the second shape factor - $dF_{2-1} \sigma T_1^4 - T_2^4$ into dA_2 . So all these are equivalent expressions which we have put down for the net radiative heat exchange rate between dA_1 and dA_2 and mind you again I keep the physical meaning of shape factor in mind.

That is important - dF_{1-2} stands for the fraction of radiation emitted from dA_1 which towards dA_2 and is absorbed by dA_2 in this case because dA_2 is black; dF_{2-1} similarly is this fraction of the radiation emitted from 2 which goes towards dA_1 . So note from the definition that $dF_{1-2} dA_1$ is equal to $dF_{2-1} dA_2$. Keep that in mind; so we have now

based on our knowledge of emission characteristics of a surface and our knowledge of intensity of radiation, that is radiation going in a particular direction.

Today, we have derived an expression for the net radiative heat exchange rate $dq_{1 \text{ to } 2}$ between two arbitrarily oriented differential areas dA_1 and dA_2 at 2 different temperatures T_1 and T_2 . We know what is the net radiative heat exchange rate between them by calculating what is emitted by one and absorbed by the other; what is emitted by two and absorbed by one, that's what we have got. In the process we have defined a very important terms called the shape factor which is the fraction of the radiation emitted by one surface which is, which flows to the direction of the other surface.