Advanced Strength of Materials
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Lecture – 7
Mohr Circle (Continued)

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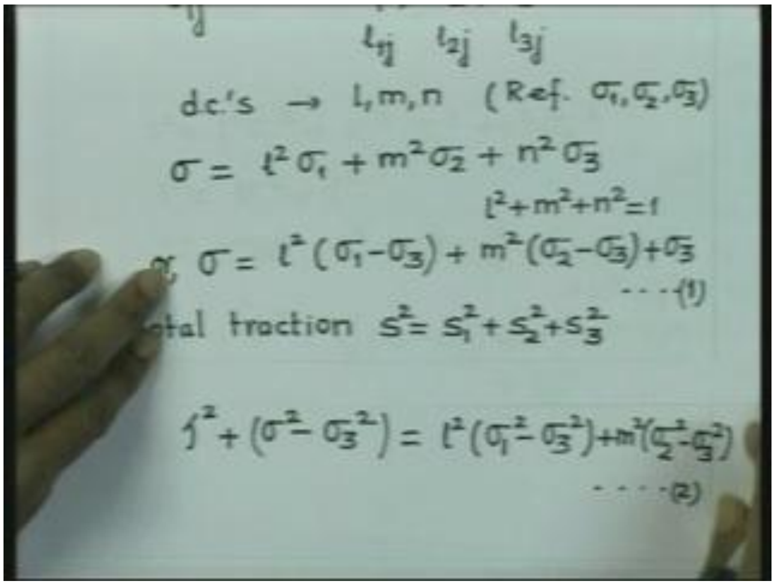

We will continue with Mohr circle in 3 D. Last time, we derived relations 1 and 2, which are shown here. Number 1 is sigma equal to l square into sigma 1 minus sigma 3 plus m square sigma 2 minus sigma 3 plus sigma 3. And then we had also tau square plus sigma square minus sigma 3 square equal to l square into sigma 1 square minus sigma 3 square plus m square sigma 2 square minus sigma 3 square. Now, would like to eliminate m square from these two relations.

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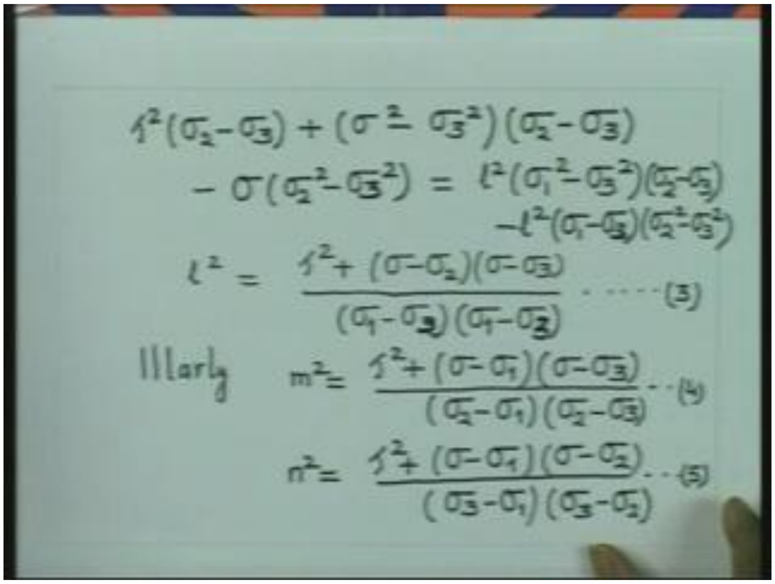

Thereby, we are going to get, so we will get now, tau square into sigma 2 minus sigma 3 plus sigma 2 square minus sigma 3 square into sigma 2 minus sigma 3 minus we will write sigma into sigma 2 square minus sigma 3 square. So, that will be left hand side. And the right hand side, you are going to have l square sigma 1 square minus sigma 3 square into sigma 2 minus sigma 3 minus l square sigma 1 minus sigma 3 into sigma 2 square minus sigma 3 square.

So, if we simplify, this can be simplified to get l square, tau square, sigma minus sigma 2, sigma minus sigma 3 divided by sigma 1 minus sigma 3 into sigma 1 minus sigma 2. So, we can see that, we will have sigma 2 minus sigma 3 canceling out and finally, we will have these simplifications possible. So, by eliminating m square from 1 and 2, we got this. And we started with elimination of m square and thereby, you got relationship between l square and m square.

We could again eliminate from the beginning m square, thereby, you will have equations symbol by l square and m square. And then again eliminating l square, we can get m square and this can be repeated for all the 3 direction cosine. So, you find that, you will get by similar procedure. You will get m square is equal to tau square sigma minus sigma 1, sigma minus sigma 3, divided by sigma 2 minus sigma 1; sigma 2 minus sigma 3. And n square tau square plus sigma minus sigma 1, sigma minus sigma 2, divided by sigma 3 minus sigma 1; sigma 3 minus sigma 2.

Let us call these relations, I will just, I want to write read correct this one as sigma 1 minus sigma 2. This is sigma 2 minus sigma 3. So, l square is given by tau square plus sigma minus sigma 2, sigma minus sigma 3. And in the denominator, you have sigma replaced by this sigma 1. Similarly, you find same thing is repeated in the case of m square and n square. So, let us write these equations as number 3, this is 4 and this one is 5. These equations are really the basis for drawing the Mohr circle.

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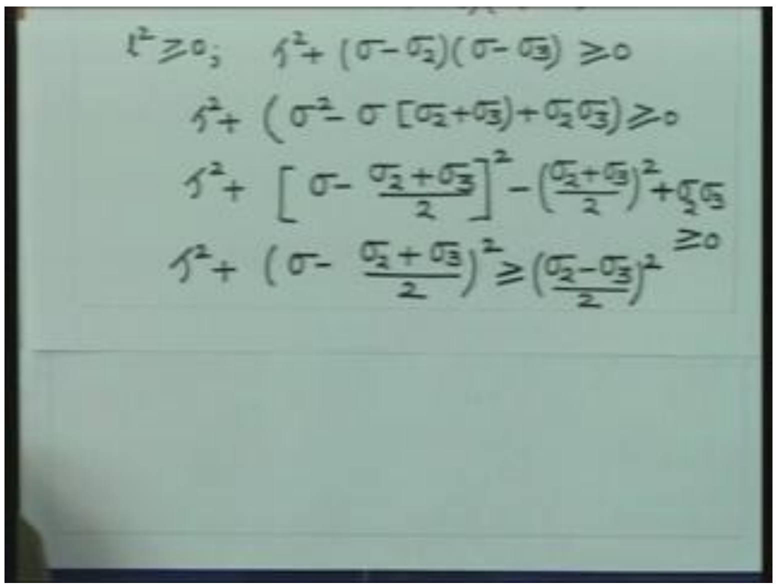

Let us take this equation 1 by 1. First of all, let us consider that, we have a situation, where in the 3 principle stresses, sigma 1, sigma 2 and sigma 3 are the 3 stresses at a point. And we assume that, sigma 1 is greater than sigma 2 and sigma 2 is greater than sigma 3. So, therefore this stresses are what are like this. Now, if you look into the relation 3, it gives you, l square tau square sigma minus sigma 2 sigma minus sigma 3 by sigma 1 minus sigma 2 into sigma 1 minus sigma 3.

Since, the 3 stresses are have been relating magnitudes like this. It is clear that, this difference is the positive quantity; this difference is also a positive quantity. And we know for sure, that l square is going to have a value 0 or something positive. So, this quantity is positive. If this is going to be positive, then it means that, the numerator here has got to be either 0 or greater than 0.

So, from the solution that, l square is greater than equal to 0, we have this tau square plus sigma minus sigma 2, sigma minus sigma 3 is greater than equal to 0. Now, you can write this thing as, tau square plus sigma square sigma into sigma 2 plus sigma 3 plus sigma 2; sigma 3 greater than equal to 0. Now, we can write this thing as, sigma 2 plus sigma 3 by 2, whole square. Then, we can subtract, we have added terms like, sigma 2 plus sigma 3 by 2, whole square. And then you will have sigma 2, sigma 3 and this is greater than equal to 0.

Now, again, we can rearrange, we have now, sigma, sigma 2 plus sigma 3 by 2, whole square. And these, we can take it to the other side and we will have now. We can write this thing as, sigma 2 minus sigma 3 by 2, whole square. So, this quantity, you can surely combine with this and you are going to get this expression. Now, this indicates an important condition, it says that the stress tau and sigma, which are acting on a plane passing through the point.

That is going to be lie on a circle of radius sigma 2 minus sigma 3 by 2, whole square. And the center is located on the normal stress axis at a distance sigma 2 plus sigma 3 by 2; I would like to explain this further. Let us consider now, we will like to draw the stresses like this; you would like to plot our normal stresses in the x direction. So, let us plot the normal stresses in this x direction. So, this is the normal stress axis, and then you would like to plot the shear stresses. So, shear stresses in this vertical direction.

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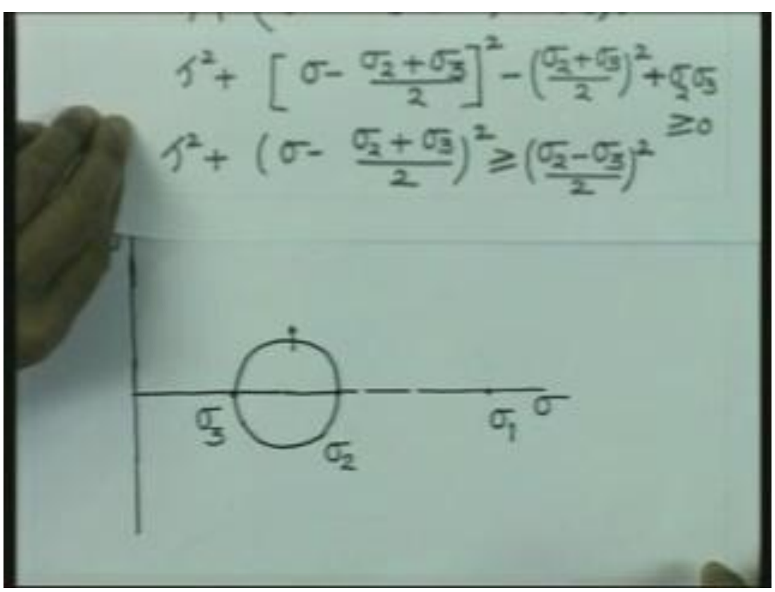

Now, what it means, let us now also write the stresses, that this is my stress sigma 1 and this is the stress, which is sigma 2 and this is the stress, which is sigma 3. So, for the planned, all the tau and sigma, that we are going to get, it is going to lie, when this is equal. It is going to lie on a circle of radius sigma 2 minus sigma 3 by 2, whole square and would center, if located on the normal stress axis at the distance of sigma 2 plus sigma 3 by 2.

So, therefore, if you consider, this is the center of sigma 2, sigma 3. And if you draw a circle with this radius distance between these two points, then will have this circle. So, this is the circle, which is having center at sigma 2 plus sigma 3 by 2, from the origin. And this radius is sigma 2 minus sigma 3 by 2. So, therefore these colleagues have indicated that, this stresses on all the planes, passing through the point is going to lie either on the circle or outside this circle.

So, this are the, we look all the point are going to lie either on the circle or outside. Now, you consider further, if you consider now the relation say, which we have obtain for m.

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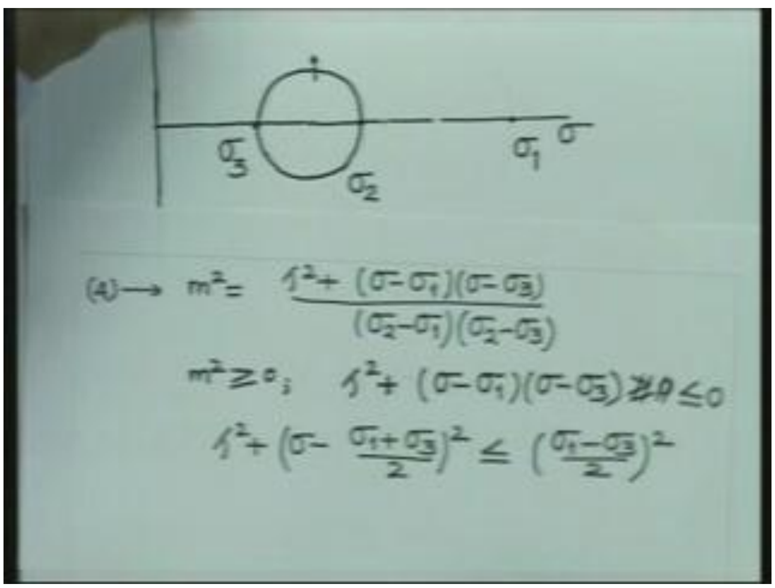

We have m given by m square, tau square, sigma minus sigma 1, sigma minus sigma 3, divided by sigma 2 minus sigma 1 and this is sigma 2 minus sigma 3. So, this is, what we had got a relation 4. Now, again if you look into, we know that, m can have value either 0 or positive values. This m square is always going to be positive, 0 or positive and this sigma 2 minus sigma 2 is here and sigma 1 is here. So, this quantity is negative and sigma 2 minus sigma 3 is positive. So, therefore this denominator is negative.

So, if this is going to be positive, that this numerator has got to be 0 or less than 0. So, therefore for m square positive. So, m square greater than equal to 0. We will have this tau square plus sigma minus sigma 1, sigma minus sigma 3 greater than equal to 0. If m has got to be this, then you find that, this has to be less than equal to 0 values, this should be like this. Since, this is negative, this have got be negative or 0.

So, now if will do little bit of manipulation on these, like the one, you have done earlier. You find that from this relationship, you will get tau square sigma 1 plus sigma 3 by 2, whole square is less than equal to sigma 1 minus sigma 3 by 2 whole square. So, this relationship, indicates again, when we have this equality. It means that, tau and sigma, on all the planes, passing through the point is going to lie on a circle of radius, sigma 1 minus sigma 3 by 2. And it is center is located at sigma 1 plus sigma 3 by 2.

So, sigma 1 is here, sigma 3 is here. So, the middle point is here. So, therefore the point is going to be, all the stresses are going to lie on the circle or within the circle.

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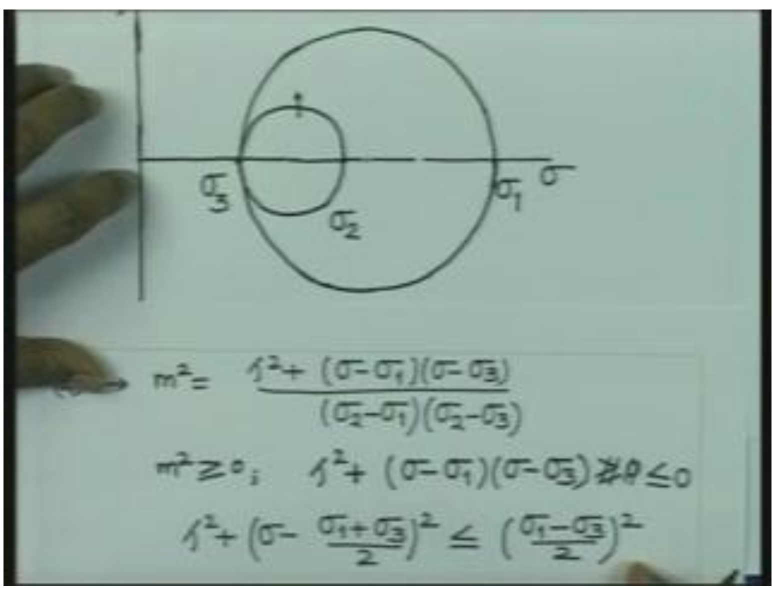

So, therefore we can now draw the circle, which is going to be showing the spread up all the tau and sigma. So, this will indicates that, all the points, all the combination for stresses. And stresses, I am going to lie on this circle, the center is here and radius is this. It is on this circle or within the circle.

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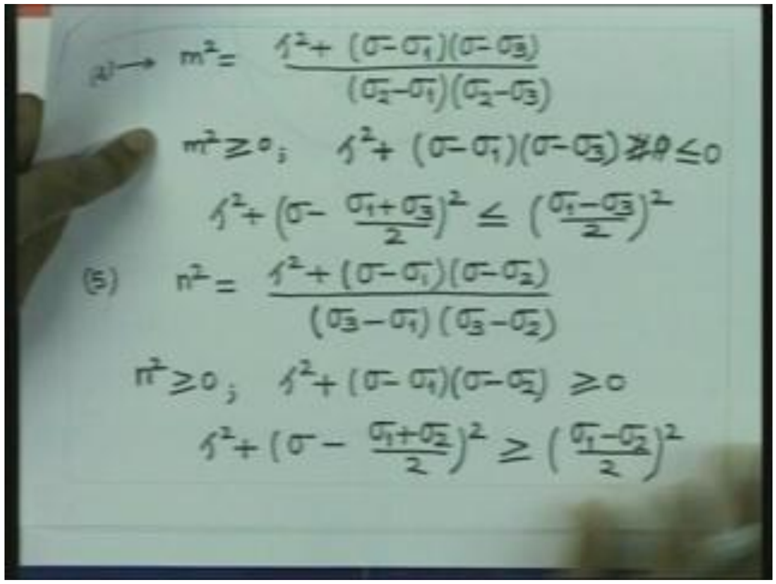

Now, will take the third relation, which was 5 and this 5 was involving stresses all in the numerator it was like this, sigma minus sigma 1, sigma minus sigma 2. And in the denominator, it was sigma 3 minus sigma 1; sigma 3 minus sigma 2. Now, this is going to, again give us, since n square is going to be greater than equal to 0. Now, sigma 3 minus sigma 1 is negative, sigma 3 minus sigma 2 is also negative. So, this is positive quantity.

So, therefore, if n square is going to be 0 or positive, the numerator has got to be 0 or positive. So, therefore this gives us, tau square sigma minus sigma 1 sigma minus sigma 2. It is going to be greater than equal to 0. And if we again rearrange, this gives us, tau square sigma minus sigma 1 plus sigma 2 by 2, whole square greater than equal to sigma 1 minus sigma 2 by 2 whole square.

So, this indicates that, when the stresses are lying on the circle, the circle would have radius of sigma 1 minus sigma 2 by 2. And the center is at the center of sigma 1 and sigma 2, so therefore, if we again consider.

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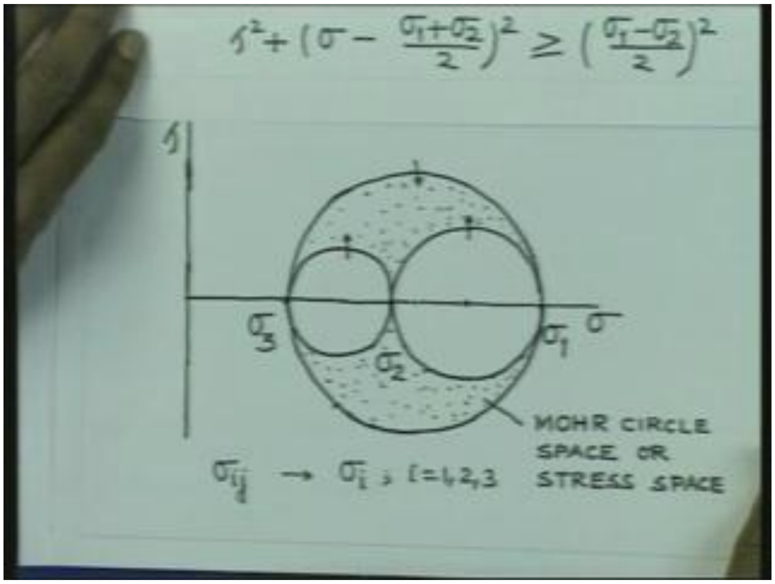

So, if we consider now our circle again. So, sigma 1 is here, sigma 2 is here, so this is the center of the circle. And then the circle which is going to give the dividing boundary is drawn in to radius of sigma 1 minus sigma 2 by 2. So, this indicates that the all the stresses are going to lie on the circle or outside the circle. So, therefore the stresses on all the planes, passing through the point is going to lie in the space bounded by the 3 circles. And this is the space, which will be occupied by all the stresses.

So, this is the Mohr circle space in 3 dimensions or stress space. So, you should be now in the position to draw the Mohr circle in 3 dimensions. What is needed then the steps would be that, you must have your stresses should be known at the point, the stress can sigma i j. Then, you calculate the principles stresses, sigma I, I is 1, 2, 3. Then, you select your coordinates sigma and tau; locate the point's sigma 1, sigma 2 and sigma 3.

Then, draw 3 circles with center, first one with center at this center of sigma 1 and sigma 2 and the radius sigma 1 minus sigma 2 by 2. Second one, you draw with the center at the middle of sigma 2 and sigma 3 and with radius sigma 2 minus sigma 3 by 2. And the third one is, having center at center of sigma 1 and sigma 3 and the radius sigma 1 minus sigma 3 by 2. This is the way; you can draw the Mohr circle in 3 dimensions.

Now, the question might be asked, how do, we find out the stresses on a plane, whose direction cosines are given with respect to the principle directions. So, let us now consider determination of sigma and tau, on plane with direction cosine l, m and n.

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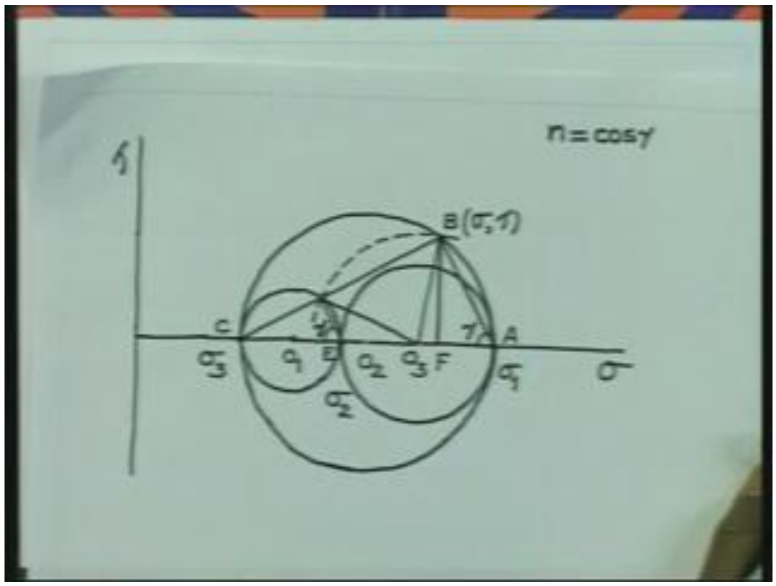

Now, for this case, we draw the Mohr circle, first of all, you select your axis, this is the sigma axis, this is the tau axis. And then we have this, say sigma 1 is here, sigma 2 is located here and sigma 3 is here. So, this is sigma 1, this is sigma 2 and this is sigma 3. Now, we draw the 3 circles. So, now, let us consider that, n is equal to cosine gamma. So, angle gamma is known. Now, you consider, draw a line at an angle of gamma, with the normal stress axis. So, you draw a line at an angle of gamma the normal stress axis. So, this angle is gamma.

Now, we draw the, join the two points, these two points. So, let us consider, this is A and this line intersect at the point B on the larger circle. And this point, let us say, which is c. Now, we would like to joint this point to this sigma 2 point or this point. Let us say, this is E. Now, let us identify this centers of the circle as, this is center O 1, this will be center of sigma 1 and sigma 3 is O 2. And this is the center of sigma 1 and sigma 2, this is O 3.

And now will do little bit of construction will try to now draw perpendicular from this point on the normal stress axis. So, this is F and this, if I consider this O 3 B, you consider joining this O

3 B and we also join these 2 points. And now we draw is, draw an arc, with this has be radius. Now, since this is semi circle, C B A is semi circle. And this is on the circumference and A C is the diameter. So, therefore this angle is 90 degree.

Similarly, you find that, E to this point is going to be normal to this straight line, because this is the diameter. So, therefore this straight line and this straight line, they are parallel and of course, this is also going to make an angle of gamma with the normal stress axis. So, therefore this is also gamma.

Now, we would like to show that, all the stresses, which can satisfy this, n is equal to cos gamma is going to lie on the circle. Now, let us consider that the stress on the plane, that we have trying to find out, their normal stress is sigma and shear space is tau. Now, I would like to show you, that all the stresses; that can satisfy this condition is going to lie on the circle in circular arc.

Now, let us consider, the triangle A, B, O, 3 and this is the triangle; that I am talking, this is the acute angle. And therefore B, O 3 square is equal to O 3 A square plus A B square minus twice O 3 A into A B into cosine of the angle gamma. Now, you can see that O 3 A is nothing but sigma 1 minus sigma 2 by 2. So, sigma 1 minus sigma 2 by 2 and this A B. This distance A C is sigma 1 minus sigma 3 and if I take the cosine of that, so that will be sigma 1 minus sigma 3 cosine gamma that will be A B. So, therefore, sigma 1 minus sigma 3 square cos square gamma is A B. And then we substitute twice O 3 as sigma 1 minus sigma 2 by 2 and A B as sigma 1 minus sigma 3 into cos gamma. So, that gives us cos square. So, that is the expression.

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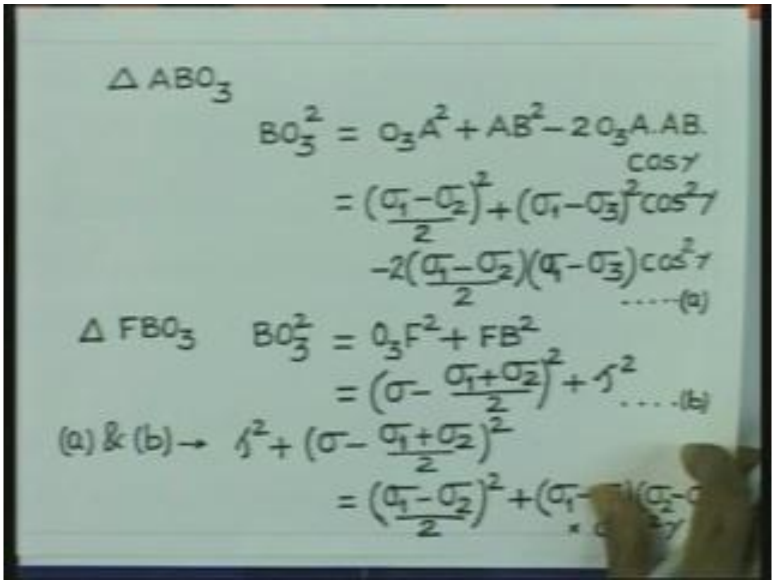

Now, let us consider the triangle F B O 3. So, this is the right angle triangle. So, we can write BO 3 square, B O 3 square is equal to O 3 A square plus A B square. And now, we can write for O 3 F, we can write for O 3 F, O 3 F is nothing but, if we consider the coordinate at this point as sigma and tau, so this distance is sigma from the origin to this point is sigma. And the center of this circle is sigma 1 plus sigma 2 by 2.

So, therefore, if we subtract, it is going to be sigma, sigma minus sigma 1 plus sigma 2 by 2 whole square and B F square is tau square. So, now, if we couple this two relationship A and B, so we can write for B O 3 square as this one. That is BO 3 square and that will be equal to the right hand side, which can be simplified to get sigma 1 minus sigma 2 square, sigma 1 minus sigma 2 by 2 square plus sigma 1 plus sigma 1 minus sigma 3 into sigma 2 minus sigma 3 cos square gamma. So, therefore coupling A and B, we have this expression.

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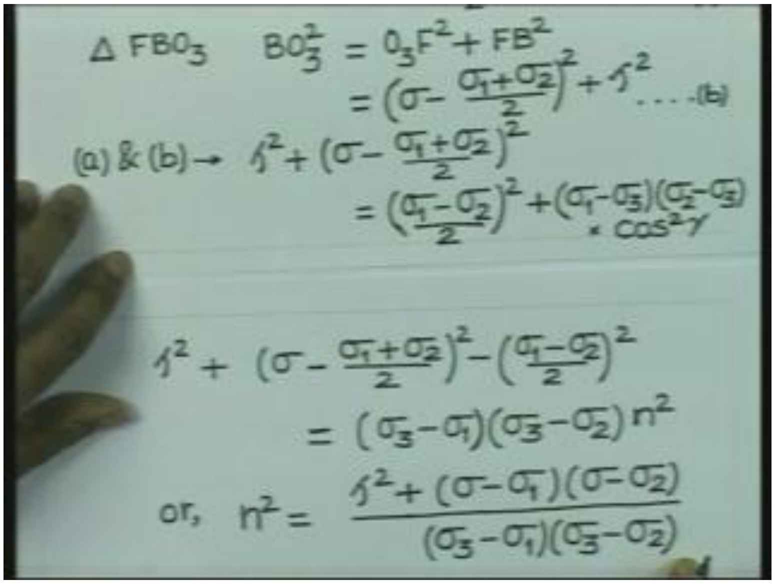

So, now this can be further simplified and you get tau square plus sigma minus sigma 1 plus sigma 2 by 2 whole square, I take this thing to the left. So, therefore the sigma 1 minus sigma 2 by 2 whole square is equal to sigma 3 minus sigma 1 into sigma 3 minus sigma 2 and cos square gamma is nothing but n square. So, therefore will get n square is equal to tau square plus sigma minus sigma 1 into sigma minus sigma 2 divided by sigma 3 minus sigma 1 into sigma 3 minus sigma 2.

So, therefore we see that we get the expression for n; that we have derived during the construction of Mohr circle in 3 dimensions. Now, I would like you to note one point, that if I consider the stress to be given by any other point on this arc. And if I consider this distance, this radius; that is also going to be given by that will form a triangle. So, if I join obviously B O 3 distance will still be given will remain the same.

So, that expression will not change and we can have this same expression of 10 starting from any other this position of the stress. So, what I am saying, that if you consider this radius, that is same as this radius. And therefore, this expression that you had derived will not change. And therefore, even if this sigma and tau is lying at this point, which is going to satisfy the same type of relationship. And we will actually have n square given by this.

So, therefore, stresses are going to lie on this arc. So, we have seen that, any point on this arc is going to give rise to finally, a relationship of this type. Therefore, all this points are ((Refer Time: 38:28)) points, satisfying the condition here. And do not, write at it, let us summarize, what we have done drawn that three circles from the given data. Then, you have taken the angle gamma here drawn this cord, then join this two points, that intersected this circle at this location. And then we have drawn a circle with radius given by this.

So, that is the circle, which is really giving all the combinations of sigma and tau, which will satisfy this relation. Now, we have not yet sure about the values of sigma and tau. So, now, we have to make use of the other direction cosine.

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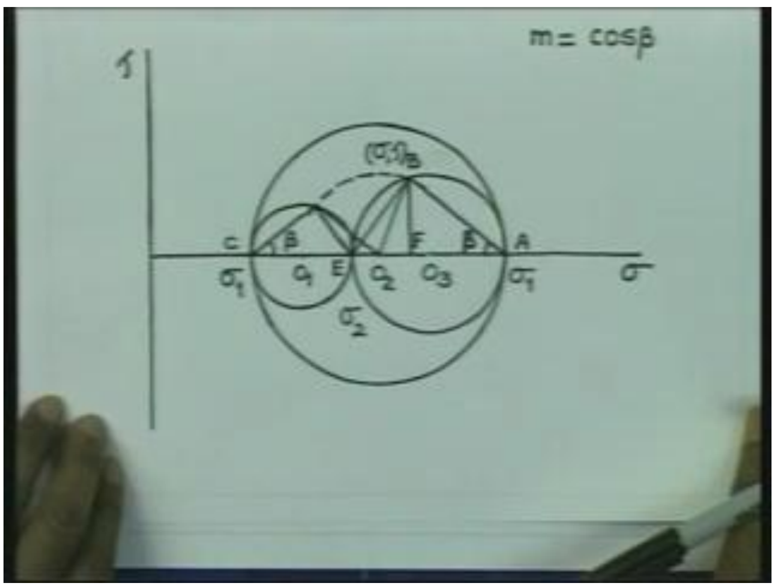

So, now let us consider the again our three circles and let us now consider the direction cosine m. And let us say that, that is equal to m is equal to cosine beta. So, therefore this beta angle can be obtained. Now, what do we do is that, we try to draw a cord with this point at an angle beta in the normal stress direction. So, therefore we draw a cord here, which is at an angle of beta. So, that is the cord will draw.

Obviously, if we consider that, this is A, this is B. Let us draw this line B to this point. Then, we have indicated this point as E. So, therefore B E and B A, there at right angle to each other. Let us, we try this stresses, this is sigma 1, this is sigma 2 and this one is sigma 1. Let us say that, this is C again. And the center of the circles has shown here, this is O 1, this is O 2. And this circle is O 3. Now, we also draw a line here at an angle of beta with a normal stress direction.

So, therefore, let us draw this line, at an angle of beta, again done normal space that direction. And now, if you draw a line joining these two points, they are going to be again at right angle. So, this is the right, this angle is 90 degree. This circular arc, we draw a circular with O 2 At the center and O 2 B as the radius. So, therefore we draw a circular arc, varying this is radius, O 2 B is the radius.

Again, this point; let us say that, the stress at this point is sigma and tau. And now, you draw a normal from this point on to the space axis. Let us say that is equal to F, the foot of the normal effect. Now, these constructions will make use of to derive expressions for m square now.

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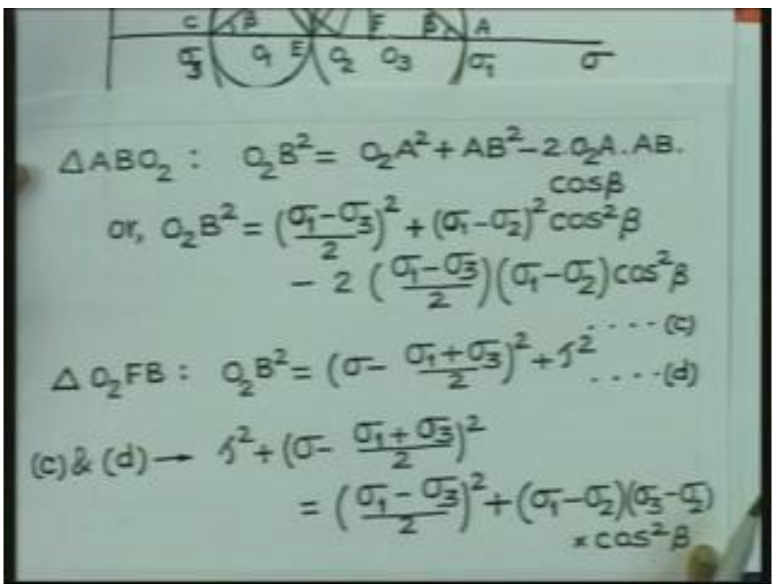

Now, let us consider the triangle again A B O 2. You consider the triangle A B O 2, from this triangle; we have O 2 B square, since beta is an acute angle. We have O 2 B square is equal to O 2 A square plus A B square minus twice O 2 A into A B into cosine beta. Now, O 2 b is nothing but, sigma 1 minus this is the center of the circle, sigma 1 plus sigma 3 by 2. So, therefore, it is going to be sigma 1 minus sigma 3.

So, therefore this distance is nothing but, sigma 1 minus sigma 3 by 2 square. And we can get A B, this is sigma 1 minus sigma 2. So, cosine component of that will be A B. So, therefore, it is sigma 1 minus sigma 2 cos square beta minus we substitute the value of O 2 A and A B; that gives us this relations. Now, again if you consider the triangle O 2 A B, consider this triangle O 2 F B, then you have O 2 B square is equal to O 2 A F. That distance is nothing but, sigma minus sigma 1 plus sigma 3 by 2 whole square and the height is F B is tau. So, therefore that is tau square.

So, please note that, this distance is nothing but, this distance is sigma, here to here is sigma and here to here sigma, 1 plus sigma 3 by 2. So, therefore, if we going to subtract, which is going to be sigma minus sigma 1 plus sigma 3 by 2 whole square plus tau square. Again, if you consider that these two relations, let us say that, this relations are C and D. Combining C and D, we get tau square plus sigma minus sigma 3 by 2 square is equal to sigma 1 minus sigma 3 by 2 whole square plus sigma 1 minus sigma 2 into sigma 3 minus sigma 2 into cos square beta.

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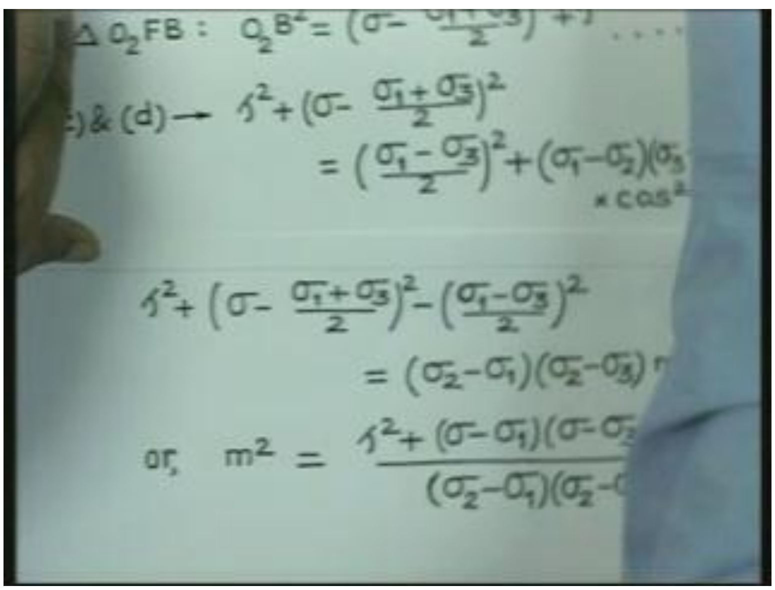

So, this can be further written as simplified form and this will give us now, these expressions. And on rearrangement, we get m square equal to tau square plus sigma minus sigma 1 into sigma minus sigma 3 into sigma 2 minus sigma 1 into sigma 2 minus sigma 3. So, therefore, what we find is that, the stress point.

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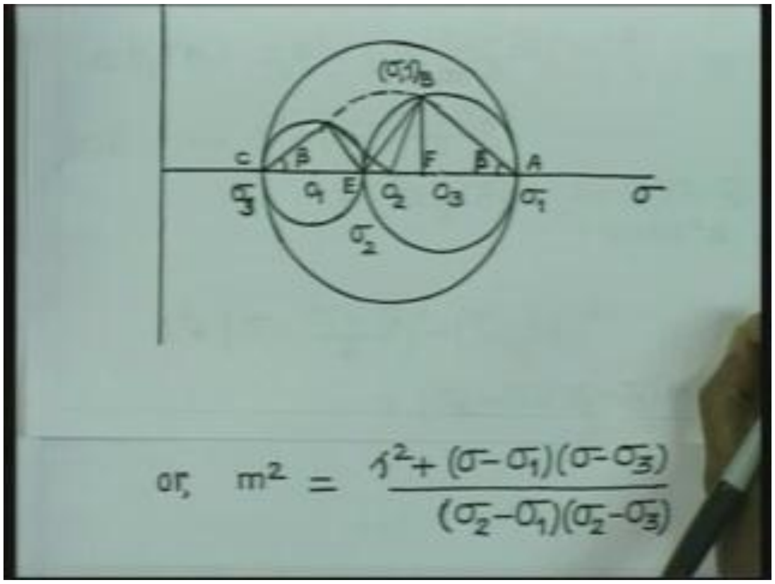

Stress point beta at bigger point, stress point B satisfies the relationship. And now, if you draw any arc, would this at the center and this at the radius, it will also satisfy the same condition. And therefore, this is the other arc on who is the possible combination of sigma and tau will lie. So, therefore, now we have got two arcs.

And if I now try to superpose these arcs on the earlier one, so will get this. I would not like to go the construction, I will show the full construction, but I draw that arc, that you are tend from the second constructions. So, I will just try to this construction, we have got from m. So, this is from m and this is what is from m. So, therefore, if I draw the arc; that we have got in the second case, it is like this. So, this is the intersection of the two arcs and therefore, that is going to be the point, which defines the stress required on the plane in direction cosine l, m and n. And it can be shown very easily. That, so I think you have got the intersection of the two arcs; that the stress.

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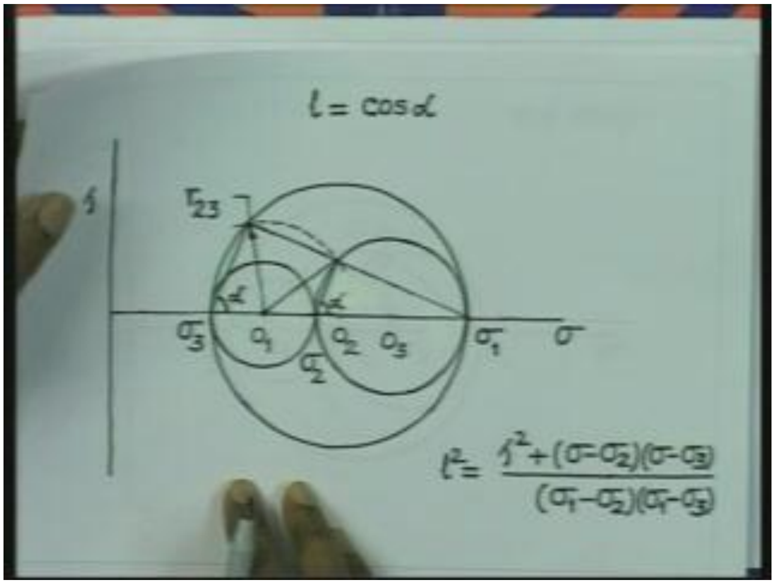

And in fact, you are going to see that, if the direction cosine l is also taken, l is also considered, l let us say, that is equaled to cos alpha. So, if you take l as cos alpha, then your construction would be like this. So, this is the circle. So, what we do is that, you draw a line here with this has be angle alpha. Then, this stress line is also going to make an angle alpha and this at the center with O 1 as the center. If you draw a circular arc of this radius; that will be the combination of stresses, which will satisfy this relationship. So, therefore this becomes another arc, which is also going to satisfy the condition or you can satisfy the weaker conditions.

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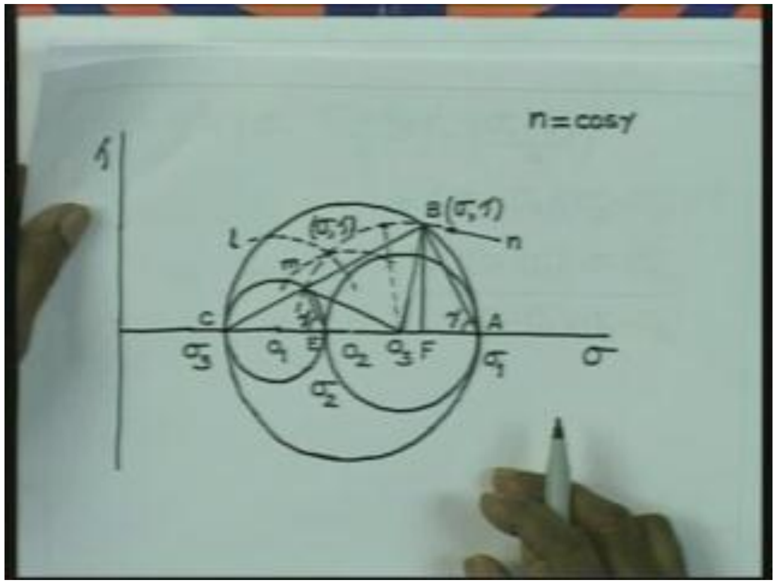

And therefore, you will find for sure, that will be tree arcs for superpose this third arc; I will also superpose it here. It is going to be like this. So, this one, we had got this first one, we had got from the consideration of m. The second one, we have got from the consideration of m. So, therefore, this is from the consideration of n and then we have got this from the consideration of m and the last one; that we have got from the consideration of l.

So, therefore all this arcs are intersecting at a point and therefore, this is the unique point, which is indicating the stress on the plane with direction cosine l, m and n. So, we can now solve problems involving determination of stresses nominee arbitrary plane, which is given to you. Let us then consider one example, wherein the state of stress was giving in terms of a tensile involving nine components. And the principle stresses and the directions, where determined.

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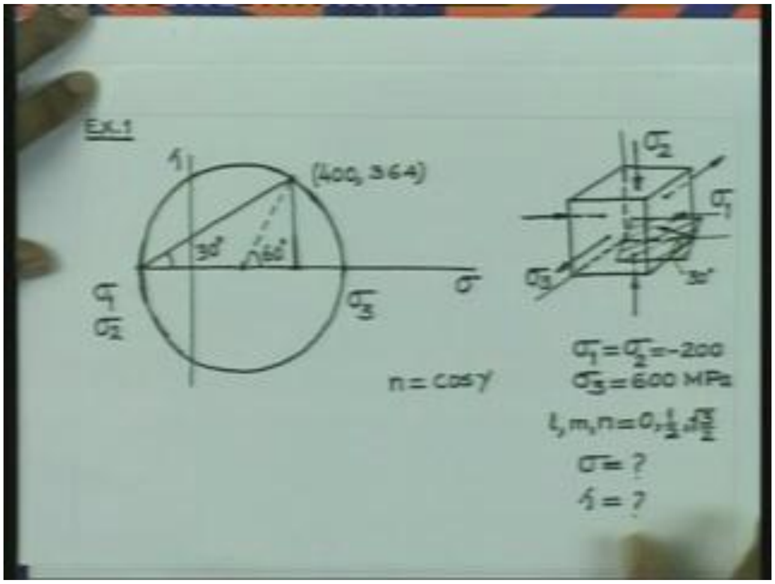

Let us consider that, this is the principle, stress direction 1, this is 2 and 3. The magnitude of the two principle stresses, sigma 1 and sigma 2 are minus 200 MPa and sigma 3 is 600 MPa. So, in this case, what is needed is that, find out the stress on a plane, which is making an angle as whose outer normal is making an angle of 30 degree with the sigma 3 directions. And 60 degree with the sigma 2 directions.

So, you are giving the plane, whose is outer normal is approximately like this. This is outer normal and that could be plane could be, which is normal to this, could be this one. So, this is the outer normal, which makes an angle of 30 degree. So, this is the angle, which is 30 degree and which is making an angle with the sigma 2 directions of 60 degree. And it is necessary to find out, the normal stress acting on this plane and also the shear space.

So, let us now draw the Mohr circle. So, you select that the axis. Let us select that scale of that, we have 1 centimeter is equal to 100 MPa. So, now this direction will have sigma 1 and sigma 2. So, therefore, this is sigma 1 and sigma 2, and then sigma 3 is equal to 600 MPa. And it is positive stress. So, therefore it will be somewhere here, this is sigma 3. Now, since sigma 1 and sigma 2, have the same value the Mohr circle, which is to be drawn with radius sigma 1 minus sigma 2 by 2 has become a point.

Now, we have to draw a circle with radius sigma 1 minus sigma 3 by 2. So, therefore, this center will be at the middle of this distance. And therefore, if you draw the circle, so this is the circle, which is with radius sigma 1 minus sigma 3 by 2. So, now if we draw in this case, n equal to cosine gamma, and if you draw a line at 30 degree from sigma 1 and sigma 2, facing sigma 3. So, therefore, this is the line. Therefore, this is the point.

In this case, again if you consider the circle with sigma 2 and sigma 3. So, if you draw a line at angle of 30 degree. That is going to be meeting the circle at the same point. So, therefore, this is the point that we are interested in or this indicates the stress on this plane, which is given by direction cosine like this. And therefore, the stress on this plane is going to be given by the normal stress is of this magnitude.

And obviously, you can calculate this distance, who can also obtain from the measurement. This comes out to be 400, normal stress is 400 MPa. And this same stress, which is given by this height, is nothing but, radius of this circle is 300. And you can note that, this angle is equal to this angle is 60 degree. And therefore, this is nothing but the radius of the circle is, 400. So, therefore, 400 into 10's sin 60 and therefore, this will give us 364 MPa. So, that could be stresses on the plane is nothing but, normal stress is 400 and the shear space is 364. And we have always, of course, made use of this at the two stress directions.