Advanced Strength of Materials
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Lecture – 6

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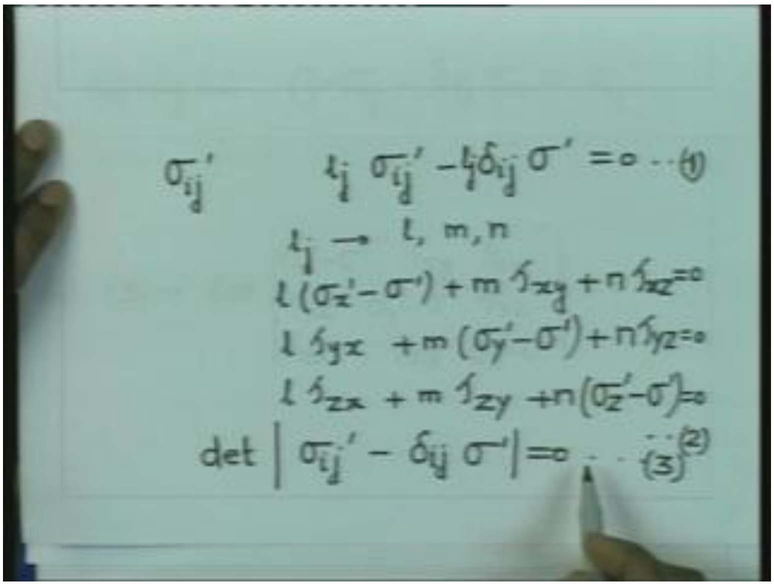

Last time we saw that the Deviatoric part of the straight sense are sigma i j dash. We will have their principles directions and also the associated stresses, which are going to be given by these set of equations. Where, the sigma dash represents the principle stress and principle directions are given by l j. And also we saw that the l j has components like l, m and n. Then, we are going to have these equations in the expanded form. And if we have to have the non zero solutions for the directed cosines l, m and n, we must have determinant of the position matrix equal to 0.

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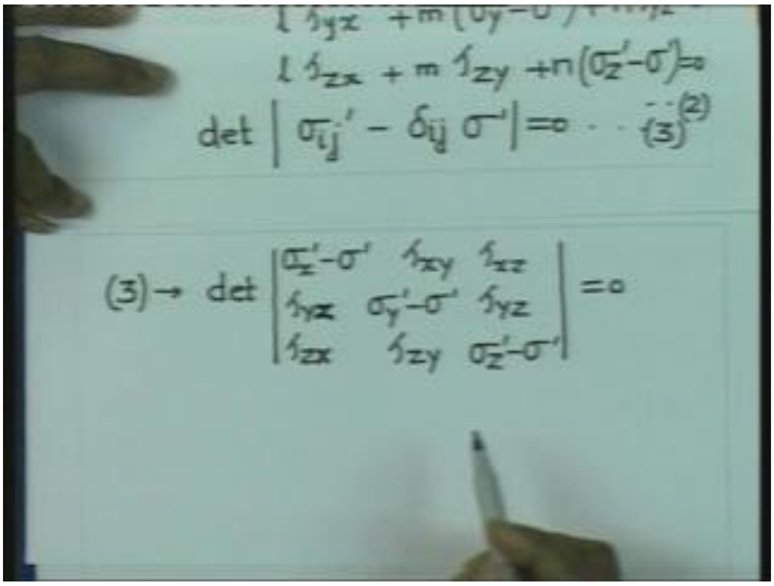

This can be written in the expanded form like this, which is very similar to the earlier case, where we have considered that the total stress. And the principle stress is given by sigma. And these stress components were sigma x sigma y and sigma z, not if the dashes.

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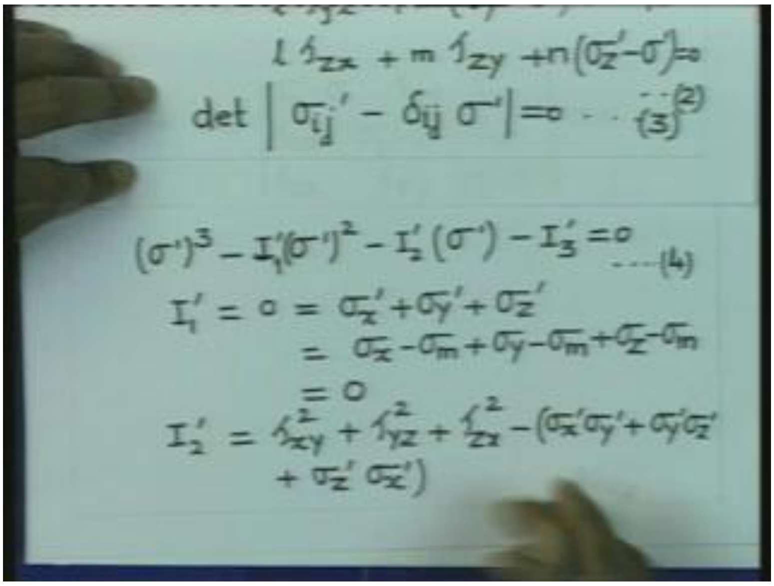

Now, this can be expanded and if we expand it, comes out to be like this. Sigma dash cube minus I 1 dash into sigma dash square minus I 2 dash, sigma dash minus I 3 dash equal to 0. This is equation number 4, where in again I 1 dash, I 2 dash and I 3 dash are stress in variance. And these are given by I 1 dash is equal to 0, because this is equal to sigma x dash plus sigma y dash plus sigma z dash. And this is nothing but sigma x minus sigma m plus sigma y minus sigma m plus sigma z minus sigma m.

Since, sigma m is equal to sigma x plus sigma y plus sigma z by 3. So, this is sum of these three stresses to cancel and therefore, this is going to be 0. Similarly, the other invariance as been formed as we have seen earlier is going to be tau x y square plus tau y z square, tau z x square minus sigma x dash into sigma y dash plus sigma y dash into sigma z dash plus sigma z dash into sigma x dash.

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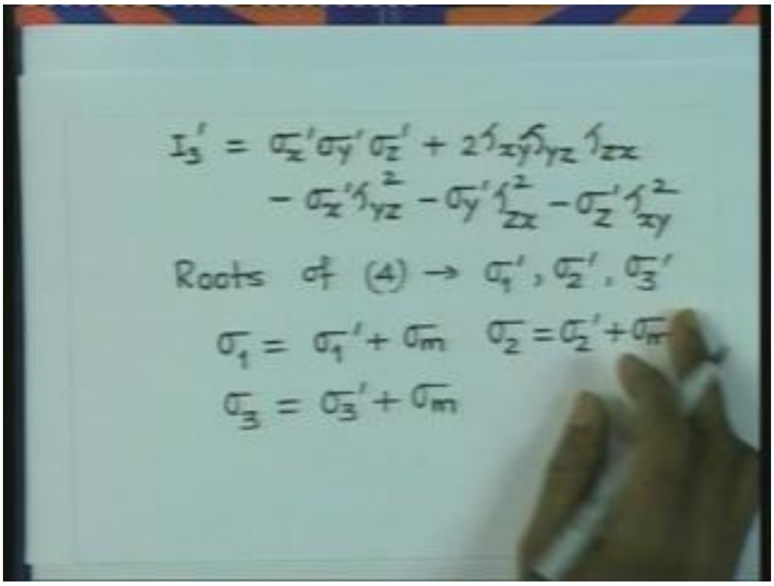

That will be second stress invariance and we can know write the other component. I 3 dash sigma x dash, sigma y dash, sigma z dash plus twice tau x y, tau y z into tau z x minus sigma x dash y z square minus sigma y dash, tau z x square minus sigma y dash into tau x y square. So, we can solve the equation number 4, using these coefficients I 1 dash, I 2 dash and I 3 dash. We are since, I 1 dash, we will get some advantage in solving this equation 4.

And finally, suppose these roots are sigma 1 dash, sigma 2 dash and sigma 3 dash. Then, we can write that principle stresses for the total stress stanza is nothing but sigma 1 dash plus sigma m. Sigma 2 equal to sigma 2 dash plus sigma m and sigma 3 equal to sigma 3 plus sigma m. You can also, after having obtained this 3 Deviatoric principle stresses, you can get back to the equation number 2.

You can get back to this equation, therein; you can substitute the value of sigma 1 dash. In these three equations, you can extract the roots l, m, n from this equation to get back the corresponding directed cosines. And it can be repeated for sigma 2 dash and sigma 3 dash. This is the way, how you can solve for the principle directions starting from the Deviatoric part of the stress components. Next, we will consider the transformation of stresses.

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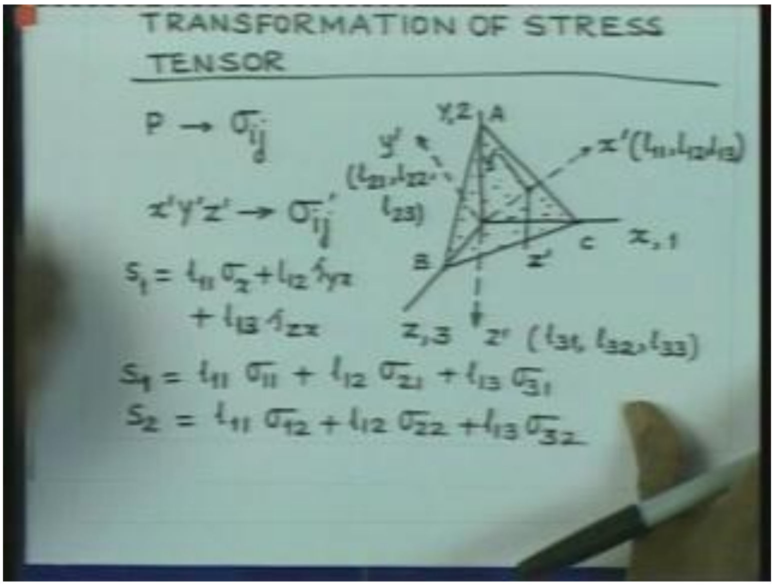

So, we will consider the transformation of stress tenses or the transformation of stresses, from one system of coordinates to another system of coordinates. Let us consider that, we have three directions like x, y and z, which you can also write as direction 1, 2 and 3. Suppose, the stress at a certain point is given by the tensor sigma i j, you have nine components of stresses in three dimensions.

Now, we would like to transform these stresses to another coordinates. Let us say that, we have another set of coordinates, which is given by x dash, y dash and z dash. So, if the stresses are given in this system x, y, z. We would like to get the system in x dash, y dash, z dash, which are arbitrarily oriented with respect to the original system. Now, in the system x dash, y dash and z dash, we would like to write the stresses by this symbol sigma i j dash.

Now, let us see, how we can go about getting these stresses in the system x dash, y dash and z dash. Let us write, that the directed cosines of the new set of axis, say this x dash.

These have directed cosines like l 11, l 12, l 13. So, this is the directed cosines of the system x dash with respect to the x, y, z. So, also, the directed cosines of this axis y dash is l 21, l 22, l 23 and the directed cosines of z dash is l 31, l 32 and l 33.

Now, if you consider the plane perpendicular to x dash. Let us consider the plane, which is perpendicular to x dash. So, this is the plane A, B, C. So, this is the plane, which is perpendicular to x dash. Then, obviously, we can see that on this plane; will have the axis Y and Z line. So, therefore, this is the outer normal to the plane. Then, these two axis y dash is going to lie and on this plane, so also, z dash is also going to lie on this plane.

Now, we can write the fraction on this plane to be given by, let us say S 1 is given by l 11 into sigma plus l 12, tau y x plus l 13, tau z x. If we make use of the indicial notation, we can write now little in the simplest form l 11, sigma 11, l 12, sigma 21 plus l 13, sigma 31. And the S 2, the other traction s going to be nothing but l 11, sigma 12, l 12, sigma 22 plus l 13, sigma 32.

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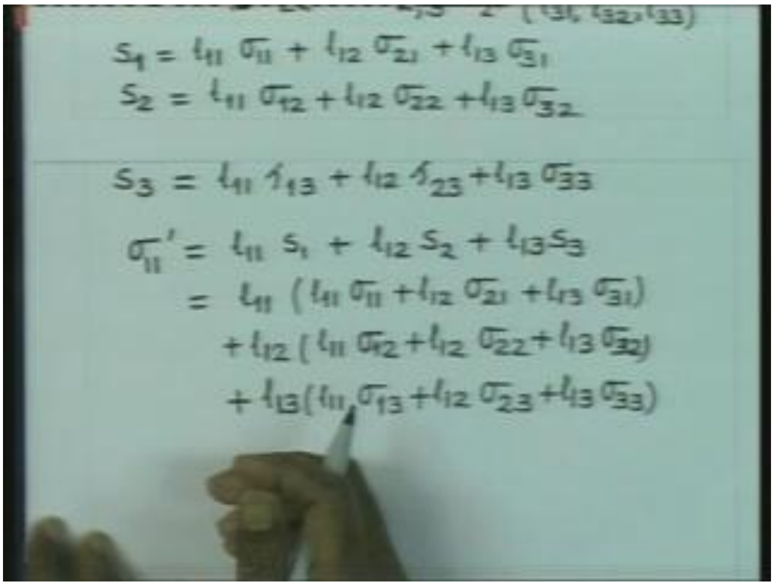

Similarly, we will have traction S 3, l 11, tau 13, l 12, tau 23, plus l 13 sigma 33. So, these are the tractions on this plane. So, we are going to have the tractions on this plane to the x direction as X is S 1, Y direction as S 2 and Z direction as S 3. Now, we can get the components of the tractions, along the x dash. So, if we try to consider the components of these tractions in x dash direction. That will give us the normal stress or that will give us the stress in the x dash direction.

So, therefore, we are going to get now, the stresses, sigma 11 dash is nothing but it is l 11, S 1 plus l 12 into S 2 plus l 13 into S 3. So, if you substitute the values of the three tractions. We are going to get now, l 11 sigma 11, l 12, sigma 21 plus l 13, sigma 31 plus l 12, l 11 sigma 12, l 12 sigma 22 plus l 13 sigma 32, plus, we are going to have l 13, l 11, sigma 13, l 12 sigma 23, l 13 sigma 33.

So, this is what is going to give us the expression for the normal stress on the plane, which is perpendicular to new axis, x dash. Similarly, we can get now, after having got the stress in this direction by taking the components of the three tractions. In this direction, we got sigma 11 dash. Similarly, if we take the components of those traction in the direction Y dash, that will give the traction sigma 12 dash.

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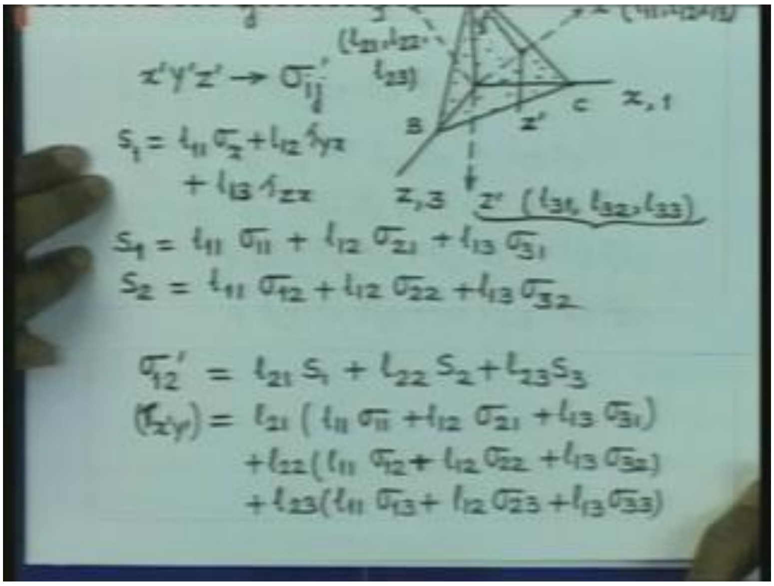

So, we can now write sigma 12 dash. So, for the convenience, I can probably write here, this is nothing but sigma x dash and this nothing but tau x dash y dash. That is the stress and this is going to be given by, will now make use of the direction cosine of this y dash which is nothing but these set.

So, we will now write, that the component sigma 12 dash is nothing but l 21, S 1 plus l 22, S 2 plus l 23, S 3. And again, if you substitute the value of the three tractions, that you have, what we find is, nothing but the expression l 21, l 11, sigma 11 plus l 12, sigma 21 plus l 13, sigma 31 plus l 22. S 2 is nothing but l 11, sigma 12, l 12, sigma 22 plus l 13, sigma 32, plus l 23 into l 11 into sigma 13 plus l 12 sigma 23 plus l 13 sigma 33.

So, we can get now the stress components sigma 12 dash. And you should have no difficulty in appreciating the fact that, if you want to find out the component of the stress in the z dash direction. We can make in the z dash direction, we can make use of the direction cosine given by this set. So, this will give us the component sigma 13 dash. Now, all these can be written very conveniently by making use of the tensor notation and it will give us finally, the total of all these components that we are going to write.

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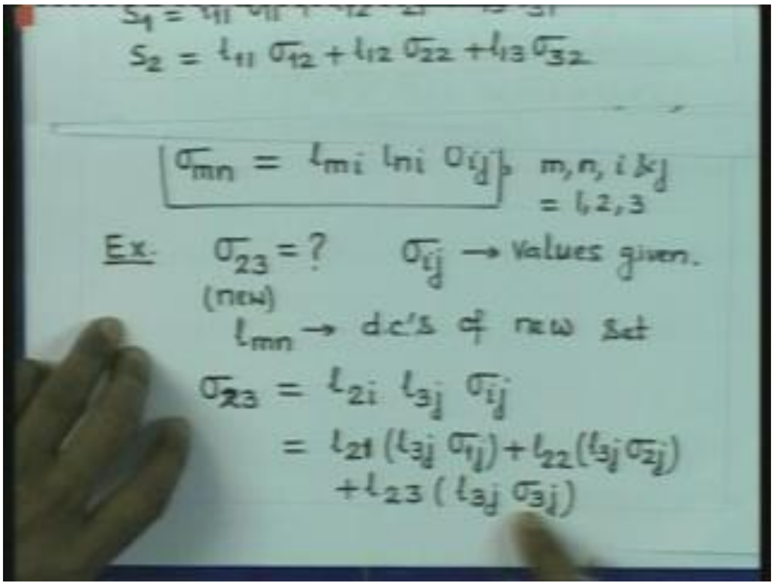

We can write in this form, which is nothing but sigma m n. Suppose, these are the stresses in the new system, X dash, Y dash and Z dash. And we have the original set given in the sigma i j, let us say and it is given in the system x, y, z. Then, we can write the connectivity between the two set, using the earlier expressions in the compact form and the generic form like this. Sigma m n is equal to l m i, l n i, sigma i j, where in all these m, n, so also i and j take of values 1, 2, 3.

So, this is the transformation rule, for the second rank tensor. So, this rule for transforming the second rank tensor, sigma m n is equal l m i into l n i into sigma i j. Now, I would like to illustrate this by considering the examples later. But, if we want to write any set, it will be like this. Suppose, if you are interested in writing. So, let us say will consider the example, we are interested in getting the new value of let us say, stress 23 and our original set is given by sigma 23. New is what, when these values are given.

So, again, we will make use of the fact that the new set will have the directed cosines given by l, m, n. Let us say, the direction cosines of the new set. So, I can now write the sigma 23 is equal to l 2 i, l 3 j sigma i j. Now, this can be further expanded like this. We will have values of i varying from 1, 2, 3, so also value of j varying from 1, 2, 3. So, this repetition of indices indicates that there is the summation implied.

So, we can now write, if i take the value of i to be first changing. So, therefore, I can write this thing as l 2i, I am sorry, l 21 into l 3j and this will become 1 j plus l 22, l 3j, sigma 2 j plus l 23 into l 3 j sigma 3 j. So, you can link it up, that these are nothing but actually the tractions. These bracketed quantities are nothing but the tractions on the plane, which is perpendicular to the directions 2.

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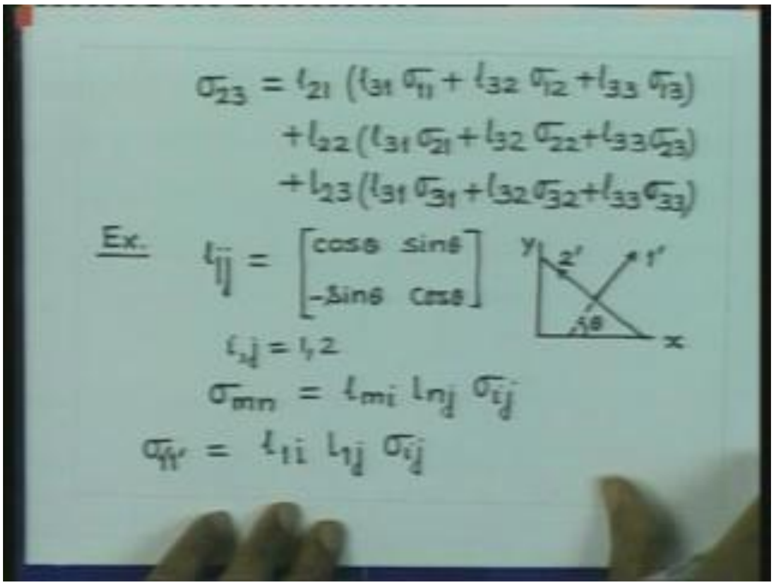

So, therefore, we can now expand it further. This is nothing but l 21 into now j can be taken as the given values 1, 2, 3. So, therefore, it will be l 31 sigma 11, l 32 sigma 12, plus l 33 sigma 13 plus l 22, l 31 sigma 21 plus l 32, sigma 22 plus l 33 sigma 23 plus l 23, l 31, sigma 31 plus l 32, sigma 32 plus l 33, sigma 33. So, you can go on writing the value for the stress components sigma 23. In the new system, from the old system, knowing the direction cosines of the new set of axis.

Now, we will like to consider another example to make you comfortable about the transformation. Let us consider the examples. Wherein, you are interested in considering the stresses in two dimensions. Let us say that, we have the stresses given in the system x, y and we will consider the new directions. Let us say the new directions are, this is the direction, new direction. And let us say, that this direction is 1 dash and this is the direction 2 dash. So, therefore, this direction 1 dash makes an angle theta with the x direction.

So, therefore, we have the direction cosines l i j, l i j is given by the direction cosines of this is nothing but cos theta sine theta. And direction cosines of this direction 2 dash is sine theta, cosine theta. So, therein, it is a two dimensional problem. So, therefore, i and j takes some value 1 to 2. Now, the transformation rule is going to be sigma m n l, m i l, n j sigma i j. Let us find out the component sigma 11. So, let us find out the component sigma 11. So, sigma 11 dash, we indicate that with the sigma 11 dash. This is nothing but l 1, 1 1i, l 1 j into sigma i j.

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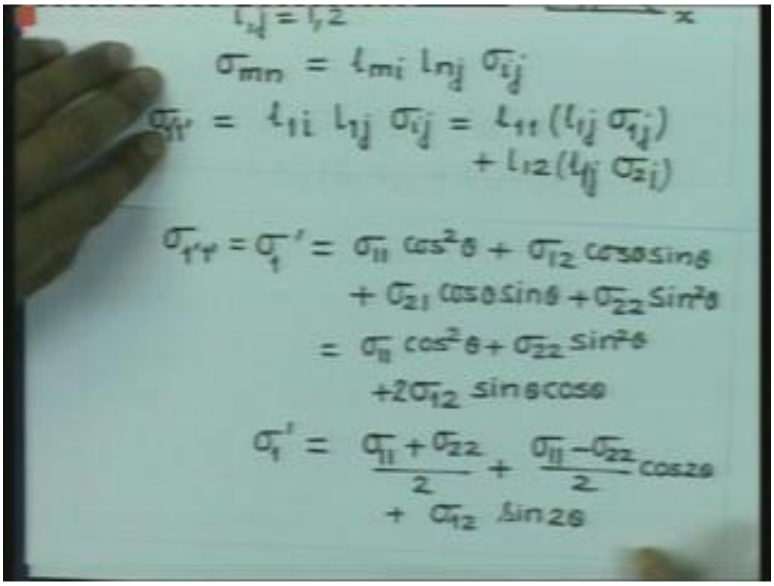

This is going to be given by l 11, l 1 j. So, we are taking the value of i to be 1. So, therefore, l 11 into l i j into sigma 1 j plus l 12, l 1 j into sigma 2 j. So, you can expand it further. So, this is equal to l 11, l 11 sigma 11 plus l 12 sigma 12 plus l 12 into l 11, sigma 21 plus l 12, sigma 22. So, now we will try to substitute the values. So, we will get now, we will simply represent the stress sigma 1 dash, 1 dash as sigma 1 dash. This is going to be now, we have l 11, l 12, l 21 and l 22.

So, if we now substitute the values, then this is sigma 11 cosine square theta plus this is cos theta, this is sin theta, so sigma 12 cos theta, sin theta. Similarly, we have this is sine theta, this is cos theta. So, therefore, it is sigma 21, cos theta, sine theta. And this is nothing but sine square theta, so therefore, sigma 22 sine square theta. So, this finally is nothing but sigma 11 cos square theta plus sigma 22 sine square theta.

Since, sigma 12 equal to sigma 21, we can write this thing as sigma 12 twice sine theta cos theta. This can be written finally, that the sigma 1 dash is nothing but if we write this cos square theta is equal to 1 plus cos 2 theta by 2 and sine square theta as 1 minus cos 2 theta by 2. Then, this will be given by sigma 11, sigma 22 by 2 plus sigma 11 minus sigma 22, by 2 cosine 2 theta plus sigma 12 sine 2 theta. So, we find that sigma 1 dash is given in this form.

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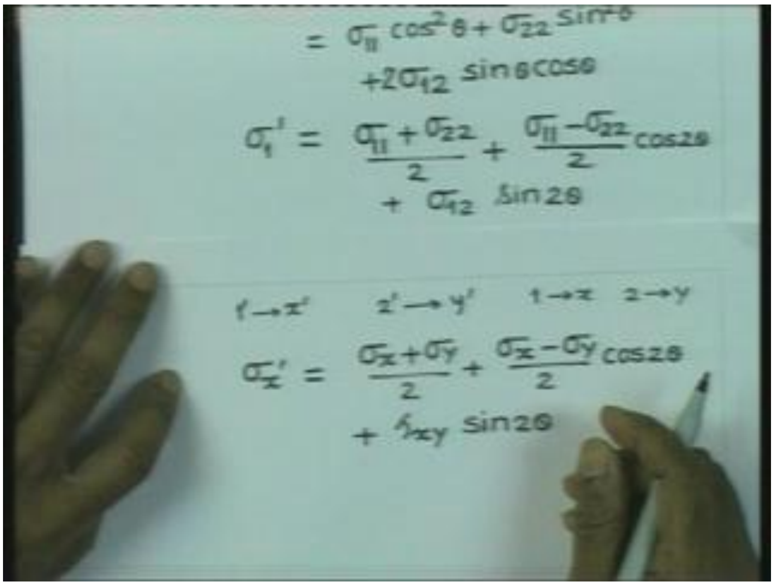

And if you want write in the x, y system. So, if represent 1 dash by x dash and 2 dash by y dash and similarly, 1 by x and 2 by y. Then, you can write the above expression in the form, that sigma x dash is equal to sigma x plus sigma y by 2 plus sigma x plus sigma y by 2 cosine 2 theta plus tau x y sine 2 theta. So, this is the transformation that you have considered in relation to more circles in two dimensions.

Similarly, if you now consider the other stress components. So, we would like to consider the component of the stress, which is going to be in the direction, 2 dash. So, it will be, this is the plane, we are considering and so this is the direction, which is outer normal. So, now we will get the component sigma 1 dash, 2 dash. This is now going to be equal to. So, it is again 1 i and n is 2 now, 2 j.

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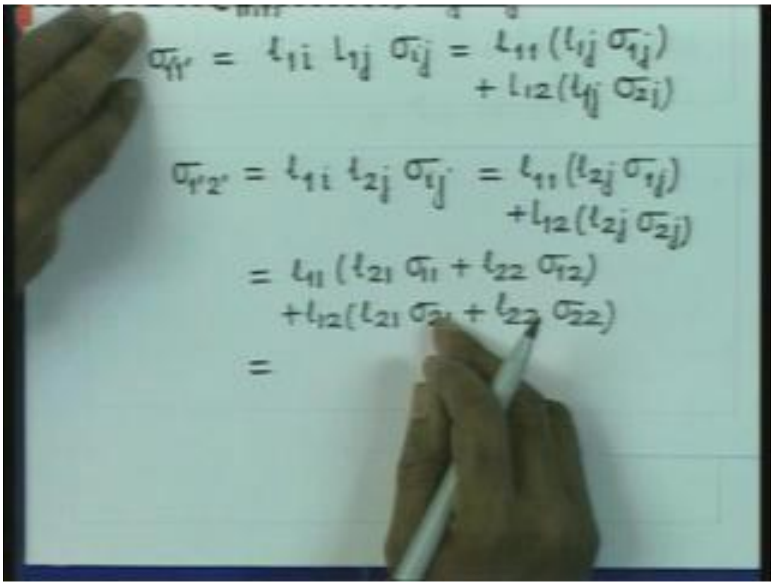

So, in fact, this dash has no meaning, we are just try to consider the new direction 1 and 2. So, this is how, we are going to get the link of the new system to the whole system. And again, we are going to have expansion of i first. So, it is l 11, l 2 j, sigma 1 j plus l 12, l 2 j, sigma 2 j. Now, we can write l 11, l 21 sigma 11, plus l 22 sigma 12, plus l 12, l 21 sigma 21, plus l 22 into sigma 22. So, all the four components of the stresses are now involved, sigma 11, sigma 12, sigma 21 and sigma 22.

So, if we now try to collate them, noting that the values of directed cosines. So, if we look into the values of the direction cosines, which we have given earlier. So, we will have now, this is cos theta, this is minus sine theta. So, therefore, it is the going to be cos theta, sine theta, sigma 11 plus this is cos theta, this is cos theta. So, therefore, cos square theta sigma 12.

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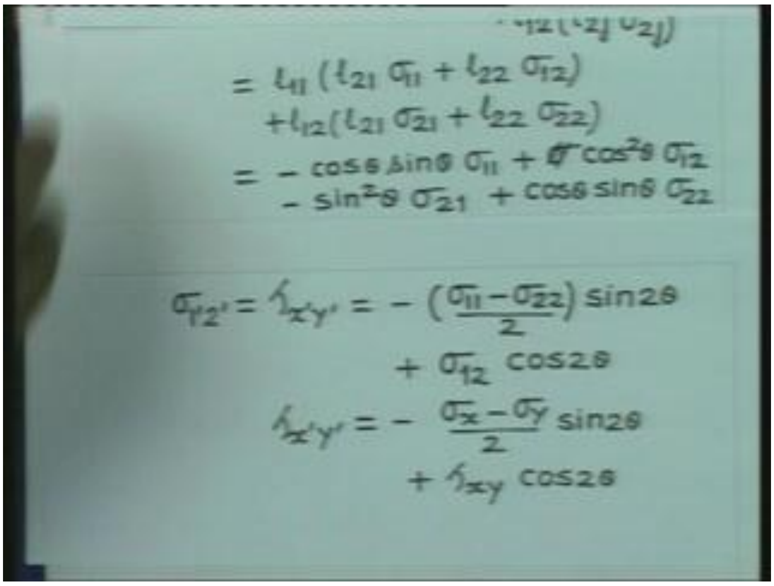

And this is again, l 12 sine theta and this is minus sine theta. So, therefore, it is minus sine square theta sigma 21 plus here this is l 12 is minus sine theta and this is cost theta. So, therefore, this is sine theta and this is cos theta. So, therefore it is cosine theta, sine theta, sigma 22. So, we can now write it, sigma 1 dash, 2 dash, which is nothing but the stress.

And this is going to be minus sigma 11 minus sigma 22 by 2, sine 2 theta plus since sigma 12 is equal to sigma 21. So, you can write this thing as sigma 12 cosine 2 theta. So, this is the expression for the shear stress in the new coordinates. And if we want to write in terms of sigma x and sigma y, this is nothing but sigma x minus sigma y by 2 sine 2 theta plus tau x y cosine 2 theta.

So, the new shear stress is like this and this is what, again you have obtained in connection more circle diagram in the two dimensions. Just finally, to look at the expression for the transformed x dash and the shear stress is like this.

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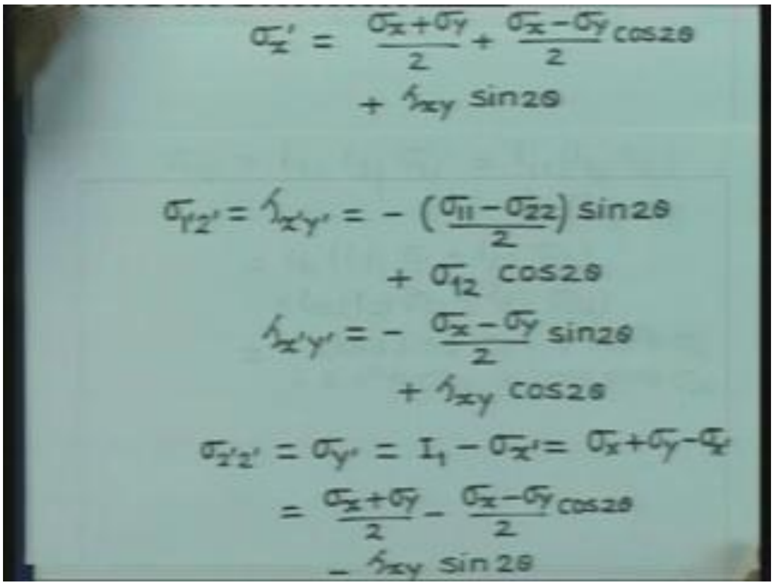

And if you are interested in finding out the stress sigma 2 dash, 2 dash, which is nothing but sigma y dash. You make use of the fact that, this is driven by the stress invariant. We can make use of I 1 minus sigma x and this is nothing but sigma x plus sigma y, this is sigma x dash minus sigma x dash. The stress invariant is same. So, therefore, one can use of the fact that sigma x dash equal to sigma y dash, I that gives these.

And finally, if you substitute the value of the sigma x dash, it is finally, come to be like this, sigma x plus sigma y by 2 minus sigma x minus sigma y by 2 cosine 2 theta minus tau x y sine 2 theta. So, therefore, that is the sigma y dash. So, finally, your new stress are give in sigma x dash like this and sigma y dash is given here and shear stress. Now, we will like to see, how to draw more circle for stresses in three dimensions. You are familiar with the most circles in the two dimensions.

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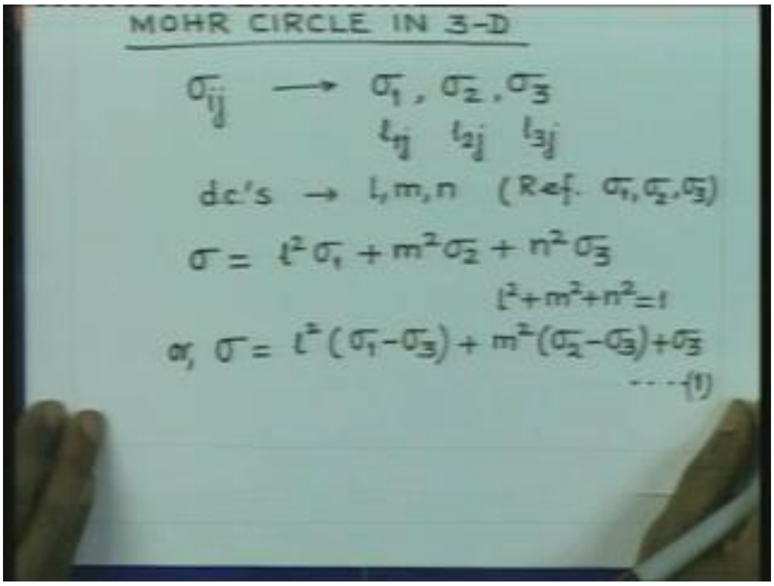

So, we will like to consider the construction of Mohr circles in three dimensions. We have the stresses at the point i j starting from the stresses. You must calculate the principle stresses, sigma 1, sigma 2 and sigma 3; you can also calculate the directions. Now, we will like to consider an arbitrary plane with direction cosines l, m and n and the reference directions are sigma 1, sigma 2 and sigma 3. So, on this plane, we will have the normal stress given b l square sigma 1 plus m square sigma 2 plus n square sigma 3.

We are also having the relation that l square plus m square plus n square equal to 1. So, if I substitute l square to 1 minus m square minus n square. That will give me sigma equal to l square sigma 1 minus sigma 3 plus m square sigma 2 minus sigma 3 plus sigma 3. So, let us consider this relation to be 1.

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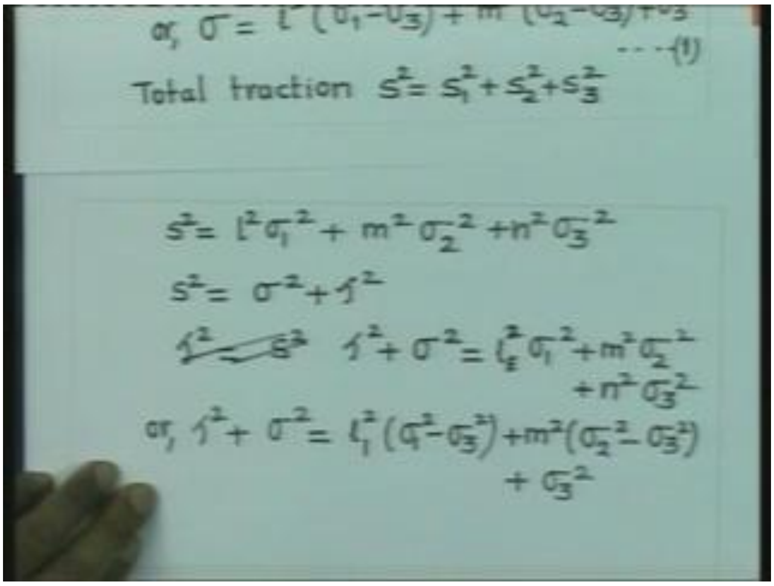

Now, the tractions on the plane l, m and n are going to be given by sigma 1, m sigma 2 and n sigma 3. So, if we now consider the total traction, which is going to be related to S 1, S 2, S 3. The three tractions on the same plane and the relationship is nothing but S square is equal to S 1 square plus S 2 square plus S 3 square. Since, S 1 is l sigma 1 and S 2 is equal to m sigma 2 and S 3 is n sigma 3.

We can write the relation as S square is nothing but l square sigma 1 square plus m square sigma 2 square plus n square sigma 3 square. These types relationship, we are also derived earlier in connection with the introduction of the maximum shear stress and also the shear stress directions.

Now, since the normal stress on the plane is equal to sigma. And if you try to consider the shear stresses on the plane to be tau, then we write S square, which is also related to sigma square plus tau square. And therefore, we will have tau square is nothing but S square. We will continue writing tau square plus sigma square is equal to l square, sigma 1 square plus m square sigma 2 square plus n square sigma 3 square.

Again, we will like to substitute the n square in terms of 1 minus l square minus m square. So, we will get tau square plus sigma square is equal to, we have l 1 square into sigma 1 square minus sigma 3 square plus m square sigma 2 square minus sigma 3 square plus sigma 3 square.

So, this we can now rearranging, tau square plus sigma square minus sigma 3 square is equal to, this is not l 1, this is l only. So, this is l square sigma 1 square minus sigma 3 square plus m square into sigma 2 square minus sigma 3 square. So, let us write this equation as equation number 2. I will show you, that these relations 1 and 2 are the basis to draw the Mohr circle in three dimensions in my next presentation.