Advanced Strength of Materials
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Lecture - 40

In this lecture, I would like to consider application of beam on elastic foundation analysis to pressure vessels.

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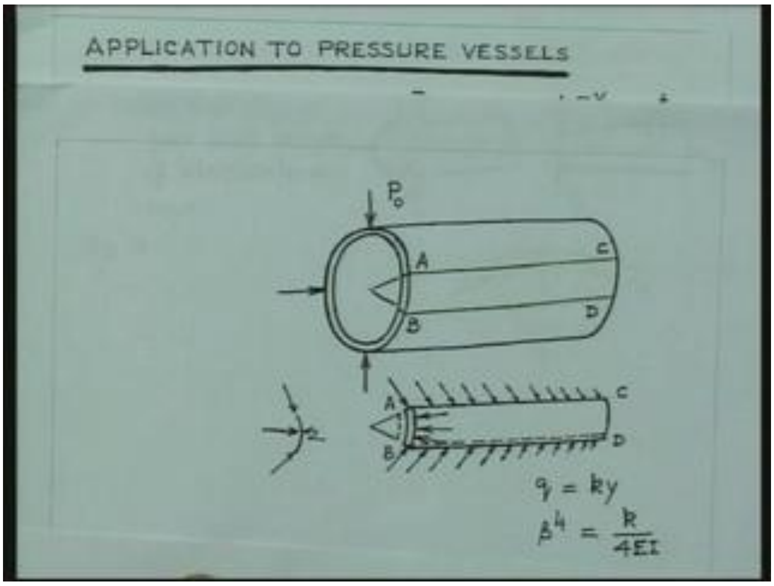

Let us consider 1 pressure vessel like this: cylindrical vessel which is subjected to some loading at the end which is radially distributed. So, consider that there is some distributed loading all over this edge and the intensity of this load let us say, that is P0. If you consider the deformation of the cylinder; obviously, we expect that this points will go towards the center and all the points on the circumference at the same will go close to the center.

And that deformation will gradually dye at the move (( )) from this edge, confine your attention to the deformation of a sector. Let us consider the deformation of the sector this sector. Let us say that this is A B C and D and it is making an angle at the center. When it is trying to move towards the center that remaining part of the vessel is going to registic. And therefore, on this edge and also on this edge there will be some forces developing which will oppose that moment and we can picturaize that situation by considering just take this segment out.

And try to draw it separately. So, if you should draw it separately like this, it will look like this that you have this sector. And now it is trying to move towards the center. So, therefore, A and B will try to move towards the center. And this movement will certainly be registered by internal forces which are going to develop on this edge and also on this edge.

So, when you to try to think of this dimensions CD very small you have now a case of a beam, which is going to be opposed from deflecting by this forces. Obviously; this forces are going to be perpendicular to this edge and it will have some component in the radial radially outward direction. So, the picture is like this that; you have these are the forces which is acting like this and this forces they are finally, going to have some resultant component in the radial direction, which is going to just oppose the displacement.

So, which is going to displace in this direction, that; will be opposed. So, therefore, you can now think of it that this, this sort of forces, are going to develop all along the length. And hence, you have a case like a beam on elastic foundation. So, therefore, this deformation of this small element under the action of forces like these can be analyze by considering a sector of the cell like a beam on elastic foundation.

Therefore, we will have the basis to find out the constants we have seen that; the resistant q we have written that as q into ky that displacement and then we have brought in some beta factor, beta factor was beta 4 was k by 4 EI where EI was the modulus of rigidity of the beam. So, let us now try to see, how we can evaluate this parameter like k and beta for a sector of a shell like this. So, we just have drawn the same picture here, the we have the force P0 per this is radial force per unit length of the circumference.

Now, if you see the 1 of the view of these wall for the cylinder you will find that under the action of the load it is going to deform like this: the displacement will be maximum here and it is going to gradually reducing. Let us consider that radius of the shell is r and thickness of the wall. I think we will some time make use of the symbol also h. So, either t or h will be the symbol for thickness. Now, we just take this sector which is making an angle at the center. Let us say that this is making an angle of phi at the center. And we will draw it we try to now look at that.

We just consider a portion here, which is drawn to a (( )) scale. So, this is the angle phi and this is the portion which is in this direction and the perpendicular direction we have taken again 1 unit 1 just 1 unit length in the length direction. So, you find that there is going to be some reaction forces coming up from the remaining cylinder in the circumferential direction.

Now we can estimate the value of this N and then it can take their component in the vertical direction to find out, what is going to be the vertical force registing the movement due to the loading which is externally applied which is P0 per unit length.

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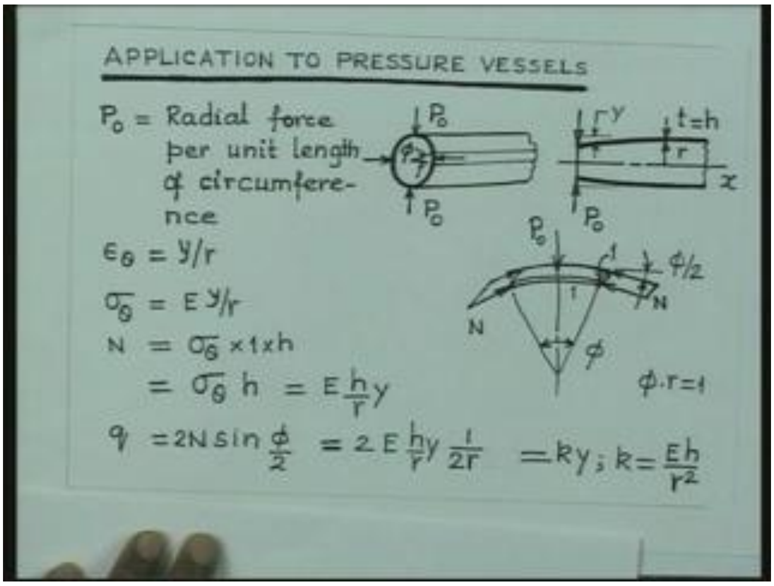

So, let us now see, that when you consider that the shell have got contracted. Let us say, that the shell has got contracted at a point which is, let us say that this is my origin at a distance x the radial movement of the wall is equal to y. And therefore, now it is asymmetric problem. So, we can write the strain in the circumferential direction, it is nothing but radial movement which is y divided by the radius of the vessels. So, that is epsilon theta.

Therefore, this strain is in the circumferential direction therefore, what is the circumferential stress, that circumferential stress is nothing but sigma theta and it must be E into epsilon theta which is Ey by r. So, that is the strain. Now, what is N N is nothing but this sigma theta acting over an area of 1 multiplied by h. So, therefore, this N which is acting over an unit length along the length, it is sigma theta multiplied by area is nothing but 1 into thickness.

If you now, substitute on this it gives you sigma theta into h and therefore, it is E h by r into y and you see that you got something which is now actually proportional to displacement. And now what is q, q is nothing but component of this forces N in the vertical direction, and therefore it should be this angle. This angle is nothing but phi by 2. Similarly, this side also angle phi by two.

So, this is phi by 2 and therefore, if I take this component which should be sin theta by 2 component and therefore, you see we can write the total vertical forces coming from this and this edge it must be nothing but N sin phi by 2. And we have 2 sides and therefore, this and this gives us, if we are considering this phi to be small and therefore, we can write now, this as E h by r and this angle y of course we have y there N. And this phi by 2 phi by 2 I can write now, phi into radius is equal to 1 because this circumferential length is 1.

So, therefore, phi into that and therefore, phi is equal to 1 by r. So, therefore, you see I have now phi by 2 is going to be nothing but 1 by 2 r. So, therefore, it is 1 by 2 r into we have that is it. I think I have put the same 2 is there and pi by 2 I have approximated this phi by 2 sin phi by 2 phi by 2 which is nothing but 1 by 2 r. So, once you simplify all this you get that, this is nothing but k into y where k is nothing but, E h by r square.

So, that is the k for the sector of the shell and hence, we have got the value of the constant which is needed for beam on elastic foundation. Now, we have something different, we have taken the case of beam. Now, we are trying to talk the we are trying to think of a case which we continuous wall is continuous all along this circumference. So, there is a difference, if you see the case of a beam.

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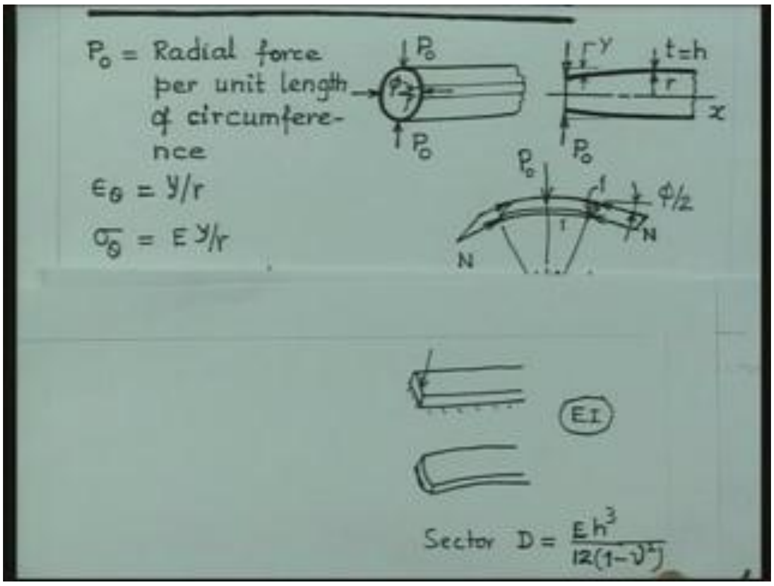

Let us see, the beam. So, if you have the beam. So, this is beam is on elastic foundation and therefore, if you are trying to press it like this, it is going to have curvature like this. So, if you look from this side is going to have, it will really deform like this, it is going to get bend and there will be anticlastic curvature. So, it is going to deform like this: it is going to deform, it will become doubly curve because of the compression there will be tensile on the top and compression at the bottom and because of that this anticlastic curvature will come.

So, this anticlastic curvature can come up in the beam is just having a finite width dimension. But in the case of the cylindrical shell this sort of curvature cannot take place freely and therefore, there is a there is extra resistance that particular sector that, we are trying to consider this sector cannot really take the anticlastic curvature and that will be prevented because of the continuity of the shell all over the circumference. And therefore, there is: 1 extra rigidity and this is same as in the case of plates we have talked about.

So, this portion is going to behave like: a portion of a plate, it is not a beam and therefore, we will have extra rigidity and therefore, what you find is that: this rigidity we have actually in the case of a beam, we express the rigidity by EI but, in the case of a sector like this: it will be just like a you have to treat it like, a portion of a plate and therefore, in the case of this sector you will find that this have got to be substituted by D and it should be the modulus of rigidity for a plate of length is equal to width equal to unity and height equal to h.

So, that therefore, it is nothing but E h cube by 12 into 1 minus nu square. So, this is the extra rigidity that comes up because of the continuity of the shell in the circumferential direction. So, in the case of beam it will be EI but, in the case of this sector, this sector of a this sector of a shell it is going to be replaced by D and it is E h cube by 12 into 1 minus nu square, where nu is the poison ratio.

So, there is some extra factor extra rigidity. So, by the extent 1 minus nu square it is it really goes into increasing the rigidity.

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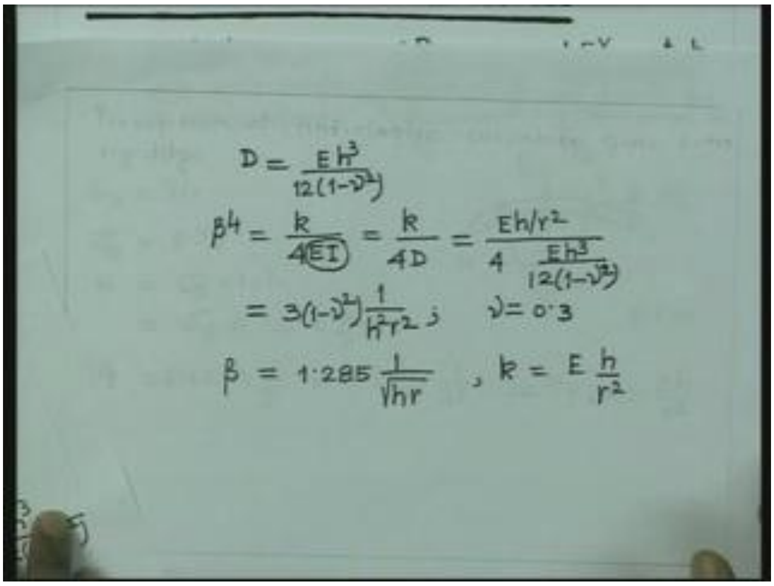

So, we can now, therefore, write that this D is equal to Eh cube by 12 into 1 minus nu square and hence, we can now write beta 4 which is the constant, which is nothing but k by four EI and this four EI is to be replaced for this case by D. So, therefore, it should be k by 4 times D and now if I substitute we have already got this thing as Eh by r square and this is now 4 Eh cube by 12 into 1 minus nu square.

So, this gives you finally, 3 into 1 minus nu square into 1 by h square r square. And if you select this nu to be given by 0.3 which is generally in the case. For all the steel it will be approximately like this. And therefore, we can now write this beta is nothing but

1.285 1 by square root hr. So, that is also, something which we have obtained for the case of this pressure vessel.

So, 2 information’s are finally, obtained beta is given by this which is related to thickness and the radius of the vessel and also we have got k which is nothing but E h by r square. So, once, this is done we can now apply the derivations that, we heard in our earlier lectures.

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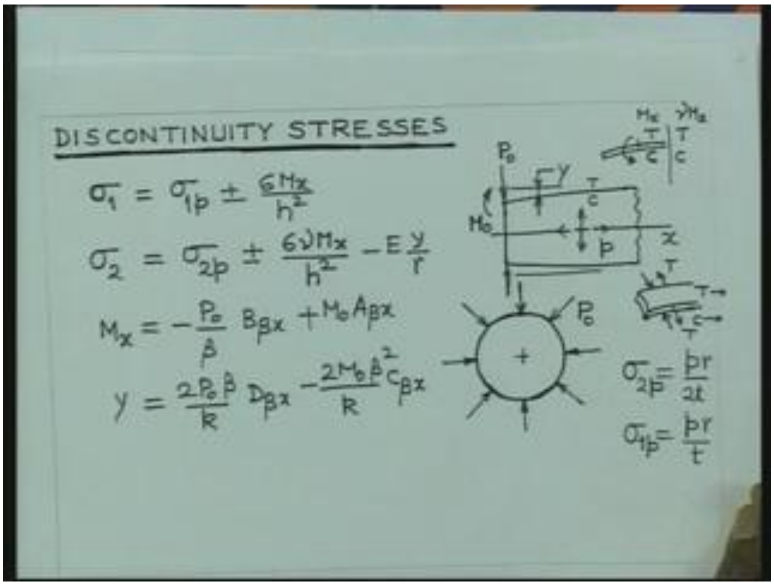

We will go for applying this thing, Discontinuity stresses. So, only we are going to have if you are going to have a junction of a vessel, which is subjected to forces like this think of a vessel which is going to have local forces like this. And because of the local forces, we are going to get some discontinuity stresses in this vessel, it also subjected to fluid under stresses. Let us say P you will get hook stress.

So, you are going to get hook stress, which is going to be we represent that thing by sigma 2 sigma 2 is equal to pr by 2t. And then axial stress is going to be pr by t. So, these are the stresses, if we did not have the presence of this force P now because of the presence of force P, we expect the deformation to take place and hence there will be discontinuity stresses. And if you consider that, the force which is going to come up, if we have continuous vessel and if we just segregate this portion from the rest then we are going to get internal forces.

Let us say P0 and also we are going to get some moment here, which is represented by let us say, M0. Then in that case local deflection is going to be y and we consider the coordinate to be like this. So, we have the picture like this: deformation picture like this. So, then in that case we are going to get stress you see that you have already seen that in a circumferential direction, we are going to get y and it is getting contracted therefore, there will be compressive stress developing in the circumferential direction you are going to have circumferential stress of magnitude pr by 2t.

And because of the compression that will be compressive stress in the circumferential direction. So, that is 1 stress because of this local forces and at the same time what you find is that since, it is bending at this section there is going to be some bending moment. And that bending moment is going to cause in the shell stresses, which is going to be tensile at the top fiber compressive at the bottom fiber. And due to the poison ratio ((…)) you will find that: the stress in the circumferential direction is going to be compression at this point tensile at this point.

So, therefore, we are going to get stresses now, some total of the stresses will be like this, we can write now sigma 2; sigma 2 is equal to sigma 2 due to pressure. So, I write this thing really sigma 2P and then we are going to get the compressive stress due to the contraction in the radial direction, which is E y by r compressive and then due to the poison ratio effect at this point I am going to get stresses in the circumferential direction whose magnitude is nothing but if you consider, the moment is nothing but nu times Mx.

Then the magnitude of the stress is going to be nu 6 nu Mx by h square and this going to be plus minus to indicate that is going to be having value at the outer fiber of some shine at the inner fiber, it is going to be of the different shine. Now, I think I will just repeat again that you see that, when it is trying to be because of this bending let me, draw that wall. Let us draw this. So, if you draw this because of the bending moment acting at this section, which is going to be like this at the section you are going to get tensile stress there compressive stress there.

So, when it is having tensile stress in the longitudinal direction in the circumferential direction it will try to contract. And that contraction will be prevented because of discontinuity of the shell. So, therefore, this in the circumferential stress, circumferential direction the stresses are going to be tensile and it is going to be compressive at this point. So, I repeat that because of the bending moment this is due to Mx the space going to be tensile and compressive there and in the circumferential direction that is effect is like this: you are going to get tensile stress there compressive stress there.

Think for a moment that when you have this shell is bending like this, bending like this, you are going to get tensile stress there compressive stress there in the axial direction this is in the axial direction let me show it that way. Now, since it is trying to elongate in this direction, it will try to contract in this direction and the material which is on the this side and on the other side, it will try to prevent contraction and therefore, it will try to again generate tensile here at the top fiber.

So, it is going to be tensile and in that bottom fiber since, it is compressed. So, the continuity of the material you try to prevent that and therefore, it is trying to actually since, it is trying to contract in the direction, it will expand in this direction and therefore, it will be prevented from contracting and therefore, we have compressive stress that is all you find that the nature of the stress will remain in the same at the top and the bottom fiber.

So, this is the stress therefore, in the circumferential direction similarly, in the axial direction we are going to have the stress due to pressure that is, this much first I am going to get bending stresses, which is directly 6Mx by h square. So, at a point like this you are going to get vessel stresses plus the vessels due to the bending arising out of the local force that is going to be ((…)) and magnitude of Mx and y they are known to us from the last time analysis Mx is nothing but P0 by beta B beta x plus M0 A beta x.

And y is equal to twice P0 beta by k D beta x minus twice M0 beta square by k C beta x and already we have got the value of beta and k in terms of the dimension of the vessel. Now, we will apply, we will really like to go for taking up the analysis of some problems using this theory.

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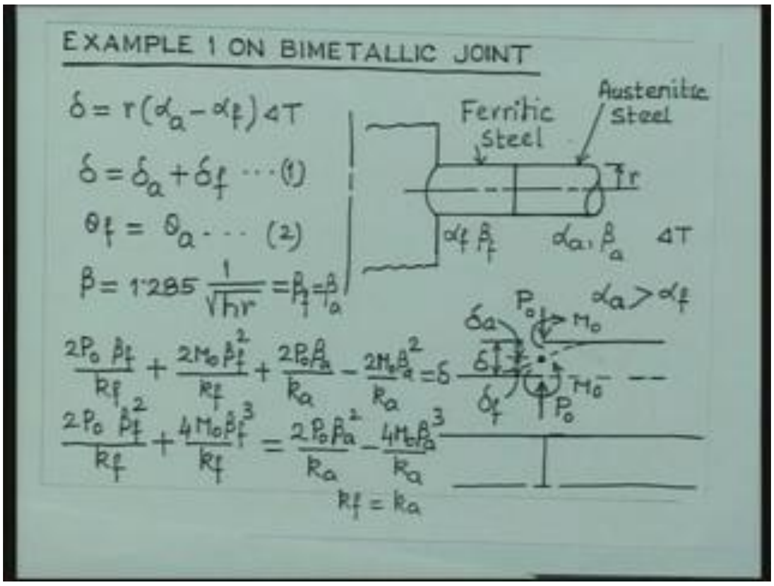

Let us, consider 1 example this is bimetallic joint think of a pressure vessel which is vertical. So, we will think of it that you have a vertical vessel here and this vertical vessel. As a nodal junction, which is like this and this portion of the pipe which is connected to the vessel is made up of Ferritic steel.

So, this is made up of Ferritic steel and this portion is made up of Austenitic steel. You will find that, the coefficient of thermal expansion and this factor of F beta f if, I consider that this thermal coefficient of expansion for this, segment let us say it is alpha f and this is beta factor is nothing but beta f. So, actually indicate it is Ferritic steel and this side we are going to have alpha a and beta a. The magnitude of beta factor depends on the depends on thickness of the vessel and the radius of the vessel.

So, therefore, I have also depends on really poisons ratio, poisons ratio for Ferritic steel and Austenitic steel can be taken to the same therefore, we can consider this beta f and beta a to be almost the same. On the other hand this Ferritic steel has a lower thermal coefficient of expansion than the Austenitic steel. So, therefore, you see if this vessel is operating at high temperature what you find that there is going to be mismatch in the expansion, this side is going to expand higher for a particular temperature raise than this left hand portion.

Therefore, there will be some mismatch in the expansion and hence forces integral. If we increase the temperature by let us, say delta t then in that case what you find is that, we draw the vessel schematically that this is the centerline of the pipeline and this is the Ferritic part of this pipe. And let us say. this is the of course, we have the this side is Austenitic part because of the temperature raise, we raise the temperature by delta a noting that alpha a is higher than alpha f. I expect this portion to expand less than this portion.

Let us say, that the expansion the difference in the expansion. I think that let me, put in this way I think let me, illustrate the problem I do it this way. Let us, consider that this is the junction and the centerline of the pipe is here because of the temperature raise let us, consider that the Ferritic part of the pipe is going to expand by this much. So, therefore, this is the expansion of the Ferritic part and the Austenitic part is going to expand by this much. So, this the expansion from the initial position.

So, therefore, there is a differential expansion between the 2 and let us, indicate that, that is equal to delta and how do you get this delta this, delta is nothing but this is going to expand by r into alpha a into delta t this is going to expand by r into alpha r into alpha a into delta t. And therefore, the differential is going to be delta which is nothing but r into alpha a minus alpha f into delta t.

So, that is the differential. And this differential is going to give raise to; obviously, some forces to come to a common position. And that force will try to take it closer to the center and the same force is going to expand this fellow outwards to come to a common position. Let's say that they are going to get some common deform position like this. So, the force which is going to act internally at this junction.

Let us, represent that there will be some radial force per unit length and the same force is going to act on the this part of the pipe which is also P0. And if you consider, that the bending moment acting at this point is equal to let us say, M0 then we are going to also get the same bending moment, it is going to be M0. So, these are the forces which are going to come up and under the action of these forces, this fellow is going to take up position like this, another action of this 2 forces this point is going to move there.

Therefore, we can now write that the, deformation of the Austenitic part of the steel under the action of P0 M0 is nothing but let us say, that is delta a and the deformation of the Ferritic part of the pipe is nothing but delta f. So, now we can write the compatibility condition and this compatibility condition is straight forward that this differential has been briefed by the deformation delta a and delta f. So, therefore, we can write now, compatibility condition delta is equal to delta a plus delta f.

So, that is 1 equation. Now, another equation we have 2 forces, we have to determine P0 and M0 these are the 2 unknowns to determine how do you do it. I we have only 1 equation now, there is another equation under the action of this force P0 and M0 this fellow has deformed like this. So, therefore, it has taken up a angle of orientation like this similarly, under the action of P0 and M0 this fellow has taken up position here.

Therefore, it is taken another angle of orientation; obviously, the angle of orientation of the Ferritic part is equal to the angle of orientation of the Austenitic part. So, therefore, we can write that theta f under the action of P0 and M0 for the Ferritic part must be equal to angular orientation of the Austenitic part. So, therefore, we have got these 2 equations and these 2 equations are sufficient to calculate the 2 forces. We can already write now, beta equal to 1.85 1 by root over hr.

So, that is common for both the pipes. So, that is also equal to beta f and is equal to beta a. Now, we can consider what is we have derived the formulae, what is going to be delta a. Delta a will be 2 P0 beta by k minus twice M0 beta by k. So, since k is same for both the materials we can now, write if you would like to actually write separately also, it is matter. So, we will write now, twice P0 beta f for the Ferritic part this and this M0 is also trying to get give the deformation in the same sense.

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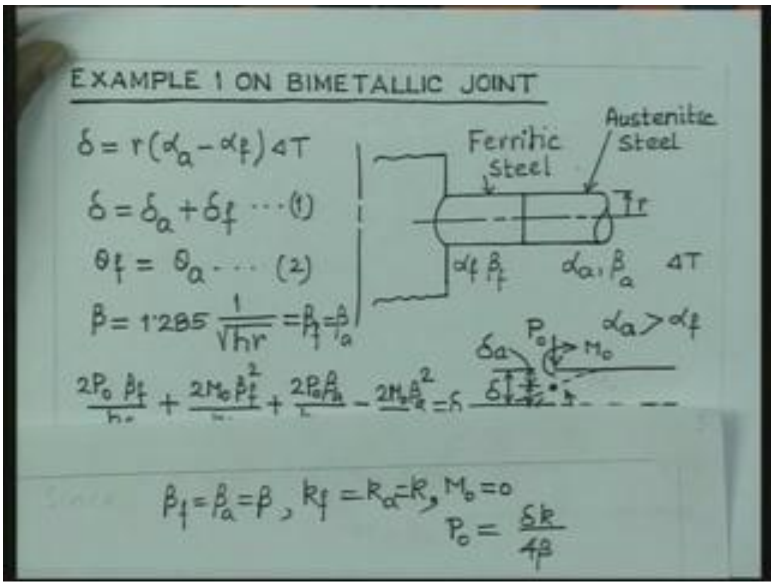

So, therefore, this is twice M0 beta f square by kf. So, that is what you have for this delta Ferritic. And delta Austenitic is going to be twice P0 beta a by ka minus twice M0 beta a by ka. So, this is delta f, this is delta a and that should be equal to delta. So, that is equation number 1 we have. Similarly, if I consider the slope produced by P0 and M0 it is nothing but twice P0 beta f square by kf by kf plus 4 M0 beta, it should be cube by kf is equal to twice P0 beta a square by ka minus 4 M0 beta cube by ka.

Here, it is square just I have missed on to. So, this is the form now look, at this what is kf kf is nothing but Eh by r square h and r are the same E is the modulus of velocity of the material E for Ferritic steel and Austenitic steel they are the same. Therefore we can take it to the that kf is equal to ka. So, that 1 simplification is possible and now if you do that what we are going to get we already got beta f equal to beta a. So, here we find that these 2 terms are equal they cancel and therefore, finally, from this equation what you get is that M0 equal to 0.

So, we can now, get the value for the constant since, beta f is equal to beta a. Let us write that thing is equal to beta .Similarly, kf is equal to ka is equal to let us say, k without in its suffix then in that case from the second equation what you find is that M0 equal to 0 and once M0 equal to 0 then from the first equation you can get P0, P0 terms out to be delta k by 4 beta.

So, that is the magnitude of the force which is going to develop at the junction of the 2 pipes and from these you can calculate the discontinuity stresses. Total stresses that is going to come up over this segment of the pipe or this segment of the pipe you can calculate. So, you can calculate the stresses in the circumferential and axial direction at this junction once you know this value of P0 you can calculate the stresses over this segment also over.

So, once you get the value of P0 then it is a question of just writing the expression for.y and Mx and then try to go back to the expressions here, which you have already shown here that, discontinuity total stresses in the vessels are obtainable from this relationship. So, which was consider little highly go. So, this gives you the total stresses in the axial direction and circumferential direction. Let us consider a case varying the joint 2 metallic joint is little different.

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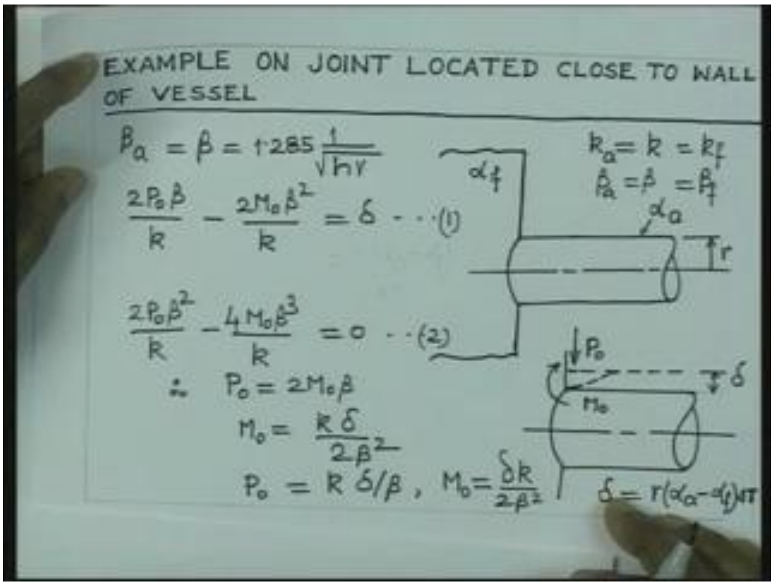

Suppose, you consider that joint located close to the wall of vessel think of the situation like this. In the earlier case we thought that the Ferritic part of the vessel will be up to this much this is also Ferritic steel, this was extended up to this then this are made up of Austenitic steel. Let us, consider now your situation that this vessel main vessel is made up of Ferritic steel and the pipe is made up of Austenitic steel.

So, therefore, the junction is now by material junction. By the argument that we heard given in the earlier case, we can write certainly ka is equal to k and that is also equal to kf and similarly, beta a is equal to let us say, beta and that is equal to beta f. Now, in this case picture is different when we are trying to consider that the vessel is operating at elevated temperature. The vessel is also expanded vessel is expanding in this direction and this is the expanding in this direction.

In fact, the expansion of this is going to relieve it not that much compare to that of the pipe. So, what you will find that, if this is the vessel portion inside that; there will be some mismatch in the expansion itself and this mismatch in the expansion again you have Ferritic steel is going to be expanding less than this portion this Austenitic part. So, this is the Austenitic part. And therefore, what you find that; there is going to be mismatch.

So, if you consider that this is our original pipe under the differential expansion the pipe is going to now, expand more and that is going to be let us say, delta and this delta we can again write this delta is nothing but it is r into alpha a minus alpha f into the temperature raise. So, that is the expansion that is the differential expansion. Now, the vessel is going to be very rigid in this direction because of this expansion, what you will find that as, if this is a rigid body and this pipe is going to expand, it will be arrested.

Finally, what you will find that; there will be forces developing at the age of the pipe to prevent its expansion here and there will be also a bending moment developing here to again cause the angular match. So, you will find that the vessel is now going to take up a deform position like this. So, the pipe junction is pipe junction with the main vessel is going to shape a going to take a shape like this.

Hence you will find that; there will be forces now, P0 M0 which are going to be different from the earlier case. So, let us try to get this thing done. So, beta a is equal to beta and that is again equal to 1.285 root over hr. Now, if you consider what is the moment under the force P0 and M0 that is calculable from the vessels that we heard earlier twice P0 beta by k minus twice M0 beta square by k.

So, this is directly from our derivation and that is equal to delta. So, therefore, the contraction in the radial direction under the action of P0 and M0 is going to be this and that is nothing but delta. Now, what about theta, this theta under the action of P0 and M0 it is going to be 0. It is going to be at almost like a cantilever end and therefore, we will find that twice P0 beta square by k minus 4 M0 beta cube by k that is the total rotation and that is equal to 0.

So, these 2 equations are sufficient to help us to find out the discontinuity forces P0 and M0. And once you do this solve this you find that P0 is now, going to be equal to twice M0 beta and M0 is equal to k delta by 2 beta square. So, these are the forces wherein of course, if we now at write it simplified form this can be written as P0 is equal to if we substitute for M0. So, this is going to be k delta by beta.

So, this delta in terms of delta, we can write that P0 is equal to this M we can actually make use of there and therefore, we can now write P0 is equal to this much. And M0, M0 will be equal to M0 is equal to delta k by 2 beta square. So, we have got this and from here if you simplified it gives you P0 equal to this and again M0 equal to this. So, these are the forces. Now, let us see what difference that this make from the earlier case that is very interesting.

That we have 2 possibilities, that we can make the junction to be add in the earlier case where in we had consider, that the junction could be like this or we have another choice that we have make the Austenitic steel to extent up to the vessel and have this junction.

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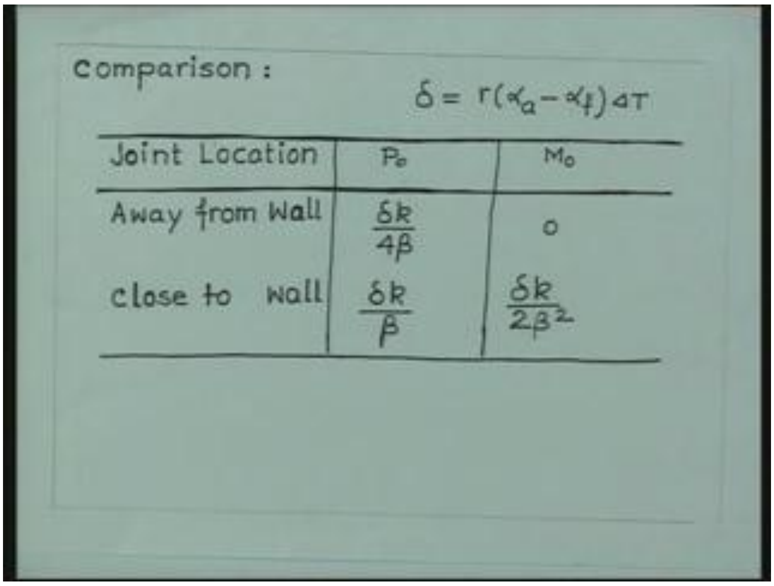

Now, look at the differences in the forces. So, if you consider the same temperature rise if you now, say that this is your differential expansion in the both the cases then you see, if you look at the joint away from the wall then you have only P0, P0 is equal to delta k by 4 beta M0 is 0 on the other hand, if you put it close to the wall delta k by beta delta k by 2 beta square. Look at this you have a magnification in the force 4 times. So, if you are locating close to the wall which is giving you a magnification in the force P0 4 times and at the same time we are going to get some bending moment which is non zero.

So this is, something very important from the point of your design and why it does it happen. In this case what happens that, when you are trying to have the junction away from the vessel then in that case you have tremendous flexibility coming from both the left wire which is made up of Ferritic steel and from the write, which is Austenitic steel. So, both are actually deformable solid in the radial direction but, when you are keeping the pipe connected to the wall directly then rigidity of the vessel in the radial direction of the pipe is very high.

Therefore, it is behaving as if the end is like connected to a rigid body and hence there is constant in the radial expansion of the pipe. And this prevention of the radial expansion always gives rise to more forces and that is what is demonstrated here. In the case of thermal stresses, thermal forces also or forces arising out of thermal expansion you will find that; they are going to always have if you trying the erase the thermal expansion and that is what is actually demonstrated in this table.

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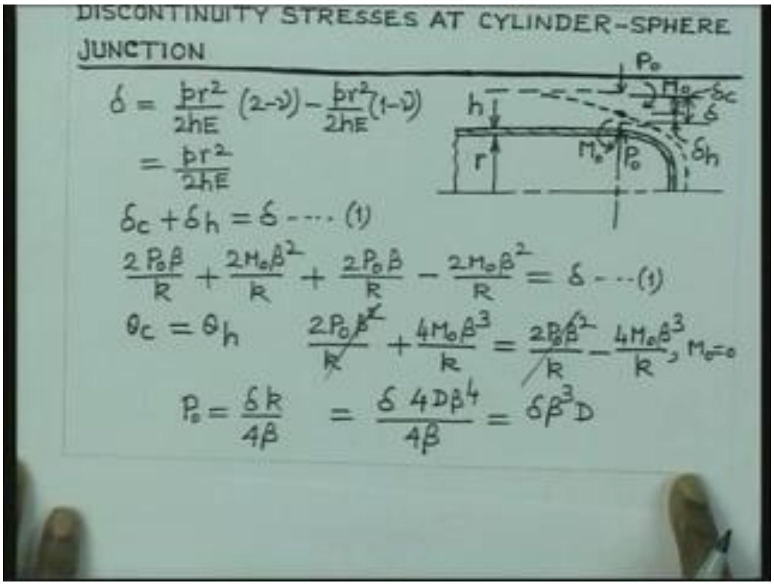

Now, you would like to consider, another example which is also very important from the point of the application discontinuity stresses at the cylinder sphere junction. You can have a vessel like this that we have a cylindrical vessel and it is having spherical head. At this spherical head and connected to cylindrical shell is going to give rise to mismatch in the radial expansion and hence local stresses will develop.

So, we will try to again see, how we can estimate the forces that develop at the junction of the head with the cylinder. Quickly if you look into the stresses in the case of cylinder axial stress is pr by 2t hoops stress is pr by t and in the case of sphere, it is pr by 2t in both cases. So, therefore, the radial expansion of the cylinder is going to be pr square by 2t e into 2 minus nu in the radial direction on the other end in the case of sphere, which is going to be pr square by 2t e into 1 minus e.

It is almost twice you see the expansion of the cylindrical part is going to be under the action of only internal pressure, it is going to be let us say, this much and the head is going to expand much less, it is almost half, it is going to be actually this much only. So, therefore, there is a differential expansion because of the internal pressure itself and this differential expansion.

Let us indicate that is nothing but delta here. And what is this delta, it must be equal to expansion of the cylinder pr square by 2hE instead of t I am writing h into 2 minus nu minus expansion of the sphere which is nothing but Pr square by 2 hE into 1 minus nu. So, that was almost twice neglecting this nu. So, therefore, you see that, this differential expansion delta is nothing but pr square by 2hE. Now, how are they going to meet you will find that because of this mismatch, there is going to be some forces developing here on the cylindrical portion which is, let us say P0 and then we are going to get also, some bending moment developing which is equal to let us say, M0 and simultaneously on the spherical head, we are going to get some forces coming by exactly opposite direction P0.

The bending moment is going to be like this M0 under the action of this P0 and M0, it will try to move up under the action of P0 M0, it will move down and finally, this junction is going to have the shape deform shape, which is going to be somewhat like this. So, they meet at a common point here and hence you find that under the action of the force P0 and M0 the cylinder is going to get deform to the extent delta c and simultaneously, the deformation of the head is going to be delta h.

So, this is the deformation of the head and this is the deformation of the cylinder. Now, we can write this delta c in terms of P0 M0, we can write delta h in terms of P0 and M0 and also we see that there is match of the slope and therefore, we will have 2 equations to solve for the 2 unknown. So, we can now, write delta cylinder plus delta h is equal to delta that is our compatibility equation and therefore, this is 1 and now we can directly write the value for delta cylinder, which is nothing but twice P0 beta by k plus twice M0 beta square by k plus delta h.

If we assume that, both the materials are same then beta and k will be same in the case of cylinder and now we can have this thing else twice P0 beta by k minus twice M0 beta square by k is equal to delta. So, that is your equation number 1. Now, I give you 1 clue why this is negative look at this here you see that this P0 and M0, P0 is trying to deform this M downward and this is also going to deform this N downwards.

So, therefore, they are actually additive action is additive rotation by P0 is downward rotation by M0 is also downward. So, they are added together here. On the other hand here P0 is trying to move this fellow up where as M0 is trying to do the other way. So, therefore, there is negative shine. Now, coming to the slope continuity will have theta c is equal to theta h and if you now try to write the value it is P0 beta square by k plus 4 M0 beta cube by k again this 2 actions are additive and that is equal to twice P0 beta square by k minus 4 M0 beta cube by k.

And in this case when the thickness of the shell, cylindrical shell and head is same you will find that all these here k is same beta is same therefore, what you find is that this term cancels to this term and therefore, from this 2 you find that M0 equal to 0.

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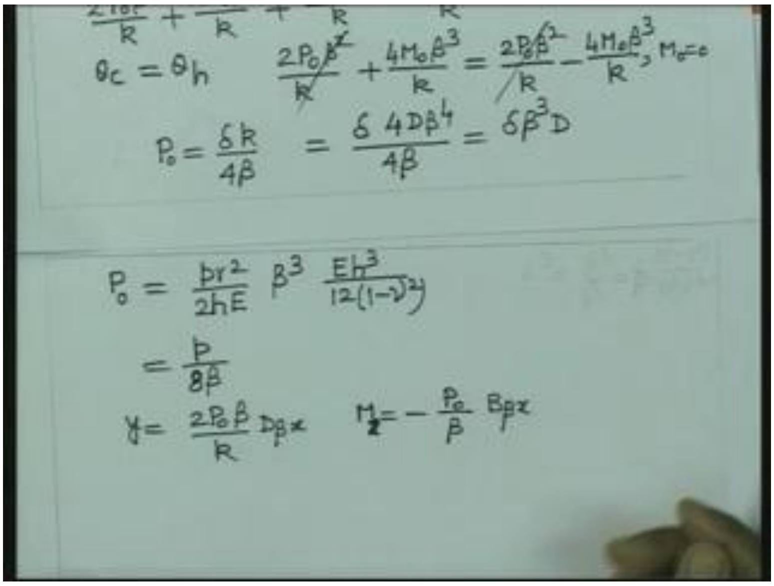

So, that is movement in 0 on the other hand you will find P0, P0 is equal to delta k by 4 beta. And this 1 once you try to put the value of k and in terms of E and all those we find that this is delta 4 D beta 4, we can write this thing k has this 1 and this is divided by 4 beta and this gives us delta beta cube B. So, the magnitude of the force is this 1. Now, under the action of the forces, it is good to see what is the variation of stress. So, we can now write P0 as pr square by twice hE.

So, I have just try to write the value of this parameters I am writing really that all this parameters beta cube Eh cube by 12 into 1 minus nu square this is the value of delta this is beta cube and D, I have written and this simplifies to P by 8 beta. So, that is the value of the force. Then in that case, we will have deflection is equal to twice P0 beta by k d beta x and M0 is equal to Mx any point Mx is equal to P0 by beta B beta x.

So, this is the variation of the movement after having obtained y and Mx around the junction, it is possible to show the variation of stresses in the cylinder and also in the sphere.

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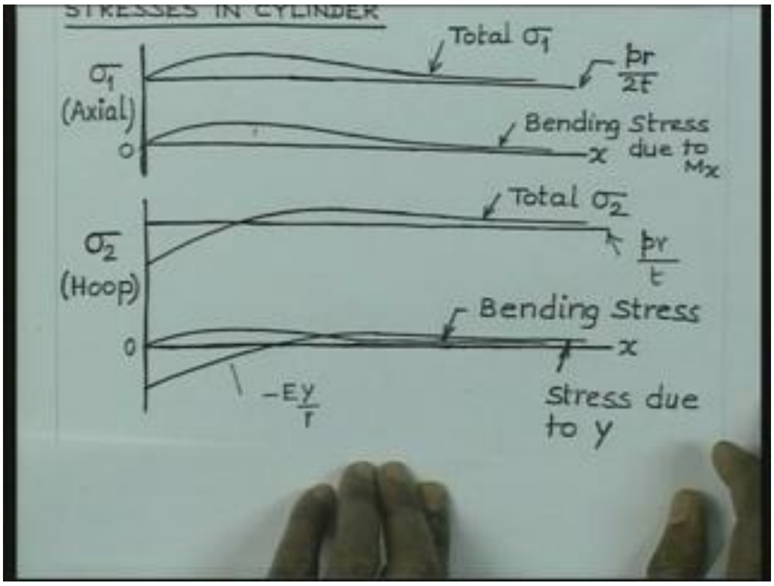

Let us, consider the variation of stresses only in the cylinder, if you consider first of all the stress in the axial direction under the action of the pressure, we will get it sigma we will get sigma 1 which is given by Pr by 2t. And then variation of y will give rise to variation of Mx and that Mx variation will give us variation of stress because sigma in the axial direction is going to be 6 Mx by 8 square that variation is going to be like this. And this is also going to have magnitude, which is tensile if you consider 0 to be there both the stresses are tensile.

So, this stress which is constant across the thickness and this is the stress which is at the top fiber and if I add this to the stress sigma 1 due to stresses then I get it local stress like this. The maximum of the stress focus somewhere ((…)) from the junction. Similarly, if you consider the stress in the circumferential direction due to pressure we are going to get this stress sigma 2 which is pr by t.

Since, the radial direction there is movement towards the center there will be compressive stress in the circumferential direction, which is of magnitude minus Ey by r and this is the variation of that stress. And this bending moment is going to give rise to stresses in the circumferential direction due to the prevention of prevention of anticlastic curvature, which is proportional to nu times nu x or its exact magnitude is 6 nu Mx by 8 square this variation is going to be like this. Look at this variation, it is very similar to that variation only that it is magnitude is lower and it is dependent on poisons ratio.

So, therefore, we have now 3 components in the circumferential direction 1 is the sigma 2 stress due to pressure, sigma 2 stress due to y, sigma 2 stress due to Mx. So, these 3 can be added and we get the total stress variation like this. You find 2 distinguish features again. That the total stress is maximum somewhere here which is away from the junction and the magnitude of the stress, at the junction is lower than the stress which will be due to pressure only.

That is the variation of stresses, at the junction can be obtained magnitude also can be calculated provided, you are giving some specific values of the pressure and the dimensions of the vessel.