Advanced Strength of Material

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Lecture - 38

We will now consider, calculation of stresses in thin shells. And these shells are such that, there is some axis of symmetry. And the wall thickness is small. They can be cylindrical, spherical, conical, toroidal. And this case, they are nothing but, solids of revaluation.

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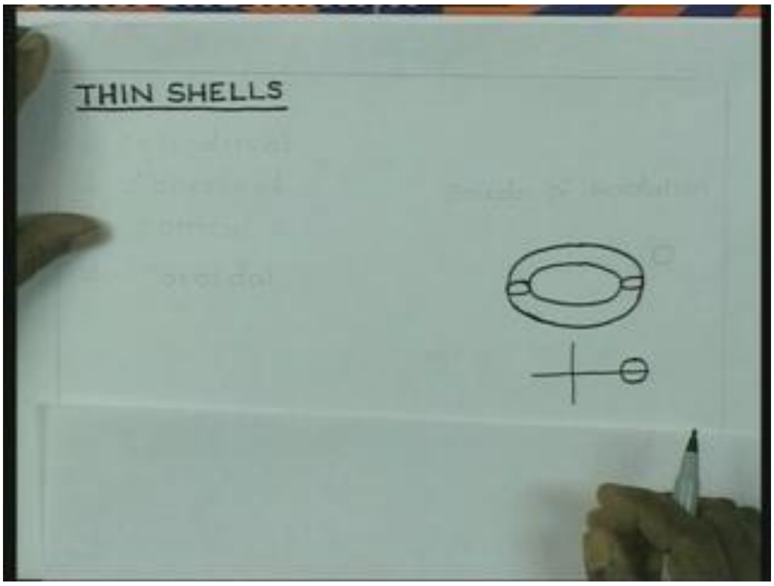

Toroidal shell, it is like this. It looks like a ring. It is like a ring. And then, the cross section here is circular. So, also cross section here is circular. So, that is the toroidal shell. And we get the generation of this shell, by considering rotation of a circle about the access of symmetric. Calculations of stresses in these shells are simpler. So, we like to consider that.

Now, already seen that stresses in thin shells which are cylindrical in shape. There are two stresses. One in a axial direction, if it is a close cylinder. And the magnitude of the stress is nothing but, PR by 2 d in a axial direction. And the space in the circumstances direction is PR by t. You can imagine, if the shell is spherical what difference will come about?

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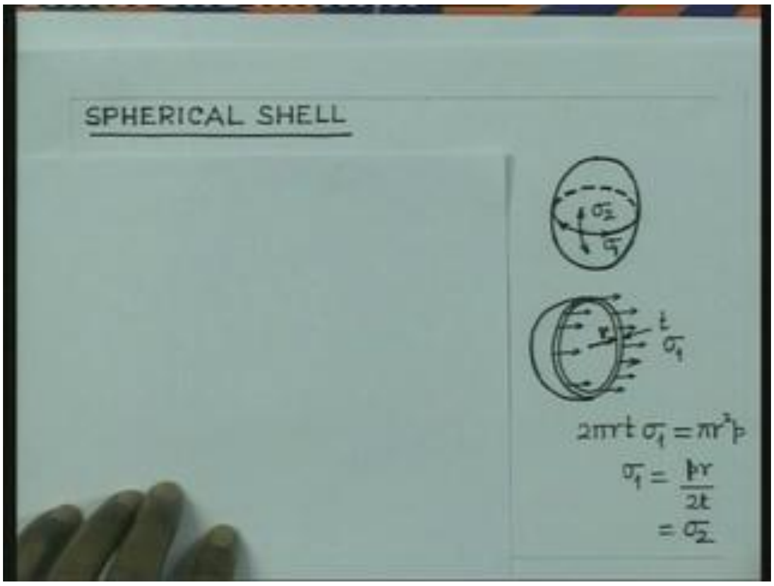

So, in the case of this spherical shell think of it, that this is the shell. And now, the shell is of thickness t. Let us say everywhere, and the radius is R. You will find that at any point here. You are going to get some principles stress in the. If you consider that, this is your axial direction. There will be some stress in the axial direction. And then, there is going to be also another stress, which is going to act in the orthogonal direction.

If you like to call it, circumstances stress that is sigma 1. So, this is that is sigma 2. If this is sigma 1 x. This is let us says axial stress. Then, that is the circumstances stress. To calculate the stress sigma 1, it is quite simple. That you take a section, vertical section and if you take the vertical section here to Laplace Scale hydra. So, this is the vertical section. You have some wall thickness. So, these stresses are going to act in this direction.

Since the shell is or small thickness. We can take the stress to be uniform everywhere. And that is, what is of intensity sigma 1? And if r is the internal radius and t is the thickness. Then, obviously the total load which is counter balance. Counter balancing the pressure here is nothing but, 2 pi r into t into sigma 1. So, 2 pi r into t that is the metallic area multiplied by sigma 1. And it is counter balancing the load acting in the opposite direction, which is acting on this circular area pi r square and pressure is p.

So, that gives us sigma 1 equal to p r by 2 t. Similarly, if you are interested in calculating the stress sigma 2, you take a horizontal to section. And it will come out to be the same. So, in the spherical shell you have the both hoops stress and the axial stress, of the same magnitude. And that is the least stress that can happen. Let us consider now, conical shape.

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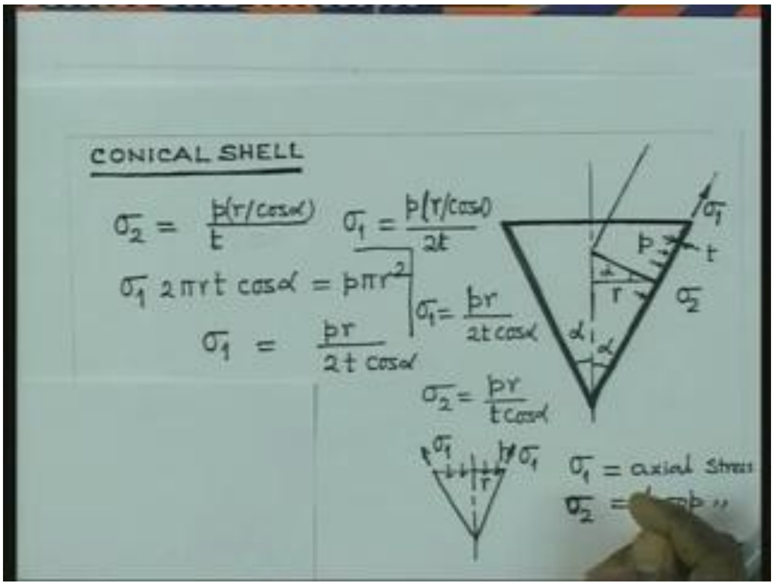

So, if the shell is like this. That we have some angle alpha, you find that many times the cylindrical shell has an in closer, which is conical. So, therefore it is necessary to calculate the stress at the wall of a conical shell. Let us say, that internal pressure of this conical shell is p. And we are interested in calculating the stresses at every point. We now consider, to this concentrate that there is a section here.

This is section of radius. Let us say r. And the wall thickness, we will consider this wall thickness to be equal to t. If you consider the vessel to be like this, just concentrated on the vessel. Say this is your normal to the wall. So, this is the normal to the wall. So, this angle is also alpha. Then, you can see this portion. If we just concentrate on this portion. Then, this portion will appear to be a cylindrical vessel of radius this.

That is enough in what? r by cos alpha. So, the pressure which is acting on the wall is the internal pressure. So, you have ((Refer Slide Time: 07:16)) cylindrical vessel of radius is equal to r by cos alpha. So, if that is the case then we can calculate the axial stress, which is going to act in this direction will represent the axial stress, by sigma 1. And the hoops stress, which is going to act in the circumference stress that is by sigma 2 hoops stress.

So, let us write that sigma 1 is axial stress. And sigma 2 is hoops stress or circumference stress. So, for a vessel with radius r by cos alpha, internal pressure p, the axial stress is going to be pr by 2 t. So, therefore if we write that. That will be nothing but, sigma 1 is equal to p r by cos alpha by 2 t. So, that is the value of sigma 1 space. And therefore, sigma 1 will come out to be p r by 2 t cosine alpha. That is the axial stress.

Now, the other stress which is the hoops stress. That will be nothing but, p r by t. So, here in we have p r by cosine alpha by t. So, that will become p r by t cos alpha. So, therefore sigma 2 is going to be p r by t cos alpha. So, these are the stresses at a point in a conical stress. So, it is going to remain constant. And you see that, if alpha is equal to 0 degree that becomes cylindrical shell. Then, you get back the formula for the cylindrical shell.

To calculate these stress, sigma 1. We can also follow another procedure. I need not use the formula that, we have derived for this cylindrical shell. You can consider now, let us say we will take a section of the vessel at this points. So, if we take a section of the vessel here, just like this. We have this radius is equal to r. And this is the stress sigma 1 acting on the material. This is sigma 1 and the pressure which is acting downward of the fluid.

So, therefore you see that. We have the force acting due to the pressure is nothing but, p into pi r square at that section. And now, this is counter balance by the metallic force shears, which is acting over an area 2 pi r into t that is the total area. And the intensity of the stress is sigma 1. And now, we must take its components here in this direction, which is cosine alpha. So, therefore we can have. Now, sigma 1 into 2 pi r into t into cosine alpha.

So, you have 2 pi r into t cosine of sigma 1 is sigma 1 cosine alpha. That gives us the equilibrium equation. And this must finally or this finally, gives you sigma 1 equal to 2 t p r by 2 t cos alpha. So, that is what we have also obtained earlier by considering it to be a cylindrical. Now, next thing we will consider a toroidal shell.

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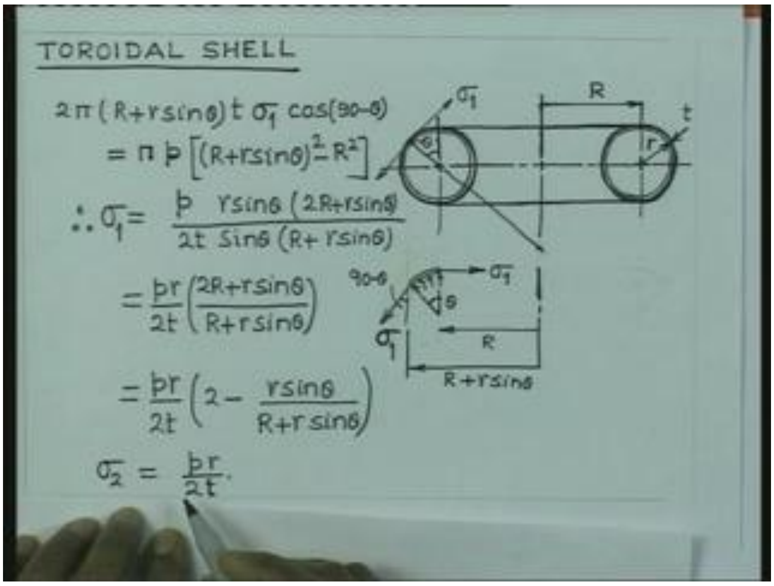

This sometimes made use of in plants. If you take this section diametric section, it is look like this. You have two rings and this is the other portion. So, it says semicircular thing, which we can taken section here. Look at the dimensions that the ring radius is capital R. And the wall of this cross sectional radius is equal to R, wall thickness is t. And the internal pressure acting on this vessel is as usual p.

I already say that, this shell is generated by evolving this circle about the access of symmetry. So, the circumferential stress is going to be acting perpendicular to the plane of the paper. And for ((Refer Slide Time: 12:52)) that if you consider the equilibrium in the circumferential direction. You will have pi r square into p is the load. Due to the fluid pressure, which is counter balance by the stresses acting over this metallic area, which is 2 pi r into t into sigma 2 is the stress.

So, therefore P r square is equal to 2 pi r t into sigma 2. And that gives, you the stress in the circumferential direction very easily to be sigma 2 is equal to p r by 2 t. That is same as in the case of cylindrical vessel. Now, we would like to calculate the stress in the axial direction. Here in, the axial direction if you consider this one this point. Then, this point will be subjected to some stresses like this. In this direction at this point, that is nothing but, sigma 1 here.

So, sigma 2 is acting out of the paper. And this is going to act tangentially to this cross section here. That is, what is sigma 1. We will calculate this stress. And let us concentrate on this point, which is making an angle of theta with the particle. And if I extend it, this will intersect here. If I now concentrate on this portion of the material, look at this. This portion of the material, we expect the stress in the axial direction.

If we consider the theta to be small, we will have stress here is nothing but, sigma 1. And this is also sigma 1. That is the stress there acting. And now, we have the pressure acting full pressure acting, which is uniform intensity P. So, our focus is on this segment only, which is making an angle of theta with the particle. If you consider now this portion, and it is whole circular portion of it.

So, therefore we are trying to just consider this revolve these thing, about the access of symmetry. So, if you just consider the revolution of this material portion, about this access. So, you get one ring short of. And therefore, the total load which is acting vertically upward. We can calculate and that load is balance by the component of this stress, acting in the vertical direction.

So, this angle is also theta. So, that is also angle. That is 90 minus theta. So, this is theta. And therefore, this is 90 minus theta. This angle is, if the particle is 90 minus theta. So, therefore the total force which is acting vertically upward. It is going to be that annular area of radius, which is nothing but, this is what is capital R? And this one is nothing but, R plus r sin theta. So, these are r sin theta is this, this term.

So, therefore R plus r sin theta. So, therefore the total force acting on this annular area which is nothing but, pi p into R plus r sin theta whole square minus R square. So, that is the annular area. And that annular area is vertical component will give of a due to vertically upward force. And that is now, counter balance by 2 pi R plus r sin theta into t. That is the metallic area. And sigma 1 is the intensity of this stress.

So, that is this stress and it is components of, it is cosine 90 minus theta. So, cosine 90 minus theta that gives you, the stress at this point. And this sigma 1 obviously, there is no contribute to the vertical direction. So, therefore once you simplify this, you get the stress sigma 1 axial stress. And this is nothing but, p r sin theta. So, it is a square minus b square form. So, you can write that. This is 2 R r sin theta divided by 2 t sin theta into R plus r sin theta.

So, that is the stress. And once you simplify this, it becomes p r by 2 t 2 R plus r sin theta by R plus r sin theta. That is the value. And we can now make it slightly differently. You can write it us, p r by 2 t 2 minus r sin theta R plus r sin theta. So, that is the axial stress in a torous. So, the hoops stress is remaining constant like a cylindrical shell. But then, the axial stress its value is going to change some point to point on the circumference of the cross section.

It is very important to note the variation of this stress. And then, you can understand what set of failure can takes place in a torous. And how they can be prevented? So, let us plot the variation of this stress sigma 1.

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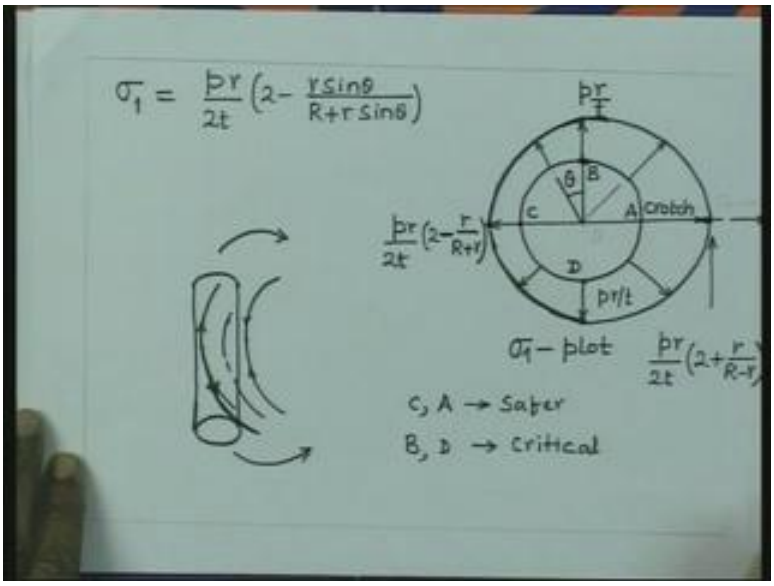

Let us write once again, p r by 2 t 2 minus r sin theta by R plus r sin theta. So, we draw the circumference of the cross section. Let us write this point to be A. The vertical point vertically top most point is B and this, for this point from this center. So, center of curvature is somewhere this side. And this bottom most point is let us say D. And our reference direction is this one. So, this is what is? theta direction.

Now, look at this if theta equal to 0. Then, we have p r by 2 t as the stress. So, therefore if we now plot, in the radial direction the magnitude of this stress. So, therefore I am going to get the magnitude of the stress here is, p r by t theta is 0. This will become, 2 will cancel, so p r by t. Similarly, if theta is equal to 180 degree. So, if you consider the exactly opposite point there, D point.

It is going to give us, exactly the same magnitude of this stress for theta equal to pi. This is again 0. Look at this, if I now consider theta equal to 90 degree. So, it is 1, so therefore p r by 2 t 2 minus r by R plus r. So, the magnitude of the stress here at this point, is going to be p r by 2 t 2 minus r by R plus r. So, here we find that the magnitude of these stress as reduced. And that is the minimum value of this stress that, we can get on this section.

So, this is the minimum stress at the outermost point. Similarly, if we find that the picture is going to gradually vary from this stress, to this minimum. So, therefore the variation is gradually it is like this. So, minimum and then it is going to pick up value. Now, let us consider theta equal to minus. If theta equal to minus 90 degree, that means you come to this point. Here in actually, sin theta is minus 1.

So, therefore it becomes 2 plus r by R minus r. So, here you find that some addition is there over 2, which is nothing but, r by R minus r. So, therefore that is actually going to give as a highest stress.So, at this point you get the highest stress. And you find that the symmetric picture, on this portion with the portion here. So, therefore the variation of stress you find here that, it is gradually increasing. And you have the highest stress at this point.

Similarly, you find the picture same picture is repeated here. So, this is this and here. This stress the reason is like this. So, therefore you see that the stress is gradually determined. That is increasing and taking up the highest value there. And this distance in fact, this radial distance we are plotting the magnitude of the stress, for any theta value. So, this is radial plat we call. So, this is the radial plot of the stresses. And everywhere, it is tensile stress.

This point is known as crotch. Point A is known as crotch point. And you find that, the value of this stress at this point is p r by 2 t 2 plus r by R minus r. So, therefore here it is highest, here it is smallest. And here, they are same and it is nothing but, p r by t. So, this is your variation of sigma 1, on the circumference of the cross section. Now, why do we expect the failure to occur? For a torous like this, where do we expect the failure to occur?

The stress is highest here. And it is lowest at this point. And it is going to be like that of a cylindrical vessel at this point B and D. So, therefore C and A in fact, C is the lowest stress. So, therefore it is a shape point. And here, it is highest stress. Unfortunately, although it is highest it is not a critical point. It is they are consider to be safer locations. Where this B and D, B and D are consider to be critical. This is something very funny.

You see that, here you have the highest stress. And we consider that to be not very critical. It is mostly you will find that torous is going to fill at this 12 bar o'clock position and 6 o'clock position. If it is filling, it will at the 12 o'clock and 6 o'clock position. It will not fill at the 3 o'clock and 9 o'clock position. Why that is so? This is something we should be able to understand. After all, we try to take a straight pipe.

Let us say, we take a straight pipe like this. And this straight pipe is bent into the form of a torous. So, it will do that, you are trying to bend it like this. So you, when you are bending it like this. You are trying to generate compressive stress in the axial direction. So, in your generating compressive stress in the axial direction, you are going to get thickening of the wall at this location. On the other hand, you find here that tensile stresses are generated at the outer fiber.

And it is going to lead to thinning of the wall layer. So, therefore you see that during the process of manufacture itself, you are going to find wall thickness to be higher. And here, it is reducing and that thickening of the wall will, take care of the extra stresses here. And here, it is getting thinning down. And therefore, here the stress is reducing. So, naturally they are taken care of. Whereas, these points are not really going to be affected to that extend.

And therefore, for such shells of configurations we will find that, these are not the critical locations. It is going to be B and D of the critical locations. So, the thinning of the wall can take care of the extra stress that is coming up here. And it is not thin, there is increasing. So, your stress is increasing, but at the same time thickness is increasing. So, therefore it will take care of. On the other hand, here it is stress is reducing wall is also thinning. So, it is not a problem matter.

On the other hand, at this location B and D it does not change mark. And here, it will become critical particularly at the top of the shell. You will find that these locations are neutral axis location. So, this location B and D comes on the neutral axis. See does not change. So, at the neutral axis there is no thinning of thickening. And therefore, these B and D points remain critical points for the torous.

Now, we will consider that such general shells which have double curvature. How do we find out the stresses in such shells? And they are also shells of revolution. So, let us consider a general shell. And then, try to derive the stresses. How are they going to be related? So, let us look into that problem.

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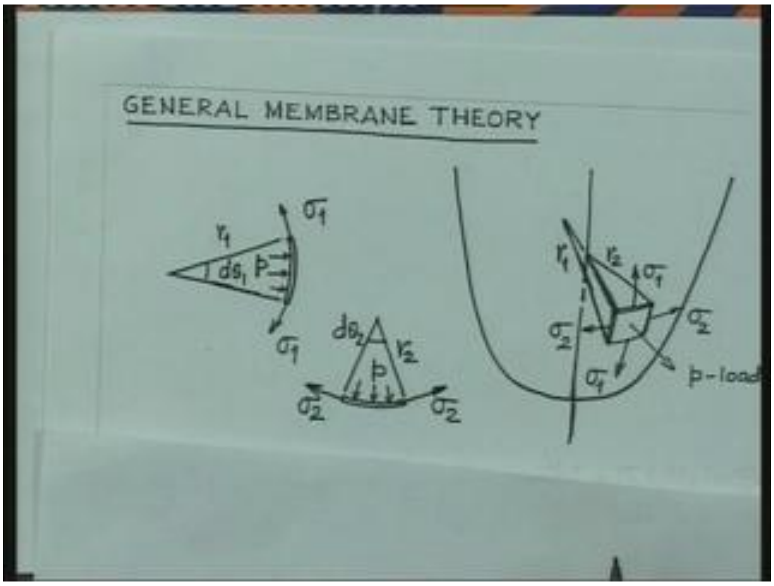

So, let us consider a general membrane. This is known as general membrane theory. We will try to consider a shell of revolution which is doubly curved. So, this is the shell of revolution. And this is the axis. Let us concentrate on an element here, this element. Let us say that, this is making an angle here at the center. And this radius of curvature is equal to r 2. So, therefore the stress which is going to act on this stress is nothing but, hoops stress sigma 2.

And similarly, the stress which is going to act on this space, that is equal to sigma 2. Now, the curvature of this edge let us consider, this curvature. This curvature of this edge is equal to r 1. So, the wall thickness let us consider, this small element will consider that thickness is uniform. So, that is the element will be concentrating upon. So, therefore there will be pressure acting all over the enough surfaces and that will act outward.

So, the pressure loading is going to act in this direction. So, if we now try to look at to look at the picture again. So, this edge is like. This is making an angle of d theta 1. The pressure is p and radius is r 1. And this is the, say sigma 1 that is the actual. So, therefore you do not have this. This is the stress, which acting on this edge is sigma 1. Similarly, if I consider this sector, this is making an angle of let us say d theta 2. This is d theta 2, the radius is r 2.

And the pressure is acting in the radial direction p. Stresses at the edges is nothing but, sigma 2. So, in the r 2 sector, we have 7 stresses sigma 2. And here, it is r 1 sector we have the stresses equal to sigma 1. Let us now try to consider the equilibrium in the radial direction.

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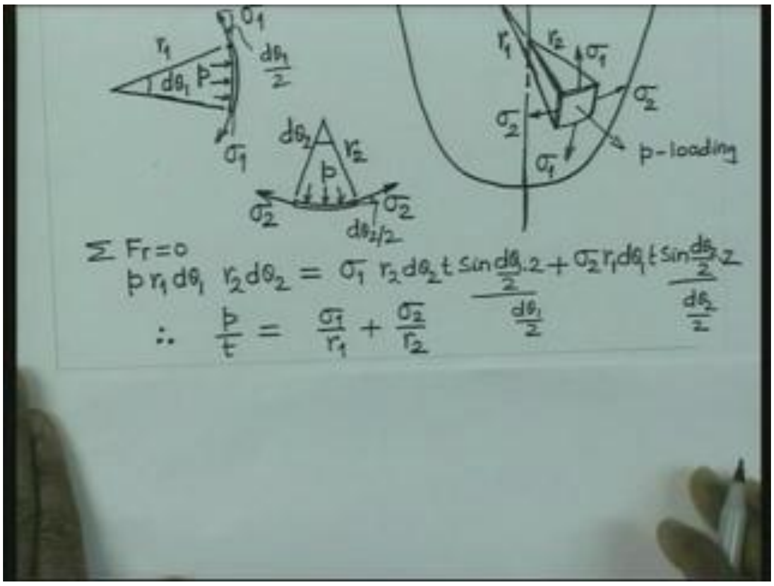

So, if you try to sum up the forces in the radial direction. So, if I now try to take this area, this length is nothing but, r 2 d theta 2. And this area is r 1 d theta 1. So, therefore the total area we can take approximately to be r 1 r 2 d theta 1 d theta 2. And the pressure p is acting readily outward. So, therefore we have p into r 1 d theta 1 r 2 d theta 2 ((Refer Slide Time: 34:05)) if it is a rectangular area. So, that is the radically outward load.

And it is counter balance by the component of these, in the central towards the center. So, therefore this angle is d theta 1 by 2. So, therefore we can now consider what is this area? This area is nothing but, r 2 into d theta 2 into t into sigma theta 1. And it is sin component sin d theta by 2 component. So, we can write now, sigma 1 r 2 d theta 2 into t. So, that this force and it is sin component sin d theta 1 by 2.

So, this angle is nothing but, d theta 1 by 2. And we have one contribution from this side, another contribution from this side. So, therefore we can multiply by 2. So, we get the contribution from these two edges. Similarly, if I try to consider this forces which are here again, it is going to make an angle here which is of d theta 2 by 2. So, I can write now sigma 2 acting over an area of r 1 d theta 1 into t and the sign d theta by 2 component.

So, sin d theta 2 by 2 component. And we have one from this side and another from that side. So, multiply by 2. So, that is therefore finally, you find that we can write. Substitute these things else d theta 1 by 2 for small angle. And these can be substituted by d theta by 2 and these 2 and that 2 will cancel this 2 and that 2 will cancel. Finally, we will find r 1 r 2 d theta 1 d theta 2 will be come.

And you find that final relationship is p by 2 p by t is equal to sigma 1 by r 1 plus sigma 2 by r 2. So, this is the general membrane formula. The two principles stresses acting at a point in a W call shell of revaluation is related to pressure and thickness by this relationship. And this relationship is also very useful, in determining the one of these stresses if you know the other stress. In more option, you will find that the axial stress can be found out easily.

Then, the other component of this stress hoops stress can be found out by making use of the formula. So, this is the general membrane theory for W call shell. We will quickly consider the applications of the formula, to some of the known vessels. Think of it, you have the cylindrical shell here.

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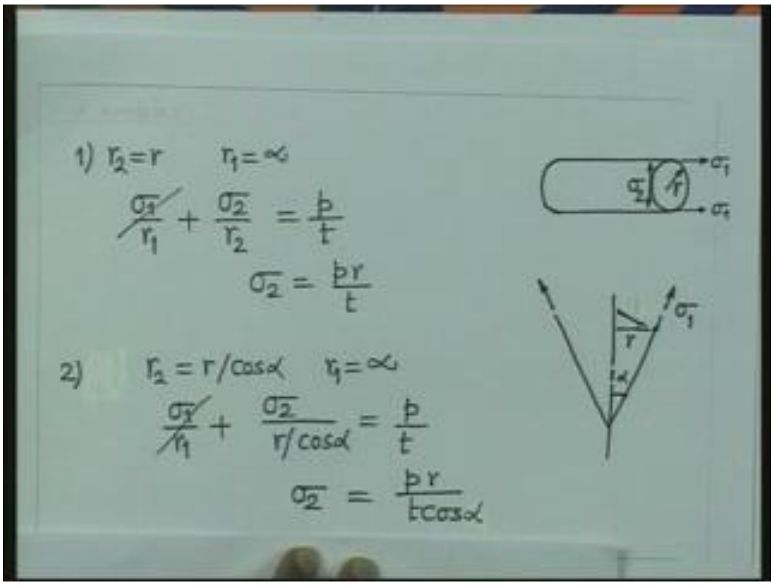

And this cylindrical shell is of radius r and the wall thickness t. We have the stress sigma 1, acting in the axial direction and circumference stress is sigma 2. And therefore, this r 1 in this case, which is infinitive and r 2 is going to be r. So, if I now apply the formula, sigma 1 by r 1 sigma 2 by r 2 is equal top p by t. And since, r 1 is infinitive this term goes off. So, you have now sigma 2 by r is equal to p by t.

And therefore, sigma 2 is equal to p r by t. So, it follows from the formula that we have derived. Let us consider this second example. Think of this, the conical vessel. Conical vessel with the vertex angle equal to alpha, at any point at a distance r. If I now look into sigma 1 stress is going to act in the wall thickness wall direction and therefore, this is sigma 1. And the radius of curvature for this direction is infinitive.

And the other stress, which is going to be hoops stress for that the radius is nothing but, r 1 by cos alpha. So, this radius is r 1 by cos alpha that is r 2. So, if I now try to substitute in this formula, sigma 1 by r 1 plus sigma 2 by r 2, which is r by cos alpha equal to p by t. Again this term turns out and we have sigma 2 equal to p r by t cosine alpha. So, that result we have already derived in a different way.

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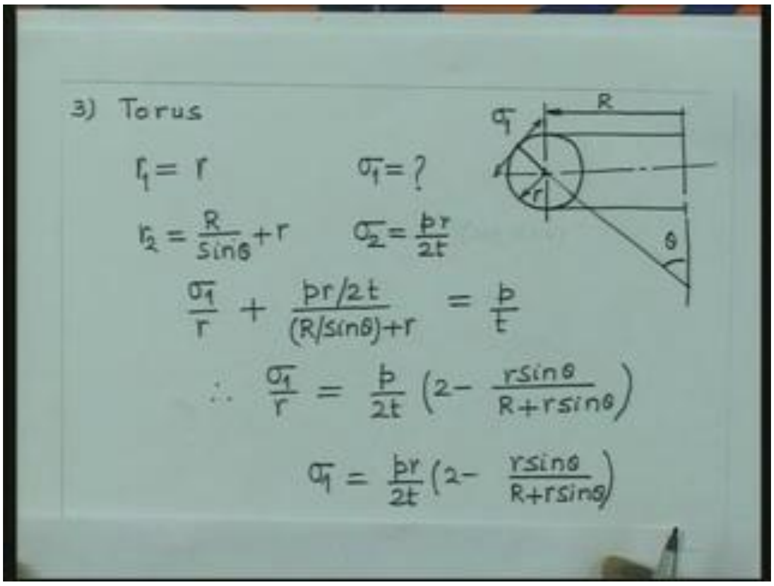

Let us consider the case of the torous. This is something very typical. So, in the case of a torous, we have the hoops stress which is going to act perpendicular to the plane of the paper. That space is going to be given by sigma 2, which is p r by 2 t very easy to determine. Now, we are interested in finding out the axial stress sigma 1 acting like this. And this axial stress, obviously the radius of curvature for that is this r.

So, r 1 is equal to r and what about the radius of curvature of the vessel in the sigma 2 direction. So, that is nothing but, this radius of curvature. This is the center of curvature for this point. And therefore, if you now consider that radius for the axial direction, that is going to be nothing but, this distance is r. This angle is equal to theta. And therefore, this is nothing but, r by sin theta capital R by sin theta plus this small r. So, therefore this radius for the direction r 2 is nothing but, R by sin theta plus r.

So, again if we try to substitute in the formula, sigma 1 by r 1 is r sigma 2 by r 2 which is this expression. And that is equal to p by 2. And then, you find that sigma 1 by r is equal to p by 2 t. If I take it to the other side, it will become 2 minus r sin theta by R plus sin theta. And therefore, sigma 1 equal to p r by 2 t into 2 minus r sin theta by R plus r sin theta. So, this formula can also be derive, considering the general membrane theory. Now, we would like to consider some problems solving, using the derivations that we have got.

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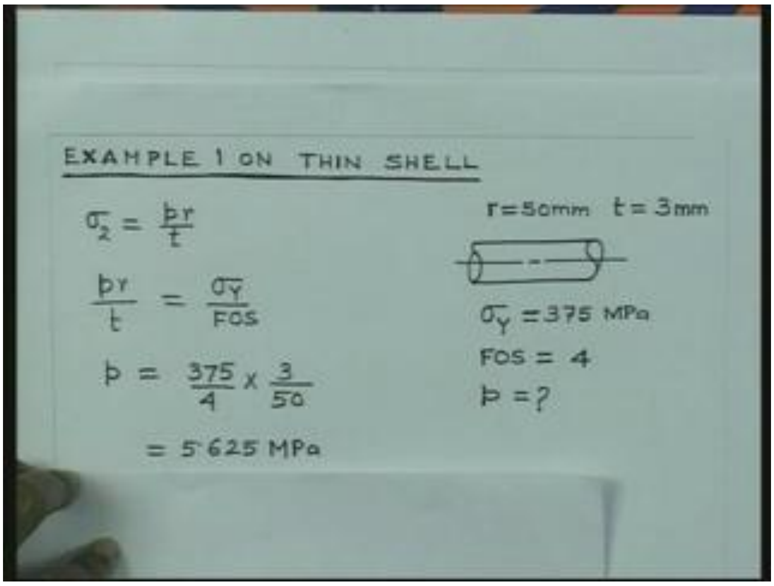

First example let us consider, on thin cylindrical shell. It is giving that, there is a pipe of internal radius 50 millimeter. Thickness of the wall 3 millimeter, the material at a ill stress 375 mega principle, factor of set equal to 4. Find out the shape pressure that, this pipe can be subjected to. So, here obviously we can use the maximum normal stress theory or maximum shear stress theory. In either case, we will have the maximum principle stress is equal to the ill point divided by the factor of safety.

So, the design equations becomes in this case obviously, we have the highest stress in hoops stress which is nothing but, sigma 2 equal to p r by t. So, sigma 2 is equal to p r by t. And that should be limited by, sigma y by the factor of safety. And therefore, once you do the calculations you get this p. That gives you now 5.625 MPa. Let us consider another example on a thin spherical shell.

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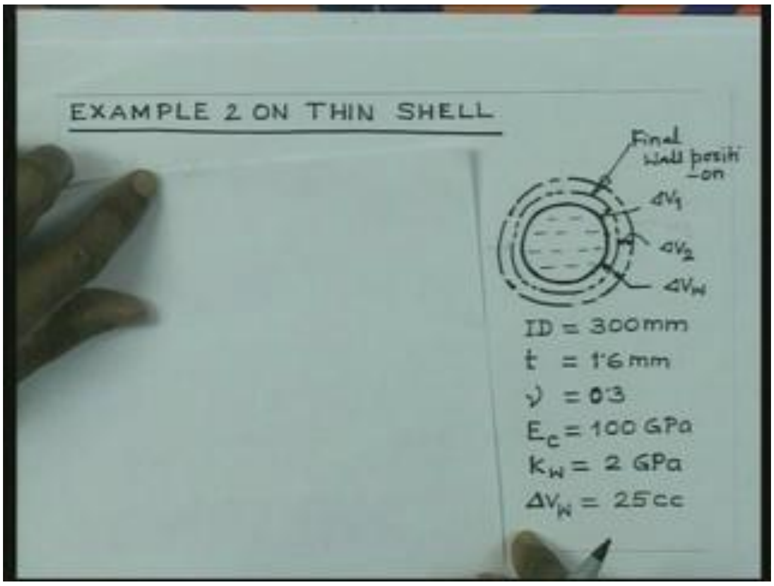

This spherical shell is with the following details. Again it is a thin shell, which is filled with water at atmospheric pressure. Its internal diameter is 300 millimeter, wall thickness is 1.6 millimeter. Material property poisons ratios is .3 and it is having modular 100 GPa. It is really made up of copper. And the bulk modulus of water is 200 GPa. Now, what is specified here? That this vessel is filled with water at atmospheric pressure.

And now, 25 cc of water you injected into the vessel. So, 25 cc of water is injected into the vessel and what is needed to be found out? Find out the pressure that develops, because of this injection of the fluid. So, fluid extra fluid is inserted into the vessel. And due to that, there will be some pressure raise in the vessel. Find out that pressure. How do you go about it? If I now look at the problem, it is like this.

We have this water already there. And now, if I think of it that I am trying to add 25 cc to it. And let us say that, at atmospheric pressure think of it that it say, spherical shell of water without any cover. And now, you try to add on to it 25 cc. And therefore, its diameter is going to be now more. And let us say that, if there is no vessel then it will try to take up outer boundary position, like this.

So, that is the therefore, this to this it is corresponds to that extra water delta V w. Obviously, the vessel is already in position. This water injection cannot be taking place at atmosphere pressure. Are you try to put in little bit of water, it will try to expand the copper vessel. And that copper vessel, you will try to compress the water inside. So, this sort of task of water will continue and what you will find finally? That when you have try to add the full 25 cc, you will find that the vessel wall is not going to really get expanded up to that position.

It is going to be somewhere in between. And it will try to compress the outer surface of the water, to the same position. So, what happens that, this 25 cc of water that we have added. It is going to expand the vessel by this much. And the pressure that develops in the vessel, that vessel will compress the water shell from this point to that point. So, therefore if we now mark it that the equilibrium vessel position is, let us say this one. So, this is the final wall position.

What you find really, that the water this is the sphere of water it got? It got compressed. Let us say that, it got compressed by some volume, which is delta V 2. And the copper vessel got expanded by this much, which is equal to delta V 1. So, it is obvious that this total volume of water that we have inserted. It is going to be nothing but, delta V 1 plus delta V 2. Now, we can calculate what is delta V 1, because the internal pressure of water will try to expand the copper vessel by delta V 1.

And the pressure which is acting on the outer surface of the water sphere, that will try to compressive compressed by delta V 2. And therefore, we can calculate both the things under the same pressure. And then, we can write the competitively condition that delta V 1 plus delta V 2 equal to delta V w.

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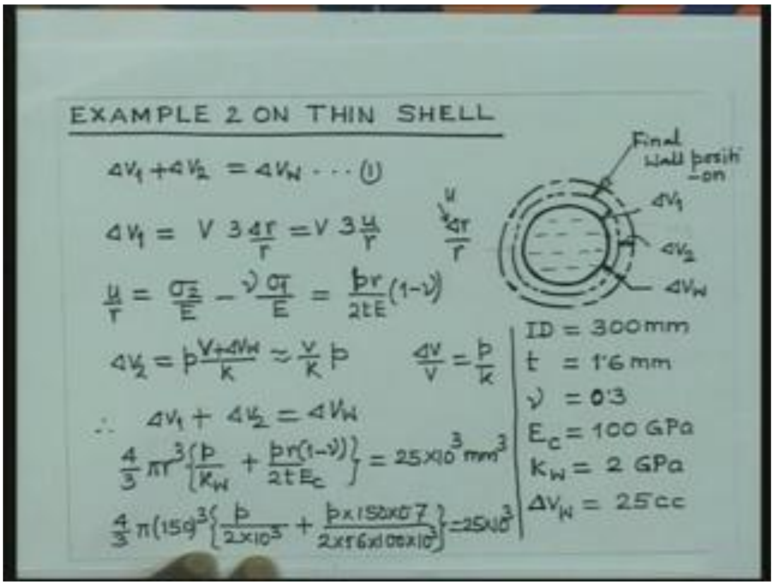

So, I have written here that, delta V 1 plus delta V 2 is equal to delta V w. So, that is the equation number one, which is nothing but, competitively condition. Now, how do you calculate? Let us say delta V 2. Delta V 2, the expansion of the let see that, this is I will write this delta V 1 expansion of the copper vessel. So, by copper vessel under the action of the pressure p, it is going to have some radial expansion.

Let us say, that radial expansion is equal to delta r. So, that delta r by r is nothing but, hoop strain. So, if that is the hoop strain the volumetric expansion is going to be 3 times of that strain. So, therefore if the strain in the circumferential direction is delta V by r. Then, the volumetric strain is going to be delta 3 times delta r by r. So, therefore the volume increases if initial volume of the sphere internal volume is D.

Then, that is the increase in the volume. So, that is nothing but, we can write this thing else radial displacement of the wall divided by r. This delta r is nothing but, again u displacement of the wall. And for a spherical vessell, you can calculate the hoops strength which is nothing but, it is u by r is given by hoops stress sigma 2 by E minus nu times axial stress by E nu times as this.

And for this spherical vessel it is therefore, both sigma 1 and sigma 2 is nothing but, p r by 2 t. So, therefore p r by 2 t E multiplied by 1 minus nu. So, therefore you got this u. And now, what is delta V 2? Delta V 2 is the compression of the water sphere. And that should be available from p by V plus delta V w. That is the total volume of the water divided by the bulk modulus. You see, you know that this delta V by V we have the formula. Delta V by V is equal to p by K. That is the bulk modulus.

And therefore, we can write the same thing here that, this delta V 2 is equal to p into the volume divided by K. And this V plus delta V w, we can write that thing approximately equal to V. And therefore, this is V by K into p. So, everything is available in terms of pressure. And now, from the competitively equation therefore, we can now write delta V 1 plus delta V 2 is equal to delta V w.

And you substitute the values, the volume of this sphere is nothing but, four third pi r cube. So, I am writing that to be common. And that multiplied by, we have the pressure of course, p by K. K bulk modulus of water plus. If you write this one it is nothing but, p r by 2 t E of copper multiplied by 1 minus nu. That is equal to V w, which is nothing but, 25 cc which is in millimeter unit. It is 10 to the power 3 millimeter cube.

So, here you have written a everything in terms of millimeter unit, Newton millimeter unit. And r is nothing but, 150 millimeter. And all the values K w 2 GPa, E c is 100 GPa, nu is .3, r is given 150. So, therefore all that constitute substitute, you find 4 by 3, 50 cube p by 2 into 10 to the power 3 plus p 150 into 1 minus nu .7, 2 into 1.6 into 100 into 10 to the power 3 is equal to 25 into 10 to the power 3.

So, that is what you have the expression to calculate p. And p, once you calculate it comes out to be 2.13 mega per square. So, that is the pressure which is going to develop. So, this is the way the sort of problem can be tackled. Now, I would like you to think about another problem, which is also very interesting. And it is solvable along similar lines.

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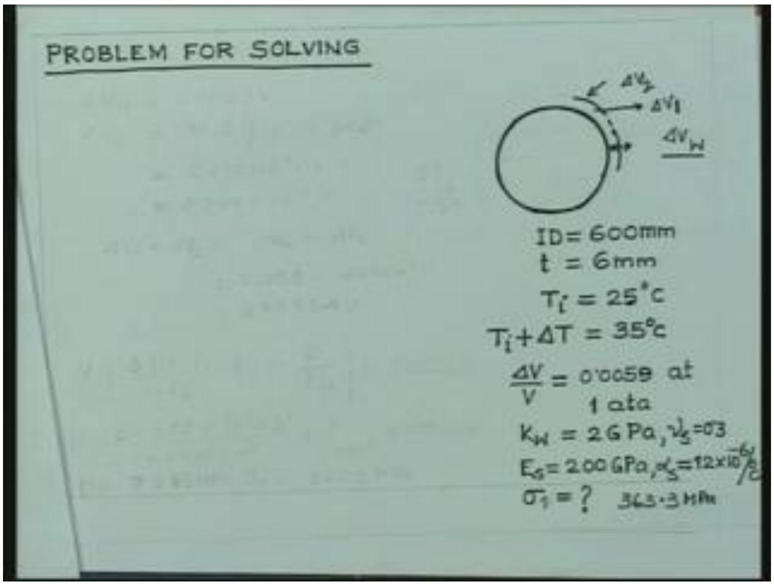

Think of yourself, that you have a vessel here. It is filled with water and its dimensions are ID is 600 millimeter, thickness is 6 millimeter. And the water is initially at 25 degree centigrade. Now, the water of the temperature of the water is raised by 10 degree. Now, you have to calculate what is the stress in the vessel? Given the information that the volumetric expansion at atmosphere in this temperature range is going to be 0.0059 unit per unit volume.

Bulk modulus of water is 2 GPa, nu s this is a steel material and it is nu s is equal to .3, E s is equal to 200 GPa. And the thermal coefficient of expansion of steel is 12 into 10 to the power minus 6 per degree centigrade. So, therefore in this case what is given here is, that you are trying to raise the temperature of water by 10 degree centigrade. Because of that raise, the volume of water is going to raise at this rate.

So, you can find out the volumetric increase of the water volume. And once you have done that, now you will find that if the vessel is allow to what are is allow to expansively. It will take up some position up to let us say this position. At the, because of the temperature raise vessel is going to expand up to this position, it will expand radically. So, there is a net change in the volume. And this volume now becomes delta V w edge, in the previous case.

So, in this case your volume increase in the water is not exactly delta V w. It is going to be that change in the volume of water minus the increase in the size of the shell, due to the temperature raise, that will become delta V w. And then, this is going to be balanced by increase in the vessel dimension. Let us say that is delta V 1. And decrease in the water volume is equal to delta V 2. So, increase in the water volume.

So, finally let us say they are in equilibrium somewhere they are. So, for that equilibrium position what you will find that, increase in the volume of shell is delta V 1, decrease in the volume of water is delta V 2. That is going to being compatibility with delta V w. So, you can solve this problem. And in this case, the answer is going to be 363.3 MPa.