Advanced Strength of Materials
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Lecture – 37

We have seen how we can calculate approximately the stress in a plate, which is simply supported at the edges. And subjected to uniformly distributed pressure loading P. Let us consider today, if we have the plate simply supported at the edges and loaded by a concentrated course at it is center.

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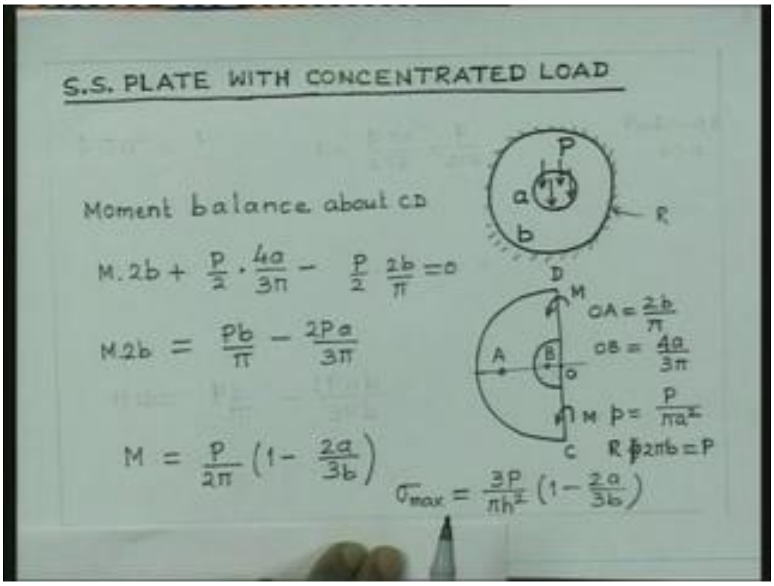

So, we will consider the plate to be like this, it has let us consider that this load P to begin with it is distributed over an area of radius at this center. And obviously, we are trying to talk about b is very large compacted to the outer radius b. We will try to see when this a sinks to 0, then we will get the case of the concentrated loading. So, let us calculate the stresses for this case by taking the approximate approach that we have considered for the simply supported plate with the uniformly distributed loading

So, now if I try to consider the again the, this is one half of the plate, this is the outer circle and this is the inner circle. Now, let us this is the point o, you consider that all along this diameter there will be some bending movement acting, whose intensity let us say is M. So, that is M bar unit length of this diameter. Now, if you consider that reaction at this suppose, which is going to be uniformly distributed and it will have some intensity that intensity can be very easily found out.

That distributed force could have a center, somewhere there. Let us consider that it is line on this radii and at a point o. Similarly, the force P, which is distributed over this circle and therefore, p by 2 will be acting over this semi circle, will have this centralized somewhere there, which is B let us say. And therefore, we can very easily established that this OA is equal to nothing but 2 b by pi, OA that distance is 2 b by pi. And OB is nothing but 4 a by 3 pi.

This intensity of bending movement, let us represented by M everywhere, the loading which is uniformly distributed over this small circle. The intensity of this loading, let us consider that the reaction force which is all over this boundary is equal to p per unit length, then in that case we can write p into 2 pi b is equal to p. Better to we will better represent this reaction at this point to be R rather than p because p let us result of uniform pressure.

So, therefore, this is R, R into 2 pi b equal to p. So, you get that intensity of the loading. Similarly, we can also find out p, which is the intensity over this smaller circle it is going to be nothing but p by pi a square both these in formations are available. Now, we calculate the, we will consider the moment balance. So, this moment balance will now take up about the diameter, let us say that this CD. So, if you take the moment balance about CD, then we have AM acting all over the distance CD, which is of length equal to 2 B.

So, you will have anticlockwise movement of magnitude AM into 2 B. We have this load, which is acting over this semicircle its magnitude if p by 2 and its moment term is 4 a by 3 pi. So, therefore, this moment is we can it is acting downward, so therefore, it is acting downward, so therefore, will have again anti-clockwise moment. So, the moment is nothing but 4 a by 3 pi minus the low, which is acting over this boundary it is also some totally nothing. But, p by 2, which can be obtain by considering that R by 2 that is R into pi b is going to p by t pi by 2.

So, therefore, that p pi 2 is the force which is acting upward. So, P by 2 and its moments are OA, which is 2 b by pi, so 2 b by pi equal to 0. So, that gives us the moment balance. And therefore, M 2 b is equal to Pb by pi minus 2 p a by 3 pi, so simplifying get this.

And now we can write therefore, M is equal to p by 2 pi 1 minus 2 a by 3 b, so that is the moment per unit length of CD. And now if you want to calculate the stress, the stress will be available by considering that maximum stress is nothing but 6 M by h square.

So, if we now try to calculate, sigma max it is 6 M by h square. So, therefore, two will cancel. So, we will have 3 p by pi h square into 1 minus 2 a by 3 b, so that is the magnitude of distance. And we are looking for a case, when the radius of this circle is 0 that will really lead to the concentrated loading at this center of the plate. So, therefore, if you do that if we just try to consider now a equal to 0.

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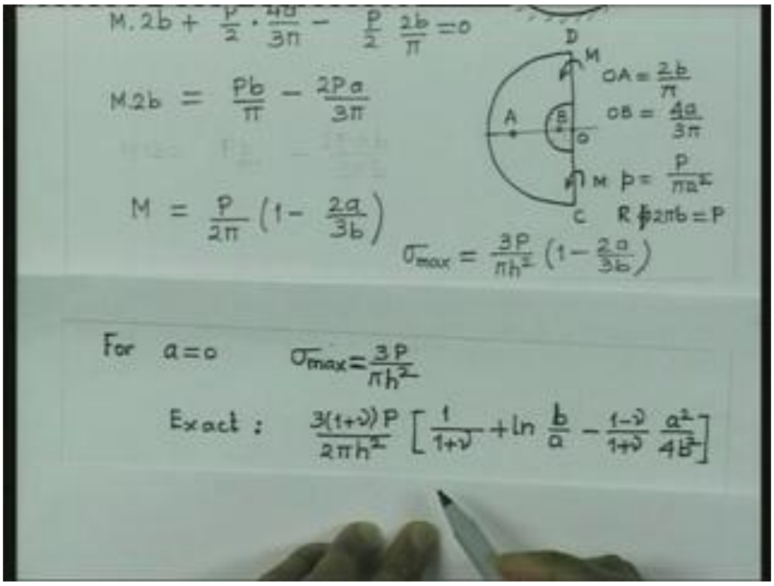

So, if we now to a equal to 0 will have sigma max is equal to 3 p by pi h square. So, that is the maximum stress, which is going to occur at the bottom fiber, it is going to be tensile stress and if it is of the top fiber it will be going to compressive stress. You might be interested in trying to know, what is the exact value of this stress if, we go rigorously for the analysis.

Then, in that case this maximum stress exact answer is nothing but 3 into 1 plus nu 2 pi h square into P 1 plus nu plus l n b by a minus 1 minus nu 1 plus nu a square by 4 b square, so that is the exact expression. So, you can see that this is what, when we have a and b both are known 0 and that compares to this exact expression like this. There is some difference, but then for approximate calculation one can make use of this formula which is derived approximately, following the state that I have talked about this one.

So, this is about the circular plate, you can go on doing similar calculations for the rectangular and square plate. So, let us try to consider the case of a rectangular plate and something rectangular plate we can get the answer for its square plate So, the plate is shown here the dimensions.

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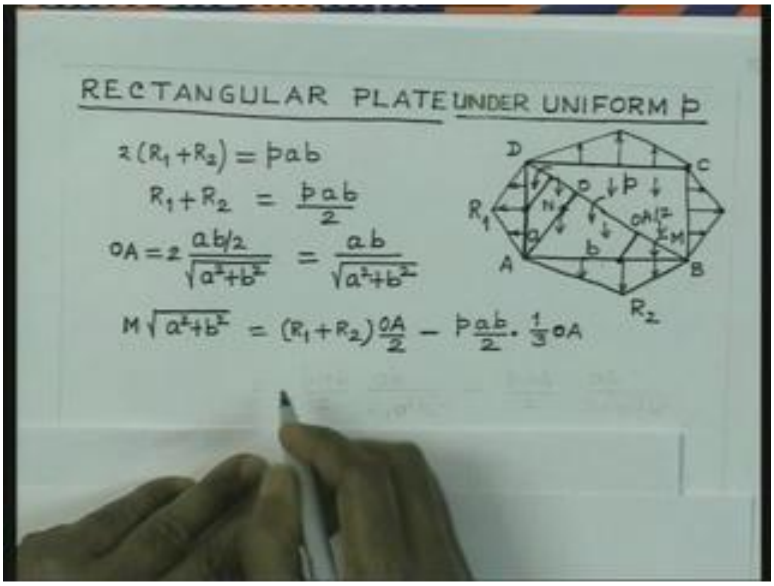

Let us consider that the height is a and this width is b. And it is subjected to uniformly distributed load. The plate is simply supported at its outer boundary and the pressure loading is P; obviously, the reaction over the boundary it is not going to be uniform as in the case of circular plate. Here, the center of loading is exactly at the center of the plate and this one, the middle points of the size their closest to the center of gravity and therefore, what you find is that.

Reaction is going to be non-uniform and it will going to vary along the boundary. If, we just assume that the reaction forces at the boundary is going to very linearly then in that case, we can consider that the reaction is going to the maximum at the center of the side. So, therefore, the reaction forces are going to have distribution like this, over this boundary. Similarly, we are also going to see some distribution of the reaction force over this side and similar thing will be repeated for the other two sides.

We are not really going to find out the maximum of these distributed loading, but then what we just assume is that is going to very linearly. Let us consider, let us indicate this plate by AB CD and let us say the total reaction at the edge AD is given by R 1 and obviously, its centralized is going to be here. Similarly, for the side AB the total reaction is equal to R 2. And therefore, we are going to have reaction here is other one reaction there is R 2.

Now, if you consider half of the plate, let us take a diagonal section we just cut the plate into two along this line BD. So, if we do that, then we will have shear forces acting on this diagonal plus, we are going to see the bending moment. Let us say that this bending moment is uniformly distributed, all over with some intensity let us say that is equal to M. So, what we have a triangular plate, where we have some shear forces acting over this line BD.

And then we have force reaction force, R 1 acting vertically upward this is also R 2 is acting vertically upward. And there will be some reaction forces over BD, which can be downward we are not really concerned about it. Now, the external loading which is uniformly distributed pressure over the triangle ABD, it is going to have its center. So, it is going to be, it will drop a normal here on this diagonal. So, at one third of the height somewhere here, we are going to get the center of the loading.

So, therefore, this ON is the distance of this CD of the distributed stress. Now, let us consider that this OA height, which can be represented we can consider that to be we can also calculate that need we not a write it explicitly. So, let us first of all write the force equilibrium. So, if it consider the force equilibrium herein that sum of R 1 and R 2 must be equal to twice R 1 plus twice R 2 must be equal to the external loading, so that is the equilibrium of the whole plate.

So, therefore, if I write the equilibrium of the whole plate in the vertical direction then we are going to get 2 R 1 plus R 2 that is equal to total downward load, which is P times ab. Therefore we have R 1 plus R 2 is equal to P ab by 2. So, that from the whole plate we get that. Now, we can calculate this height OA, OA we can consider that by considering the fact, that this area of the triangle is nothing but ab by 2. So, therefore, that is the area of the triangle.

Now, if you consider area of this triangle, again it should be equal to half DB into OA and DB is nothing but square root a square plus b square. So, therefore, from that relationship we can calculate the height OA. So, OA is nothing but area of the triangle divided by length DB which is nothing but a square plus b square. And that half, half into that will give us now 2 here. So, therefore, that is the value and therefore, this height OA is nothing but square root a square plus b square, so that is the height.

Now, you go for considering the equilibrium, about the line DB. So, if you now consider the equilibrium. So, you will have anticlockwise movement, which is M to the intensity multiplied by the total length is a square plus b square of the diagonal DB. So, therefore, that is the total moment in the anticlockwise balance direction, it is counterbalance by the moment produced by R 1 about this length DB, R 2 about this line.

And at the same time distributed pressure acting downward to the point 10, so therefore, we can now write that this R 1 and R 2; obviously, you see that this CG is going to be CG is here. That CG height is going to be nothing but half of these side similarly CG of this R 1 is going to have height, which is again going to be half of this height. So, therefore, for both the cases we have height equal to nothing but h by 2, so this is nothing but this OA by 2.

So, therefore, this is OA by 2. So, also this is also equal to OA by 2, so I would like to repeat that. So, therefore, we have R 1 plus R 2 OA by 2 that is what is acting in the clockwise direction. And this load, load is nothing PAB by 2 because P is the intensity and area is AB by 2. So, therefore, P ab by 2 is the total downward load and its distance is ON which is one third of OA. So, therefore, that is the movement and this movement is going to be downward and will settle anticlockwise. So, therefore, this gives as the moment equilibrium about DB, and finally, if you substitute the value of R 1 and R 2, R 1 plus R 2 is nothing but Pab by 2.

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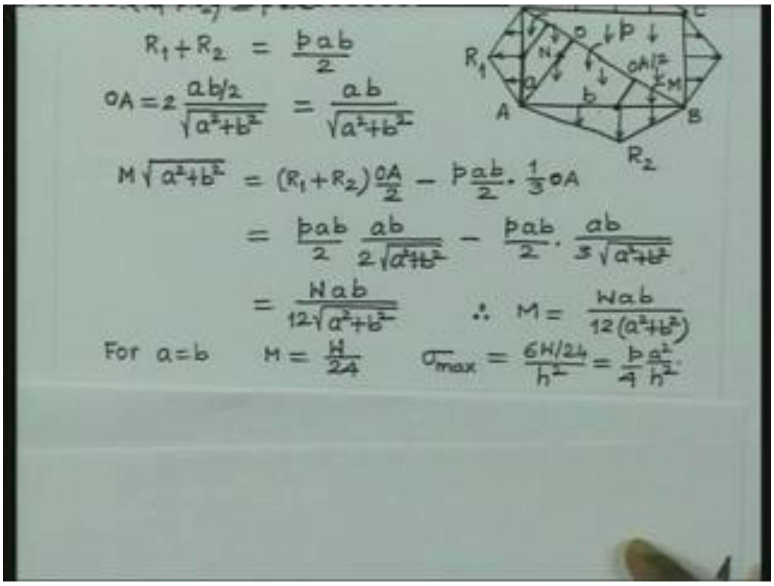

So, you have Pab by 2 ab by 2 into a square plus b square minus Pab by 2 into ab, this is 3 a square plus b square. So, that is the value and once you substitute, you know that Pab is nothing but if you consider the total pressure is equal to W therefore, we can write now. This is nothing but W ab, it is simplifies to 12 square root a square plus b square. So, that is the, this is the total load due to P. And hence M intensity per unit length, DB is nothing but W ab by 12 a square plus b square.

So, that is the moment per unit length and obviously, you see that unit is also balanced because this is going to be (Refer Time: 22:38) meter square this is going to cancel meter square, but then we can write this thing else Newton meter per meter, so now if I consider square plate. So, for a equal to b we have M equal to W by 24 and therefore, sigma maximum is equal to 6 W by 4 by h square and this is nothing but P by 4 a square by h square.

So, again it is a good approximation. So, you can calculate by this simple approach the approximate stress acting at the, it is going to be really critical at the center and therefore, you can calculate that critical at the critical location what will be stress acting. Now, we would like to consider some problems solving, you have taken up number of cases. So, let us now consider some applications. So, as to consolidate whatever we have derive, so for.

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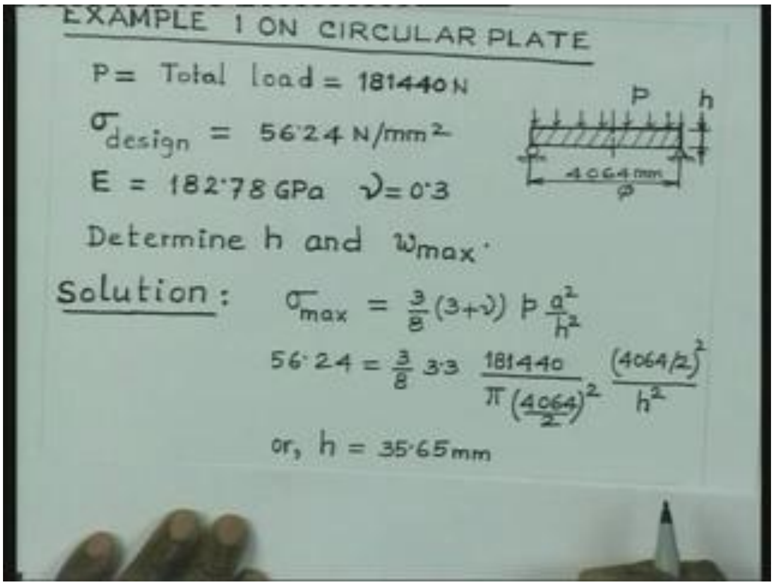

Let us now consider one example, on the design of a plate, it is a thick plate of uniform height h for thickness h, its diameter is 406 per millimeter, the total load which is uniformly distributed, which is given by 181440 Newton. As in specified that the design stress of the plate is 56.24 Newton per millimeter square and E for the material 182.78 GPa, it is steel. And poisons ratio is point 3, what is needed is that calculate the thickness h and the maximum deflection of the plate.

So, therefore, that is the problem. So, let us now go for calculations and in fact, this is going to be direct application of the formula that we have derived. So, now let us look at the solution, if we just take the expression that maximum stress in the case of a circular plates subjected to uniform loading and simply supported at the edges, the maximum stress at this center and its magnitude was 3 by 8 3 plus nu P a square by h square.

So, here in what is given is that you have to get P from the total load and the diameter and a is already specified and sigma maximum it happens to be limited by the design stress and therefore, only unknown is h, so you can calculate. So, now we can try to plug in the values. So, we have this thing as 56.24 and that is equal to 3 by 8 and this 3 plus point 3 it gives us 3.3 And P, P is nothing but total load capital P, so 181440 Newton.

And now if we divide by the area which is nothing but pi D by 2 whole square, so that gives us the pressure in Newton per meter square. And a square is 4064 by 2 whole square divided by h square, so that is simple substitution. And here in this cancel should this one so therefore, the calculation becomes simple and if you now calculate the thickness, thickness comes out to be 35.65 millimeter. So, the thickness of the plate has got to be this much.

Now, second thing we have to calculate the maximum deflection of the plate and the deflection is obtainable from the pressure, poisons ratio, modular’s of elasticity and also the dimension of the plate. So, if you do that, first of all the formula that we have derived for the maximum deflection it was, right at this center of a circular plate, so that maximum deflection if we write….

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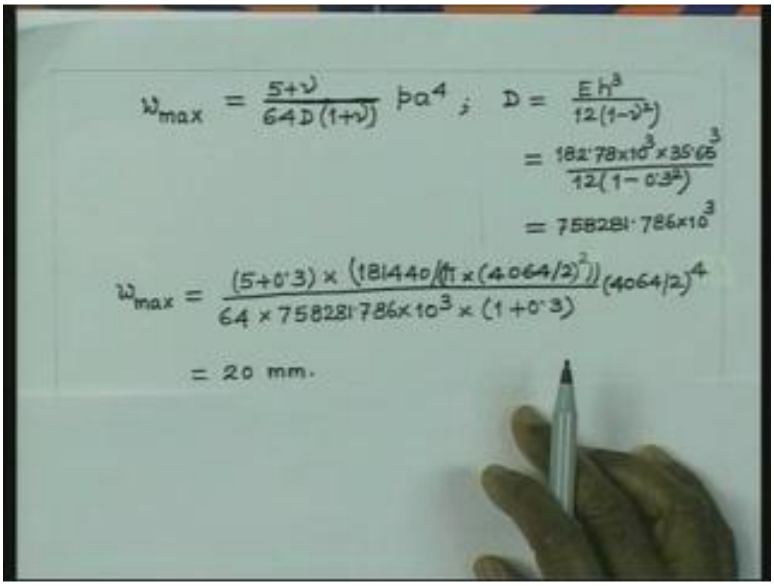

W maximum is equal to 5 plus nu by 64 D 1 plus nu P a raise to the power 4. This D is nothing but modulus of rigidity and therefore, you should calculate this value, rests of the values are all given. So, therefore, let us now calculate this D. So, if you consider this D, D is nothing but Eh cube by 12 into 1 minus nu square. So, it is different from that of a plate and its value is E h cube by 12 into 1 minus nu square.

So, let us substitute this value here 182.78, if you want to keep it in Newton per millimeter square unit then it will be simply 10 to the power 3 and this is 35.65 cube divided by 12 1 minus 0.35 square. And this gives us 758281.786, 10 to the power 3, it is a Newton millimeter units. If we now substitute the values, we have 5 plus 0.3 for nu and this P is given by total load divided by the area, which is pi r square. And again r is nothing but this raise to the power 4 and you have 64 D value is given here, 1 plus nu is

1 plus 0.3. So, once you simplify this it gives us value to 20 millimeter. Now, we would like to consider another example, wherein the same plate subjected to the same total load, but then it has a circular cut out at the center of some radius, which is one fourth of the outer radius.

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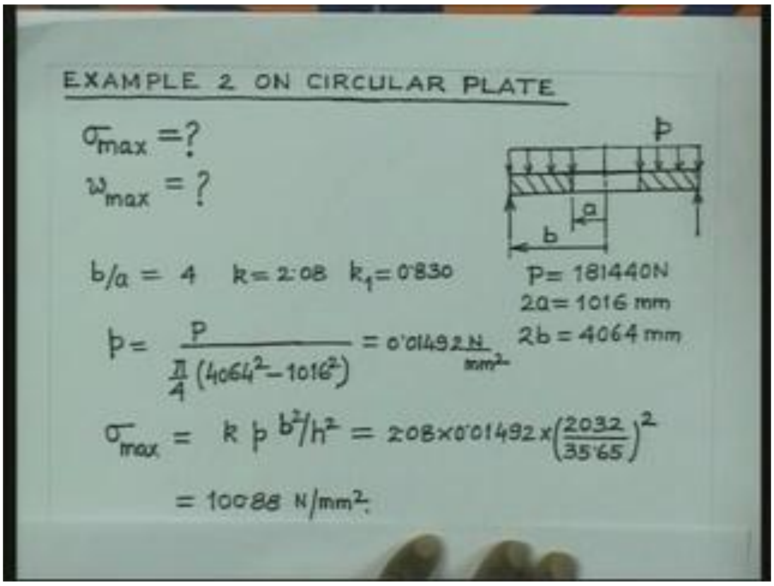

So, the problem is shown here, this is the plate the cut out at this center is of radius a and outer radius is b, the load is uniformly distributed, total load out of this uniform distribution is 181440 Newton. And as I said that this inner radius is one fourth of the outer radius. Now, let us see what happens to the maximum stress and the deflection. So, these are the two things to be calculated.

To solve this problem, we do not want to go for deriving the formula, we will make use of the vessels, which are available in the tabular form take the data directly from the table and do the calculations. In order to make use of the tables that we have consider in a earlier lecture, we need the value of the ratio b by a. So, if you have the ratio b by a in this case, is 4 then the constants which are associated with the calculation of the stress and the deflection.

So, this is the constant, which is associated in the calculation of stress. So, therefore, this is b by a is equal to 4 and therefore, consulting the table we get the constant k as 2.08 and similarly the other constant which is associated is 0.830. P is now total load divided by pi by 4, 4064 square that is this square minus 1016 square. So, that gives us value equal to 0.01492 Newton millimeter square.

The maximum stress is given by this formula, small k into P b square by h square. So, here h is nothing but 35.65 that we have got in the earlier case. So, that we make a, we have to make use of and you have to calculate this maximum stress. So, therefore, this is equal to 2.08 into 0.01492 b square is nothing but 2032 and the height of the plate is 35.65, so b by h square.

So, if we do that, it gives us a value of 100.88 Newton per millimeter square. Remember the value in the earlier case, we had the maximum stress was 56.24 Newton per millimeter square, just by having a hole at the center of radius one fourth of the outer radius, we get the stress amplified by almost two times. So, that is the amplification in the stress, now let us see the value of the maximum deflection. So, if you calculate the maximum deflection again it is given by the standard formula.

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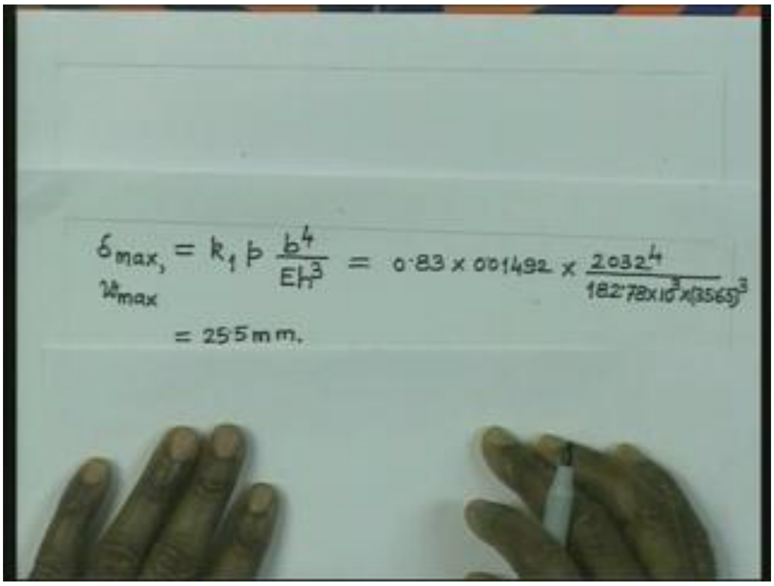

Delta maximum or it will be W maximum is equal to k 1 into p b 4 by E h cube. So, W maximum is k 1 p b 4 by Eh cube. So, if I substitute the value that is the value of b and this is the value of E 182.78 into 10 to the power 3 and this is 35.65 cube. And on simplification we get this thing as 25.5 millimeter, the deflection in the earlier case we had only 20 millimeter. Now, the deflection is increase to 25.5 millimeter.

So, this is a very simple OA to go about finding out the deflection and the maximum stress provided to configuration falls within the cases which are tabulated. Now, we will try to consider a different problem, it is not necessary that we have to have uniform thickness, the formula that we have derived they can be applied sometimes it is also possible to apply to situations, which are different.

Consider for example, many cases in application, if you find that a plate is made up of one plate of uniform thickness and it is welded to ((Refer Time: 38:48)) at the bottom or at the top and therefore, short cases are also treated like uniformly thick plate and calculations can be done. So, we like to show one particular case, the problem is like this 6096 millimeter diameter nuclear core support plate is composed of a solid plate of 50.8 millimeter thickness to this plate.

And orthogonal ((Refer Time: 39:24)) consisting of 254 millimeter deep. And 50.8 millimeter wide waps are integrally welded the ((Refer Time: 39:38)) dimensions are 254 millimeter deep 50.8 wide, waps are of upto 50 millimeter dimensions. It supports uniformly space nuclear fuel elements of total weight of 1451520 Newton. Assuming the structure to act as a simply supported plate with uniform x reaction determine the flustered rigidity D, bending stress at the top of the flat cover and bending stress at the bottom of the ((Refer Time: 40:15)) of the wide. So, these are the things to be calculated just to have a picture of the configuration which is like this.

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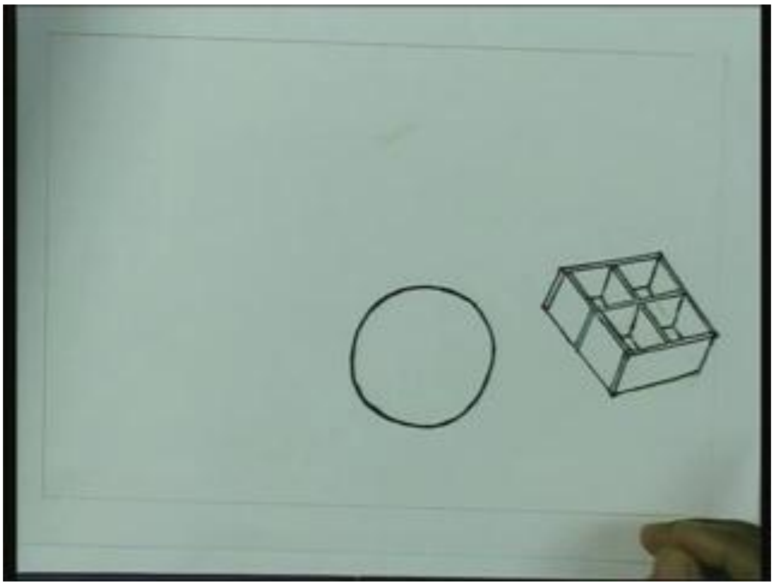

That you have a plate and there are grillage like this. You have stiffener of uniformly thick plate and these are space that a particular pitch distance, similarly you have cross grillage also they are plate also of this much height and this thickness, so this sort of grillage. So, think of it that this plate is placed on top of this grillage and they are welded everywhere. So, that becomes say monolithic structure and this sort of problem can be also solve be considering the formula that you have derived. So, I will give you the procedure to solve it.

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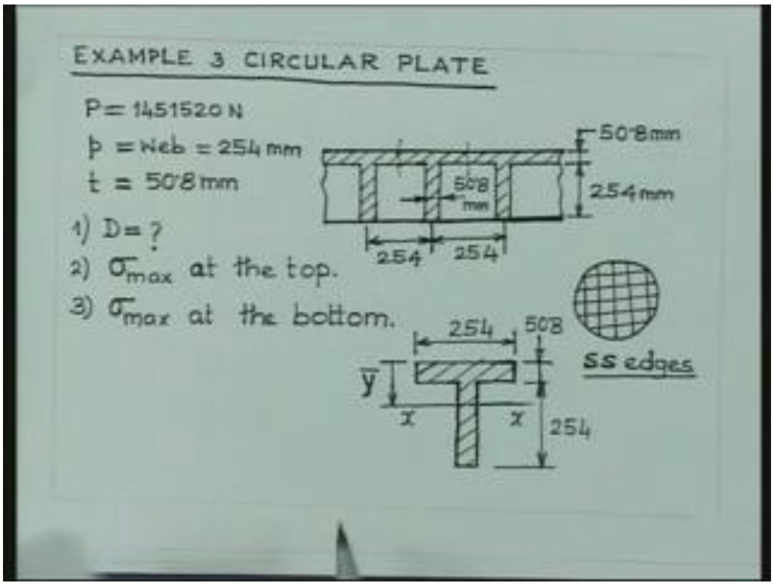

So, if you look into this problem, what you have this that you have a plate of thickness 50.8 millimeter. And then we have grillage of height 250 millimeter and thickness is equal to 50.8 millimeter and this peach is 254 millimeter. So, now if you try to consider a portion of the plate, which is 254 millimeter let us say then it looks like this. The section of the plate is going to look like a T. Now this becomes the dimension of the section, rather than uniform thick plate and we can do the calculation considering I for this section.

Other data are that P is equal to 1451520 Newton that is the load, total load acting on the top And peach is of course, 254 millimeter and thickness of the basic plate is 50.8 millimeter. Now, the problems to be solved that what is the D value of modulus of rigidity, maximum stress at the top, maximum stress at the bottom, so before we do that it is necessary to calculate the I for this section, you treated as this is the section of the plate. And therefore, we can now calculate the location of the CG all the dimensions are known.

We can calculate this CG location of this section and I will like to just the brief here. So, if we calculate the CG distance y bar that is you can calculate by considering the movement of this portion about this base, moment of this portion about this base. And that divided by the total area of these two segments.

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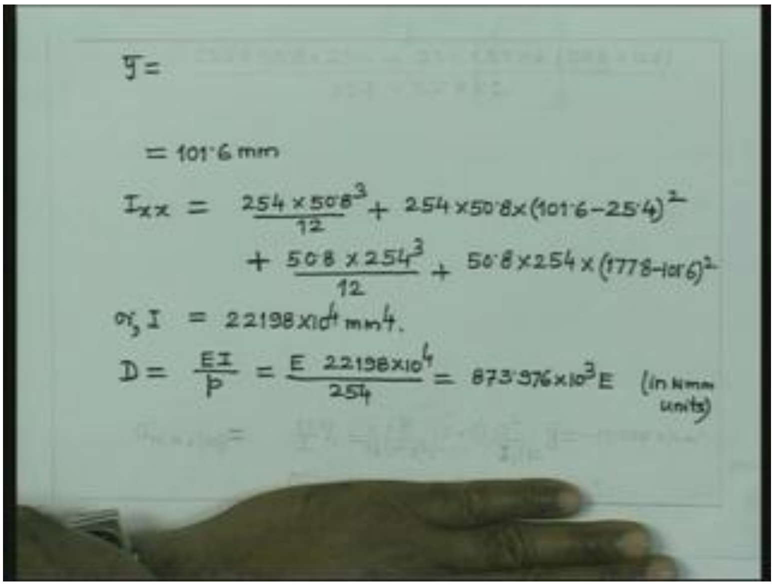

If we do that will give us y bar as 101.6 millimeter. So, therefore, the CG is located at a distance of 101.6 millimeter of the top. Now, if I know that I can now calculate the moment of inertia of this section about Ixx, so if you calculate Ixx. We can now consider this segment, which is of dimension 254 height 50.8. So, therefore, we will have the moment about its own CG parallel to x axis is 254 into 50.8 cube by 12.

So, that is the moment about the x is passing through this point parallel to xx then we apply the parallel axis theorem. So, therefore, it is 254 multiplied by 50.8 multiplied by now the distance, this distance is 101.6 and this is nothing but 50.8 by 2, so therefore, if you now take the difference 101.6 minus 25.4 that is the distance between the two CG. So, therefore, this is the contribution of the parallel axis theorem

Similarly, the ((Refer Time: 44:39)) we can consider now it is half thickness 50.8 its height is 254. So, therefore, b h cube by 12 that gives us the moment of inertia about the

CG of this ways somewhere there. And then if you apply the parallel axis theorem. So, 50.8, 254 is the area and the distance is nothing but the total distance we can consider that if I consider this total distance is going to be 50.8 plus half of this height, which is 127

So, that total gives us 177.8 minus 101.6 square. So, therefore, that gives us the contribution of the parallel axis theorem. So, on simplification we will simply write I rather than Ixx this is nothing but 22198 into 10 to the power of 4 millimeter raise to the power 4. So, that is the moment of inertia. Now, this D is nothing but EI by P, Pth distance. So, it is moment of inertia over a unit length and we are considering a Pth distance of 254.

So, therefore, now we can find out this D. So, therefore, this is E into 22198 into 10 to the power of 4 divided by 254. So, this gives us 873.976 into 10 to the power of 3 E and it is in Newton millimeter units. So, we have to keep everything in Newton millimeter. So, that is the value of D. Now, how do we calculate the stress at the top and bottom. Now, we can apply the formula that stress is nothing but M maximum moment by I into y 1 distance of the fiber.

So, therefore, what it means is that if you are interested in calculating the stress at the top (Refer Time: 47:04) what you need to do is that you find out the maximum moment and that divided by I multiplied by y that will give us that will give us the stress. Now obviously, this I, this I that we have calculated it is for a pitch length of 254 and this I should be calculated per unit length.

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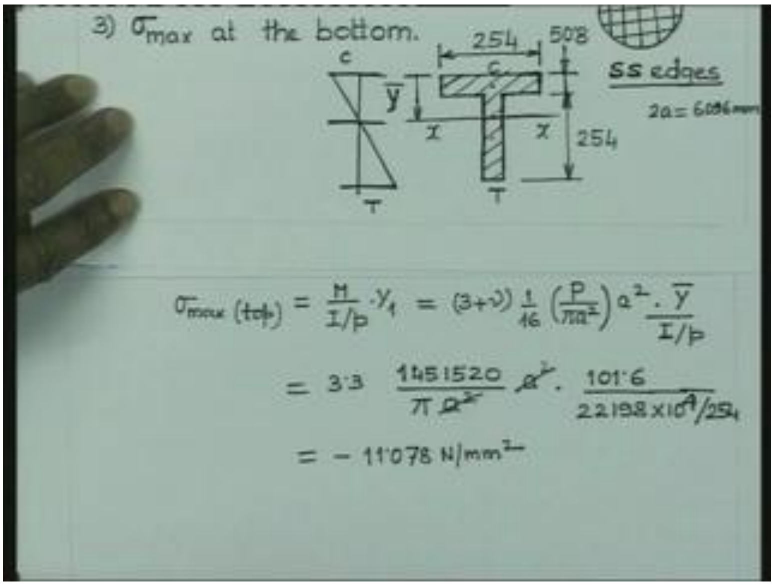

So, therefore, if we can write now this maximum stress for this problem is going to be at the top is nothing but M this I per unit length. So, therefore, I by P into Y 1 because this moment is given per unit length, so therefore, this has got to be also per unit length. Sigma maximum is equal to M by I by P into Y 1. So, if you are calculating the stress at the top fiber you must think of distance Y 1 of the top fiber from the centralized location.

And M is moment per unit length. So, this I is also to be express as moment of inertia per unit length. So, now we can write the value of the moment is nothing but 1 by 16 P a square into 3 plus nu. So, therefore, we have 3 plus nu into 1 by 16 pressure distribution, which is capital P by pi a square. So, that is the small p and that multiplied by a square that is the expression for the moment per unit length.

And now we can write Y 1 is nothing but Y bar and this I by P will I by P. So, if we now substitute the value it is 3.3 and this is this load is equal to 1451520 by pi a square, we can cancel this a square with this a square, so we need not substitute. And this is 101.6 millimeter and this value we have 22198, 10 to the power of 4 and this P is nothing but 254. So, these are the values.

And once you calculate this, this simplifies to minus 11.078 Newton per millimeter square. So, this is the stress at the top fiber, if the stress at the top fiber is this ((Refer Time: 50:28)) there is a linear variation. So, you have linear variation of stress from the top to the bottom, in the case of simple bending.

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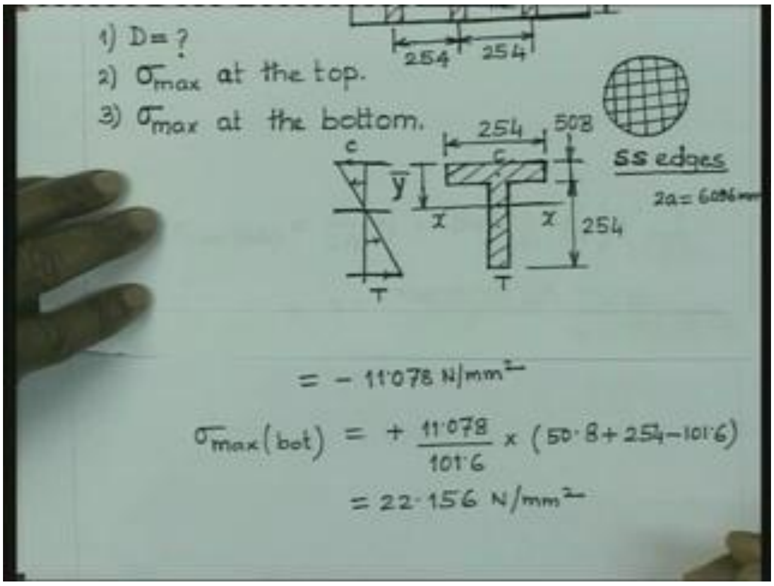

And therefore, you can calculate the stress at the bottom fiber this is nothing but 11.078 divided by 101.6 multiplied by it is 50.8 plus 254 minus 101.6, this gives you value equal to 22.156 Newton per millimeter square. You expect naturally the plate to be subjected to tensile stresses at the bottom under the action of the load at the top (Refer Time: 51:29) and compressive stresses to come up at the top. So, therefore, we have got those stresses. So, you can solve the problem of this type where, the thickness is not uniform by following procedure like this. You can consider problems of solving yourself. So, look into the problem of this type.

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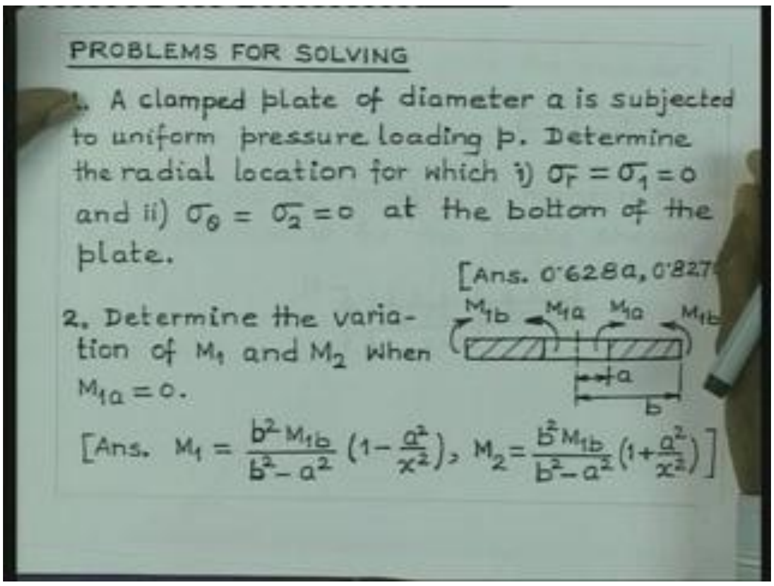

A clamped plate of diameter a is subjected to uniform pressure loading P. Determine the radial location for which, sigma r which is nothing but sigma 1 equal to 0. And sigma theta equal to sigma 2, 0 at the bottom of the plate. So, therefore, determine the location where, sigma 1 is going to be 0 and sigma 2 is going to be 0. Obviously a plate which is subjected to loading at the top, you know that for a clamped plate the stress varies from 0 from positive to negative.

So, there is a 0 somewhere in between. So, those are the locations to determine. So, answer is given here. Similarly, you have a plate here, which is subjected to bending moment M 1 a per unit length at the inner edge M 1 b per unit length at the outer edge. Determine the variation of M 1 and M 2 from R equal to a to E equal to b, under the condition when M 1 a equal to 0. So, it is a plate which is subjected to uniform bending moment at the outer circumference, find out the variation of the bending moment in the radial and circumferential direction.

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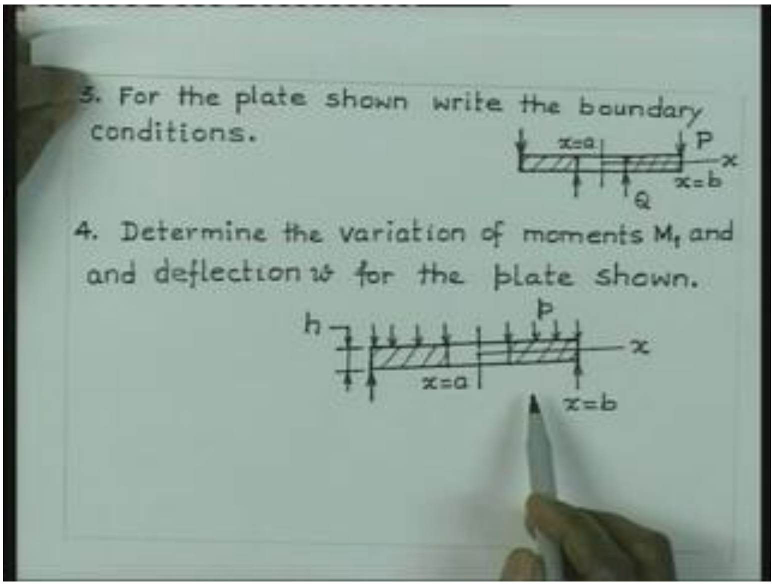

Similarly, you can consider a plate which is loaded like this, the radii of the plate is x equal to a to x equal to b. The loading at the outer edge is P per unit length and loading at the inner edge is Q per unit length. For this plate write the boundary conditions. For a plate like this, where it is supported at the edges and distributed loaded who are the annular area, which is from x is equal to a to x is equal to b. Determine the variation of moments M 1 and deflection W for the plate shown. So, you can follow the steps that you have given you earlier and get the answers to these problems.