Advanced Strength of Materials
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Lecture – 36

We will consider comparison of stresses in simply supported and clamped plates under uniform stresses.

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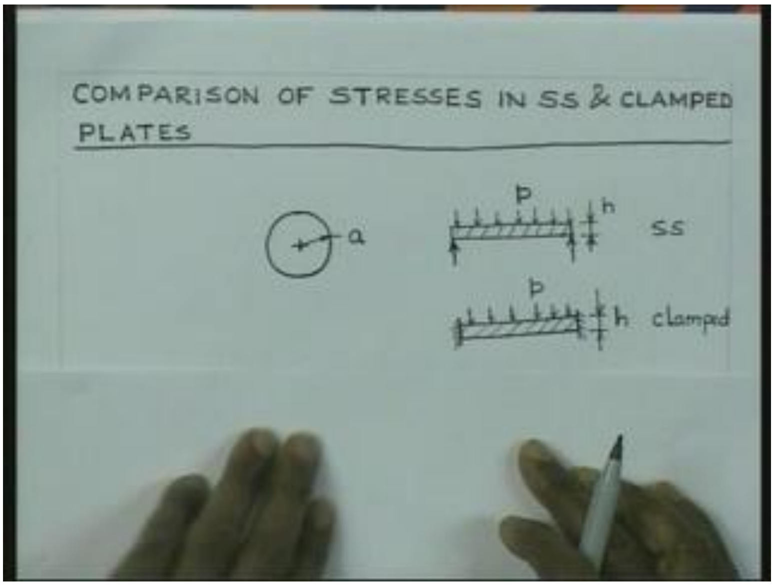

These are the 2 typical plates they are circular in shape and then we have the n conditions like this in the case of simply supported. And we have the n conditions shown here for the case of clamped plate. The radius is a height is p for thickness of the plate is h. So, and pressure is p. So, we would like to now show you the stresses acting in the radial and tangential direction in the 2 cases.

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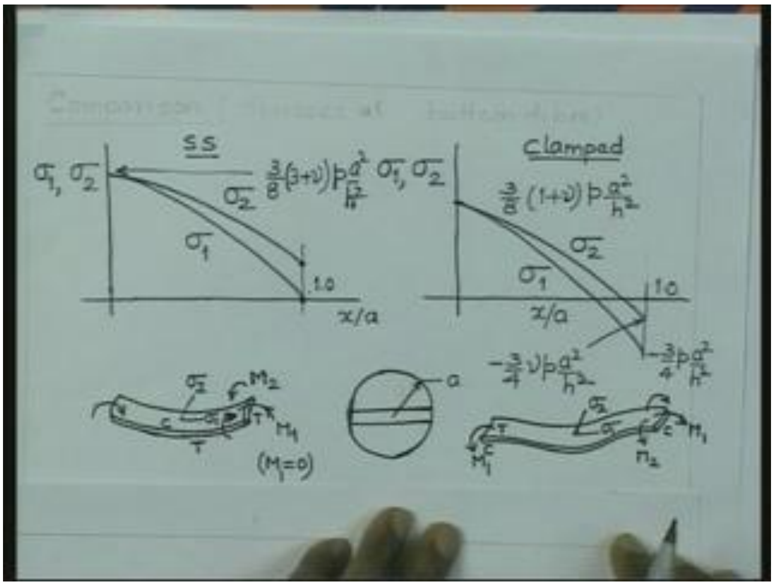

So, we will plot now we will consider the stresses to be plotted in the vertical direction radial direction x is plotted in the horizontal direction. So, we will have sigma 1 sigma 2. And for the simply supported plate the variation is the variation of sigma 2 is the variation of sigma 1. Similarly, for the case of clamped plate, this is the ratio, is this is the extreme outer radius. So, therefore, it is unity here.

Similarly, if you consider this case the variation sigma 2 like this. Sigma 1 is this again this x by a is unity at the outer radius. The magnitude of this stresses if you write for this case the maximum stress which occurs at this center is 3 by 8, 1 plus mu p a square by h square. And the value of stress sigma 1 is compress is 3 by 4 p, a square by h square and this magnitude of sigma 2 is minus 3 by 4 nu time's p a square by h square.

The value of stress here this value will write it there it is 3 by 8, 3 plus nu p a square by h square. At the stress sigma 1 is 0 and sigma 2 will have some value here. Now, note this that the stress at the center in the clamped plate is much less it is if you forget about the value of nu which is less than 1, you have the stress in the case of simply supported plate almost 3 times that of the stress in the case of clamped plate.

So, higher spaces are going to develop at this center of the simply supported plate. And if you remember the deflection the maximum deflection is also going to be high or rather higher in the case of the simply supported plate. Now, let us try to understand why the stresses are going to be in this case the stress here is going to be now positive. On the other hand these stresses are going to be compressive in the case of clamped plate.

So, this can be understood to in from physics of the problem. So, for that lets consider again the plate. Now, we will focus our attention on a portion of the plate a strip which is oriented close to the center. If you consider this to be a simply supported plate this plate this segment is going to deform like this. So, this is the direction of stress sigma 1 and this direction corresponds to sigma 2. Since the plate is bending like this we would expect tensile stresses to develop all along and that is what is here sigma 1 is all along tensile stress.

And since, at the edge of the simply supported beam the bending movement is 0. So therefore, the bending movement in this direction M is 0 in this case. But little inside it will be positive, but it is going to have a direction like this similarly here the direction like this that is the direction of M 1. Now, since the tensile stresses are developing at the bottom edge. Compressive stresses are developing at the top fiber or tensile stresses are developing at the bottom fiber and compressive stresses are developing at the top fiber. Because of the anticlastic curvature this will try to bend like this. But, we are trying to keep the plate all the time tasks to the support.

So, therefore, the support is now going to apply some movement which will be acting like this which will try to undo that curvature. So, therefore, this is what the movement is; which is going to come up at the outer edge on the plate and this is what is going to be the direction of M 2. And in our convention this is positive movement and hence this movement is going to produce now compressive stress, it will produce the tensile stress at the bottom and compressive stress at the top.

So, therefore, you find that tensile stress is going to develop at the bottom fiber and that is what we have found here that, it is positive stress. So, once again because of the anticlastic curvature this is going to bend like this, but it has got to be flat to be intact with the support. Support is going to apply some movement which is going to be I am doing this curvature and that movement has got to be directed like this. And if this is the movement acting here, we must have tensile stresses developing at the bottom fiber and compressive stresses developing at the top fiber.

And we are plotting the stresses here at the bottom fiber of the plate and therefore, sigma 2 is positive as expected. Now, let us try to see the difference in the case of clamp plate which I have already stress in my last lecture. Let us again look into so that, you can see the comparison between the 2. In this case is portion at this center is going to deform like this.

So, here again this from this center we can show our direction sigma 1 and this is direction sigma 2. For this case, the movement that is acting at the edges it is clamped and therefore, there will be non 0 movement at the edges. And this movement is going to be like this. So, this is the direction of movement M 1 and under the action of this movement M 1 we are going to get tensile stresses at the top and compressive stresses at the bottom.

So, therefore, tensile stresses in the direction in the direction sigma 1, we are going to get compressive stress at the bottom. And that is what is your sigma 1 stress is compressive here. Now, if this is the movement acting the steep will try to have anticlastic curvature like this. So, it is going to be curving with the curvature center curvature at the top. And since, this curvature cannot be permitted, because which has got to be in contact with this support this have got to be undone. The support is the apply movement which is going to be now in this direction M 2.

This is negative this is opposite compare to what we have seen here. And therefore, under the action of this movement we are going to get now, compressive stresses at the bottom tensile stress at the top. And therefore, we expect sigma 2 to be compressive and that is what we have got here. So, this is how that the stresses are going to be different in the 2 cases. We can have plates of different geometry and also support conditions and they are all going to be mate in practice. So, you would like to consider some geometries for further elaboration on the procedure.

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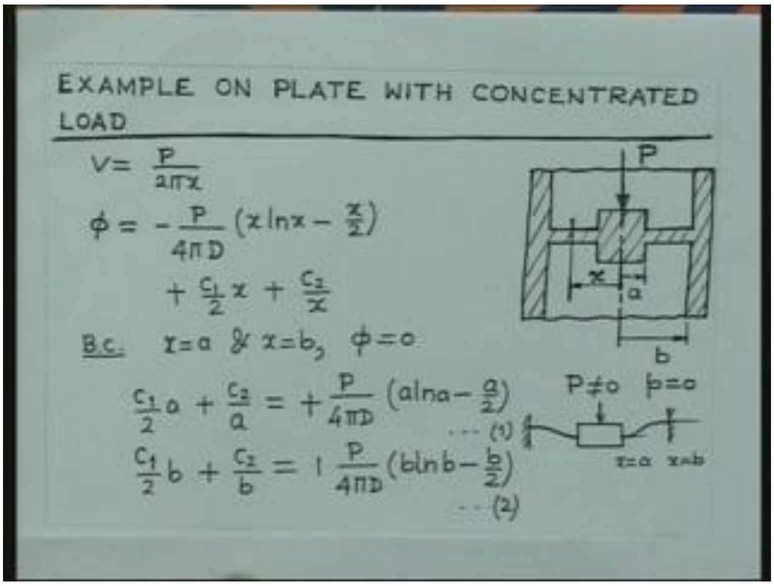

So, let us now consider 1 case wherein the plate is with concentrated load at this center. So, this is the geometry that will focus upon that we have a cylindrical sale and then we have the wall there it can be closer and there is 1 boss. And on the boss and external load P is acting. And in this case, the dimension here is a and the internal radius is b. So, we can consider since this dimension is very large compare to this height of the plate of thickness of the plate. We can consider that the condition here is nothing but a clamped condition.

Similarly, this dimension is so very large compare to the height or thickness of the plate here and therefore, you can consider that this is also this is also an edge with clamped conditions. Now, for this problem therefore, when it is deforming it is going to deform like this schematically, we will have the deformation looking like this. So, this P is now not 0 and small p which is uniformly distributed pressure that is 0.

So, let us try to go on the derivation of deflection and also calculation of for the stresses in this case. As usual, we will consider a radial portion of the plate which is at a distance x will consider that portion. And therefore, you can now consider that area since, we are interested in finding out the shear force per unit length of this circumference on this circle will have 2 pi x into V that must be equal to the total loads. So, therefore, V equal to P by 2 pi x.

Now, we will have phi the relationship that you have derived last time they are will substitute small p equal to 0. And hence, we will get the expression for slope phi which is nothing but P by 4 pi D into x lnx minus x by 2 plus c 1 by 2 x plus c 2 by x. So, we have obtained this phi making use of this solution that we had derived in the last lecture. There we have simply knock out the expression involving small p and we are left with this expression for phi. Now, what are the boundary conditions in this case?

So, let us look into the boundary conditions; obviously, in this case this is x is equal to a and this is x is equal to b. What we have is that slope here is 0 here also it is 0. So, you can write the boundary condition to be at x is equal to a and x is equal to b phi is equal to 0.

Now, if we put value we are getting now from this relationship c 1 by 2 a c 2 by a is equal to P by 4 pi D a lna minus a by 2. So, let us say that is equation number 1. So, we simply substituted x is equal to a and then since phi 0 we have taken this part on 1 side and the other part to the right.

This could be plus similarly, we can write c 1 by 2 b c 2 by b is equal to P by 4 pi D b lnb minus b by 2 and let us consider this as equation number 2. So, we have 2 constants of inclination c 1 and c 2 they can be resolved easily.

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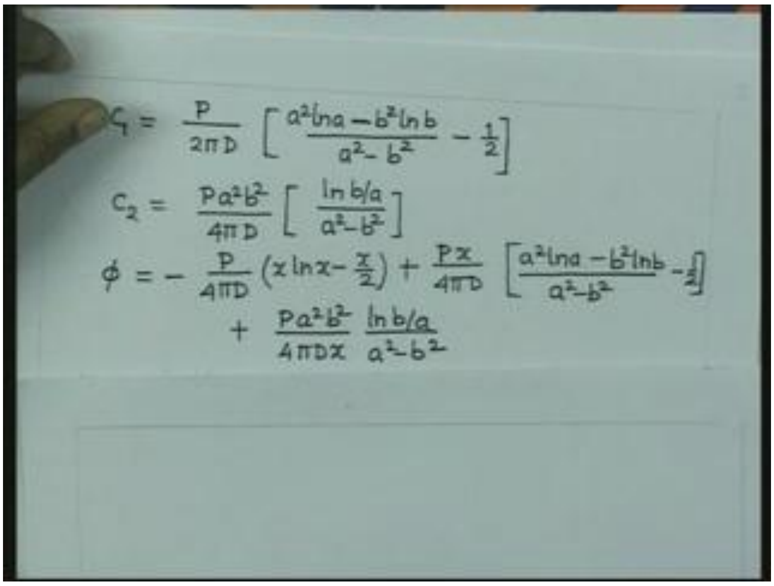

After solving these 2 equations we get the constants as follows: P by 2 pi D into a square lna minus b square lnb divided by a square minus b square minus half that is c 1. Similarly, c 2 is given by P a square b square by 4 pi D into lnb by a divided by a square minus b square.

So, we can substitute the value of c 1 and c 2 back into the expression for phi. Finally, we get phi in the form minus P by 4 pi D x lnx minus x by 2 plus P x 4 pi D into a square lna minus b square lnb divided by a square minus b square minus half. So, that is parts involved in c 1 and c 2 is P a square b square by 4 pi D x ln b by a a square minus b square. So, you have got now phi and now we can calculate the bending movement which is necessary for calculation of stresses.

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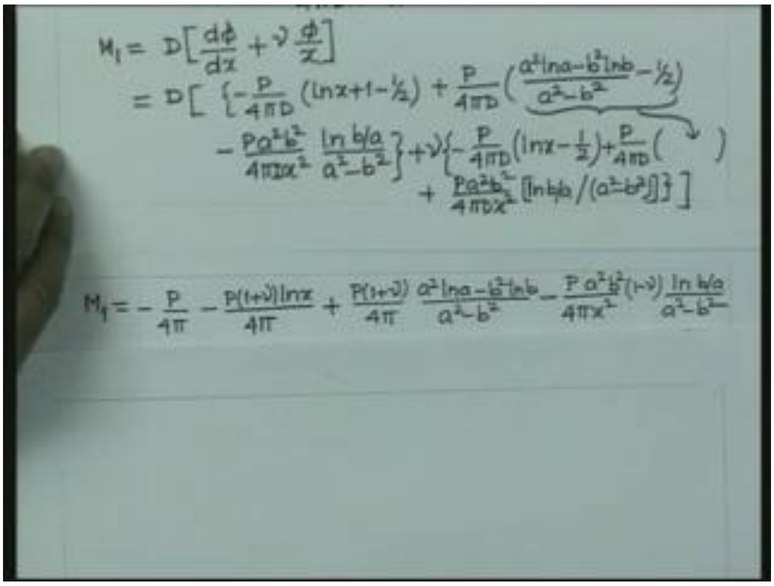

So, it calculate now bending movement bending movement is given by M 1 is given by modules of rigidity D into d phi dx plus nu into phi by x. So, once we do that if we substitute the value we get the following. So, it will give us lnx plus 1 minus half plus P by 4 phi D a square lna minus b square lnb a square minus b square minus half.

Then we have minus P a square b square 4 pi D x square ln b by a by a square minus b square plus nu times pi by x that will give us minus 4 pi D lnx minus half plus P by 4 pi D into. It will be really this expression repeated here so I will not repeat that will come. And then will have another portion P a square b square by 4 pi D x square ln b by a by a square minus b square that this all expressions. So, this is M 1 similarly we can we can really have some simplification for this 1 for this expression and we can write it little in any tough form.

So, we write that it in simplified form from cancellations are cancellations are possible. So, you can write now M 1 is equal to minus P by 4 pi P 1 plus nu lnx by 4 pi. So, that is the term which we get starting from this the term here that 1 we can take out. And then P into 1 plus nu by 4 pi a square lna minus b square lnb by a square minus b square minus P a square b square by 4 pi x square 1 minus nu ln b by a by a square minus b square. So, this is the expression for M 1 similarly M 2 will be given by D into pi by x plus nu times D pi D x.

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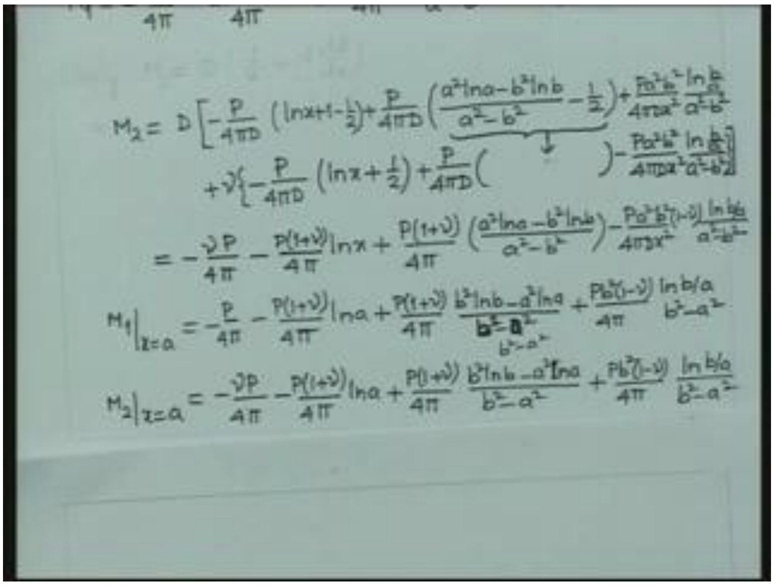

And if you substitute the value of phi and D pi D x then after simplification, you can get M 2 this also looks very similar. So, we have the portion from pi by x and then we have the following for D pi D x nu times D pi D x gives us nu into minus P by 4 pi D lnx plus half plus P by 4 pi D into again we will have this expression coming here. So, you can above write same expression and then minus P a square b square by 4 pi D x square ln b by a minus a square.

So, this is the expression for M 2 and once you simplify this, this looks like the following minus nu P by 4 pi minus P into 1 plus nu by 4 pi ln x plus P 1 plus nu by 4 pi into a square lna minus b square lnb by a square minus b square minus P a square b square by 4 pi D x square into 1 minus nu ln b by a a square minus b square. So, finally, if you calculate the stresses if you want to calculate the stresses at the location R equal to a, you can evaluate the bending movements.

So, substituting x is equal to a in the 2 expression here and in this expression you get the bending movements which are critical. So, we will get now, M 1 at x is equal to a it is P by minus P by 4 pi minus P 1 plus nu by 4 pi lna plus P into 1 plus nu by 4 pi b square lnb minus a square lna minus a square by b square plus P b square 1 minus nu by 4 pi ln b by a it is b square minus a square. Where this minus sign we are just trying to observe. So, therefore, it is so this should be also b square minus a square.

This we have we have change the position of the terms. So, therefore, this should be b square minus a square. So, this is b square minus a square. Similarly, M 2 at x is equal to a is equal to nu P by 4 pi minus P 1 plus nu by 4 pi lna plus P into 1 plus nu 4 pi. And. In fact, we have the same term here.

So, this expression b square lnb minus a square lna by b square minus a square plus P b square into 1 minus nu by 4 pi ln b by a by b square minus a square. So, you have the expression for the bending movement at the location x is equal to a given by these 2 expressions and you can see that the bending movement is higher in that direction 1. So, this is P by 4 pi and this is nu P by 4 pi nu is less than 1, and therefore this M 1 is more significant than M 2.

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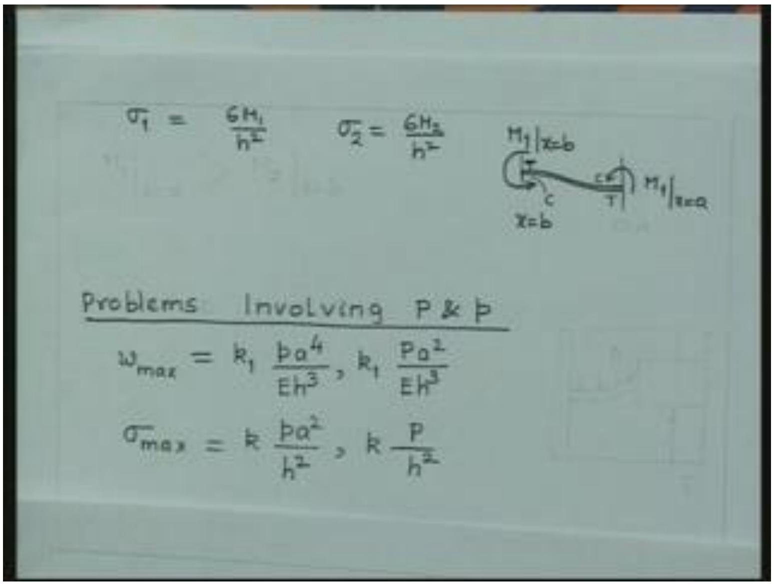

So, therefore, now we can calculate the stresses by using the formula which is nothing but sigma 1 is given by 6 M 1 by h square and sigma 2 will be 6 M 2 by h square. So, look at the deformation of the plate again that we are going to get now, the bending moment acting like this. So, the bending moment is M 1 is acting like this. That x is equal to a it is going to act this way and at x is equal to b it is going to act like this. See the shape of the plate is going to be like this, and therefore this curvature can be generated only when the bending moment is like this.

Similarly, this curvature can be generated only when the bending moment is acting in this direction. So, therefore, this is going to be M 1 at x is equal to b. So, we are expecting stresses to be at this center we are expecting tensile stress to come appear compressive stress at the top fiber. And we are going to have exactly opposite tensile stresses at the top fiber and compressive stresses here at the bottom fiber. So, this gets changed.

You can consider plates of various geometry we will have also problems of this type; you can have a plate which is having a cut out at this center like this it is. It can be supported at the edges it can be clamped at the edges there can be distributed pressure over this portion. Let us say that the x is equal to a to b: there can be distributed loading, there can be concentrated loading acting at the inner edge and many combinations are possible.

So, all the problems can be very easily solved by following the procedure that we have given in the last lecture and in the lecture today. Many problems of this type have been solved finally, what you find is that the stress is going to be related to the distributed pressure P or concentrated load P and it is possible to write this stresses in this form. So, for general problem you can write like this that we have; we are going to have problems involving P or pressure P.

So, we will have problems involving various geometry and concentrated loading of distributed pressure. Now, it is in all the cases 1 thing will become clear we have of course, we have derive this sort of relationship in the last lecture. We will find that the maximum deflection is going to be of this from there is a constant multiplied by P. If it is distributed pressure dimension of the plate a raise to the power 4 divided by E h cube.

So, that the type of form which we will always get and of course, this k is going to observe the constant related to the person stress here Similarly, if it is the case of the concentrated load then we will have another constant and then which is going to look like this P a square by h cube. And the stress, you can see the stress here is going to be propositional to h inversely propositional to h square. And we can write in general form that sigma maximum is equal to some constant k Pa square by h square and it will have the dimension of the pressure.

Similarly, if it is a concentrated loading, then it is going to look like this P by h square. And this P has a dimension per unit length. So, with this set of constant can be these are the expressions for W max and sigma max under the various conditions. We can the people of solved this type of problem and they have given the data in a tabular form. Now, it is not necessary to derive all these formula and calculate the maximum deflection or the maximum stress these data can be directly taken from the tables and calculation can be done.

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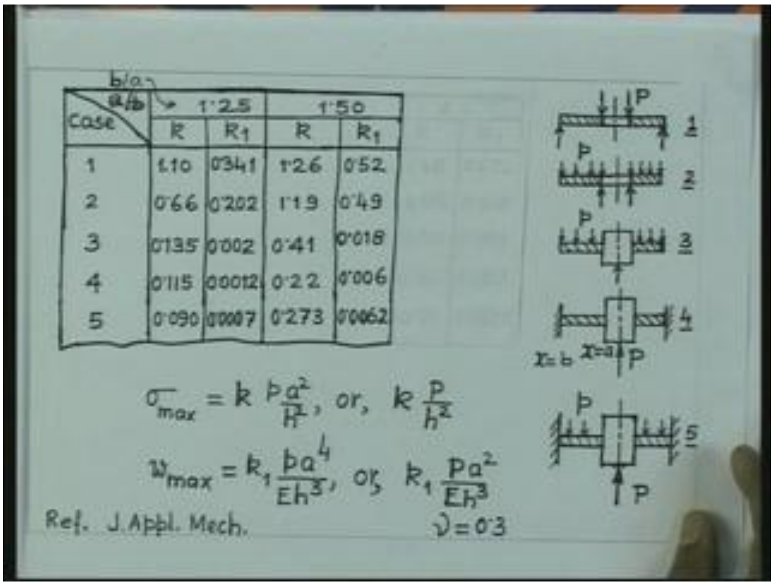

So, we will like to look into those tables. We will consider now the various geometry of the plate and supporting conditions I have shown here 5 cases, you have a plate here which is having a circular cut out at this center. Let’s say this is of radius equal to a and the outer radius is b. So, we will maintain that nomenclature we have P load acting at the inner edge of the at the inner at the edge of the cut out and it is simply supported that the ends.

You can have the situation like this, this supported along this circumference of the cut out and the pressure loading is acting this is the second case. Third case is that you have the support at this center through the boss and the distributed loading is coming up like this. And here we have the plate clamped at the edges and the loading is acting at this center through the boss and the boss outer radius is equal to a.

And here it is very similar to that case, but we have stress at the concentrated force here plus the pressure acting at the top. So, for various ratios of these radii b by a you can calculate the constants and we would like to give you those constants. What I am trying to give you here is that you can calculate sigma maximum for the case of distributed pressure use taking the constant k or k for the concentrated loading.

Similarly, if you want to calculate the deflection maximum then you collect the k 1 from this table corresponding to that geometry that you have. We have given the 5 cases which are shown here and this b by a ratio. So, this should be b by a. So, this is b by a ratio b is larger radius. So, b by a ratio we have given in the row here in 1 case we have 1.25 another 1.50, there can be other values also. Now, for the case 1 what we have when b by a is 1.25 then k is 1.10 k 1 is 1.341. And if b by a is 1.50, then k is 1.26 k 1 is 0.52. Similarly, for the case 2 you have the values of this constant for that 2 radii ratio and similarly for the case 3 4 and 5 the data are given.

These are solved in the middle of the ninetieth twentieth century and these are all available in the journal of applied mechanics and they are also available in some reference books. So, it is not necessary to going for deriving all these relations at this stage you can make you of these constants and calculate the maximum stress and also maximum deflects. So, this gives you the rigorous calculation for the deflection and stresses in plates of certain geometry.

Many times at designer want to like to do simplified calculations and if these calculations are to be done people will try to take simpler approaches just to gates some approximate idea. And therefore, I would like to give you some approximate methods of calculation of stresses in the case of circular plates maybe circular plates sheet cut outs and try to understand how this calculations can be done.

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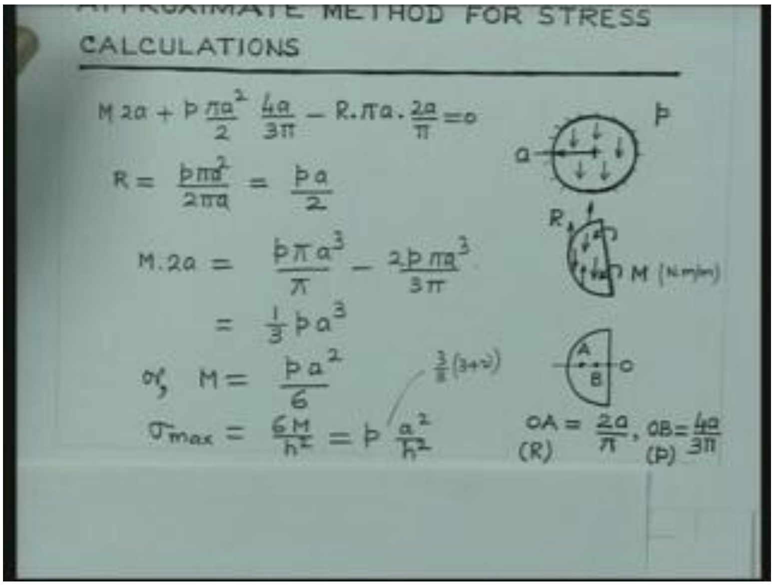

Let us take a case for approximate calculation of stresses in a circular plate which is simply supported at the edges; subjected to uniformly distributed pressure and let us say it is of dimension equal to a. If I consider that reaction at this support is uniform then; obviously, that reaction R which is going to come up all over this are components can be calculated. And now if we take a diametric section then on that section we are going to have bending moment plus shear force stress.

Now, we are more concerned about the bending moment on this section we are going to have bending moment coming up. And this bending moment we can assume that they are uniform they done have any variation in the radial direction. Although, you have seen in the exact calculation there is a variation, but here in we can assume that they are going to remain constant all over the diameter. And now, if we consider that AM is the intensity of this moment distribution rather you can think of it, it is nothing but moment per unit length of this diameter.

So, therefore, it is going to be in units Newton meter per meter. The center of gravity of the distributed pressure over that semi circle is going to lie somewhere here. If we consider that this is the other perpendicular direction then this center of gravity of this pressure is going to lie at a point somewhere at b. And this OB distance it can be easily calculated it is going to be nothing but 4 a by 3 pi, where a is the radius.

Similarly, if you consider that R is the distributed load over the edge of the plate then this distribution could have a center of gravity at A which is going to be at a distance of 2 a by pi. So, now if we take the moment of all the forces about the diameter, that will; help us to calculate the magnitude of this moment. So, we can write the moment equilibrium equation. So, what we have is M into 2 a is the total moment in the anticlockwise direction.

Now, this load P is also going to give anticlockwise moment and that is nothing but P into pi a square by 2 that is the total force acting downward and its moment term is 4 a by 3 pi that the moment you to the distributed pressure. And now, R is going to counteract this moment and this is going to be now R and its acting over a length of pi a semicircle and is at that a distance of 2 a by pi. So, therefore, this is the moment equilibrium.

So, if we consider now this moment equation we now have also some of the values here R is nothing but P into pi a square by 2 pi a total vertically down or load and therefore, this is nothing but P a by two. And once you substitute all these you find that this M into 2 a is nothing but P pi a cube by pi minus 2 times P pi a cube by 3 pi. And on simplification what you find is that this is 1 by 3 P a cube and therefore, or moment M is nothing but P a square by 6. So, that is the intensity of the moment on the diameter everywhere.

Therefore, the maximum stress is going to be given by six M by h square and this is nothing but P a square by h square. If you remember the stress for the simply supported plate of this type that also nothing by, but 3 by 8 into 3 plus nu into P a square by h square. So, we had a multiplier here in the exact formulation it was 3 by 8 into 3 plus nu. So, this 9 by 8 if you forget nu 9 by 8 it was approximately 1. Through approximate calculations you have got that as unity.

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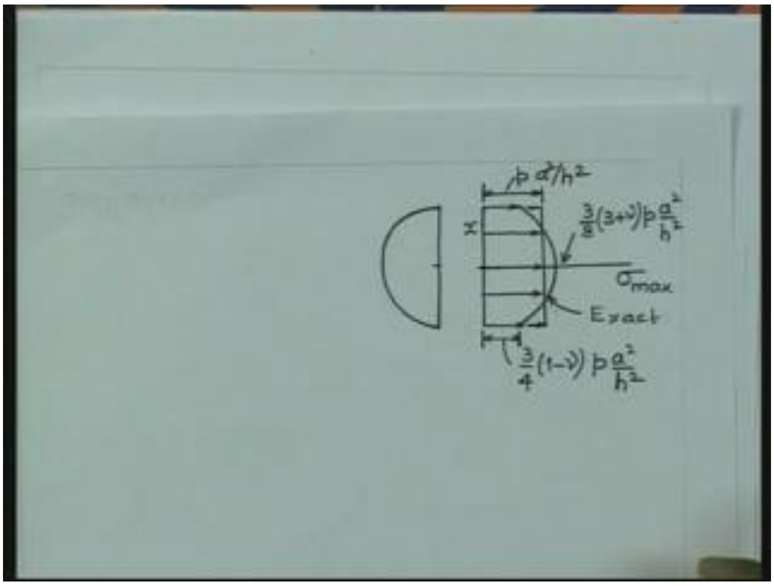

Now, I will show you a completion of the exact phase distribution with this approximate value that, we have calculated for the whole diameter. So, let us have this plate this is the center now let us plot the distance in the vertical direction. So, this is my x direction and this is the direction where I would like to plot sigma max. So, if you now consider the exact phase its tensile everywhere and the variation is symmetric above the center. And the approximate value that we have obtained is nothing but.

So, this is the approximate value that we have got P a square by h square and this value is nothing but 3 by 8 3 plus nu P a square by h square. And this is our exact distribution and this magnitude here which is at the outer edge which is much lower this is nothing but 3 by 4 into 1 minus nu P a square by h square. So, it is really that approximate calculation can do for a rough estimate at this the difference. We will consider some more cases in the next lecture, where we would like to consider plates; where the plate is subjected to concentrated loading. And we would also like to look into the case of rectangular plates subjected to uniformly distributed load and simply supported that the edges.