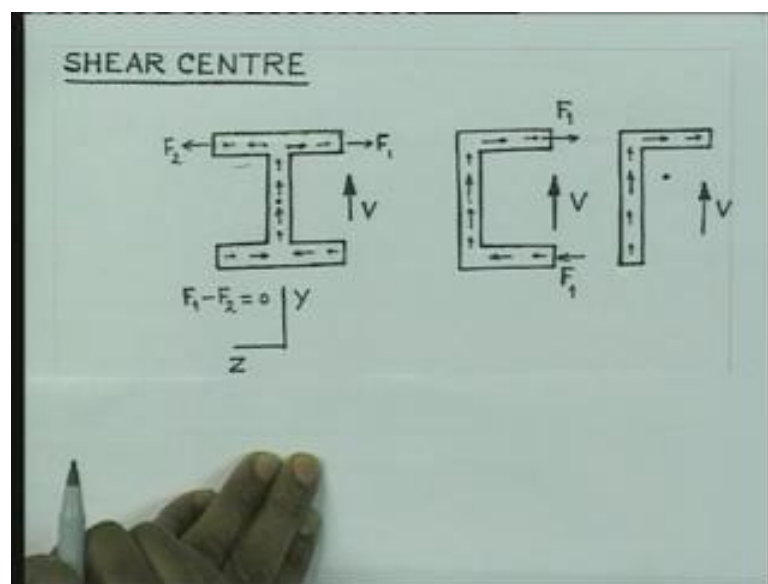


**Advanced Strength of Materials**  
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**Lecture – 33**

We will talk about shear centre of a section. This is very important in connection with bending of beams. Beams can be made up of cross section of various types.

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You can have a cross section of I type, C type, L type. When, you try to consider bending of such beams of such cross section. Suppose, a shear force is acting on the section like this, then you are quite clear. That the shear stresses that acts, due to  $v$  are going to be like this. We will have maximum shear stress near the centre of the web. And then it will gradually reduce in magnitude, as we go away from the centre portion.

Similarly, in the flanges we are going to see maximum shear stress, around the junction of the flange and the web; and it will gradually reduce to 0 there. This stress is also going to reduce to very small magnitude here and there. Similarly, on this side we are going to have shear stresses, located oriented like this and for this bottom plane, it is going to be exactly mirror reflection. So, this is how the shear flows are going to be on the cross section.

Now, if you consider a beam of this type of section, subjected to a shear force like this, then in that case, this is the cg of the cross section. Again you will find that around the cg. We are going to have maximum shear stress and then it is reduced to small magnitude at the junction of the flange and the web. And then in the flanges, we are going to get shear stresses, maximum there and it is reducing to 0 at this point. On the other side it is going to be like this, for the section of this type, if the shear force is acting parallel to this. Then in that case, it is cg is somewhere there, then we will have maximum shear stress like this. And it will reduce in magnitude to 0 at the bottom ((Refer Time: 03:20)). And similarly it is going to reduce from the maximum at the cg to this point. And on the flange we are going to get shear stresses like this.

Now, look at this difference. If you consider this type of section, you see that the magnitude of the shear stress, acting at this point is same as the magnitude here. Magnitude of the shear stress here is same as the magnitude there. So, therefore, if you now sum up the shear stresses over this portion, you will get some resultant force acting in these directions.

Let us say, that if  $F_1$  and similarly, if you sum up the forces for this segment you are going to get some magnitude, which is going to be let us say  $F_2$ . And you will find that for this symmetric section,  $F_1$  is equal to  $F_2$ . And therefore, you do not have any net force acting in the horizontal direction. So, what you really have that, if the shear force is acting parallel to the y axis. In this case you find that  $F_1$  minus  $F_2$  is equal to 0. So, therefore, there is no net force in the horizontal direction, z direction.

On the other hand, if you consider this case really you see that you are going to get some total of this giving rise to a force, which is of magnitude equal to. Let us say  $F_1$  and the same force we are going to get here, which is going to be now oriented in the direction opposite and it is also a ((Refer Time: 04:54)). So, what we find is that, we have a force  $V_x$  coming out as an internal reaction, due to the external force. And that is certainly distributed like this, so this accounts for that.

And now, what we find here that you have now two forces acting on the flanges, which are going to give rise to resultant force 0 in the horizontal direction. But, there is a couple here. There is a going to be a couple acting like this and which will try to rotate the cross section. On the other hand in this case, since  $F_1$  is directed like this,  $F_2$  is directed like

this. Similarly, we will have  $F_2$  directed like this and  $F_1$  directed like this. So, therefore, they are all going to cancel, there is not going to be any rotation.

Same is the picture here, that if I apply a force parallel to the y axis then in that case my stresses are going to be due to this  $V$  is like this, but there is a set of stresses acting on the flange which remains unbalanced. So, there is net force acting in this direction, now and it is also going to give some moment. So, we have to see that this moment is not there in the cross section, otherwise it will give rise to rotation of the cross section.

And we will show that if the plane of the loading passes through a point of the cross section, typical point of the cross section. Then there will not be any rotation of the cross section. So, such a point for the cross section is known as shear centre. So, shear centre is a point typical for a cross section. If the plane of loading passes through that point there is not going to be any rotation of the cross section. So, before I go into that I would like to show you, what is going to happen in practice. So, let us now try to consider some demonstration of these phenomena.

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Now, consider this case, we have a section here, you see it is a c type section. So, I am holding it at this point fixed.

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Now, if I try to put the, this is a section. This is one of the axis of symmetry for a node out.

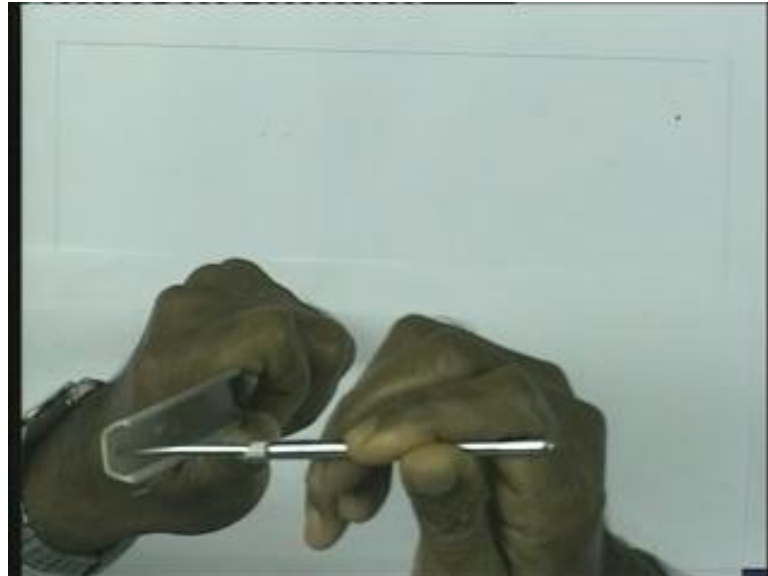
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And this is the other axis, which is not symmetric. So, therefore, it has one axis of symmetry. You will find this type of problem more often coming up for sections, which has one axis of symmetry or no axis of symmetry. In the case of I section you have both the axis of symmetry you do not have this problem. But in a C section like this or L

section. In this case you have only one axis of symmetry and you will have this problem. If the loading is parallel to the axis which is not an axis of symmetry.

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So, let us see, that if I now try put a bending load like this, this is the point. So, I am applying the load, you see that if I apply the bending load like this, on the cross section. There is no rotation of the cross section.

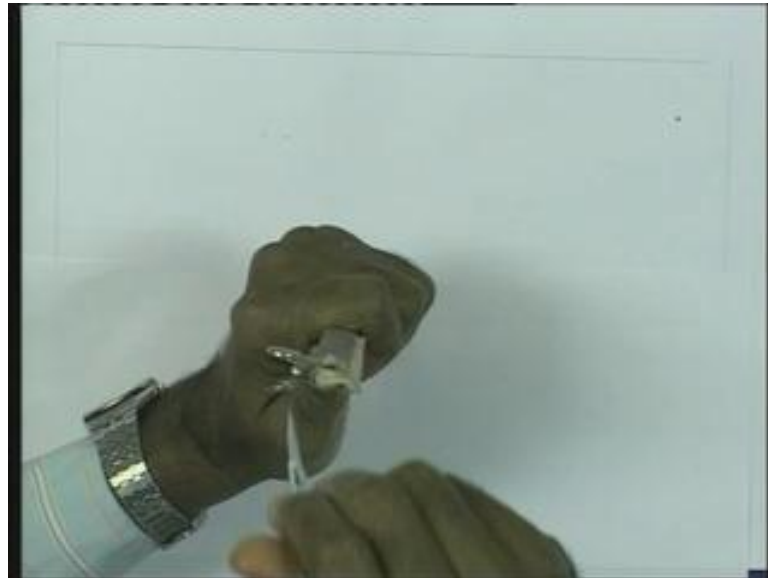
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On the other hand, if I try to bending load, apply the bending load, which is parallel to the other axis which is not an axis of symmetry. I am applying the load parallel to this

section or this segment you see that it is rotating. So, this causes rotation of the cross section. So, this rotation comes up due to bending I do not apply any twist, but then it rotates. I will show you another example, which is going to show, what is shear centre about. So, consider this case, so I have really like this.

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I have a section here, this is again made up of plastic. You see here, this is the section, which is like a c section. I have put some paper inside that is all. So, it does not have any stiffness.

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Now, therefore, this is one of the axis of symmetry and this is the other axis, which is not a symmetric axis. So, now I put a load there.

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If I put a load there on the cross section, you see it is twisting, it is twisting. I put a load there at this point it gets rotated.

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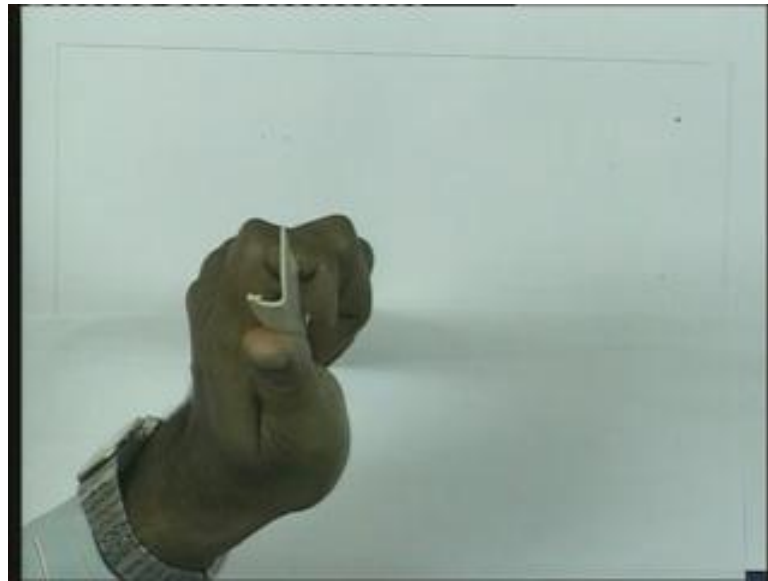


But, if I shift the point of loading parallel to this wave and you see it does not rotate, so my plane of loading, which is this plane. So, that plane of loading is parallel to this side and this plane intersects the axis this axis at a point that is called the shear centre of the

cross section. So, intersection of the loading plane with the bending axis is called the shear centre. So, this is the bending axis by this load and therefore, intersection of the loading plane with the axis of bending is the shear centre of the cross section

So, it lies outside the cross section, you see it is somewhere here. It is at a distance. So, this is for the section, which has one axis of symmetry. Let us look into another example, look at this example.

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Here, this is L a section, so this L section. Now, we do not have any axis of symmetry here. Now, if I put the loading parallel to this edge.



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Let us say I put this load there. You see this section rotates. It rotates.

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And if I shift the point, you see here the rotation has diminished. So, therefore, if you come to this corner, it does not rotate.

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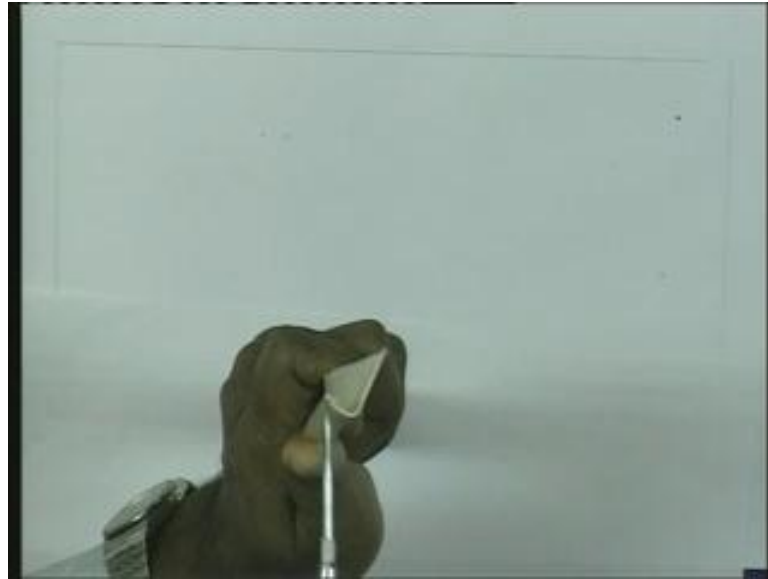
Similarly, if I now put the load parallel to this side, you see if I put the load it has twisted. You see it has twisted, but if I put the load here it does not twist. Let me show it again, so you see it gets twisted.

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But, then if I put the load very close to the corner, you see it is close to the corner parallel to this it does not get rotated. So, this is the phenomena.

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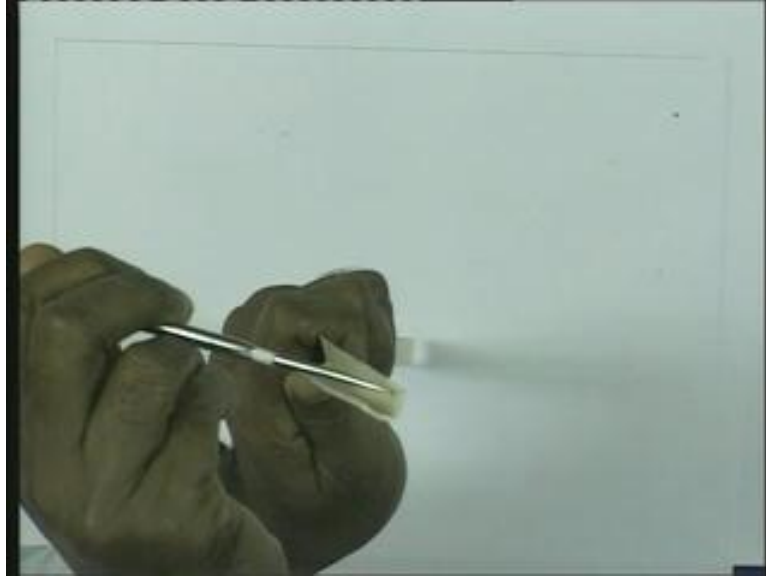
I will show again, you see it gets rotated. But, then the moment I put the load very close to the corner, this one you see or rather let me put you see here, the rotation is not very significant. Like this.

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So, may be you can see this thing better here, may be here it may be better.

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See, I put the load there, it gets rotated, it gets rotated.

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But, then if I put the load here, it does not get rotated.

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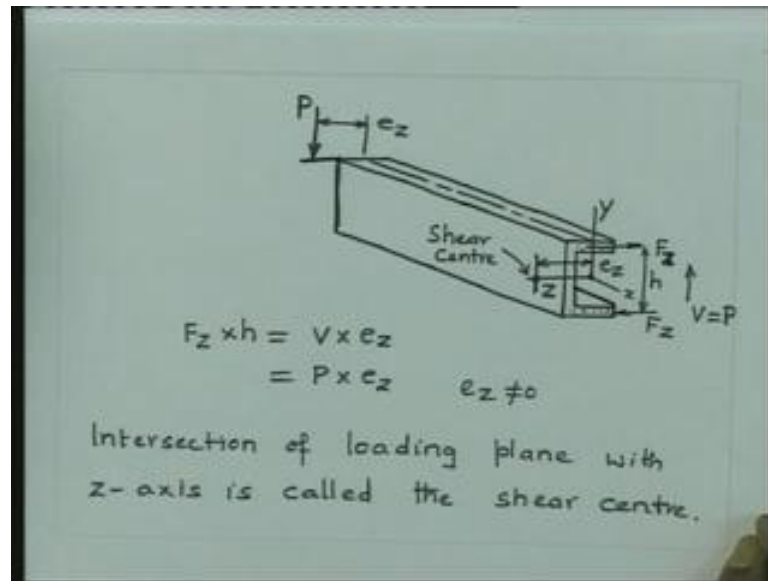
Similarly, if I put the load here, it gets rotated, rotated.

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But, then if I put the load there, it does not get rotated. So, that is the point, which will be related to shear centre. So, let us try to do some calculations on this determination or locating this shear centre. How do you go about it. Let us get back to our C section.

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So, we have a beam with C section here. So, this is the beam we have. Now, we are, what we have shown you is that the axis of the cross section are as follows. This is your  $y$ . This is  $z$  axis. Now, if you put a load parallel to this  $y \times z$  plane. So, parallel to  $y \times z$  plane. Let us say that we put a load here, somewhere there, you just think of it that we have put some member, welded some member and we are putting a load there. Then in that case, this is going to be at a distance.

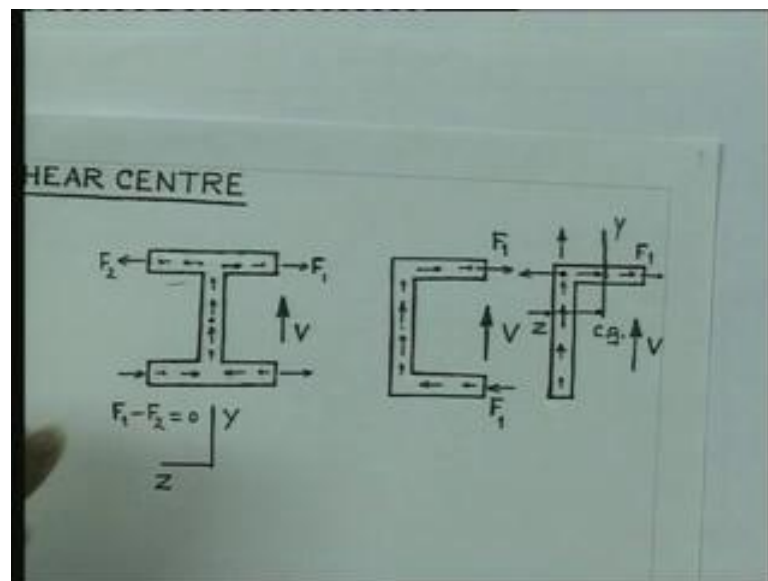
So, this is the plane,  $y \times z$  plane. So, this is  $x$  direction is here. So, this is  $y \times z$  plane and this load is acting at a distance of  $e_z$  from this  $y \times z$  plane. So, if that is the case, you are going to have now shear force acting like this vertically up. So, that is the shear force  $v$  and this shear force  $v$  for this case, it is certainly is equal to  $P$ . Now, because of this shear force, what you get now that there will be a force acting, due to the shear stresses acting on the flange on this flange we have some shear force which is coming up.

Some of the shear stresses let us say  $F_1$  or if you like to write parallel to  $z$  axis, so therefore, it is  $F_z$ . Similarly, the shear stress acting on this flange, it is going to be  $F_z$ . So, now if we consider the moment of this forces  $F_z$  then I am going to get, if the height of the flange is equal to let us say  $h$ . So, from here to here let us say it is  $h$  then  $F_z$  into  $h$  is the moment which is clockwise, which we do. And now if I consider that load is acting at this point then I won't have any moment produced by this force  $P$ .

On the other hand if I shift it here, it will produce a moment about this plane which is going to be  $P$  into  $z$ . So, therefore, that can keep a balance. So, there is an unbalanced moment and to keep it in balance, we must shift the loading point away from this location and therefore, since  $v$  equal to  $P$ . We can write  $v$  into  $e z$  and that is equal to  $P$  into  $e z$ . So, this load must act at a distance  $e z$ , which is not equal to 0. So, this  $e z$  not equal to 0.

Now, if you consider the plane containing the load, which is parallel to a plane,  $x y$  plane that is going to intercept the  $z$  axis at a distance here. So, it is going to intercept here and this distance is again  $e z$ ,  $e z$  and this is known as the shear centre of the cross section. Shear centre, so it is the intersection of loading plane with  $z$  axis is called the shear centre. So, I will write this, intersection of loading plane with  $z$  axis is called the shear centre.

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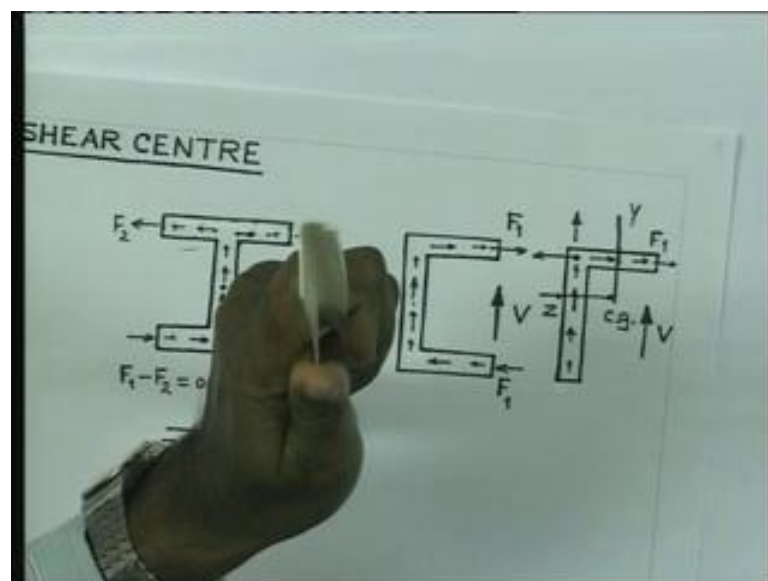
You can imagine that, if this cross section is not having any axis of symmetry. So, if you consider this cross section. So, consider now loading acting parallel to this edge we must have, this is the cg cross, cg of this section then; obviously, I expect a distance of this plane of loading, so as to keep the forces in the movement in balance. Similarly, if I consider the force acting parallel to this leg, then I also expect a distance of the plane of loading from this and therefore, in both the cases, we are going to get some distance either from this cg.

So, these are the axis let us say  $y$   $z$  then when I have a load parallel to this leg, then I will have some location of the plane of loading with respect to  $cg$ , which will intersect with  $z$  axis that will give us the location of shear centre with respect to the that sort of loading. Similarly, if I consider loading in a plane parallel to this, I will have some distance away from the  $cg$ .

So, therefore, those two will finally, you will find that in this case, you are going to have this intersection point. So, this plane and this loading plane they are going to intersect at this point. So, you must have for complete balance, what you need is that this force if it is to be balanced, you must have a loading plane passing through this type of location. And then if you have to keep this force in balance, when I am applying the load then I must have this as the plane of the loading.

So, I repeat again, when I am trying to bending by applying load on this or load parallel to this leg, then I must keep the balance of this force. So, therefore, it should pass through this line, loading plane must pass through this line. When I am applying the bending load parallel to this leg, then I must balance this force. So, therefore, the loading plane must pass through this line and therefore, the intersection here will give the shear centre for the cross section. So, here you will find for angle cross section, shear centre is always located at the corner.

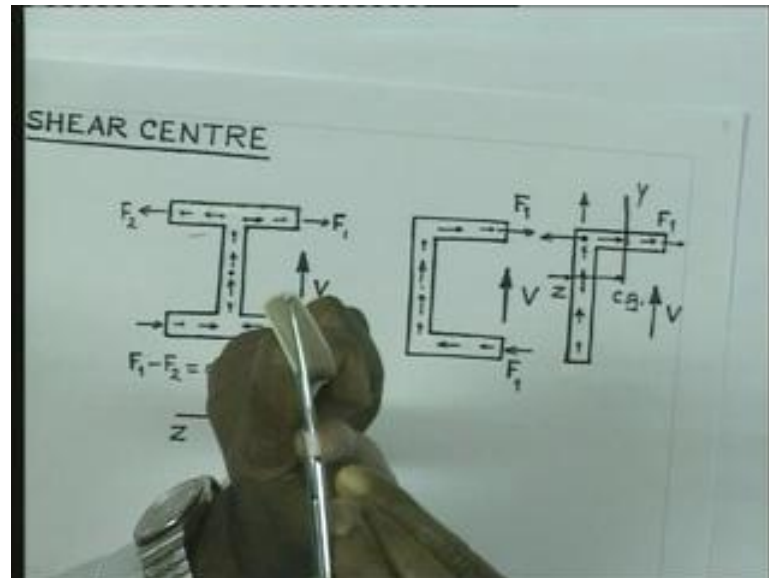
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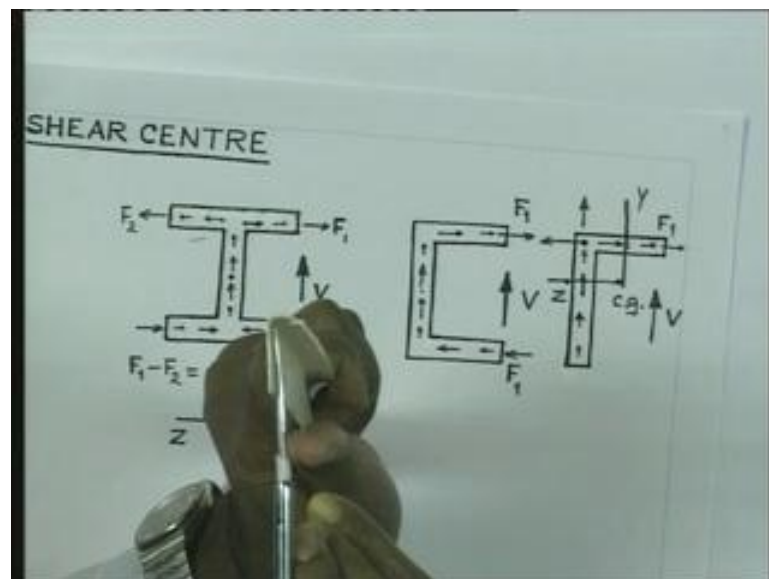
And that is what we had shown here that for this particular case; once more let us look into this that here. If you see that this is the cross section, which is not exactly right angle unfortunately.

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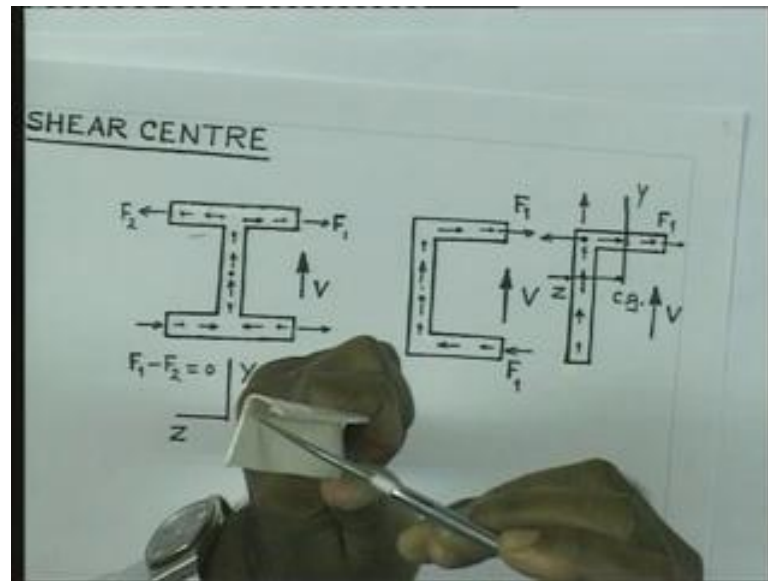
Let us consider this one. So, this one you see here, if I apply the load it does not get twisted.

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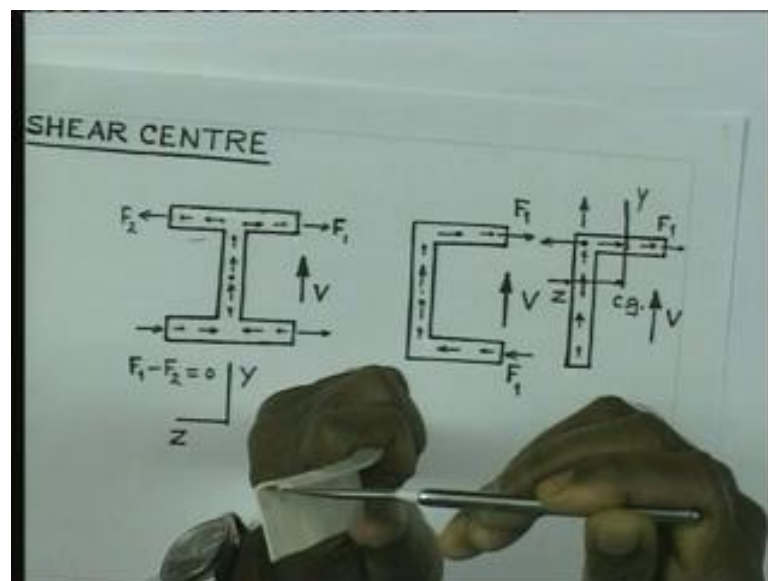
But, then if I apply the load it does not get twisted to that extent. So, let me see, you see that I am applying the load it gets twisted. But if I apply it does not get twisted.

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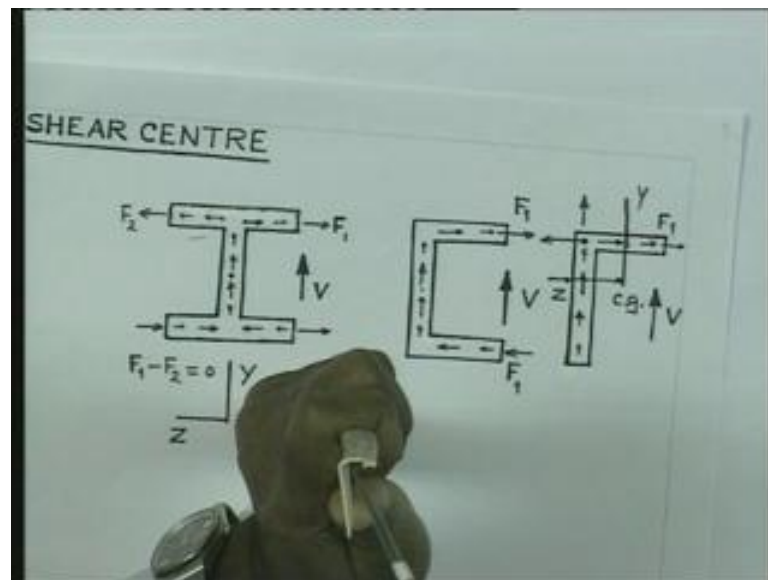
Similarly, if I consider the load applied like this it gets twisted.

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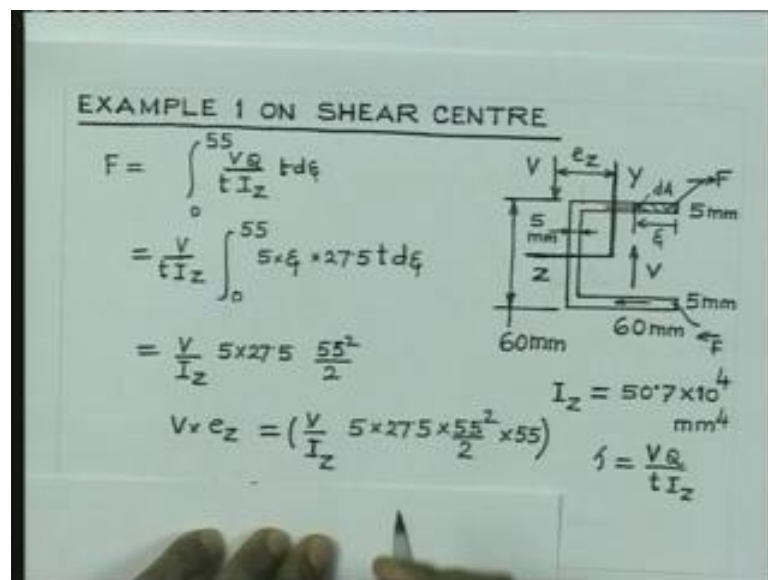
But, then if I apply the load at the junction, it does not get twisted.

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So, therefore, this corner is the shear centre of this cross section. So, now we will solve some problems to have further understanding of this one.

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So, let us consider example 1 on shear centre. We have a cross section given here. The flanges are of dimension 60 millimeter. Web is also of dimension 60 millimeter and the wall thickness is 5 millimeter everywhere, and so therefore this is 5 millimeter and so also this is also 5 millimeter. Now,  $I_z$  for this cross section has already been calculated

and it is  $50.7 \times 10^4$  millimeter raise to the power 4. So, what is needed here is that we have to calculate, this shears the position of the shear centre.

So, loading plane is parallel to y axis. So, therefore,  $v$  is acting like this and therefore, this shear stresses should be oriented in the flanges like this. Now, let us first of all find out, the shear force acting on this arm and also on this arm. Obviously the magnitude of the shear force here and here will be equal. So, if you want to calculate the shear force acting on this arm.

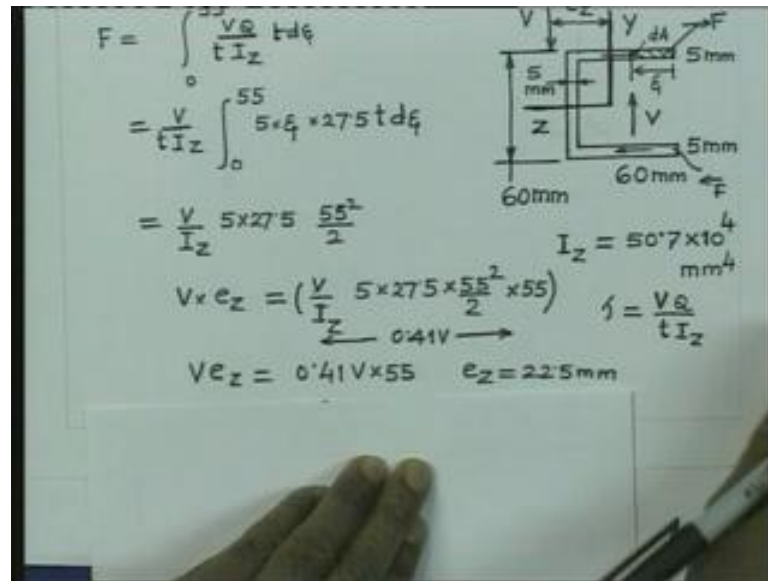
Let us consider a point at a distance of  $z_i$  ((Refer Time: 22:31)), and therefore you remember that the shear force, shear stress at this point is going to be given by if you consider  $\tau$ , this  $\tau$  is going to be  $v$  is the shear force multiplied by the moment, half moment of area, half moment of this area about the neutral axis. So, therefore, let us represent that by  $Q$ . Half moment of this hatched area,  $VQ$  by thickness into  $I z$ . So, therefore, that is the formula, this you have derived in the first course.

Now, if I want to integrate, I have to integrate let us consider this junction to be at a distance of 55 millimeter from this. So, therefore, the total force is going to be nothing but  $\int_0^{55} v Q \cdot t \cdot I z \cdot dA$ , so if you consider this  $Q$  is of course, this area. And now if I consider an elemental in the area  $dA$ . So, this is  $t \cdot d z_i$ . So, that is equal to  $v$  by  $I z$   $\int_0^{55}$  and it is of course  $t$  you can keep there,  $5 \cdot d z_i$ . So, this area, this is  $5 \cdot d z_i$  and its height is nothing but  $30 \text{ minus } 27.5$ ,  $27.5 \cdot d z_i$ .

Keep this  $5 \cdot d z_i$  is the area, so therefore, we will have this also there,  $t$  is there. So, therefore, finally this  $t$  cancels, and finally what we are going to get. This as  $v$  by  $I z$   $5 \cdot 27.5$  and integration of this thing will give us  $55$  square by  $2$  and therefore, that is the force. Now, if we consider that this shear centre is to be located somewhere at a distance of  $e z$  from the y axis.

Let us say this is  $e z$  then we can write that moment of, so this if this is  $v$ , this is also  $v$ . So,  $v$  into  $e z$  that will counter balance the moment due to the force acting on these two arms and therefore, this is nothing but we can write  $F$ . So, this  $v$  by  $I z$  into  $5 \cdot 27.5$   $55$  square by  $2$  and this height is nothing but  $55$ . So, therefore, this is nothing but balanced. So, the force  $F$  is acting on this arm like this,  $F$  and on this arm it is like this. So, therefore, they produce a moment, which is counter acting with this moment.

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Handwritten calculations for the shear center of an L-section:

$$F = \int_0^{55} \frac{VQ}{tI_z} t d\xi$$

$$= \frac{V}{tI_z} \int_0^{55} 5 \cdot \xi \times 27.5 t d\xi$$

$$= \frac{V}{I_z} 5 \times 27.5 \frac{55^2}{2}$$

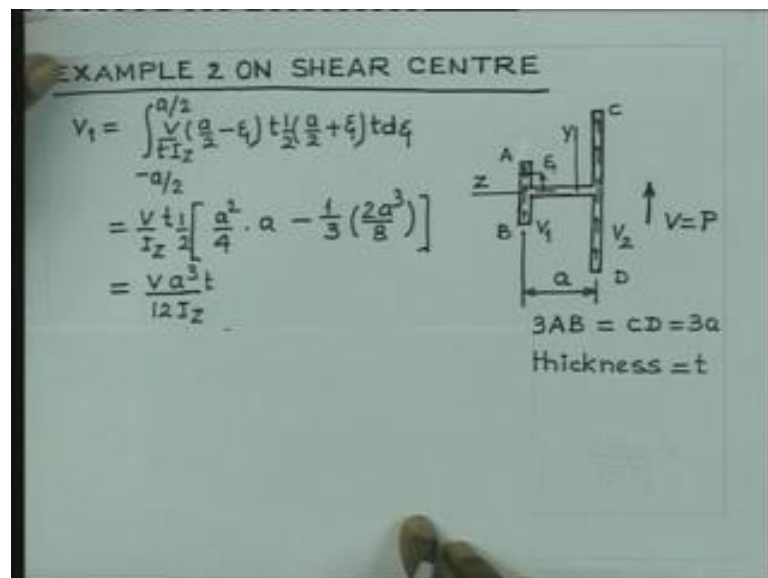
$$V \times e_z = \left( \frac{V}{I_z} 5 \times 27.5 \times \frac{55^2}{2} \times 55 \right) \quad \therefore e_z = \frac{VQ}{tI_z}$$

$$Ve_z = 0.41V \times 55 \quad e_z = 22.5 \text{ mm}$$

Diagram of an L-section with dimensions: vertical leg 60mm, horizontal leg 60mm, thickness 5mm. The centroid is marked with Y and Z axes. The shear center is at a distance  $e_z$  from the Y-axis. The moment of inertia  $I_z = 50.7 \times 10^4 \text{ mm}^4$  is noted.

And therefore, we can now simplify, It gives us  $V$  into  $e_z$  is equal to  $0.41v$  multiplied by 55. So, therefore, this is nothing but  $0.41v$  and hence  $e_z$  is equal to 22.5 millimeter. So, the plane of loading cannot be wide plane, it must be at a distance of 22.5 millimeter from the  $y$  axis then only the twisting of the cross section can be prevented. So, this is one example, to show how we can go about calculating the position of the shear centre of your section, which has one axis of symmetry.

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Handwritten calculations for the shear center of a Z-section (Example 2 on Shear Centre):

$$V_1 = \int_{-a/2}^{a/2} \frac{V}{tI_z} \left( \frac{a}{2} - \xi \right) t \left( \frac{a}{2} + \xi \right) t d\xi$$

$$= \frac{Vt}{I_z} \left[ \frac{a^2}{4} \cdot a - \frac{1}{3} \left( \frac{2a^3}{8} \right) \right]$$

$$= \frac{Va^3t}{12I_z}$$

Diagram of a Z-section with dimensions: flange width  $a$ , web height  $a$ , thickness  $t$ . The centroid is marked with Y and Z axes. The shear center is at a distance  $e_z$  from the Y-axis. The shear force  $V=P$  is applied. The dimensions are given as  $3AB = CD = 3a$  and thickness  $= t$ .

Let us consider another example. So, this cross section is shown here. It is like  $t$  and the orientation is like this, that you have one flange oriented vertically and this is another flange, but then this is the wave, which is a line with the  $z$  axis. And this  $AB$  and  $CD$  they are related this is 3 times  $AB$ . So, therefore,  $3 AB CD$  is equal to  $A$  and this is equal to  $3 A$  and this is equal to  $A$  and the distance of the two legs  $AB$  and  $CD$  is equal to  $a$

Let us consider that this particular cross section is subjected to shear force like this. So, if it is subjected to shear force like this, then we are going to have shear stresses coming up. So, it will be like a parabolic distribution. So, we will have maximum magnitude of the shear stress coming up at the centre and it will reduce to 0 at the top edge. Similarly, it will reduce to give at the bottom edge.

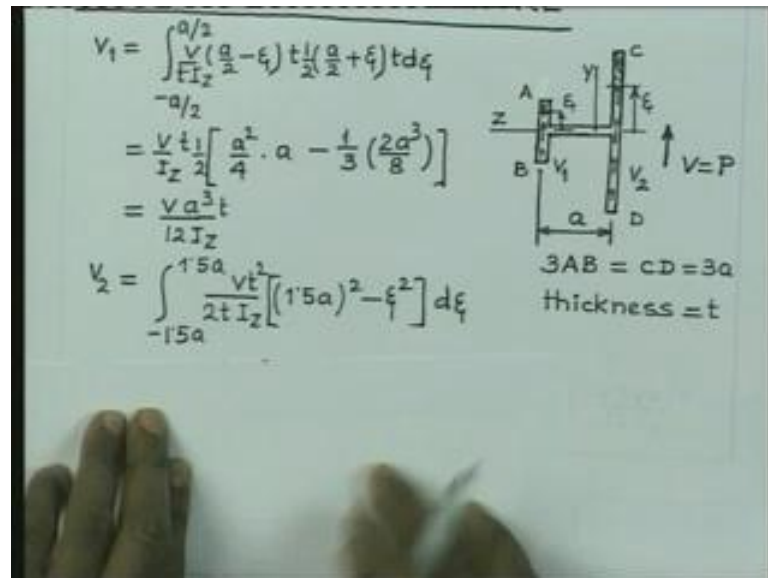
And on this side, we are going to have again this also is going to be gradually reducing to 0 at the top edge, starting from maximum of the centre, same in the picture and we go to the bottom edge. So, let us consider that some total of these forces is equal to  $v_1$  and some total of these forces is equal to  $v_2$ . Now, in this case, it is obvious that we have this  $y$  axis is  $a$ , this is not an axis of symmetry, but  $z$  axis is an axis of symmetry. So, the plane of the loading must be offset with respect to  $y$  axis, so as to remove the twisting of the cross section under bending load.

So, in order to do that let us, now consider the force  $v_1$  coming upon this arm  $AB$ . So, we consider a section or fibre at a distance of let us say  $z_i$  or  $z_i$  let's say from the neutral fibre. And now we would like to consider, the shear stresses at this level, which will be given by  $v Q$  by  $t I z$  that is the shear stress. And therefore, we can write now  $v_1$  to be integration of this shear stress, which is nothing but  $v$  by  $t I z$  and  $q$  is nothing but  $a$  by 2 minus  $z_i$  into  $t$ , that is the area and its cg is at a distance of half  $a$  by 2 plus  $z_i$ , that is the cg distance and then we can consider that integration is equal to  $d z_i$ .

So, that is the area and  $v q$  into  $d a$ . So, we will also have  $1 t$  coming here. So, finally, what we find here is that we will have, the integration limits are minus  $a$  by 2 to plus  $a$  by 2. So, please note that this is the shear  $v$  by  $t I z$ , this is, this is nothing but  $q$  is from here to this expression and this is  $d a$ , so now if I just substitute here, so  $1 t$  will cancel. So, we have  $v I z$ . It is half and this is a form a square by 4 minus  $z_i$  square, it is easily integrable.

So, we will have a square by 4 into a minus integration of  $z_i$  square is one third  $2a$  cube by 8. So, that is the integration and it gives us value equal to  $v$  by  $12a$  cube  $t$  into  $I_z$ . So, that is the shear force acting on the leg A B. Similarly,  $v_2$  can be calculated, so you can take again a fibre at a distance of  $z_i$  and take the  $q$  corresponding to this portion and hence you can calculate the sum total of the forces which is  $v_2$ .

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The image shows a handwritten derivation of shear forces  $V_1$  and  $V_2$  in an I-beam, along with a cross-sectional diagram.

**Derivation for  $V_1$ :**

$$V_1 = \int_{-a/2}^{a/2} \frac{V}{I_z} \left( \frac{a}{2} - \xi \right) t \left( \frac{a}{2} + \xi \right) t d\xi$$

$$= \frac{V t}{I_z} \left[ \frac{a^2}{4} \cdot a - \frac{1}{3} \left( \frac{2a^3}{8} \right) \right]$$

$$= \frac{V a^3 t}{12 I_z}$$

**Derivation for  $V_2$ :**

$$V_2 = \int_{-1.5a}^{1.5a} \frac{V t^2}{2 I_z} \left[ (1.5a)^2 - \xi^2 \right] d\xi$$

**Diagram:** A cross-section of an I-beam is shown. The top flange has width  $3a$  and thickness  $t$ . The web has thickness  $t$  and height  $3a$ . The total height is  $3a$ . The top flange is labeled A (top-left), B (bottom-left), C (top-right), and D (bottom-right). The web is labeled B (bottom-left) and D (bottom-right). The shear force  $V = P$  is applied upwards at the center. The coordinate  $\xi$  is measured from the center of the web. The distance from the center of the web to the edge of the flange is  $a$ . The shear force  $V_1$  acts on the flange, and  $V_2$  acts on the web.

$V_2$  is equal to minus  $1.5a$  to  $1.5a$  and this is  $v t^2$  square  $2 t I_z$  and it is  $1.5a$  square minus  $z_i$  square  $d z_i$ . So, it is simply we try to replicate that form and then once you calculate this.

(Refer Slide Time: 33:01)

The image shows a whiteboard with handwritten mathematical work. At the top, there is a formula for shear stress distribution: 
$$= \frac{Vt}{2I_z} \left[ 2 \cdot 25a^2 \cdot 3a - \frac{1}{3} 2(1.5a)^3 \right] = \frac{27Va^3t}{12I_z}$$
 Below this, there is a diagram of a cross-section with two vertical arms. The left arm has an upward shear force  $V_1$  and the right arm has a downward shear force  $V_2$ . The distance from the left arm to the central axis is labeled  $z_1$ . To the left of the diagram, the following equations are written: 
$$V_1 + V_2 = P = V$$
 
$$V_2 a = V \times z_1$$
 Below the diagram, the derivation for  $z_1$  is shown: 
$$\therefore z_1 = \frac{27Va^3t}{12I_z} \cdot \frac{a}{V} = \frac{27a^4t}{12I_z}, I_z = \frac{2}{3}a^4t$$
 
$$= \frac{27}{28}a.$$

It comes out to be  $\frac{Vt}{2I_z} [2 \cdot 25a^2 \cdot 3a - \frac{1}{3} 2(1.5a)^3]$ , which on simplification becomes  $\frac{27Va^3t}{12I_z}$ . So, we have seen the shear force acting on the two particle arms.  $V_1$  on the left and  $V_2$  on the right and obviously, the external force is going to be balanced by sum of this  $V_1$  and  $V_2$ . So, we can now write that schematically, we have  $V_1$ . This is the cross section schematically and we have  $V_1$  acting here and  $V_2$  acting there.

And as I said that our external force is acting vertically and therefore or it is acting on a vertical plane and so this sum total of this two will be passing through somewhere here. And we must see that the external loading is also passing through the same point or it must act on a plane, which must pass through the same point. So, if the external loading is acting on a plane. Let us say somewhere like this, this is the external load. Then in that case, let us consider that it is acting at a distance of  $z_1$  from the left arm.

Then, we must see that the moment is balanced. First of all we have  $V_1$  plus  $V_2$  is equal to  $P$  which is nothing but total shear force. And now we must see the balance, moment balance also. So, therefore, we can now write, if I take the moment about this point we are going to have  $V_2$  into  $a$ , this distance is  $a$  is equal to  $V$  into  $z_1$ . Now, we have already got  $V_2$  values,  $V_2$  we have already obtained as  $\frac{27Va^3t}{12I_z}$ , so therefore, this  $z_1$ .



$Z_1$  is equal to  $27 \times a \times t$  by  $12 I_z$  into  $a$  by  $t$  and this gives us  $27 a t$  by  $12 I_z$  and  $I_z$  for this cross section is approximately equal to  $7 \times 3 a \times t$ , a cube into  $t$ . And therefore,  $Z_1$  comes out as  $27$  by  $28 a$ . So, it means that the plane of loading must through a point, which is closer to the longer arm, so that the moment balance will be there.

So, we have now go two cases and we have seen that the location of the shear centre, we did nothing other than the orientation of the external load. And we have been able to find out the location of the shear centre. So, the shear centre is a property of the cross section, given the orientation of the loading, it is a property of the cross section. That is why always we try to find out the location of the shear centre without any consideration to the loading point. We simply consider what is the orientation of the loading and try to calculate it. Now, I will try to consider one more example and I will show you that the calculation can be done in a little simpler fashion. So, let us consider this example.

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**EXAMPLE 3 ON SHEAR CENTRE**

Determine the location of shear centre.

$$I_{max1} = \frac{V \times 50 \times 10 \times 50}{10 \times I_z}$$

$$I_{av1} = 1250 \frac{V}{I_z}$$

$$I_{max2} = \frac{V \times 20 \times 10 \times 20}{10 \times I_z}$$

$$I_{av2} = 200 \frac{V}{I_z}$$

$$V_1 = 50 \times 10 \times 1250 \frac{V}{I_z} \quad V_2 = 200 \times 200 \frac{V}{I_z}$$

$I_z = 170 \times 10^4 \text{ mm}^4$

So, it is a section which is like C, but then we have some flanges also on the other side. It is not exactly C type section and wall thickness everywhere is 10 millimeter and this height is 100 and 10 and this is 60 and this flange length is equal to 20 millimeter. The cg of the cross section is somewhere here and this  $z$  axis is an axis of symmetry,  $y$  is really the axis which is not a symmetric axis.

And therefore, loading in plane is parallel to  $y$  will give rise to or will have to be offset with respect to the cg ((Refer Time: 38:58)) to eliminate the twisting of the section. So, we would be interested in trying to find out the location of the shear centre, when the loading is on plane parallel to  $y$ . So, we have the shear force acting like this. Now, for this section, you must first of all find out the shear stress direction, which is critical. It may not be that obvious.

So, if shear source is acting like this then in that case, the shear stresses are going to be it is going to have maximum value there. And it is going to reduce at the away from the centre point and we expect the flange stresses to be maximum at this junction and it will reduce as you move away from it. So, therefore, this is how it will become 0 there and exactly similar picture will exist here, it is 0 there.

Now, about this flange it is again the shear stress is going to have maximum value here and it will be 0 there. And on this flange it is going to have maximum value at this point and then it will be 0 there So, here you have a shear flow that is why, shear flow is very important. The force  $v$  is acting therefore, the shear flow must be like this and for this also it has got to follow that it should be in this direction, that flow analogy is very, very important.

So, we have the total flow coming here, it is getting branched out into this and here in this flow is coming in this flow is coming, this is also coming, this gets added up to this flow. So, that shear flow analogy is very useful in these applications. Now, the sum total of the shear force on this flange is going to be let us say  $v_1$  and since it is symmetric I would also have sum total of the force coming up on this is  $v_1$ ; obviously, this will be directed to the left and this will be directed to the right.

And on the other hand, let us consider the sum total of the force that comes up from this flange is equal to  $v_2$ . It is directed to the left and the sum total of the force it will come up from here. It is again going to be  $v_2$ , but it is going to be directed to the right. Now, the distribution of the shear stress is going to be linear, I think you remember that the shear stress is going to vary linearly on the flange like this. And same is the picture here also it is going to be linear and on the web, it is going to be parabolic.

So, let us now find out what is the maximum shear stress. So, if you consider, this particular point here which is at a distance of 50 from this edge. So, the maximum shear

stress this one, which we indicate this thing by  $\tau_{\max 1}$ . Then this  $\tau_{\max 1}$  is equal to  $v \cdot q$  is nothing but first moment of this area about the  $z$  axis, that area is nothing but  $50$  into  $10$  and its distance is nothing but it is  $45$  plus  $5$  is again  $50$ . So, this is the this is nothing but  $q$  and divided by thickness of the wall by  $I_z$

So, this gives us the value, some value and the average value is going to be half of it. So, therefore, let us say this is the average value of the shear stress on this flange. And let us say that this is  $\tau_{\text{average}}$  is equal to, it is half of this  $\tau_{\text{maximum}}$  and it comes out to be  $1250 v$  by  $I_z$  because it is  $2500 V$  by  $I_z$  half of that is  $1250 I_z$ . Now, let us consider this one, this arm here it is again it is linear. So, I will draw it here, this variation is again linear it is like this.

This is let us say  $\tau_{\max 2}$ . So,  $\tau_{\max 2}$  is equal to it is nothing but  $v$  multiplied by  $q$  of this divided by  $t$  into  $I_z$ . So, therefore,  $v$  area first moment is  $20$  into  $10$  and it is at a distance  $20$ , wall thickness is  $10$   $I_z$  and therefore, this is  $4000$ , this is  $400v$  by  $I_z$  and therefore, the average value  $\tau_{\text{av } 2}$ , that is the average here, that average is equal to  $200 v$  by  $I_z$ . So, we can now easily calculate, the total shear force  $v_1$  which is nothing but average shear force multiplied by the total area which is  $50$  into  $50$  is this distance and into  $10$ .

So, therefore, we can now write that the total force  $v_1$  is nothing but  $50$  into  $10$  multiplied by the average shear stress, which is  $v$  by  $1250 v$  by  $I_z$ . Similarly,  $v_2$  the total of the force here,  $v_2$  is nothing but  $200$  because this  $20$  into  $10$  is  $200$  is the area multiplied by  $200 v$  by  $I_z$ . So, that gives us the two shear forces. Now, we have to see that the moment produced by  $v_1$  and  $v_2$  are balanced by having an offset location of the loading plane, so what we have to do now.

This  $v_1$  acting on this flange on this flange they are producing a moment, which is clockwise and this  $v_2$ , it is going to produce a moment which is anti-clockwise. So, net we are going to have a clockwise moment and if we have to balance that our loading plane must be somewhere this side, with respect to  $cg$  and therefore, we have to calculate that distance  $e_z$ .

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Determine the location of shear centre.

$$I_{max1} = \frac{V \cdot 50 \times 10 \times 50}{10 \times I_z}$$

$$I_{av1} = 1250 \frac{V}{I_z}$$

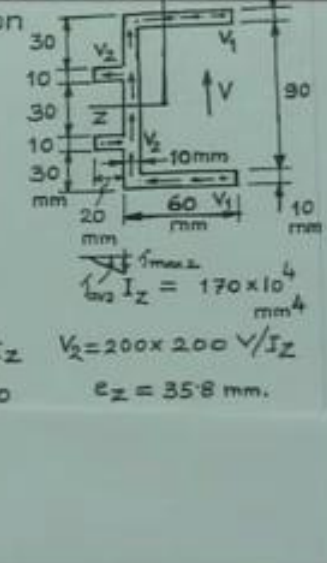
$$I_{max2} = \frac{V \cdot 20 \times 10 \times 20}{10 \times I_z}$$

$$I_{av2} = 200 \frac{V}{I_z}$$

$$V_1 = 50 \times 10 \times 1250 \frac{V}{I_z}$$

$$V_2 = 200 \times 200 \frac{V}{I_z}$$

$$\therefore V \cdot 100 = V_2 \times 40$$

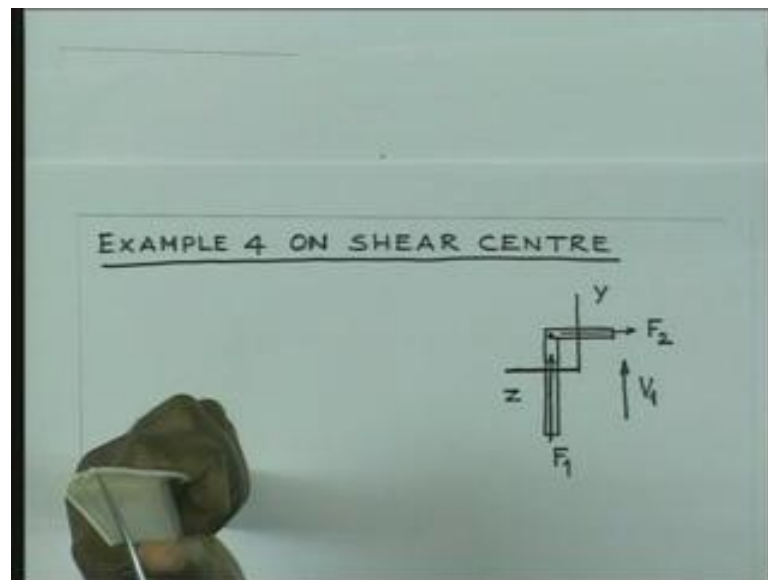
$$e_z = 35.8 \text{ mm.}$$


The diagram shows a C-channel cross-section with the following dimensions: flange width 50 mm, web width 20 mm, flange thickness 10 mm, and web thickness 10 mm. The total height is 90 mm. A vertical shear force V is applied. Shear flow V<sub>1</sub> acts in the flanges and V<sub>2</sub> acts in the web. The shear center is located at a distance e<sub>z</sub> from the center of the web. The moment of inertia I<sub>z</sub> is given as 170 × 10<sup>4</sup> mm<sup>4</sup>.

So, now I can write the balance equation  $v$  total shear force multiplied by  $e_z$  is equal to  $v_1$ . And this distance from this to this middle is nothing but 100, so that is the moment. And  $v_2$  is the other force and here the distance is it is again from here to here is 40. So, therefore, that is the equation to calculate the distance  $e_z$  of the loading plane and if you substitute this values of  $v_1$  and  $v_2$  here. On simplification you get  $e_z$  equal to 35 point 8 millimeter.

So, this is how you can find out in a little simpler fashion, the shear centre. Now, once again I want to reinforce the point, that the shear centre is a very special location of the cross section. And that point must be noted, while trying to consider the loading in bending of the beam. Now, how do you go about finding out the shear centre, in the case of section, angle section, that is something I want you to reflect upon, so how do you find out.

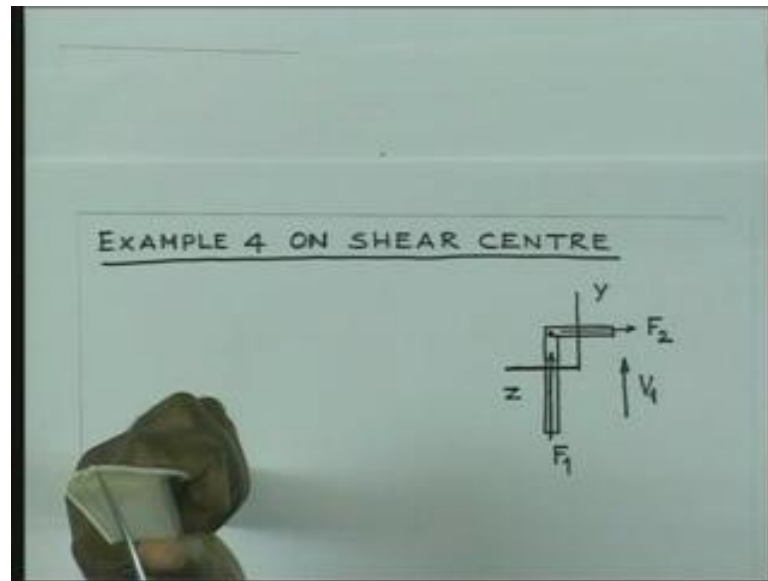
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Say let us consider, this example, example 4. Now, we are trying to talk about section of this type. Let us say that this is, these are the two axis here, which are both are unsymmetric for the cross section. Now, if I am trying to apply a load, let us say apply a load parallel to  $v_1$  then in that case, I have to see that the forces which are going to come up here, it must be in balance. So, therefore, what we try to do, herein actually you see that it is going to give rise to force here, on this leg and force here on this leg.

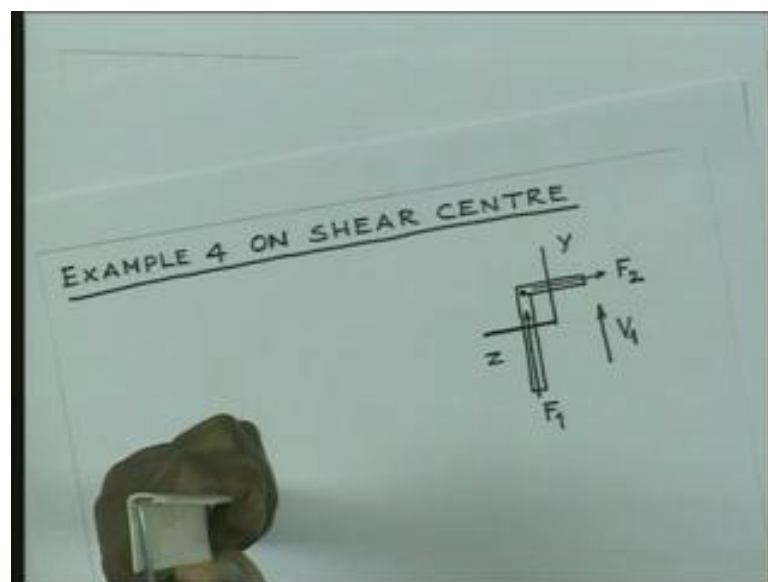
So, therefore, let us say that the shear force here is  $F_1$  and  $F_2$ . So, this  $F_1$  and  $F_2$  would have an intersection point there. And if this force passes through this intersection point then you will avoid twisting of the cross section. So, let us look at this demonstration again.

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So, what I said that this is my cross section, this is my cross section. So, I have this is this and the other arm is this one. So, if I am trying to apply a load parallel to this it is twisted.

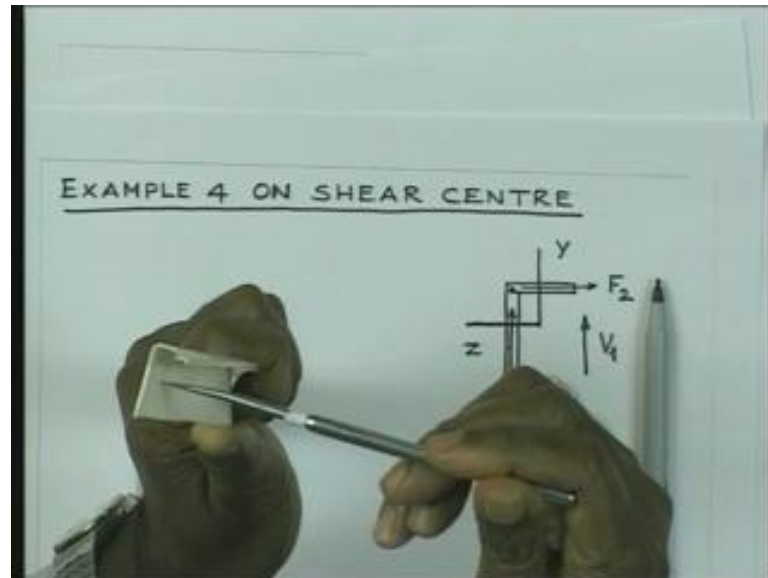
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But, if I make to pass it through this point it does not get twisted. So, therefore, we must actually see that this load passes through the point of intersection here. Similarly, if you are now trying to consider. Lets the loading plane is like this, which is parallel to the other arm. So, then in that case again I am going to get forces like this  $F_2$  and  $F_1$ . Of

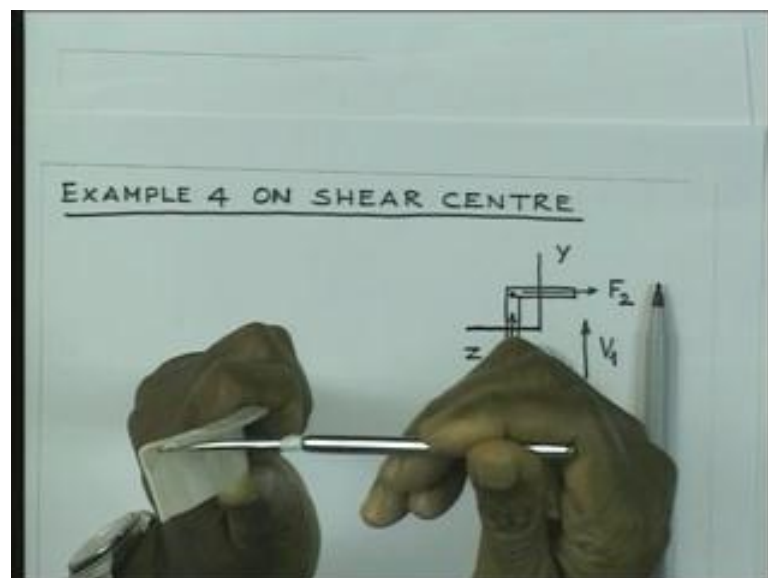
course the magnitudes will be different and if I have to balance the moment, I must see that this line of action passes through again through the common point. So, let us look at this. So, what I am trying to say now I am applying force parallel to this.

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So, therefore, I am applying force parallel to this, you see it gets twisted.

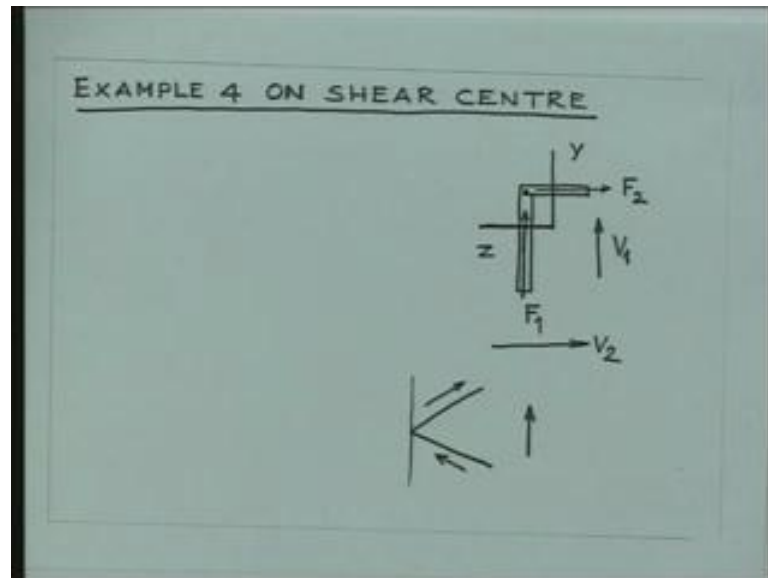
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But, then if I apply the force which is close to the intersection point it does not get twisted it gets twisted. But, if I apply the force, so that one it does not get twisted, so

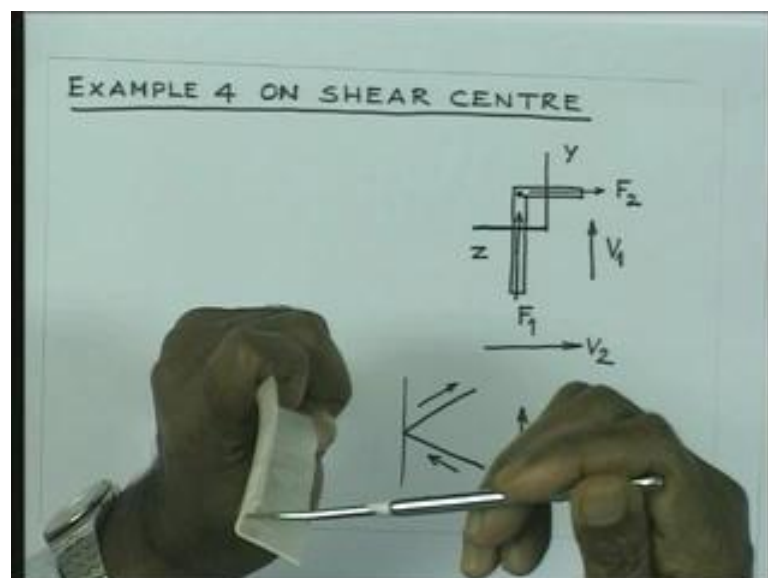
therefore, for such L sections, always you will find that the intersection point is the shear centre.

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Even if you have a section like this, if you have a section like this, here again if you have a vertical force acting like this. The shear force stress is going to act this way and then their intersection point is here and you will find that, the shear centre is going to be passing through this point. So, this I think is you can easily establish. So, this is the section, now we are trying to keep it at an angle.

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So, I am keeping it at right angle and therefore, now if I put the load passing through this point it does not get twisted. Its orientation remains that is it. So, you can consider solving examples by considering problems, which are given in L ((Refer Time: 53:06)) at one strength of material.