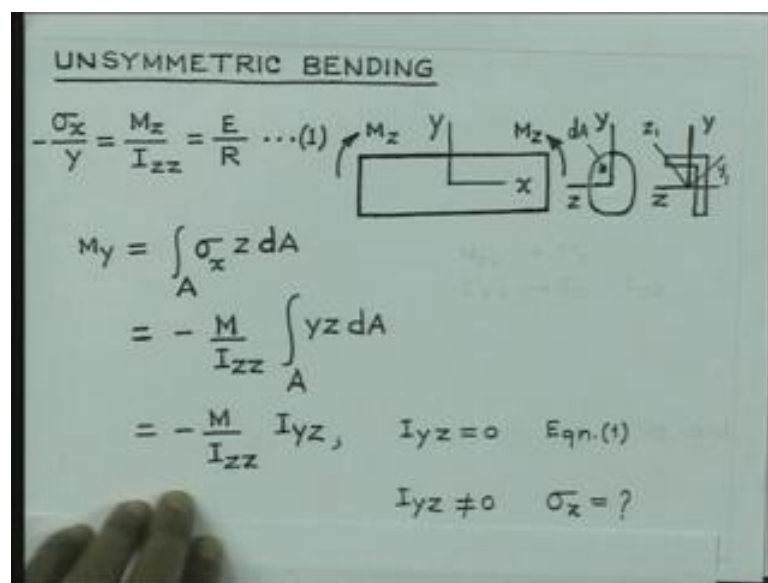


**Advanced Strength of Materials**  
**Prof. S. K. Maiti**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Bombay**

**Lecture – 32**

Today we will talk about un symmetric bending. We have already studied bending of beams with symmetric cross sections and also when the loading was in the plane of symmetry.

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So, let us consider a situation like this; that you have a beam of uniform cross section. The cross section is like this and this y axis is an axis of symmetry. The beam was bent by applying moment in z like this. Then we have already derived the stress formula, which gives you the stress distribution in the cross section which is like this; minus sigma x by y is equal to Mz by Izz equal to E by R. Where sigma x is the stress acting at any point on the cross section in the x direction. Y is the distance of the point from the z axis. Mz is the movement; Izz is the moment of inertia of this cross section above the z axis. E is the modulus of elasticity of the material; R is the radius of curvature.

Now, let us try to consider the moment of these stresses which are acting on this cross section, about the y axis which is the axis of symmetry. If we consider the moment this is let us say that this is an area dA. So, on this area if x axis is out of the board, then we have this stress sigma x acting there. So, therefore, the force is sigma x into dA and the

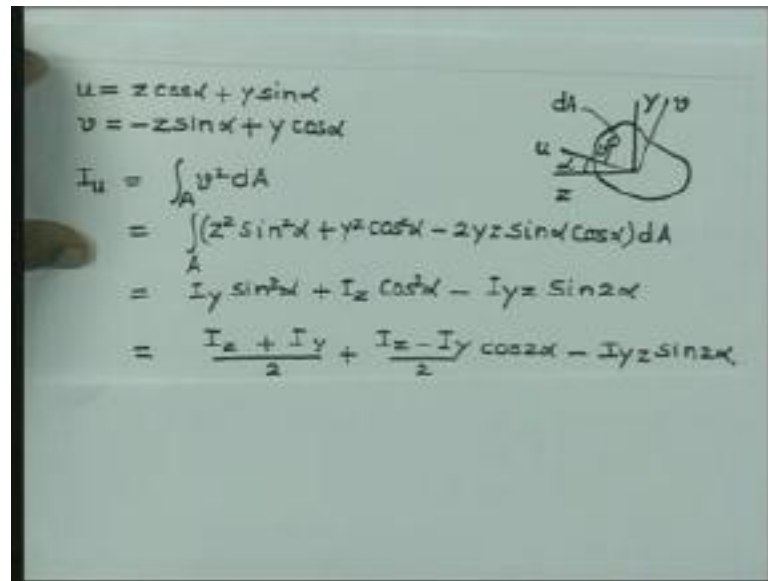
moment about the y axis will be  $z$  into  $\sigma_x$  into  $dA$ . So, if we integrate that this as the moment and if you do this integration we get the value  $M$  by  $I_{zz}$   $yz$  integration over the whole area. And you can recognize that this particular integration is nothing but the product moment of inertia.

So, we can write this thing as minus  $M$  by  $I_{zz}$  into  $I_{yz}$ . We assumed, in the derivation of this stress strain formula that y axis is an axis of symmetry there by meaning that; this product moment of inertia  $I_{yz}$  is equal to 0. So, that was the underlying assumption in arriving at the equation number 1. Suppose you have a situation  $I_{yz}$  is not equal to 0. Then in that case  $\sigma_x$  cannot be determined by this formula and we have to find out a different route to arrive at this stress distribution on a cross section. This can happen, when you have a cross section like this. Think of a cross section, where in we have the y axis, y axis and z axis located like this. And this is the centroid of the cross section and in this case, if you try to load the beam in the xy plane, then you will find that you are not going to have  $I_{yz}$  equal to 0. And hence we will have difficulty in calculating the stress distribution in the cross section and it will be improper to calculate the stress by this formula alone.

So, that is the object today, that is  $I_{yz}$  is not equal to 0 for a cross section and if you have bending of the cross section, how do you find out the stresses? You know that, to do that 1 way to go about is find out the directions. Let's say,  $y_1$  and  $z_1$  whereby we will have  $I_{yz}$  equal to 0 and then work with those 2 directions to find out the stresses. I will again repeat, suppose you have a cross section like this and you have loading in the xy plane. Then in that case if you work with this system you are not going to have  $I_{yz}$  equal to 0 and you cannot find out the stress distribution using this formula. Alternatively, what you can do is that, for this cross section you can find out directions. May be we would have a direction like this.

This is let's say  $y_1$  and there will be another orthogonal direction let us say this is  $z_1$ . So, this  $y_1$ ,  $z_1$  are the 2 directions which are going to give rise to product moment of inertia equal to 0. And then in that case, we can certainly work with this sort of formula to arrive at this stress distribution for a particular load. So, let us find out the 2 principle directions. They are called principle directions of the cross sections like  $y_1$  and  $z_1$ .

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$$\begin{aligned}
 u &= z \cos \alpha + y \sin \alpha \\
 v &= -z \sin \alpha + y \cos \alpha \\
 I_u &= \int_A v^2 dA \\
 &= \int_A (z^2 \sin^2 \alpha + y^2 \cos^2 \alpha - 2yz \sin \alpha \cos \alpha) dA \\
 &= I_y \sin^2 \alpha + I_z \cos^2 \alpha - I_{yz} \sin 2\alpha \\
 &= \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\alpha - I_{yz} \sin 2\alpha
 \end{aligned}$$

Let us make a start with a general cross section like this; that the beam section is like this. This is the direction y and this is v. Now, let us consider that we have 2 directions u and v which are arbitrarily oriented at an angle alpha with the z directions. Then in that case, from geometry we can write that u is related to y and z coordinates of a point by this formula, u equal to z cosine alpha plus y sin alpha.

Similarly, v is equal to minus z sin alpha plus y cosine alpha. Now, if I try to consider the moment of inertia about the axis u, then we have to take v square into dA integrated over the whole area. So, therefore, moment of inertia about the axis u is nothing but integration of v square dA over the whole area. So, this is the elemental area which is dA and this is the distance which is v. So, then in that case it is v square dA to be integrated. And if I now substitute the value of v, what we have got here, then we have. So, it is z square sin square alpha y square cos square alpha minus twice yz sin alpha cosine alpha dA. And this we can write now, we have sin square alpha which is constant which is not very of integration. Then we can write this z square dA integrated over the whole area will give us Iy.

So, therefore, this is nothing but Iy sin square alpha. Similarly, y square dA integrated will give us Iz. I simply write it Iz rather than Izz. It is also Iy rather than Iyy. So, this is Iz cosine square alpha and yz dA integrated it gives us the product moment of inertia. And therefore, we will have Iyz and 2 sin alpha cosine alpha we can write as, sin 2 alpha.

So, that is the expression for  $I_u$ , and now we can write  $\sin^2 \alpha$  as  $\frac{1 - \cos 2\alpha}{2}$  and  $\cos^2 \alpha$  equal to  $\frac{1 + \cos 2\alpha}{2}$  by using that we can rewrite the above expression as  $I_z + \frac{I_y}{2} + \frac{I_z - I_y}{2} \cos 2\alpha - I_{yz} \sin 2\alpha$ .

This type of expression, you have come across certainly in connection with the Mohr's circle for stresses in 2 dimension, but note that, the expression for  $\sigma_x$  that you have or  $\sigma_x'$  that you have obtained in terms of  $\sigma_x$  and  $\sigma_y$  they were little different. So, there were you certainly have noted that there was a positive sign here, but it is different in this case.

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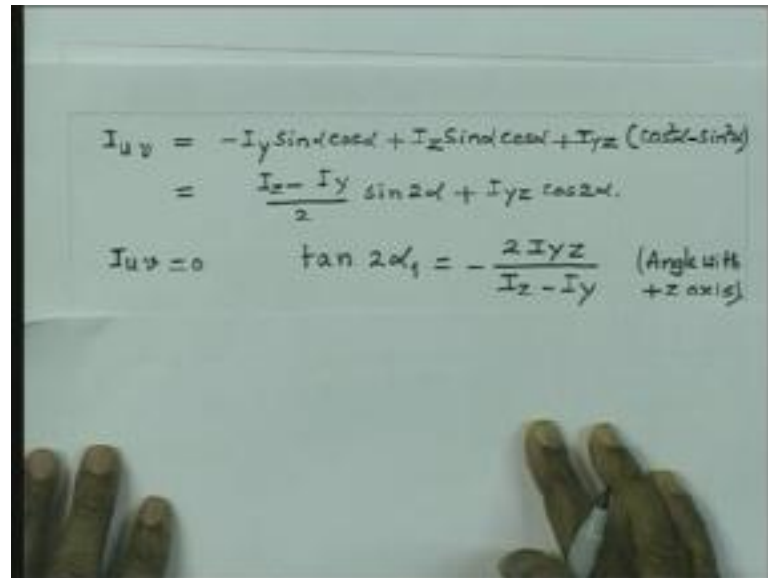
$$\begin{aligned}
 &= \int_A (z^2 \sin^2 \alpha + y^2 \cos^2 \alpha - 2yz \sin \alpha \cos \alpha) dA \\
 &= I_y \sin^2 \alpha + I_z \cos^2 \alpha - I_{yz} \sin 2\alpha \\
 &= \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2} \cos 2\alpha - I_{yz} \sin 2\alpha \\
 I_u &= \int_A u^2 dA = \frac{I_z + I_y}{2} - \frac{I_z - I_y}{2} \cos 2\alpha + I_{yz} \sin 2\alpha \\
 I_u + I_v &= I_z + I_y \\
 I_{uv} &= \int_A uv dA = \int_A (z \cos \alpha + y \sin \alpha)(-z \sin \alpha + y \cos \alpha) dA
 \end{aligned}$$

It is not exactly the replication of the formula that you had in connection with the Mohr's circle. By similar procedure, I can now write moment of inertia about the  $v$  axis is equal to integration of  $u^2 dA$  over the whole area. And this gives us  $I_z + \frac{I_y}{2} - \frac{I_z - I_y}{2} \cos 2\alpha + I_{yz} \sin 2\alpha$ . And you can see now that, if you add  $I_u$  plus  $I_v$  then we are going to get these terms canceling and you finally, find that  $I_u$  plus  $I_v$  is equal to  $I_z + I_y$ .

So, this is like sum of the 2 moment of inertia remains constant irrespective of this relation of axis. It is like a stress invariant the principle moment of inertias sum always remains constant. Now, let us try to see what happens to the product moment of inertia. So, if you now consider the product moment of inertia of the cross section with respect to

the axis u v, then we would had  $I_{uv}$  which is nothing but integration of the quantity  $uv$   $dA$  to be integrated over the whole area and this on substitution of value of u and v. So,  $z \cos \alpha + y \sin \alpha - z \sin \alpha + y \cos \alpha$   $dA$ .

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$$I_{uv} = -I_y \sin \alpha \cos \alpha + I_z \sin \alpha \cos \alpha + I_{yz} (\cos^2 \alpha - \sin^2 \alpha)$$

$$= \frac{I_z - I_y}{2} \sin 2\alpha + I_{yz} \cos 2\alpha$$

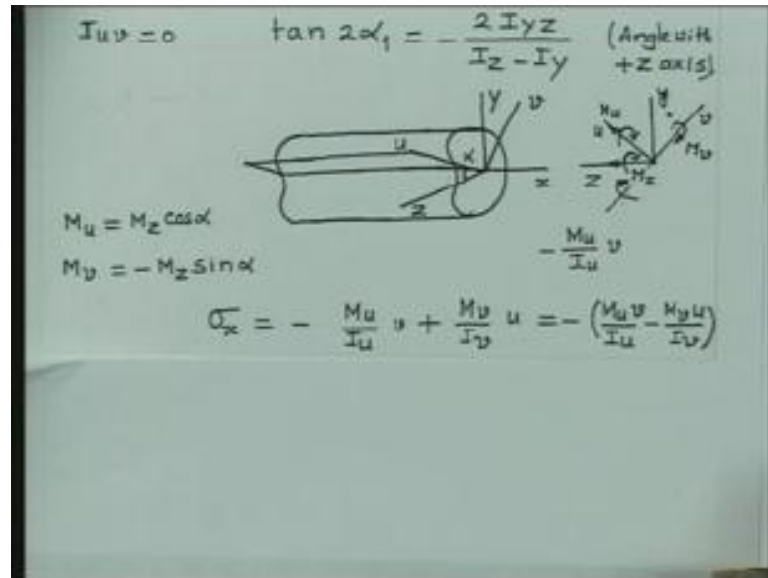
$$I_{uv} = 0 \quad \tan 2\alpha_1 = -\frac{2I_{yz}}{I_z - I_y} \quad (\text{Angle with } +z \text{ axis})$$

So, that is the product moment of inertia, and now if we try to simplify this it gives us  $I_{uv}$  is equal to minus  $I_y \sin \alpha \cos \alpha$  plus  $I_z \sin \alpha \cos \alpha$  plus  $I_{yz} \cos^2 \alpha$  minus  $I_{yz} \sin^2 \alpha$ . So, this is the value of  $I_{uv}$  and this we can now write in terms of  $\sin 2\alpha \cos 2\alpha$ . So, this will be  $I_z$  minus  $I_y$  by 2  $\sin 2\alpha$  plus  $I_{yz} \cos 2\alpha$ . This is the product moment of inertia. Now, what we are looking for, we are trying to look for 2 axes u and v such that the product moment of inertia will be 0.

So, we can now write the condition for  $I_{uv}$  is equal to 0. So, that will give us the direction  $\alpha$   $I_{uv}$  equals to 0. That gives us directly the direction, let us say, that is  $\alpha_1$  direction. So, therefore,  $\tan 2\alpha_1$  is equal to minus 2 times  $I_{yz}$  by  $I_z$  minus  $I_y$ . So, that is the angle  $\alpha_1$  which u makes with the positive z axis. So, therefore, this is nothing but angle with positive z axis. Now, we are in a position to get the direction  $\alpha_1$  which will make the product moment of inertia 0. So, these are directions which are (( )) principle directions of the cross section.

Now, it is easy to work with the formula, formula that we have derived to find out the bending stresses acting on a cross section. So, just I would like to show you here a particular example.

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Let us say that we have a beam which is of this type. So, let us say, that this is the beam. Here in our x axis is located like this. Y is located here and this is our z axis. Now, for this cross section we can find out the direction alpha 1 or the x angle alpha here which gives us the direction u and the corresponding orthogonal direction is v. So, these are the principle directions of the cross section and note that this is the, this is 1 plane, this is the x u plane. On this plane if bending moment is acting then we can use the stress formula straightway

I repeat that, if you have the loading let's say acting on the x z plane, then in that case, you find that this x z plane is not really a plane of symmetry. On the other hand, if the loading is acting on the u x plane, it now, becomes a plane of symmetry because  $I_{uv}$  is equal to 0. So now, in general what we will have is that you are going to have a bending acting on the cross section. Let us say, this is the y direction and this is my z direction and this is the series the cross section.

Now, you will most often, will give in the bending moment let's say, about the z axis. So, this is the bending moment, let us consider its direction to be like this as positive so, this is  $M_z$ . Now, this  $M_z$  we have the direction u which is this direction and the direction

$v$  is this 1. Now, this moment vector can be resolved into 2 components without any difficulty into a moment vector along this direction. So, therefore, that will be nothing but  $M_z$  into  $\cos \alpha$ . So, that gives you the value of  $M_u$ . Similarly, if I resolve the  $M_z$  component in this direction, which is minus  $v$  direction. We will probably get a component like this.

This is the component in this direction of  $v$  axis and therefore now, from this  $M_z$  we are in a position to get  $M_u$  as  $M_z \cos \alpha$  and  $M_v$ . Let us consider our convention, that it will be positive in this direction. That is our positive right hand screw rule. So, therefore, this moment that is acting on the cross section is negative. So, therefore, in this case you can straightaway write that  $M_u$  is equal to  $M_z \cos \alpha$ . So, where in I am saying that this angle, this angle of inclination is  $\alpha$  where in at. Let us show that, this angle here is nothing but  $\alpha$ .

So, therefore,  $M_u$  is nothing but  $M_z \cos \alpha$ . Similarly,  $M_v$  is equal to minus  $M_z \sin \alpha$ . So, I am trying to say that it obeys right hand screw rule. So, if you rotate a right hand screw it will go in that direction. So, therefore, that is my positive direction and similarly, in this case I also have the right hand screw rule. If the screw moves in this direction that is a positive bending moment. So, therefore, that is the rule I have adopted and the component of this will be really like this; it is exactly opposite. So, therefore, a negative sign comes in.

Now, if you consider these 2 moments  $M_u$  and  $M_v$ , they are acting on the planes of symmetry of the cross section and therefore, without any difficulty you can apply the flexure formula that you have already derived that is  $\sigma_x = \frac{M_y}{I_y} y$  equal to  $\sigma_y = \frac{M_x}{I_x} x$ , that formula cannot be directly adopted to calculate the stresses on the cross section. So, we will like to first of all look into a case, how to find out the principle cross sections, principle axis of the sections. A given section then how do you go about calculating the stresses in a particular case. So, let us look into some examples now, so, as to elaborate the calculations that we had just now worked.

Now, I think we can even straightaway write, let me continue on this point on this cross section once more. See that here if I now, get back to this cross section again. I have  $M_u$  acting so,  $M_u$  is going to give me stresses which is going to be given by  $M_u$  and this  $M_u$  divided by  $I_u$  multiplied by distance of a point in terms of the of stresses. This that your

Mu is trying to that is the Mu moment, and therefore, my formula would be  $\sigma$  by  $\sigma$  by  $g$  is equal to minus  $\sigma$  by  $I_e$  minus  $\sigma$  by  $v$  is equal to  $M_u$  by  $I_u$ . So, therefore, if I use that formula it will be  $M_u$  by  $I_u$  into  $v$ . That gives me stress and note that this particular moment is going to cause compressive stress in the segment where  $v$  is positive.

So, therefore, this is compressive stress. So, therefore, my  $\sigma_x$  is going to be minus  $M_u$  by  $I_u$  into  $v$  and similarly,  $M_v$ . Look at this  $M_v$  by  $I_v$  into distance of a point from the  $(( ))$  axes that is in terms of  $y$  that will give us stress. So, my stress would be nothing but  $M_v$  by  $I_v$  into  $u$ , that is the stress. Now, note that for this sort of bending moment you are going to get positive stress tensile stress for  $u$  positive. And it is going to give rise to compressive stresses on the bottom half of this section. So, therefore, we have tensile stress due to  $M_v$  and compressive stress due to  $M_u$ .

So, finally, my stress on the cross section is going to be given by this formula; minus  $M_u$  by  $I_u$  into  $v$  minus  $M_v$  into  $u$  by  $I_v$ . So, that is the total stress at any point on the cross section. So, to calculate the stresses on a section of this type what are the steps? First of all, you should be in a position to calculate the 2 principle directions of the cross section  $u$  and  $v$  for which  $I_{uv}$  is 0. Then from the given bending moment on the cross section, you try to find out what are the magnitudes of the bending moment acting on the axis  $u$  and  $v$  and then try to make use of the flexure formula to get at the total stresses.

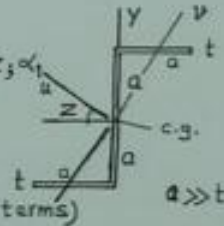
So, we will now try to illustrate this sort of derivation by considering some example. First we will see the example, where in we are interested in finding out the principle axis of the cross section then we will apply it to calculate stresses.



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**EXAMPLE 1**

Determine  $I_y$ ,  $I_z$  &  $I_{yz}$ ,  $\alpha_1$



$$= 2 \left[ \frac{at^3}{12} + at(a^2) \right] + \frac{t(2a)^3}{12}$$

$$= 2a^3t + \frac{2}{3}a^3t = \frac{8}{3}a^3t$$

(Neglecting  $t^3$  terms)

$$= \frac{2at^3}{12} + 2 \left[ \frac{ta^3}{12} + at\left(\frac{a}{2}\right)^2 \right] = \frac{a^3t}{6} + \frac{a^3t}{2} = \frac{2}{3}a^3t$$

$$I_{yz} = at\left(-\frac{a}{2} \times a\right) + at\left(+\frac{a}{2} \times -a\right) = -a^3t$$

$$\tan 2\alpha_1 = -\frac{2I_{yz}}{I_z - I_y} = \frac{\frac{2}{3}a^3t}{\frac{8}{3}a^3t - \frac{2}{3}a^3t} = 1 \quad \alpha_1 = 22.5^\circ$$

So, let us consider an example here, so, the section is like this; it is z in inverted z section. And dimensions are as shown here, very thin section or rather the wall of the section is very thin. The flange is very thin so, also the web is also thin and everywhere the thickness is equal to  $t$  and this dimension semi height of the wave is  $a$ . This is the cg of the cross section here, cg of the cross section. And what is needed is that determine  $I_y$ ,  $I_z$  and  $I_{yz}$  and also  $\alpha_1$  that is the direction corresponding to the principle axis. Now, you should be able to have some guess about the principle directions. Let us look into the figure again. You see here, this is a cross section. We have to find out the 2 directions for which product moment of inertia should be 0.

Now; obviously, we must have the orientation such that some of the portion of the section should lie in the positive segment and some of the portions should lie in the negative segment then only it is going to work out. So, therefore, most likely I will have a orientation of 1 of the axis to be like this and the other 1 also just orthogonal. So, we will try to see that; so, I expect in this case that my 1 principle direction will be oriented like this. So, that some of the portion of the material will be in the positive segment and some will be in the negative segment and other axes will be like this. So, let us try to calculate first of all the moments  $I_y$ ,  $I_z$  and  $I_{yz}$ .

So, if I consider  $I_{yz}$ , to calculate  $I_{yz}$ . I can consider it to be consisted of 3 segments. One is of dimension let's say  $a$  and  $s$ . Note that this flange is also having a dimension  $a$  and

this flange is also having a dimension. So, therefore, I can consider approximately that this portion is nothing but of dimension rectangular segment of dimension  $a$  and  $t$  and it is at a distance of  $a$  from the  $z$  axis. So, I can use the Parallel axis theorem and now, the orientation or the position of this arm and this arm is same with respect to the  $z$  axis. So, I can write the moment of inertia of this segment about  $z$  axis as  $a^3 t$  by twelve that is the moment of inertia about its own axis therefore; that means, axis passing through the cg of this segment.

Now, I apply Parallel axis theorem, that is,  $a$  into  $t$  in the distance of this from the  $z$  axis is a square. So, therefore, I have  $a^2 t$  into a square; so, this is the moment of inertia of this portion about the  $z$  axis. So, now, same expression will be obtained for this. So, I can write that for the 2 segments it is going to be 2 times this 1 and now, this portion which is of dimension  $2a$  and of thickness is equal to  $t$ . So, I will have  $t^3 2a$  by 12. Now, this particular problem has been given with the condition that  $t$  that this  $a$  dimension  $a$  is very large compared to  $t$ . So, therefore, we can ignore this  $t^3$  cube compared to a cube.

So, therefore, I can now write this portion can be neglected and therefore, I will have  $2a^3 t$  over the this part and this is going to be plus  $2$  by  $3a^3 t$  cube. And this added up together it gives us  $8$  by  $3a^3 t$  cube. So, that is neglecting  $t^3$  cube terms; similarly, if I now go in for calculating the moment of inertia about the  $y$  axis. First of all I will have this wave. Wave is going to be bus  $2at^3$  by 12 and 2 flanges, about their own centre row  $y$  axis is  $t^3 a$  by 12 plus area is  $a^2 t$  and that cg is at a distance of  $a$  by 2 from the  $y$  axis. So, therefore, this give us the value together and these 2 when we simplify we can again here is a  $t^3$  cube term involved.

So, therefore, we can neglect this and we will have then  $a^3 t$  by 6 plus  $a^3 t$  by 2 and therefore, we will have  $2$  by  $3a^3 t$  cube, this is quite straightforward. Now, let us try to look into the product moment of inertia. That is something probably you may not be very familiar with. So, let us consider now  $I_{yz}$ . Now, as usual we will go for considering the 3 segments. So, the product moment of inertia for this section can be gotten by considering again this segment and we can take the area of this segment and multiply by the cg coordinates of this segment with respect to the coordinate.

Similarly, for this segment I can write the area multiplied by the cg coordinate with respect to the origin of the whole. Origin here or the cg of the whole cross section, And

so, is the case for this segment also we can consider the area multiplied by the coordinate of the cg of this segment. Now; obviously, this portion is having cg which is coincident with origin. So, therefore, that does not contribute anything. So, we are going to get only contribution from this wave and this wave. So, we will now write if I consider now, this 1 then my area is  $a t$  and x coordinate of this cg is  $-a/2$ . So, therefore, it is  $-a/2$  and the distance from the z axis is  $a$ . So, therefore, it is  $-a^2 t/4$ .

So, that is the x cg y cg. Similarly, for the other portion of the wave we will have for the other portion of the flange. So, it is  $a t$  and here in now, the distance again is actually  $a/2$ . So, therefore, it is  $a/2$ . It is now  $+a/2$ . And this distance is now negative. So, therefore, it is  $-a^2 t/4$ ; therefore, when you add up this both are negative and we get now,  $-a^2 t/2$  and therefore, it is  $-a^2 t/2$ . So, the product moment of inertia is  $-a^2 t/2$ . Now, we are interested in finding out the section, set of axes which will make the product moment of inertia 0; that means, we are interested in calculating  $\alpha_1$  direction.

So, if we just invoke the formula,  $\tan 2\alpha_1$  is equal to  $-2I_{yz}/(I_z - I_y)$  which is equal to  $-2(a^2 t/8)/(8a^2 t/3 - a^2 t/2)$ . So, this minus minus becomes plus.  $8/3 - 1/2 = 13/6$ . So,  $\tan 2\alpha_1 = 13/6$  which is equal to 2.166, and therefore  $\alpha_1$  is equal to  $22.5^\circ$ . Now, let us try to see how good are our guess. We were, we now get the direction  $u_1$  to be like this; which is oriented at an angle of about  $22.5^\circ$ . Therefore, this is my direction  $u$  and; obviously, the other direction  $v$  is like this. This is the other direction  $v$  and you see as you were expecting that 1 of the principle direction is to be oriented like this.

So, that a portion of the material lie in the negative half and a portion lie in the positive half and we have got exactly as we expected. Let us now, continue further on this. We have got now the direction  $u$  and  $v$ . These are principle directions for the section. It will be nice to calculate the value of  $I_u$  and  $I_v$ . So, it is a concern of simply applying the formula and getting the value.

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$$\begin{aligned}
 I_u &= \frac{I_z + I_y}{2} + \frac{I_z - I_y}{2} \cos 2\alpha_1 - I_{yz} \sin 2\alpha_1 \\
 &= \frac{5}{3} a^3 t + a^3 t \frac{1}{\sqrt{2}} + a^3 t \frac{1}{\sqrt{2}} = \left(\frac{5}{3} + \sqrt{2}\right) a^3 t \\
 &= 2.357 a^3 t \\
 I_v &= I_z + I_y - I_u = \frac{10}{3} a^3 t - 2.357 a^3 t = 0.976 a^3 t
 \end{aligned}$$

So, let's do that, So, we have  $I_u$ ,  $I_z$  plus  $I_y$  by 2 cosine 2 alpha 1 minus  $I_{yz}$  sin 2 alpha 1. So, that is the formula we derived. So, it is a question of just substituting for 2 alpha 1 as 45 degree. So, once you do that you finally, get the values. I think I will not go through the intermediate states; I will give you directly the value here. So, finally, what we get here as five by 3 a cube t plus a cube t1 by root 2 plus a cube t1 by root 2 and this is equal to 5 by 3 plus root 2 a cube t. So, that is the moment of inert  $I_u$  and this gives you value equal to 2.357 a cube t for  $I_u$ .

Similarly,  $I_v$ ,  $I_v$  naught go through the similar formula. I can make use of the fact that  $I_v$  is equal to  $I_z$  plus  $I_y$  minus  $I_u$  and therefore, we have 10 by 3 a cube t minus 2.357 a cube t. It gives us 0 point 976 a cube t, that's the value of  $I_v$ . So, we get both the principle moments of inertia. So, these are the steps to calculate the direction of principle axes of the cross section and the moments. The moments of inertia, let us now, try to go for calculation of stresses.

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**EXAMPLE 2 ON STRESS CALCULATION**

$I_z = 9.44 \text{ cm}^4$     $I_y = 2.58 \text{ cm}^4$   
 $I_{yz} = 2.813 \text{ cm}^4$

$\tan 2\alpha_1 = -\frac{2 I_{yz}}{I_z - I_y}$   
 $= -\frac{2 \times 2.813}{9.44 - 2.58}$   
 $= -0.82$   
 $2\alpha_1 = +140.64^\circ, \alpha_1 = 70.32^\circ$

$I_u =$   
 $= 1.574 \text{ cm}^4$

So, we will consider now, example on calculation of stresses. The problem is shown here, we have a cantilever beam which is fixed at this end where the cross section is like inverted L. We have the web of 5 centimeter and the flange is of length 3 centimeter and the thickness of the wall is 0.5 centimeter. These are the axes z and y and P is the force acting on the cross section which is 1 kilo Newton. So, I have shown the cross section again here and already calculation has been done about the cg location which you could do very easily. It is going to be located at a distance of 0.75 centimeter from this edge. And it is going to be located at a depth of 1.75 centimeter from the top fiber, this will be cg location.

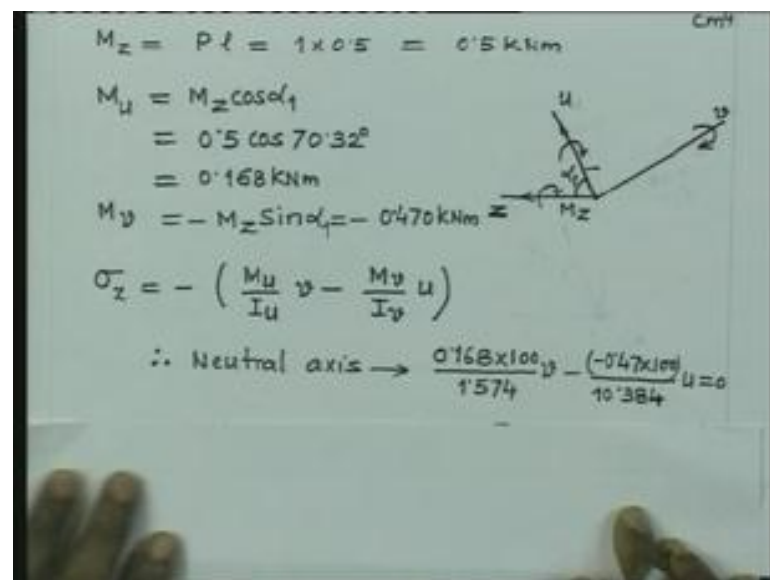
Now, we have to calculate as usual the directions, let us say we will try to consider that this is I. My u direction and similarly this is my v direction. So, we will calculate the directions of find out the directions u and v we make use of the formula once again, that the value of the moments of inertia are given here,  $I_z$  equal to 9.44 centimeter raise to the power 4.  $I_y$  equal to 2.58 centimeter raise to the power 4 and product moment of inertia is 2.813 centimeter raise to the power 4.

I must tell you, that the product moment of inertia can be positive and negative, but never ever, these moments of inertia will be negative. So, that is a very typical of this product moment of inertia that it need not be always positive. So, now, the direction of this axis u which is  $\alpha_1$ . So, this is angle I am referring to  $\alpha_1$ . So,  $\tan 2\alpha_1$  is

equal to minus twice  $I_{yz}$  by  $I_z$  minus  $I_y$ . And this gives you minus 2 times 2.813 divided by 9.44 minus 2.58 and this gives us a value of minus 0.82, which means, that  $2\alpha_1$  is equal to 140.64 degrees and therefore,  $\alpha_1$  is equal to 70.32 degrees.

So, therefore, this angle is 70.32 degrees. And you can calculate this  $I_u$  and  $I_v$ . So, we will straightaway  $I_u$  which is given by  $I_z$  plus  $I_y$  by 2 plus  $I_z$  minus  $I_y$  by 2 cosine  $2\alpha_1$  minus  $I_{yz}$  sin  $2\alpha_1$ . So, therefore, if we substitute the value, I will skip the steps. So, this gives us  $I_u$  equal to finally, 1.574 centimeter raise to the power 4. So, that is the value of  $I_u$ . And we can calculate  $I_v$  from the invariants condition.

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Handwritten calculations and a diagram showing the transformation of moments and the neutral axis equation.

$$M_z = P \cdot l = 1 \times 0.5 = 0.5 \text{ kNm}$$

$$M_u = M_z \cos \alpha_1 = 0.5 \cos 70.32^\circ = 0.168 \text{ kNm}$$

$$M_v = -M_z \sin \alpha_1 = -0.470 \text{ kNm}$$

$$\sigma_z = - \left( \frac{M_u}{I_u} v - \frac{M_v}{I_v} u \right)$$

$$\therefore \text{Neutral axis} \rightarrow \frac{0.168 \times 100}{1.574} v - \frac{(-0.47 \times 100)}{10.384} u = 0$$

The diagram shows a coordinate system with  $u$  and  $v$  axes. A moment  $M_z$  is applied along the  $z$  axis. The angle  $\alpha_1$  is shown between the  $z$  axis and the  $u$  axis.

So,  $I_v$  is equal to  $I_z$  plus  $I_y$  minus  $I_u$  and therefore, this is 9.44 plus 2.518 minus 1.574 and this gives us 10.384 centimeter raise to the power 4. So, these are the principle moments of inertia. Now the problem, what is the problem? The problem is that for this particular case. So, look at this geometry for this particular case, I am trying to apply a force here, like this. And we are interested in calculating the maximum stresses, bending stresses in the cross section. Think for a while, that you have this is trying to this particular force is trying to bend the cross section about that  $z$  axis.

So, it is trying to do like this; it will go bending like this. And there will be stresses acting there, obviously; we expect tensile stresses to be occurring in this region and compressive stresses at the top fiber region. Now, we have to find out the extreme stresses. So, we have now located the principle direction. So, one of the principle

direction is this 1, which is at an angle of 70.32 degree in the z direction. So, therefore, this is your u this is v.

Now, what we need to do; that this force P is going to give a moment of magnitude point 5 into P say, kilo Newton meter, about the z axis located, somewhere here. And that moment of must be resolved in the 2 directions u and v as  $M_u$  and  $M_v$ . Then we can apply the flexure formula to calculate the stresses and that is what we are going to do. So, here  $M_z$  is P into length of the member and it is positive moment because see it is going to give a anti clockwise moment and therefore, it will be positive. So, 1 into 0.5 and hence it is 0.5 kilo Newton meter is the bending moment.

Now, this bending moment I have tried to resolve it here. Look at this; we have this that is the  $M_z$ . This is direction z and now we have the direction here. This is the u direction and this angle is equal to alpha 1. Alpha 1 direction and similarly, we have the other direction which is v direction. So, now this angle is 90 this moment  $M_z$ . I can write that  $M_u$  is going to be  $M_z$  into cosine alpha and similarly,  $M_v$  is going to be nothing but minus  $M_z$  sin alpha. So, I repeat  $M_u$  is going to be nothing but  $M_z$ .

So, the component this 1 is going to be simply  $M_z$  cosine alpha 1. And the component which is going to be in this direction it will be  $M_z$  sin alpha 1. It will be in this direction, but my positive direction is this. And if you consider our moment to be positive this way then it will become a negative. So, I now have,  $M_u$  is equal to  $M_z$  cosine alpha 1 and therefore, this is 0.5 cosine 70.32 degrees and this is nothing but 0.168 kilo Newton meter. Similarly,  $M_v$  is going to be. My positive v direction is this, but this moment is going to be pointed this way.

So, therefore, it must be negative and therefore, it is  $M_z$  sin alpha 1 and this gives us minus 0.47 kilo Newton meter. So, these are the moments, Now, we have to locate for this cross section first it is good idea to locate the neutral axis. There will be a line passing through the cg of the cross section on which there will be no stress. So, now if I write the expression for stress,  $\sigma_x$  which you have derived little while ago. This is nothing but  $M_u$  by  $I_u$  into v minus  $M_v$  by  $I_v$  into u. So, that is the general form and now, from neutral axis what we should had, that  $\sigma_x$  would be equal to 0. So, therefore, for neutral axis, we must have we substitute the value 168 and I consider it to be Newton

centimeter kilo Newton centimeter. That's why I multiply by 100 and this is 1.574 v minus and this value is minus 47 into 100 into u divided by 10.384 equal to 0.

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$$\tan \beta = \frac{v}{u} = - \frac{0.47 \times 100}{10.384} \times \frac{1.574}{0.168 \times 100} = -0.424$$

$$= -\tan 22.98^\circ ; \quad \beta = -22.98^\circ$$

$$u_A = x \cos \alpha_1 + y \sin \alpha_1 = -0.75 \cos 70.32 + 1.75 \sin 70.32 = 1.395 \text{ cm}$$

$$v_A = -x \sin \alpha_1 + y \cos \alpha_1 = 1.395 \text{ cm}$$

$$\sigma_A = - \frac{M_u}{I_u} v_A + \frac{M_v}{I_v} u_A = - \frac{168}{1574} \cdot 1.395 + \frac{(-47)}{10.384} \cdot 1.395$$

$$= -20.13 \text{ kN/cm}^2$$

So, that is the line which is going to have no stress acting at any point on it. And this line is going to be oriented; therefore, would have positive u direction given by beta. So, we can now, write from this itself. So, we can write now, v by u which is let us say tan beta. So, tan beta is v by u and this gives us a value. 474, 470 multiplied by 100 divided by 10.384 multiplied by 1.574 divided by 0.168 multiplied by 100 and this is nothing but minus 0.426 and this is minus tan 22.98 degree. So, beta is nothing but minus 22.98 degree.

This is something very important. So, let us consider the cross section again, we have got like this. This is the cross section; we have u v directions like this. Now, the neutral axis is going to make an angle of minus. So, this is my direction u. This is v and it is going to make an angle of minus 22 point the magnitude is got as 22.98. So, therefore, this is the angle. So, this is the angle 22.98 degree. So, this is the neutral axis of the cross section. So, this is very important. That on this line you find for this cross section.

So, if I now look at this cross section, I is a line here which is oriented like this. Let us say that this is the line. For this line if you consider any point on this line there is no stress and hence what you find now, because of the loading you are going to get compressive stresses on the upper half of the plane and bottom half you are going to get



tensile stresses. So, you are going to get tensile stresses to be generating somewhere here. Compressive stresses to be generating somewhere there. So, therefore, you can now calculate the space at this for this point from the neutral axis.

This is the furthest point, because my neutral si bar is going to be located like this. This we have done So, you can find out this is the furthest point. Similarly, this is the furthest point. You can calculate the stresses and that at those points. So, quickly I will try to give you the stress at the point A. So, for that you need to calculate  $u$  of A which is nothing but  $z \cos \alpha + y \sin \alpha$  and this 1 if you calculate after substitution of the value it is 1.339 centimeter. May be I will just write these coordinates. So, that you get the idea.

This is  $0.75 \cos 70.32 + 1.75 \sin 70.32$ . Now, I can let us quickly see this what I am trying to say. I am trying to now, find out the coordinate of this point. It is at a distance of minus 0.75. So, I have written minus 0.75 and it is at a distance of 1.75 from the  $z$  axis. So, 1.75 So, that is what I have written here. So, this is  $u_A$ . Similarly, if you consider  $v_A$  it is nothing but  $-z \sin \alpha + y \cos \alpha$ . So, once you substitute the value it comes out to be 1.25 centimeter. So, these are the coordinates, now if I if I am interesting in calculating the stress at A it is going to be  $\mu_y I_u$  into  $v_A$  plus  $\mu_z I_v$  into  $u_A$  and we substitute the value now.

That is going to be  $16.8 \text{ kilo Newton centimeter} / 1.574$  into 1.295; that is  $v_A$  plus  $-47 \text{ kilo Newton centimeter} / 10.384$  multiplied by 1.395 which is  $u_A$ . So, once you do that it gives us a stress of magnitude 20.13 kilo Newton per centimeter square. So, this is the stress which is compressive as we expected. The stress at this point of the cross section is expected to be compressive, because this is the neutral axis. We expect compressive stresses to be there. Similarly, you can calculate the stress at the point here, which is let us say Q and we have calculated this stress at Q. It comes out to be 23.0 kilo Newton per centimeter square. I think it will be good idea for you to calculate yourself, the stress at the point Q and see that this is going to be equal to 20.3 kilo Newton per centimeter square.