Advanced Strength of Materials
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Lecture – 30

Today, we are going to talk about determination of thermal stress distribution. I have already mentioned earlier that, components of power plants process equipments and also components of internal combustion engines has subjected to high temperature and therefore, they will give rise to thermal stresses. So, it is not only important to calculate the stresses due to the mechanical load, it is also important to calculate the stresses, which come out or which arise due to temperature distribution in the body. We have already derived stress strain relationship involving both mechanical and thermal loading, today we would like to see how we can go about finding out thermal stress distribution, when some temperature distribution in the body is giving.

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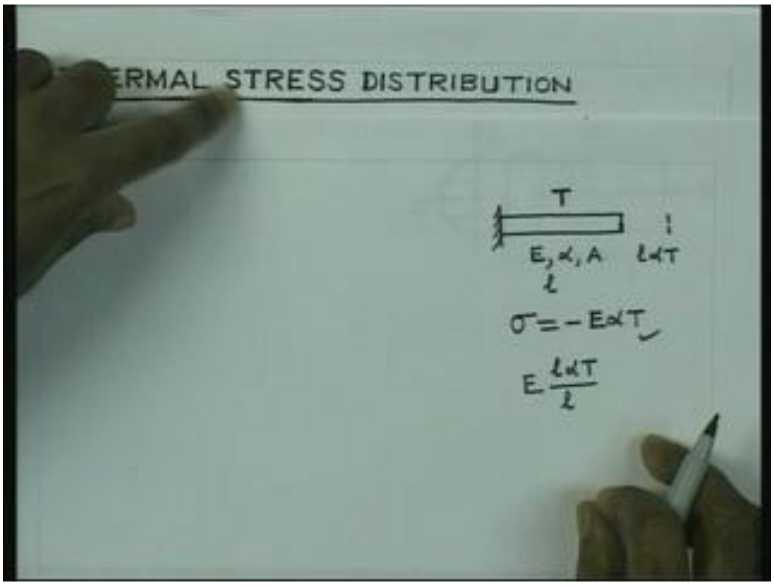

We will start with very simple considerations; if we consider a component which is uniform in cross section. Let us consider that, it is very slender if we consider that the material properties are modulus of elasticity E alpha is the thermal coefficient of expansion and a is: the cross sectional area. Now, if we consider that the component has 1 of the ends fixed. If we raise the temperature of this component above the room temperature by let us say t then it will expand, if we allow the free expansion to take place it is going to expand by a distance equal to alpha if l is it is length then the expansion is going to be up to let us say this much. And this will be l alpha t where in the length of the component is l.

Suppose, we prevent the expansion of this end to not to take place, it is not to take place then in that case we will have the thermal stress or we will have the compressive stress developing in the component which is nothing but minus E alpha T. This we have already indicated earlier just quickly you can see that, the we are preventing the expansion by an by an extent l alpha t and therefore, this strain is this much and hence the stress is going to be E into this which is nothing but e alpha T. And since, we have not allowed to expand; in fact we have tried to compress it to this length therefore, the stress is going to be compressive.

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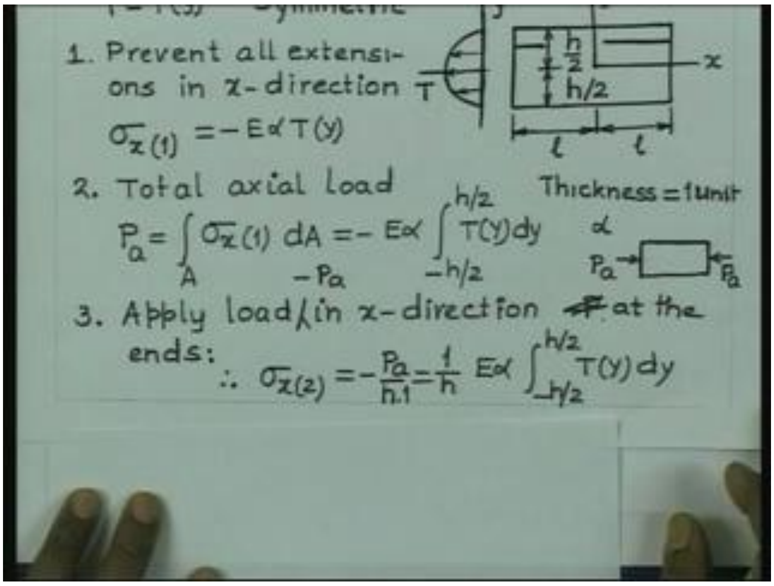

So, with this consideration let us, try to consider very simple configuration how we can find out the temperature distribution. So, consider a component which is rectangular of length 2 l height is h. And it is cross sectional area is uniform for convenience let us consider that, the thickness of this rectangular plate is equal to 1 unit. If we consider that; it has a temperature variation in the vertical direction which is given by this and.

Let us, consider that this temperature distribution is T, T is equal to a function of y and this function for the moment. Let us consider that it is symmetric. Let us assume that, we try to prevent the expansion each of the fiber say if fiber somewhere, at this level is going to have some temperature T, and therefore it will expand by an extent 2 l alpha T. Where in alpha is the thermal coefficient of expansion. Now, if we prevent the expansion of that fiber. So, what I am saying that we are considering a fiber at this level and this fiber is prevented from expanding.

Therefore, if we prevent the expansion of the fiber there will be a stress which is going to be minus E alpha T where T is the temperature at that level. So, therefore, we now consider that this gives rise to. So, we are trying to prevent the expansion and that will do for all the fibers and therefore, if we prevent. So, we prevent all the fibers to extend in the x direction. And therefore, there will be a stress. Let us indicate that stress to be equal to sigma x and we will write 1 and this is going to be equal to E alpha T and since T is a function of y let us write.

So, therefore, you have now, got a situation that there is going to be some axial forces coming up, because of this stresses acting at the ends and if we calculate the total axial load arising out of this sigma x1. So, total axial load would be let us say, that is pa and it is integration of the product sigma x1 dA and this is nothing but minus E alpha minus h by 2 to plus h by 2 TY dy into 1. So, therefore, dA is d y into 1. So, therefore, it is dy.

So, in a fact what we find now that, in this case we have a problem like this, we have now got the same body and it is subjected to some axial forces of magnitude equal to given here and it is trying to compress the body. And we have also have the temperature of the body varying in the y direction given by this relationship. In fact, we do not have any external load at the free end. And therefore, we have to if we have to get the body corresponding to this 1 we must apply the negative load at the ends and.

Therefore, if we apply negative load, at the ends of magnitude equal to Pa. It is going to give rise to stresses which is of going to be of magnitude equal to Pa by 2 times h into 1 and therefore, that is the stress which is going to come up, because of the application of Pa in the opposite direction. So, we can write the stresses arising out of this axial. So, first of all stresses arising out of this axial stress. So, let us apply. So, apply load in the x direction at the ends and we can write that apply minus Pa.

Therefore, the stress that we get out of this is sigma x2 let us say, this sigma x2 is nothing but it is height is h only, so therefore it will be Pa by h into 1. So, it is p h into 1. And therefore, it is 1 by h E alpha minus h by 2 to plus h by 2 TY dy. So, that is the stress which is due to the application of the tensile load at the ends and this tensile load is equal to the load which has come up, because of the prevention of the extension of all the fibers in the x direction. So, we have now got a case that my component is now having no load at the ends, but it is having a temperature variation TY in the y direction and therefore, the total stress that we are going to get out of it.

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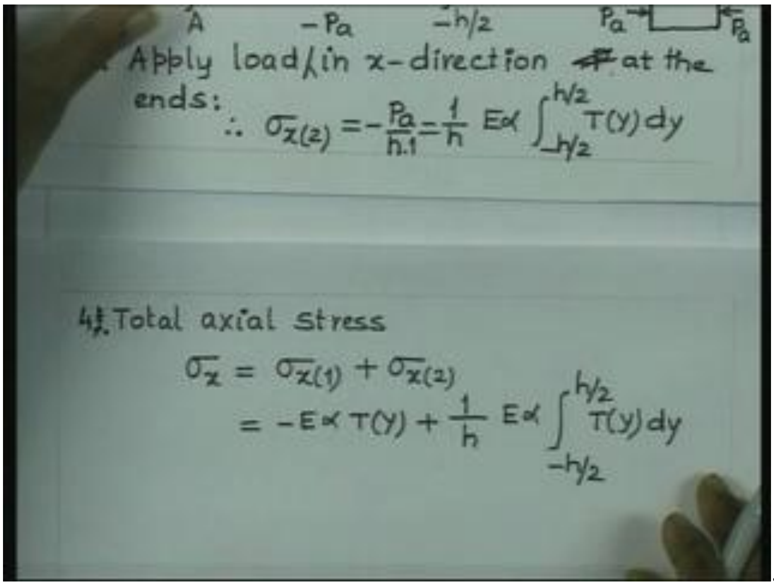

So, total stresses that; will be obtained by adding the 2 components. So, therefore, that is sigma x total we will write sigma x only is equal to sigma x1 plus sigma x2. And this is therefore, on substitution of the values E alpha TY plus 1 by h E alpha minus h by 2 to plus h by 2 TY dy. So, this is total stress that comes up in the component which is having a temperature variation of TY in the height direction.

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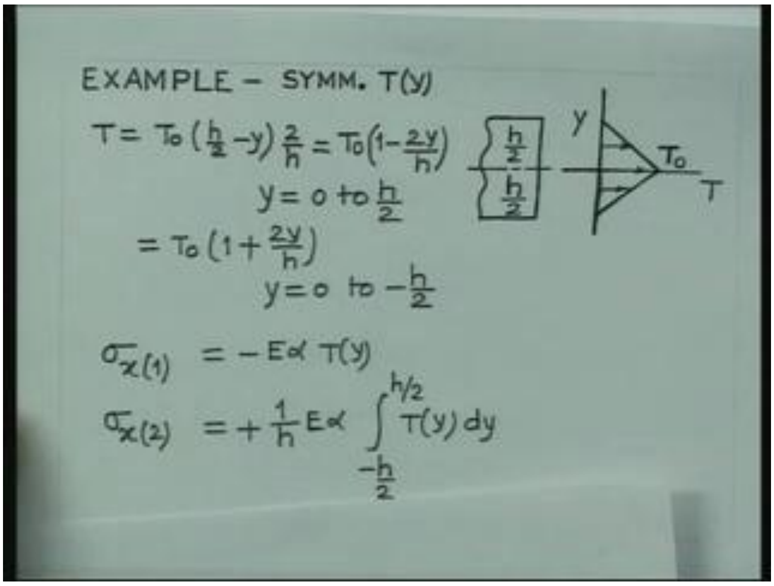

So, we would like to consider some examples to illustrate this; this particular procedure of calculation of stresses. So, let us consider 1 case here temperature distribution is given by the symmetric function TY which is shown here, we plot the temperature in this direction t and this is y direction, and therefore you have temperature let us say, this is positive temperature means that you have raising the temperature. And that is having the maximum value at its center their half magnitude is equal to t 0.

So, therefore, we can write now that this temperature variation is nothing but T 0 h by 2 minus y 2 by h which is nothing but T0 1 minus 2 y by h. So, this is the variation which is in the upper half. So, therefore, y is equal to 0 to h by 2, similarly we can write the variation to be given by T0 into 1 plus 2 y by h that is the variation in the bottom half. So, y equal to 0 to minus h by 2. Now, if we calculate the value of the stress sigma x1, it is going to be E alpha Ty and if I calculate, now the value of sigma x2. So, sigma x2 plus 1 by h E alpha integration TY dy.

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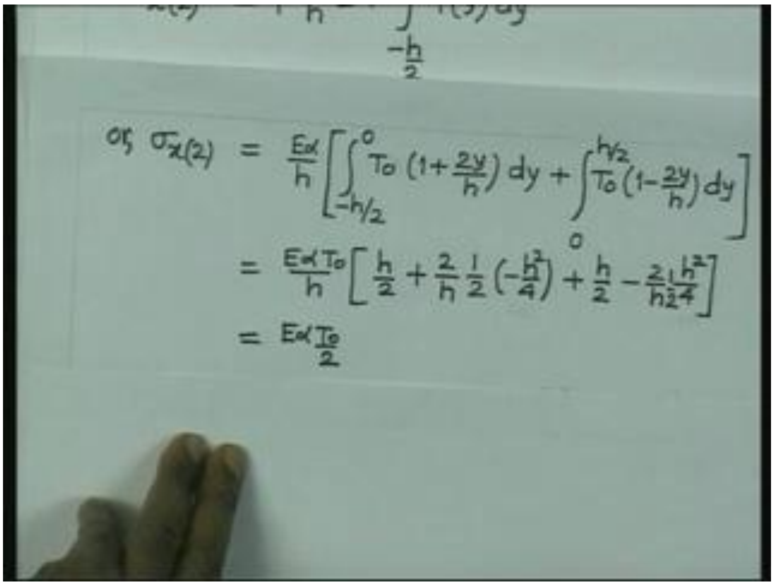

So, let us go for integrating. So, we will have integration from the bottom half therefore, sigma x is equal to sigma x2 is equal to E alpha by h it is minus h by 2 to 0 1 plus 2y by h dy plus 0 to h by 2 t 0 1 minus 2y by h dy. So, now if we do the integration it gives us E alpha by h T0 and this will give us h by 2, this is 2 by h 1 by 2 minus h square by 4 plus h by 2 minus 2 by h there will be 1 factor half here and its is h square by 4.

So, this gives us therefore, this is going to give us minus h by 4; this is also minus h by 4 that will make it h by 2. So, 1 of them will cancel and then this h b 2 will be there and it will have a cancellation with h. And finally, we are going to have E alpha T0 by 2 that is the stress due to the second part. Now, we can write the total stress by adding up the 2 components and therefore, we have sigma x is equal to sigma x2 plus sigma x1 which is nothing but E alpha T0 by 2 minus TY.

So, we can write now that: this is equal to. So, therefore, we will have 2 different expression for the 2 half's and first we write 1 t 0 by 2 T0 1 minus 2 y by h. So, this is for y equal to 0 to h by 2. And for the second half e alpha T0 by 2 minus T0 1 plus 2y by h and this is for y equal to 0 to minus h by 2. So, these are the variations of stresses in the 2 halves.

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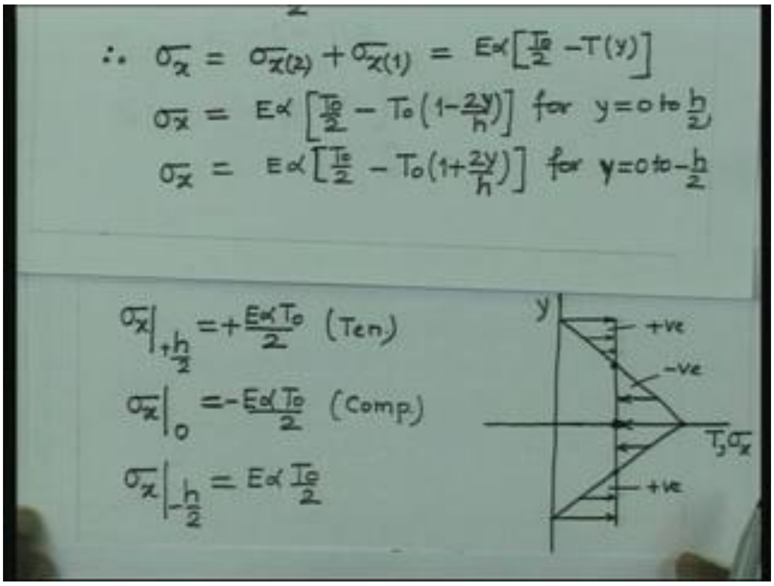

Now, if we go on plotting this. Let us try to see how that, it will look like we will consider the variation of the stress in the height direction. So, we will plot temperature here and coordinate in this direction. And therefore, what you have is that you have the temperature variation, which is the triangular variation and now what we find from the expression that we have derived, if we try to take the value of y equal to h by 2.

So, this cancels and the stress is going to be e alpha T0 by 2 it is positive. So, therefore, you can have now sigma x at h by 2, if you consider this value then in that case this stress is now E alpha T0 by 2. So, therefore, there is a tensile stress at the top fiber. Now, if we calculate the stress for y equal to 0 it is going to give us e alpha minus T0 by 2. So, therefore, sigma x at y equal to 0 is minus E alpha t 0 by 2. And this is you can see that; it is a compressive stress.

Similarly, if you consider the stress at the bottom fiber you have to put here y equal to minus h by 2. So, this cancels out. And therefore, we are going to be left with only this 1 again it is same as this is same as the stress at the top fiber E alpha T0 by 2. So, if you consider the temperature. This is the half of half the maximum temperature. So, if we now consider a if we draw a line here it gives us the idea about the stresses to certain scale if you plot the sigma x here then this height is nothing but T by 2.

The stress, if we just consider this constant E alpha by 2 to be constant then; obviously, we can write that this is the magnitude of the stress sigma x at this point and it is going to gradually go down to a value equal to a 0 here. And therefore, this stress is gradually reducing like this and even if you try to see the stress, it is going to be 0 at the middle of the top half that is at h by 4, I think look at this.

If we now look at this expression at let us say, h is equal to y equal to h by 4. So, it will be h by 4 means; it will be half and therefore, it will be half will be remaining here. So, therefore, T0 by 2 we will cancel with T0 by 2 and therefore, this stress at the middle of the top half that is at h by 4, it is 0 it is again 0 here same is the picture at this point and we have also the stress at the bottom fiber which is tensile and it is going to vary like this. On the other hand if you try to see the stress at the center it is compressive.

So, therefore, this height is T by 2 therefore, you can see that to a certain scale this is the stress in the center fiber central fiber. And therefore, we have this stress variation given by this is the variation of the thermal stress. So, therefore, here it is positive variation, here it is negative variation and here again it is positive. So, this is the 0 line and you have seen you can see that this is the temperature variation. So, you must get it confirmed with the expectation why it is that; we are going to have compressive stresses at the center and tensile stress at the outer.

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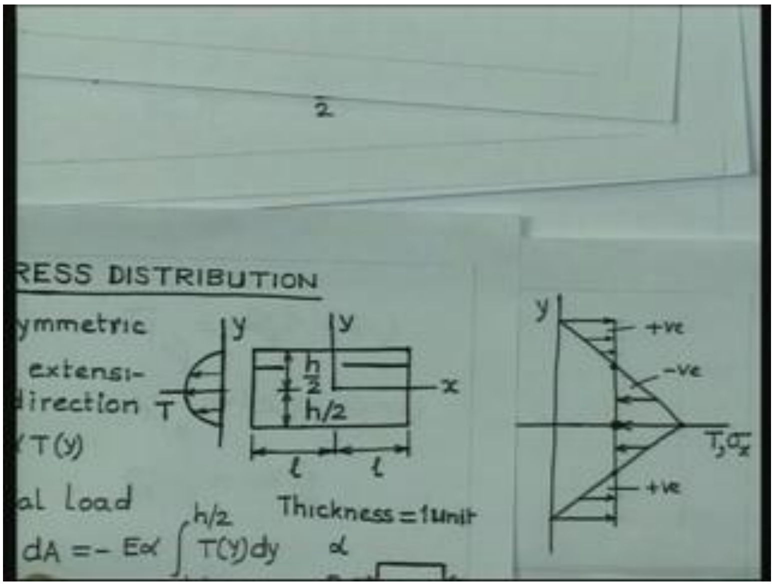

So, I want you to look at this carefully if you now look, at this component you have temperature 0 here and as you move towards the center you are going to have increase in temperature. So, therefore, if you now consider a fiber which is lying, here this fiber is trying to expand, because of the high temperature and the other fiber since, it is at a lower temperature it is not going to expand. So, therefore, it will try to this fiber will try to exert a force on this fiber to expand.

Therefore, there will be stretching effect on it, but since this fiber is not expanding on the other hand this fiber is expanding this fiber will try to apply a force on this fiber not to extend. And therefore, it will be subjected to compressive stresses. So, therefore, what you find is that; you expect under this type of temperature distribution tensile stresses to develop at center compressive stresses to develop at the center. Again I would like to repeat.

If you consider, the fiber at the center that fiber is trying to expand on the other hand, if fiber at the outer position are not going to expand and therefore, the extension of this fiber will be prevented by the outer fiber. On the other hand since, this fiber is expanding it will try to induce an extension in the outer fiber which is not expanding, because of the temperature distribution and hence we are going to see compressive stresses towards the central zone. And compressive stresses: at the central region and tensile stresses: at the outer region.

So, this gives the temperature distribution stress distribution to certain scale and it conforms with the expectations. Now, let us consider another example and we can find out this thing without doing much of a calculation.

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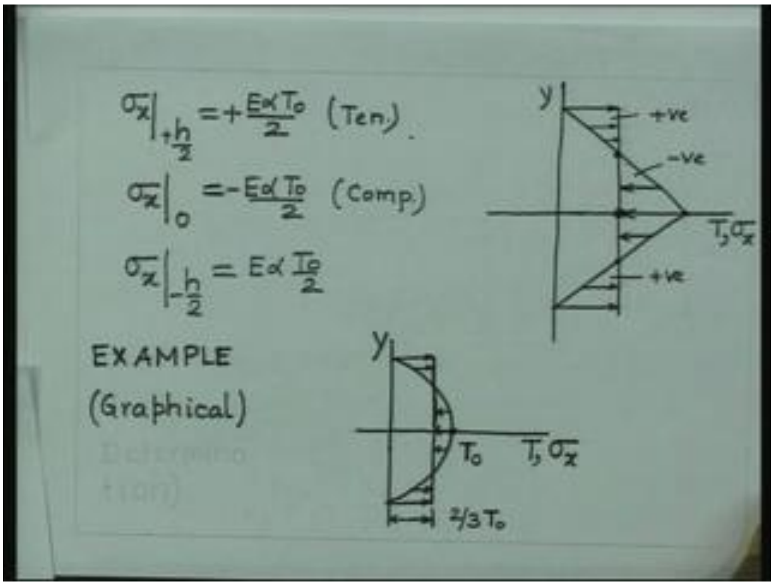

Let us, consider that we have a parabolic variation. Let us say that this is a temperature variation here and this height is equal to T0, then in that case we can again see the fact that the central fibers are going to be expanding and the outer fibers are not going to expand and therefore, we will have compressive stresses in the central zone and tensile stresses at the outer zone. Now, the average value of the temperature in this case is nothing but two-thirds T0.

So, therefore, it is 2 by 3 T0 and hence if we draw a line here at a distance of two-thirds T0 from the origin then in that case this is going to give us the order of magnitude of stresses at the outer fiber. So, it is going to gradually vary and it will reduce to 0 and as you move towards the inner region, we are going to see compressive stresses coming up in this region and at the same time we will have tensile stresses at the bottom fiber.

So, this is to a scale this to a scale e alpha by 2 the stresses. So, this through a scale is nothing but the stresses in the x direction. So, this is also one way of finding out the temperature stress distribution graphically. Now, you can varieties of distribution of temperature, which are symmetric I would like to just present some cases and then we can immediately find out the temperature distribution without doing much of calculation.

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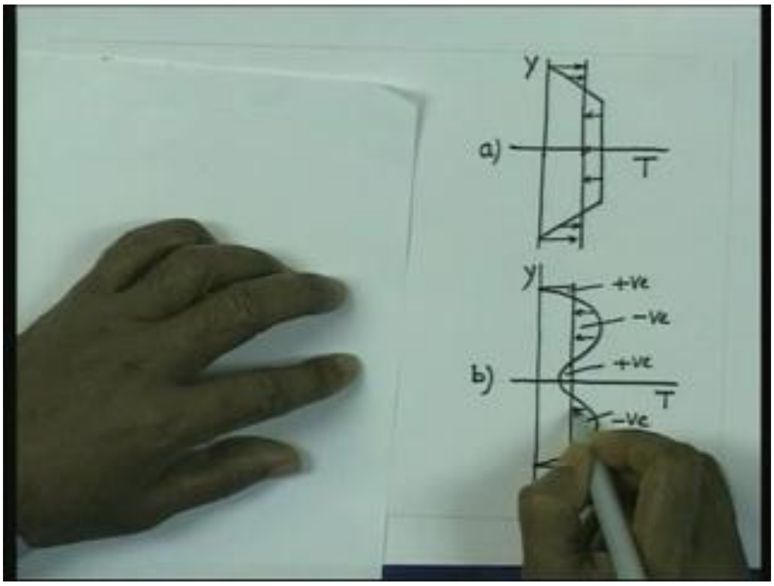

So, let us consider a variation of this type here, it is linear then there is a constant temperature at the center and then again it is linear. So, in this case 2, if we find out the average height that will be somewhere here and then we will have this variation of the stresses over the process. Similarly, think of a case which is like this. So, here again the height is average height is this much and hence we are going to get stresses which is going to be in this region, it is going to be tensile stresses.

Here it is negative and again it is going to be positive and here it is negative in this portion it is again positive. So, this stresses will be negatives. Now, it is wise to see or why it is, so that it can be determined graphically by this type of approach. So, you must have that question that how did I get it? So, let me just try to explain if you get back to our relationship.

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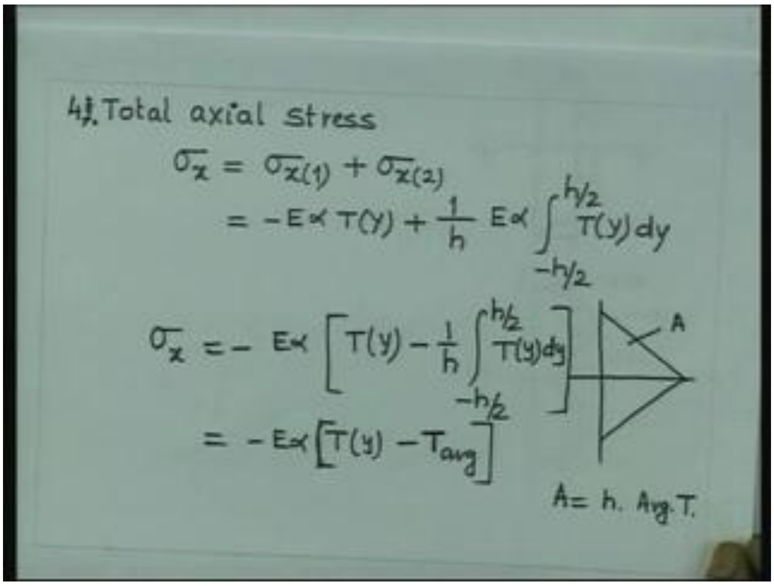

If we get our total stress total stress is going to be given by it is minus E alpha TY plus this expression multiplied by this divided by h. Now, if you consider the; this particular integral it gives us the area of the temperature variation varia enclosed by the temperature with the y axis. And therefore, if we this is nothing but the total area. So, this is nothing but the total area here and if, we divide by this height that gives us the average height or the average height.

Therefore, we can now get the area to be given by this base h into average T. So, that is the area which is nothing but this. So, we have this area divided by h is nothing but it is average temperature of course, if we just forget about this quantity for the moment. So, this is nothing but given by the average temperature and you have a constant here. So, you can see that we can write here sigma x is equal to…

If I take this thing out then it is nothing but TY minus 1 by h integral h by 2 to h by 2 TY dy. So, this is nothing, but the area and therefore, it is e alpha TY minus this gives us nothing but the average temperature. So, that is the reason we have been trying to get this thing graphically by considering simply the approach that you have the average temperature here average temperature and when we try to take the difference between the actual temperature and the average temperature that give us the stress at that level. And of course, with the negative sign, with a negative sign here that sigma x is equal to minus this and therefore, you get the right value.

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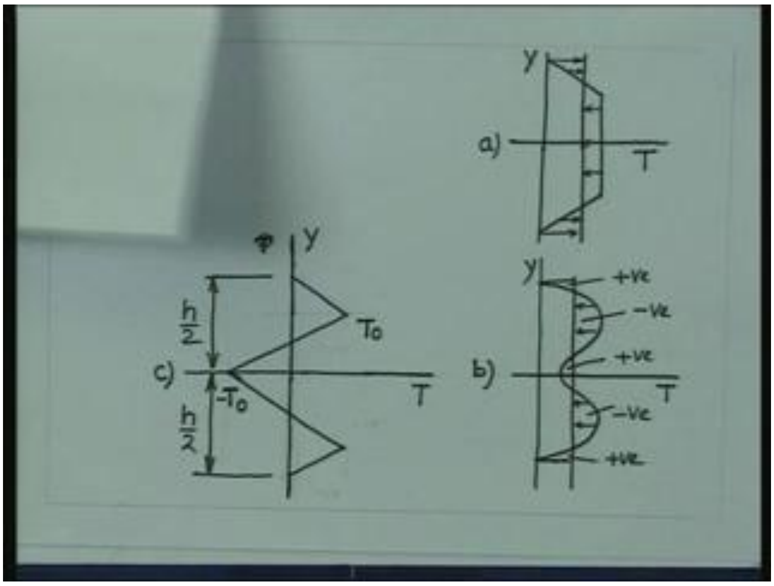

So, you can handle the cases with symmetric distribution very easily I would like you to now consider 1 example, for solving the temperature distribution is shown here the height is from here to here. Therefore this distance this distance is h by 2. So, also here h by 2 you have to consider that; this is case number c and the temperature is given here that this is T0. So, also this value is also T0 of course this is negative T0.

So, you are required to find out the temperature variation in this case thermal stress, find out the thermal stress in this case. So, I will leave it for you to consider the determination. So, we have seen how the thermal stresses in a rectangular plate can be determined provided the temperature variation is given to you in the height direction. The distribution that we have considered that is: symmetric about the central line it is not necessary that the temperature distribution has got to be symmetric in practice it can be unsymmetric and arbitrary.

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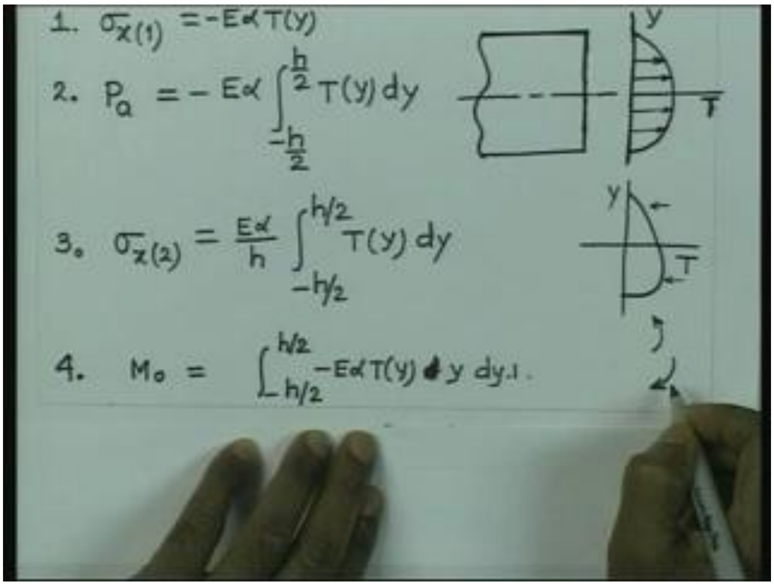

Let us, consider how we can find out the thermal stresses, if the distribution is not symmetric rather, it is asymmetric. So, what I am trying to say now, that you have the distribution of temperature which is little biased. It is more this, in this bottom half. So, may be the does not give the picture clear cut. So, therefore, what I am trying to say that we have a temperature distribution like this and this is the, as usual first of all we try to allow the free expansion of each of this fiber. Therefore, we will get the stresses, if we allow the free expansion there wont be stresses if you prevent the expansion, there will be compressive stresses. So, we indicate as usual by minus E alpha TY. So, by preventing the expansion we get that stress. Now, if we calculate the total force arising out of this compression is going to be given by let us, again represent the force to be Pa. And therefore, it is given by minus E alpha minus h by 2 to plus h by 2 TY dy.

Now, in order to annul the effect of this force we have to apply exactly opposite force; that means, we have to apply a tensile force. And hence the stresses that is going to come up due to this application of the force Pa in the opposite direction is nothing but E alpha by h, h by minus h by 2 to plus h by 2 integration of TY dy. Here you would find that the stresses that are going to coming up, because of the compression.

So, you are going to have compression at each level and, because of the asymmetry the sum total of the compression from the top half will not cancel with the sum total or rather will not balance with the sum total of the compression in the bottom half. So, there will be less force coming up, because of this sum total than this 1, and therefore there will be a moment produced about the center line.

So, I repeat that since, the stresses are not distributed symmetrically about the central lying you are going to have some moment produced, because of the unbalance. So, let us consider what is the moment. So, that moment is, so we have the stress and it is acting on a area of dy into 1 and it is at a distance of y. So, therefore, that is the moment produced, and therefore this is the moment and hence, if this moment is going to be acting in a direction. Let us say that this produces a moment like this, we have to cancel this moment. Therefore, we will apply a moment, in the opposite direction and the stresses due to this moment acting at the end can be calculated by simple bending formula.

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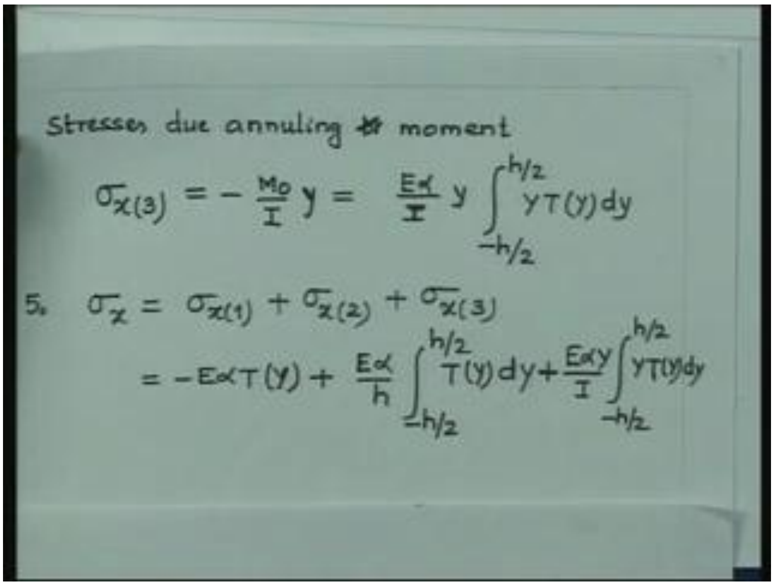

Therefore, the stresses which are going to be produced, because of this moment is nothing but sigma x3, let us say that is sigma x3 and it must be equal to minus m 0 by I into y where I is the moment of inertia of the cross section. So, this is simply application of the bending formula. So, I will write to write that this sigma x3 is nothing but stresses due to annuling moment and hence, we can write this thing as E alpha by I. And this y is out of the integral and therefore, we have the n 0 is y TY dy.

So, that is the third component of stress, and hence in the unsymmetric problem, we are going to get finally, the total stress sigma x is sum up of all the 3 contribution. So, sigma x1 plus sigma x2 plus sigma x3 and if we want to write in terms of our temperature field then it is E alpha TY plus E alpha by h minus h by 2 to plus h by 2 TY dy that is: the uniform part. And then we have E alpha y by I minus h b y 2 to plus h by 2 y dy y into TY into dy. So, this is the total stress which is going to come up at any fiber of the problem that we are giving.

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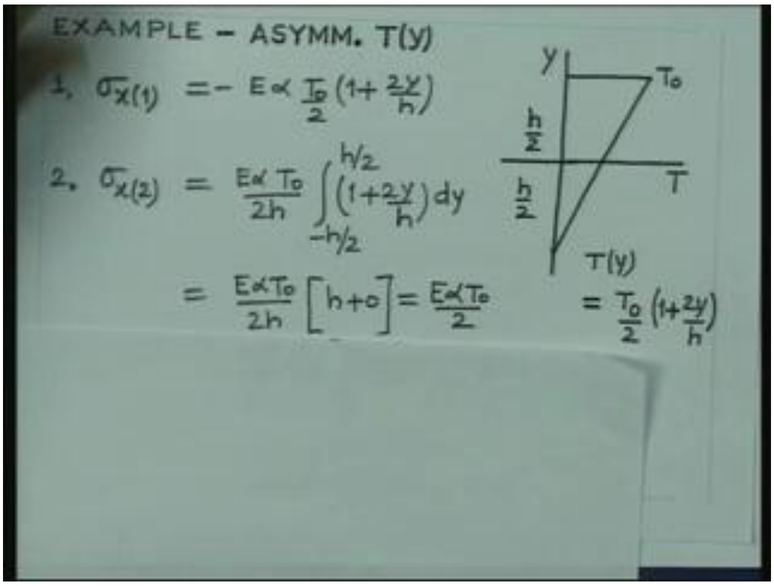

Now, let us try to consider 1 example, to illustrate this case. So, we take up the example of triangular distribution of stress in the height direction. So, we have we plot the temperature here and y axis is this direction. And let us say that, this is the maximum temperature there and the other side, this is the temperature T0 here and we have the height h by 2 h by 2. So, if we first calculate the; this temperature can be indicated as TY is equal to it is t 0 by 2 1 plus 2 y by h.

So, that is the variation of the temperature T0 by 2 1 plus 2 y by h. So, we can now write the stress sigma x1 as minus E alpha T0 by 2 into 1 plus 2 y by h this much. Similarly, the second part sigma x2 is going to be E alpha T0 by 2 into h integration of this 1 plus 2 y by h dy and of course it is for minus h by 2 to plus h by 2. So this 1, if we do the integration. So, this is a this is going to be, since it is symmetric this will not contribute only this will contribute it will give us h. And therefore, it is equal to E alpha T0 by 2 h h plus 0 and that is equal to e alpha t 0 by 2.

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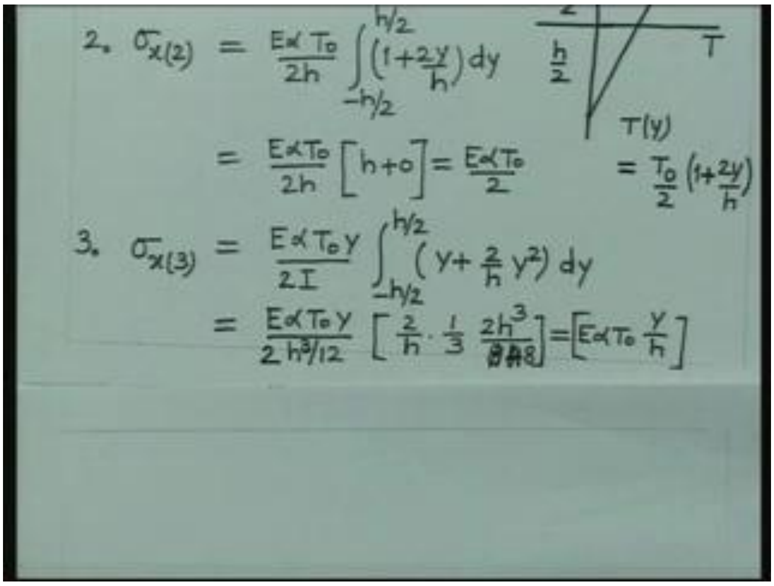

Hard compart, which is sigma x3 it is e alpha t 0 y by 2I and integral minus h by 2 to plus h by 2 y plus 2 by h y square dy. So, if we do the integration it comes out like this E alpha T0 y by 2 h cube by 12. And it is 2 by h one-third and it is 2 h by this will be 8. So, this is 8. So, therefore, if we simplify all that it goes out we have e alpha t 0 y by h. So, this is the third part.

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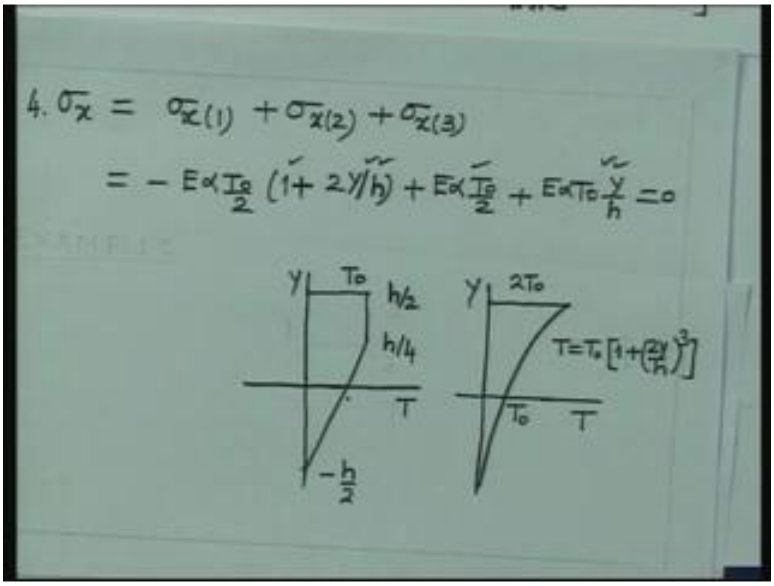

Now, if I put all this together sigma x is equal to sigma x1, sigma x2 plus sigma x3 and let us, substitute the value this is E alpha T0 by 2 1 plus 2 y by h then second 1 is E alpha T0 by 2 plus E alpha T0 y by h you find that, this is going to cancel with this first term and the second 1 is going to cancel with this term, and finally the total stress is 0. So, this is a very typical situation of linear distribution in the case of linear distribution of temperature in the height direction of in a rectangular plate you are not going to get any thermal stresses.

It will simply bend with uniform curvature. So, I would like you to consider, the example like the following for solution. Let us, consider that we have a temperature variation like this, where in this is T0 and this is 2 times T0. And therefore, this is quadratic and T is equal to T0 1 plus 2 y by h cube. So, that is: the variation of the temperature and please find out the thermal stresses.

Similarly, another example you can consider, say this is T0 here and this at this height which is h by 4. So, this is h by 2, this is h by 4 and this is minus h by 2 and the value here you can calculate this is 0. So far this temperature distribution you calculate the thermal stresses and you will find that the thermal stresses, in this case are not going to be 0 at the various levels.