Advanced Strength of Materials
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Lecture – 27

Last time we considered Griffith theory of brittle fracture, where in it was considered that during the crack operation, there is energy conversion from strain energy form to surface energy form. Unlike thermodynamic processes this energy conversion is irreversible because, the energy which gets converted to surface energy from strain energy cannot be reverted back to strain energy.

In general for the case of brittle material once the crack propagation starts. It is very difficult to stop it, that is why the onset of unstable crack extension is important and it is generally the onset which becomes the matter of concern in the case of brittle material. And you have seen last time, that we can find out the critical load capacity of a component from the formula which was given by square root e times Gc divided by pi a.

Where e is the modulus of elasticity g is the fracture resistance, Gc is the fracture resistance of the material and a is the crack length. And we have seen that, this relationship can be used in 2 ways. Given the crack dimension 1 can find out the load bearing capacity of a component provided the materials fracture resistance Gc is known. On the other hand, if the load on the component is given then it is possible to find out the amount of damage or crack that can be tolerated.

Now, we would like to consider simple examples to show the application of this Griffith's theory. Let us now consider example 1 here in it is given that we have 2 materials, 2 rocket motor case materials. We have to select one given that, design stress is sigma Y by 1 5 modulus of elasticity is 200 Gpa and the materials are Low alloy steel where in sigma Y is equal to 1200Mega Pascal and Gc is equal to 24 kilojoules per meter square.

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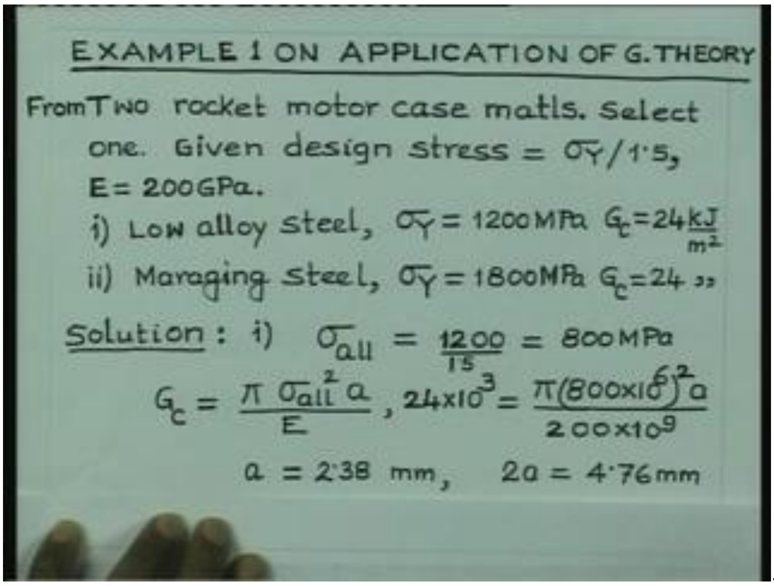

Similarly, we have Maraging steel, where in sigma Y equal to 800 MPa and Gc is equal to 24 kilojoule per meter square, now to solve this problem. Let us, first consider the material number 1. So, in this case the allowable stress or design stress is going to be given by the yield stress divided by the factor of allowance or factor of safety1 5. So therefore, it gives us 800 MPa. Now, from the relation says that Gc is equal to pi sigma allowable square by multiplied by a by e.

So, this follows from the relationship sigma is equal to e Gc by pi a square root of the whole expression. So, from that it has come Gc equal to pi sigma allowable square into a by E, where all the values are given this is given as 24 into 10 to the power 3 Newton per meter square. And then we have pi into 800 into 10 to power 6 square into a by e which is200 into 10 to power 9.

So, once we simplify we gets a value, we will get a value of the crack length as 2 38 Millimeter and therefore, if it is internal crack, the size is twice that and it is 4 76 Millimeter. So, this is what is the crack size, that is permissible or that will lead to brittle fracture in the case of material number 1.

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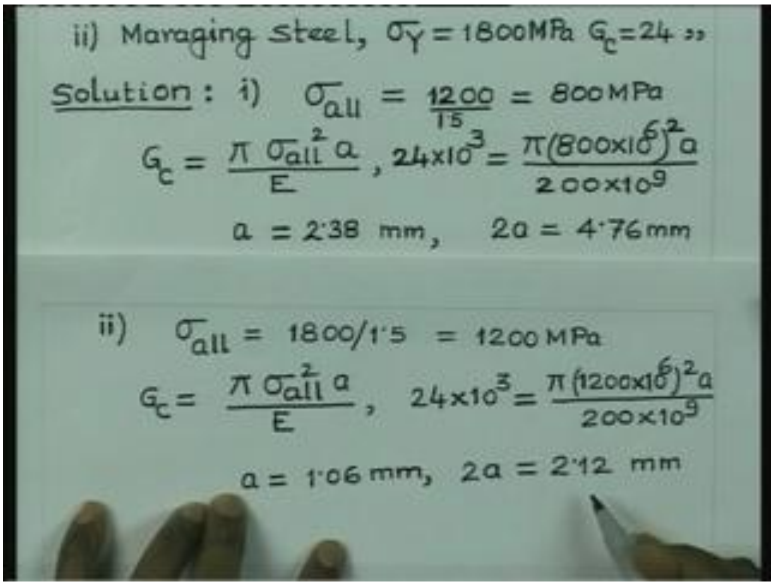

So, if we consider the material number 2 in the case of the material number 2. Here in the allowable stress is again given by sigma Y by 1 5 and it gives us 1200 MPa and again Gc is equal to pi sigma allowable square into a by E here in again Gc is 24 into 10 to power 3. And sigma allowable is 1200 into 10 to power 6 Newton per meter square and E is 200 into 10 to power 9 once, we simplify we get crack length a as 1 06 Millimeter.

And therefore, the total crack size 2a is equal to 2 12 Millimeter. Now, you have 2 possibilities: in the first case the crack size which can lead to brittle fracture is 4 76 Millimeter. And the second case: it is 2 12 Millimeter. So, what is the crack size that you are going to select? It is generally preferable to see that, you have a larger permissible crack size for triggering the larger crack permissible crack size.

Or you have a larger crack size which is going to lead to catastrophic failure. So, in this case you have the material number 1 whose yield point is less, it is going to give you a larger crack size and therefore, this is the material which is preferable. So, for this application the first material that is low alloy steel is the 1 1 should go in for…

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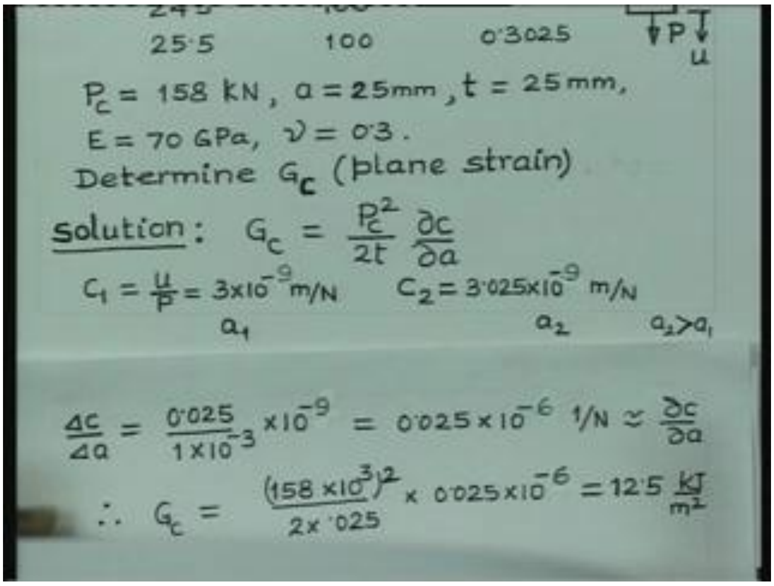

Now, we would like to consider another example sort of on the calculation on the Griffith theory here in we are interested in trying showing you how the fracture toughness can be calculated from experimental observations. It is given here that a specimen in this configuration was tested and the following data were collected the data are observation from the crack length the load and the displacement u of the load point.

So, when the crack length is 24 5 millimeter load is 100 kilo Newton there was a displacement of0 3millimeter similarly, when the crack length is 25 5 millimeter the load is 100 kilo Newton and the displacement is 0 3025 millimeter .Now, fracture test was conducted and it was observed, that the fracture load Pc is equal to 158 kilo Newton and the crack length at the point of failure was 25 millimeter, thickness of the specimen is 25 millimeter modulus of elasticity of the material is 70 Gpa, poisons ratio 0 3 what is needed here is that, determine the fracture toughness.

And this 1 is the will indicate that plane strain fracture toughness I will show you, what is the difference between the plane stress and plane strain later. You remember we have derived just in the last lecture that, the fracture resistance is going to be given by pc square by 2t into delta c delta a. So, we know here the fracture load and we know the thickness of the specimen, now it is a question of getting the data delta c delta a.

How do we get this data that is the point which is to be looked into now, what is given here that you have been given the load and the displacement, and since the behavior of the material is linear elastic. Therefore, from this load and the displacement 1 can calculate the compliance or stiffness if we take p by u that will give us stiffness and if we take u by P that will give us compliance.

So, if we calculate the compliance in the 2 cases which correspond to 2 different crack length. Let us see what are the compliances? So, in the first case we have the compliance c1 is equal to u by p which gives us 3 into 10 to power minus 9 Newton per meter. Because, u is point 3 millimeter and p is 100 kilo Newton. Similarly, for the second case when, the crack length is 25 5 millimeter if we calculate compliance, it is going to be 3 025 into 10 to the 10 to the power minus 9 Newton meter per Newton.

So, C2 is 3 025 into 10 to power minus 9 meter per Newton. So therefore, we have 2 compliances now these 2 compliance corresponds to, this corresponds to crack length a1 and the second corresponds to crack length a2. Here a2 is greater than a1. So, the compliance in the case of the larger crack length is higher. So, if we calculate the by finite difference, the difference in compliance and then take the ratio delta c delta a.

We get the following, so we can calculate now delta c, delta a which is difference between these 2 divided by difference between a a1 a2 and a1 So, that gives us this and it come out to be 0 025 into 10 to power minus 6 per Newton. And we can take this to be equal to delta c delta a, since if we again try to consider the plot c versus a. It is going to be something of this type what is give here that you have been given crack length a1 crack length a2.

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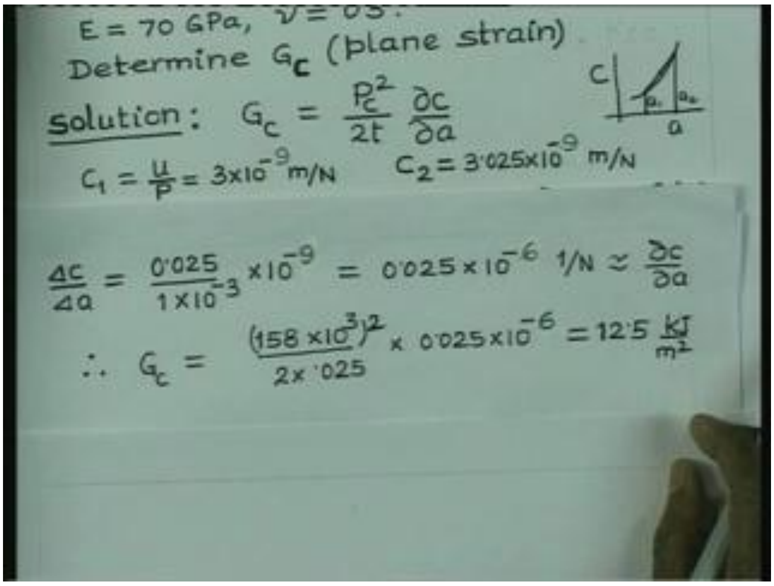

So, this is a1, this is a2 what we are trying to do is that we are trying to approximate this portion by a straight line and taking the slope to be given by this, particular slope of this line over the whole span. Fortunately, the value of the crack length a at which the fracture has taken place is lying within these 2 crack lengths. So therefore, we can take the slope between these 2 locations to be the value, which is indicating delta c delta a. So, if we now get back to this relationship again Gc, Gc equal to Pc square by 2 t into delta c delta a, and here in a we have already obtained the value of delta c delta a.

Pc is give as 158 kilo Newton, t is 25 millimeter. So, if we plot all these values finally, we get fracture toughness as 12 5 kilo joule per meter square. So, this is really the way the experiment is done. So, in this case it is specifically given that the compliance variation between the 2 crack length is linear, but it need not be linear it can be varying gradually.

Then, in that case you have to collect data between the 2 points number of values of the compliance between these 2 points get this curve and then you can take exactly the slope corresponding to the crack size at which the fracture as occurred. So, this is how you can go for calculating the fracture toughness from the experimental observations. Griffith theory has been concerned with the brittle materials. You know that, brittle materials undergo fracture or undergo deformation without any plastic deformation taking place.

But most materials, that are used for engineering construction particularly in metallic materials. They are not brittle like, glass phosphates or composites, some of the composites. So, therefore, this theory of brittle fracture become of limited utility in the case of ductile materials like, metals. So, after the theory was pro founded, it was really questionable about the application in the case of metallic materials, which shows plasticity.

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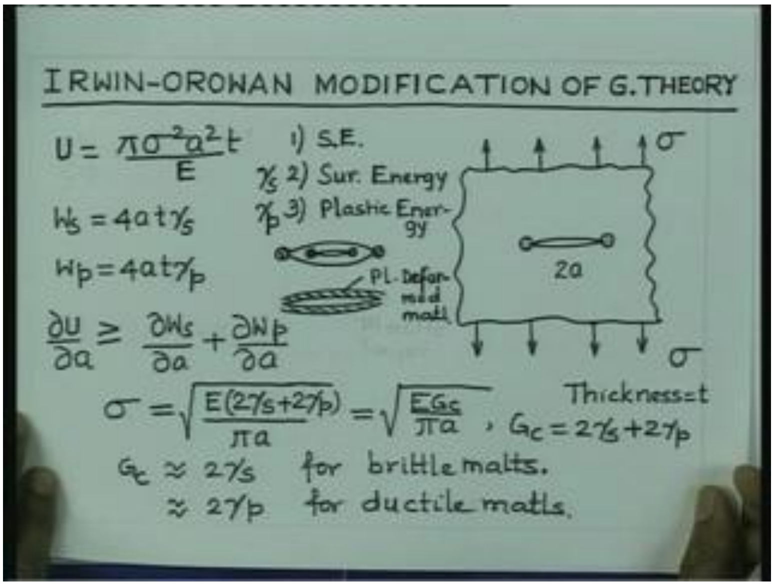

So, a modification to Griffith theory was proposed. So, that the theory can cater for some amount of plastic deformation taking place during the crack propagation this is known as Irwin Orowan Modification of Griffith theory So, this is Irwin Orowan Modification of Griffith theory, so we would like to talk about this Irwin Orowan modification now. If you consider a plate loaded like this and assume that there is a crack of size 2a.

At the load the material by external loading at the crack tip there are very high stresses and therefore, the material will give rise to plastic deformation at the crack tips. So, before the crack propagation can take place, there will be some plastic deformation at the crack tips since, these are the highly stressed region. So, therefore before the crack extension takes place there is going to be plastic deformation.

So, if we now consider suppose that you had begun with a small crack size like this, before the crack extended they are wall plastic deformation at the crack tip. So, this process has occurred at every stage of the crack extension. And once the crack has become larger like this, then also further extension has can take place after the plastic deformation has occurred at the crack. So therefore, this process continued starting from very small crack length.

Now, as the plastic deformation has taken place around the crack tip and after the crack propagation, this plastically deformed material will re will remain in deform, will have some permanent deformation. And the energy which goes into deforming this material plastically, they are absorbed by the material, they are lost in the material and therefore, we cannot recover it.

So, if we now look at that when, the crack has got extended to a size like this. We can imagine that the material at the around the top edge and the bottom edge of the crack; that means, a portion of the material here and a portion of the material on the other side, they are plastically deformed. So, these are plastically deformed material. So, plastically deformed material.

You can appreciate that, the plastic deformation requires some amount of energy. Whatever, energy goes into it cannot be gotten back like, the surface energy when, the crack extension takes place the surface energy is absorbed by the surfaces and we cannot get it back. So, if we now consider the energy conservation here or the different types of energy that are involved in this crack propagation. There are 3 forms of energy now.

We have already seen that, there is strain energy which is going to be the source of energy and then we have also seen that there is surface energy. Which is like the sink, surface is like the sink. And therefore, this energy goes into the sink is surface energy. Then, now we have another form of energy coming up is the plastic energy like, if you consider that the thickness of the plastic zone around the crack edges are more or less constant.

Then, the amount of energy which goes into creating these plastically deformed zones can be considered to be constant. We will consider that surface energy quantum is constant. So, and so we can assume, that the plastically deformed material around the crack edge has absorbed some quantum energy it’s it is actually absorbed at a definite rate. So, if you now consider the energy absorption rate per unit area.

Then, we have 2 quantities now surface energy which is given by gamma s and then we will write that there is also some energy per unit crack area for plastic deformation which is gamma p. So, if you have a crack of size 2a in a material of thickness t then in that case the total area of the crack surface is nothing but two at the top and 2at at the bottom.

So, therefore, the total area is 5at and the surface energy that is absorbed is nothing but 4at into gamma s. So, therefore, this surface energy is equal to 4at into gamma s. So, that is the surface energy we have. Similarly, if we consider the plastic energy, plastic energy is equal to 4at into gamma p. So, this is the plastic energy which has been absorbed by the material.

Now, the strain energy strain energy was given by u or reduction in strain energy was given by it was pi sigma square a square into t by E. So, last time we did this calculation the reduction in strain energy from no crack in the material to a crack of size a was given by this 1.So, if the energy is to be balanced during the propagation of the crack we have some energy available at a particular rate, some amount of energy is available for the, for the suppliance of the surface energy and plastic energy.

And therefore, the energy availability is given by delta u delta a. So, delta u delta a is the rate of energy available for the creation of surfaces. And it is going to be supplying the energy absorption rate which is due to delta w delta s and delta u delta w pa. So, therefore that energy is going to be absorbed as delta w delta s delta a plus delta w delta p delta a. So, this is the source of energy and this is the sink, this side is representing the sink and that is representing the source. And therefore, the unstable crack propagation will take place only when, we have the rate of availability of energy is more than the rate of absorption of energy. So, this is the condition for unstable crack extension when, there is plastic deformation at crack t.

And in the limiting case we can write that delta w delta a is equal to delta ws delta a plus delta wp delta a. So, if you do the simplification we will get finally, that this load at which the crack propagation will begin is given by E into 2 gamma s plus 2 gamma p by pi a. And this can be again written in the form that we did last time, that this is nothing but EGc by pi a, where in we have Gc equal to, Gc equal to 2 gamma s plus 2 gamma p as we assume the surface energy per unit area is constant.

So, also we have assumed here that the plastic energy per unit crack area is equal to is also constant. So, therefore this indicates the materials, material property this much of energy is needed for the crack extension or for unit crack extension. This is a material resistance to fracture in general, what you find is that this Gc is dominated by the value of surface energy. So, this is approximately equal to 2 gamma s for brittle materials where the plastic deformation is negligible.

Similarly, for ductile material this fracture resistance is dominated by gamma p. So, therefore Gc is equal to 2 gamma s, 2 gamma p for ductile material. So, this modification of Irwin and Orowan pepped the way for application of the Griffith theory for ductile materials. Although, Griffith and subsequent modification of Griffith criteria, Griffith’s criterion enabled application of fracture mechanics to brittle materials and also some ductile materials.

It was not in a position to explain fracture in terms of the stress and strain around the crack tip. The classical thinking of fracture was based on stress and strain at the crack tip. In the case of Griffith theory, the fracture wall defined in terms of the global energy balance, the energy balance in the whole body. It did not attach any special importance to the stress strain feel around the crack tip.

So therefore, new ways of looking at brittle fracture was tried and the aim was to explain brittle fracture in terms of stress and strain at the crack tip. This was, this was given by Irwin at a stage which is much later. So, we will like to consider now this Irwin’s stress intensity factor based approach. So, this give rise to what is known as stress intensity factor approach and this was given by Irwin.

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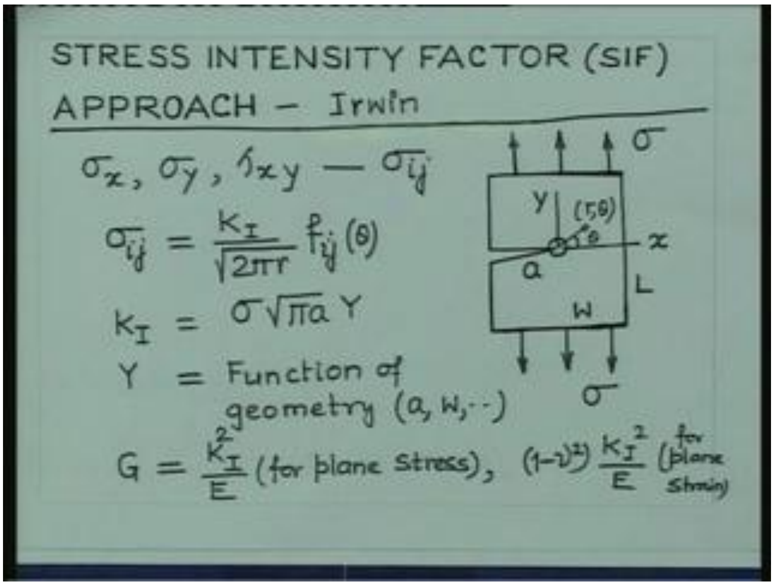

What Irwin did is that he consider the crack in the material and he analyze the stress and strains around the crack tip under the action of the loading and in a 2 dimensional problem like this. If the situation conforms to plain stress, then the stresses that are going to come up are nothing but 3 components: which are sigma x, sigma y and tow xy. So, let us represent them that these are nothing but the sigma ij stress.

He analyze the or rather obtain the solution for the stresses around the crack tip in a very small region around the crack tip here. And he showed that, this stresses sigma ij is given by an expression of the type KI by root 2 pi r into f ij theta. Here r is the coordinate of a point in polar coordinates is r theta. So, r and theta is understood this is the angle theta and this KI is a parameter, which is related to the geometry of the body crack dimension and the crack dimension.

So, this KI is a it has given in this form particularly for this type of situation. It was given in this form sigma into root pi a multiplied by a constant where this constant Y is a function of geometry of the body. So, if you consider the length L and this width is w and the crack size is a. So, therefore this is a function of a, w etcetera. And he also showed that, this factor KI is related to Griffith energy release rate, strain energy release rate. By this type of relationship KI square by E for plane stress, for plane stress and it is given by 1 minus nu square into KI square by E for plane strain. So, therefore the this parameter which expresses the influence of the geometry of the body and the loading on the stress field is related to the strain energy release rate of Griffith.

So, therefore, the stress field around the crack tip is characterized by a parameter KI which expresses the influence of the geometry of the body and the loading. So, this is a very important parameter .And at the same time, it is also clear from this relationship. That as 1 approaches the crack tip along any radial direction as r tends to 0, the stresses suites to infinity. So, therefore, the stresses are infinity at the crack tip as 1 could expect.

So, this parameter there is a similarity in the stress field that as r tend to 0 stress suites to infinity and this is actually, considered to be a stress singularity feature. And the order of the stress singularity is considered to be a square root singularity. We have already, seen that according to Griffith criterion fracture is given by, fracture condition is given by G equal to Gc, so that is the fracture condition.

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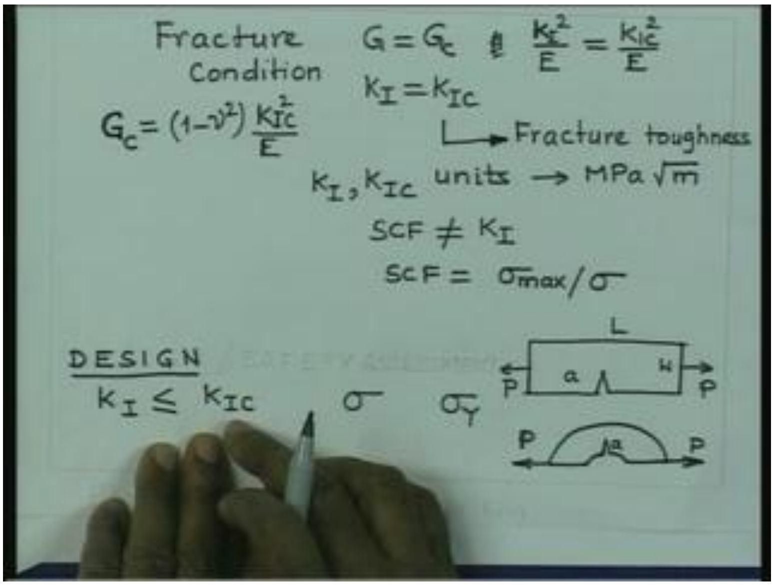

So, at the point of onset of fracture strain energy release rate reaches a critical value. Now, if G is given by Ki square by E if at the point of onset of fracture G reaches a critical value it is obvious, to expect that this KI also reaches a critical value. So therefore, at the onset of fracture KI square by E is equal to KIC square by E. So therefore, this is G and that is Gc. So therefore, at the point of onset of fracture we have KI equal to KIC.

So, this parameter it also indicates that at the point of onset of fracture, the stress field around the crack tip has a critical distribution. Since, this KI is equal to KIC at the onset of fracture. The fracture is associated with a critical stress strain around the crack tip. So, please note that in this case the fracture condition has been given in terms of the distribution of stress test over a zone rather than at a point.

So, Fracture triggers when KI equal to KIC and this KIC is considered to be now, Fracture toughness. So, we have seen that this KI reaching a critical value KIC leads to brittle fracture. And since, KIC is the term which is in involved in the stress field around the crack tip here. So, the fracture is associated with a critical stress strain field around the crack tip. Thus, the fracture condition was given in terms of stress strain condition at the crack tip.

This why it is stress intensity factor? Here in you see that this term is appearing at amplitude term this is a function of theta and this remains constant for a particular r. So therefore, this is expressing the influence of the geometry and loading of the, on the body, on this stress field and it is acting as an amplitude term that is why it is called as stress intensity factor. How you determine this KIC we have already got the relation that Gc equal to 1 minus nu square by KIC square by E for plane strain.

So, once you know the value of Gc experimentally from this relation we can find out the value of KIC. What are the units of KI and KIC? KI and KIC they have the same units. While ki indicates the amount of severity existing at the crack tip and KIC indicates the materials resistance. So therefore, they are 2 different quantities in the sense that, this is related to the loading and geometry and this is related to the material property.

Now, what are the units? If you look into the expression for KI it is sigma into root pi a into Y, sigma is in Newton per meter square and this is in meter. So therefore, its unit is units of stress multiplied by root meter. So therefore, units in the SI system is nothing but MPa root meter. You should not confuse this thing with Stress concentration factor which you have already come across Stress concentration factor is not the same thing as this stress intensity factor KI, because the stress intensity factor is given by the ratio of 2 stresses. In fact, it is ratio of maximum stress by the average stress.

So, it is a dimensionless quantity where as stress intensity factor or Fracture toughness as you need which is MPa root meter. How do you go about applying this approach to design? So, or in safety assessment, so if you consider the application of this thing in design. Consider that, you have a component like this a plate loaded with some external loading P and its the dimension are all given or a sector like this. It’s is subjected to loading, P and there is a crack here and the all the dimensions are specified. Now, you may be required to assess the safety of this component for a given set of conditions.

So, first thing you need to have the Fracture toughness of the material that is KIC or KIC and at the same time you should be able to find out the stress intensity factor existing at the crack tip under the action of loading .And this stress intensity factor can be calculated analytically, or it can be calculated or it can be calculated by numerical methods or the experimental methods.

Sometimes they are also, available in the handbooks, so you collect the data for the stress intensity factor KI. Let’s say from the handbook now, for assuring safety or assessing the safety it is necessary to see that, this stress intensity factor existing at the crack tip is less than equal to KIC. So, K1 is less than K1C where in the K1 is stress intensity factor and KIC is Fracture toughness. So, therefore, if that is the condition then the component is going to be safe. And if stress intensity factor exceeds the value there is going to be brittle fracture.

So, similar is the case here you have to find out the stress intensity factor from the handbook or by calculations and you have check whether, the stress intensity factor is less than fracture toughness KIC I think you could also look into the analogy in the classical approach, in a classical approach what you had is that, you calculated a stress representing the severity at the most critical point.

And that was evaluated by considering stress analysis or using certain formula or using certain handbook involving stress concentration factors and you compare this stress with the yield point of the material for elastic failure. So, here in this sigma Y is closely parallel to KIC since, sigma Y is a material property this KIC is also material property and the stress sigma is a quantity which is related to the level of loading and geometry.

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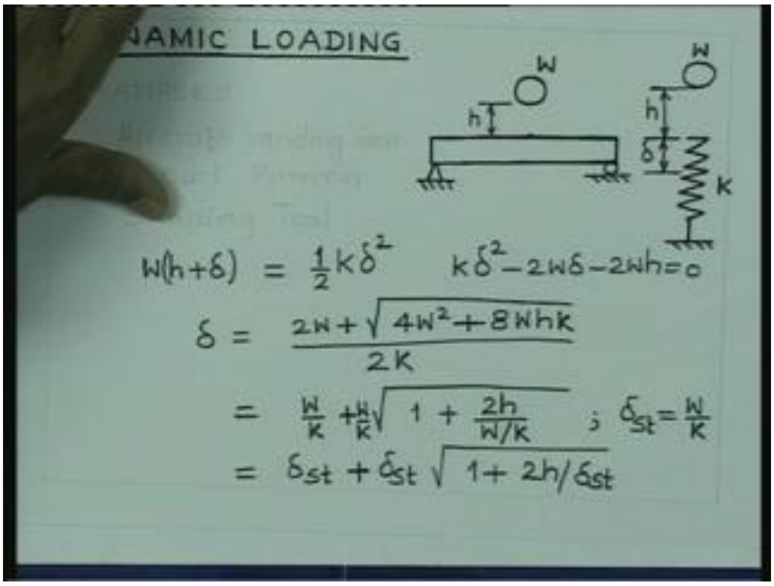

Similarly, here stress intensity factor is also related to level of loading and the geometry. Now, I would like to give you something which is useful in dynamic loading situation You would have come across or you have seen situations where in the loading is loading on a component is not gradual. It is suddenly, applied think of the case of aircraft landing on the run away.

So, as soon as the aircraft touches the ground there is going to be sudden load on the landing here. Similarly, if you think of the impact hamens which are used for forging operations there in the moving dye with the static dye and there is sudden load acting between the 2 dyes. Similarly, in the case of shaping machines the tool comes with a speed and suddenly touches the material there is sudden loading on the shaping tool.

So, these are the situations where you find that load is not applied gradually, as we generally consider to be the case. So, it is necessary to see what type of action generates in the material. Or what is the type of deformation that is going to come up in the material under the action of the sudden loading. So, let us consider an analogy here now think of a beam here. This beam is now applied with a I mean a load is acting on the beam like this.

So, that we have a weight w which is at a height h and it is sudden suddenly, dropped on the beam and if this particular weight was applied gradually on the beam we would have got some deflection of the beam at the center. Now, if I allow this weight to be dropped from a height h then there is going to be a deflection is going to be different. So, the question is that this really due to the fact that there is sudden loading on the beam and hence, there is a difference.

Similarly, if you have a spring here if it is dropped from a height then in that case the deformation that you are going to see in the spring is going to be different from what you would see if the same load is we applied quasi statistically or gradually. In the state of quasi static application you apply the load in steps in small step and you apply you allow the material to reach equilibrium at every stage of loading. And finally, we arrive at the final loading and there the load deflection diagram is going to be a linear variation.

So finally, you get the triangular area for the energy calculation, but in such cases you find that there is going to be a different situation. And therefore, it is important to calculate the deformation when the load is suddenly applied. Now, the calculation of the load in the 2 cases can be done in the similar way. So, let us consider the simple case of this spring which is now subjected or which is loaded by dropping a weight w from a height h.

Now, let us consider that under the action of these loading finally, the minimum or the maximum descend of the weight after its velocity becomes 0 is equal to delta. So therefore, we would like to calculate this delta which has resulted from the drop of this weight from the height h. So, the potential energy at this stage was something when, it has been dropped by this height h plus delta. It has lost the potential energy to the extent w into h plus delta.

Then, this work done has gone into deforming the spring which has deformed by an amount equal to delta. And therefore, the energy stored in the spring after it has deformed by delta is half K delta square. So, therefore, from the conservation of energy what we can write is that this energy w into h plus delta is equal to half K delta square. And therefore, this gives us a equation which is going to look like this K delta square minus twice w into delta minus twice w into h equal to 0. And once, you write delta in terms of k and w it come out to be like this.

Delta is equal to it is 2w plus we will not write the minus because, that will be a smaller value into 4w square minus 8whk by 2K. So, if you simplify all this we are going to get w by K plus 1 plus 2h by w by K which is nothing but if we represent that if the weight w are applied statically on the component then we would have got the deflection delta static and which is nothing but w by K. So, therefore, we can write now that this is nothing but delta static plus here we will have also w by K. So, delta static into 1 plus twice h by delta static.

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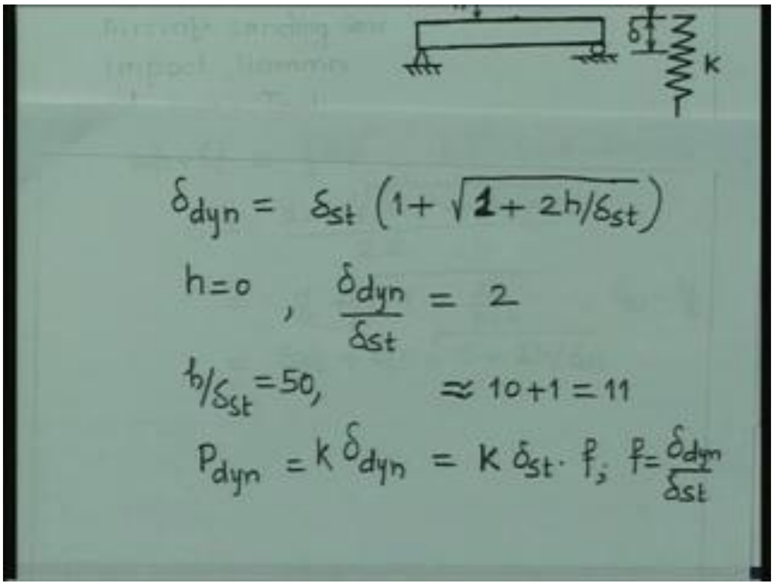

So, the dynamic deflection if you represent that this delta that come up because, of the drop is equal to delta dynamic is equal to delta static multiplied by 1 plus square root square root 1 plus 2h by delta static. So, we find that the dynamic deflection is always going to be more than the static deflection. And you find that when h equal to 0 delta dynamic, by delta static is equal to 2 when h by delta static is equal to let's say 50.

Then, we have this ratio equal to approximately equal to it is going to be equal to it is going to be almost equal to 10, 10 plus 1, so it is 11.So, thus you see that depending on the height of the drop the magnification is going to take place even if we have the weight just dropped from 0 height or suddenly applied, we are going to see a magnification in the deflection by 2.

Then therefore, the loading that is going to be acting on the component when it is, suddenly applied P dynamic. So, that P dynamic is equal to it is nothing but this factor delta into K into delta dynamic. And therefore, it is equal to K into this factor if we write that is nothing but delta static multiplied by f, where f is equal to delta dynamic by delta static.

So, you find that the load which is going to act on the component is going to be always greater than 2 times even, if it is having 0 height you are going to get twice 2 times magnification. And if its height is more than that, your magnification is going to gradually increase. So, therefore, this dynamic loading is very severe. And therefore, while considering the design of the component under dynamic loading 1 might take into account the action of dynamic loading by invoking a factor of type which is given by f here which we have indicated by f.

Then, it is nothing but the ratio of the dynamic deflection by a static deflection. So, if it is in a case of a beam which is on, which upon which a weight is dropped you can calculate the static deflection from this beam formula. And then you can consider the formula for the calculation of the delta dynamic given by delta dynamic equal to delta static into 1 plus square 1 plus twice h by delta static that will give you the dynamic deflection.

And you can also, calculate the dynamic load which is going to be subjected to the beam given by K into delta dynamics. So, that is many a times the dynamic loading can be accounted for by invoking a factor like f, which indicates the ratio of the dynamic deflection by static deflection.